

# Harnessing Random Receiver Cache in Erasure Interference Channels with Feedback

Alireza Vahid

Department of Electrical Engineering  
University of Colorado Denver

Denver, USA

alireza.vahid@ucdenver.edu

**Abstract**—We study the capacity region of two-user erasure interference channels with random receiver-end side-information and delayed channel state knowledge at the transmitters. We present a new set of outer-bounds on the achievable rates when each receiver has access to a random fraction of the message intended for the other receiver. The outer-bounds reveal the significant potential rate boost associated with even a small amount of side-information at each receiver. The key in deriving the bounds is to quantify the baseline entropy that will always become available to the unintended receiver given the intermittent connectivity, random available side-information, and causal feedback. We will also present the achievability of these outer-bounds under certain conditions.

**Index Terms**—Random side-information, interference channel, packet erasure, channel state information.

## I. INTRODUCTION

Sharing the same medium among different users comes with challenges, such as signal interference, and opportunities, such as multicast gains. There are more subtle opportunities that have been somewhat ignored. For instance, wireless users may overhear parts of the other users' signals and this available random side-information can be capitalized on to boost network capacity. This model differs from the known results wherein a mechanism is used to populate the local user memory, also known as cache [1]–[4]. Unfortunately, predetermining what needs to be placed at each user's local cache may not be feasible in practice due to privacy issues, lack of centralized decision-making, and mobility of the wireless nodes. We thus focus on the benefit of random receiver side-information in wireless systems, and we investigate how to harness this random knowledge to enhance network throughput.

To provide fundamental capacity results with distributed transmitters, we focus on the two-user erasure interference model first introduced in [5] for which a comprehensive set of results with no receiver side-information has been reported in [6]–[8]. In this model, each wireless link may be active or inactive (down) according to some Bernoulli process, and these processes may be correlated across users. This model has been shown to capture intermittent communications in massive machine-type systems and high packet failure rate in mmWave communications. The randomness in the available receiver-end

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cache is generated by independent erasure processes, and the transmitters are aware of which portion of their own messages is available to the unintended receiver. We further assume the transmitter become available of the network topology with unit delay, a suitable model for mmWave and machine-type communications. As the topology is captured by whether each link is active or not, this latter assumption can be thought of as the delayed channel state information at the transmitter (delayed CSIT) model.

The random receiver cache model is also motivated by the unreliability of the feedback channel. More precisely in [9], [10], we assumed an intermittent feedback channel model and when the feedback links are inactive, the transmitter would only know the statistics of the side-information available to the receivers. Moreover, in [11], [12], we assumed a one-sided feedback channel wherein one receiver does not provide any feedback, which results in a similar side-information knowledge at the transmitter.

Our contributions are two-fold. We present a new set of outer-bounds on the capacity region of the two-user interference channel with altering topology and channel state feedback. The first step in the derivation of the outer-bounds is to quantify the baseline entropy that will always become available to the unintended user regardless of the communication strategy. In particular, the key is to incorporate the a priori side-information at each receiver's local cache, the altering topology, and the delayed channel feedback into our analysis. Next, the outer-bounds are derived by using a genie-aided argument to convert the channel into a one-sided interference channel and then, the bounds are obtained by applying the baseline entropy inequality discussed above. The outer-bounds of course recover those known previously in the literature for the no-cache and full-cache (when the entire message of each user is available to the other one) scenarios. Interestingly, the outer-bounds suggest even a small amount of side-information may drastically improve the capacity region as we will discuss later in the paper.

We then investigate under what conditions these outer-bounds can be achieved. We provide two sets of conditions. First, we show for “strong channels” and “small cache” sizes (to be quantified in the main results), we can achieve the sum-capacity with symmetric channel parameters. Second, we

identify a subset of these conditions for which the entire outer-bound region can be achieved and thus, characterizing the capacity region in those cases.

**Summary of Results on Interference Channels with Altering Topology:** The interference channel with altering topology or the erasure interference channel (EIC) was first introduced in [5], where it was referred to as the “binary fading” model, to generalize the erasure channel to incorporate interference from other transmitters. The capacity region of the two-user EIC with output feedback was reported in [6] followed by a comprehensive set of results covering the capacity region under delayed and instantaneous CSIT with or without output feedback in [7]. The model and the results were shown to be a good representative of mmWave packet communications [8], [13], [14] and topological dynamics of wireless networks [15]–[18]. The model was also proven valuable in studying the impact of channel correlation [19], [20] and local delayed knowledge [21], [22] on the capacity of distributed wireless networks. Interestingly, the capacity region under the no CSIT assumption and arbitrary erasure probabilities remains open, and the best known inner and outer bounds were reported in [23] with alternative proof in [24], [25], echoing the famous “W-curve” result of [26]. The model has also been used to study the stability region of interference channels [27], [28] where newer coding techniques compared to the study of the capacity region were reported. Finally, this model was adopted in [29], [30] to investigate the coexistence of critical and non-critical IoT services.

The rest of the paper is organized as follows. In Section II, we present the problem setting and the assumptions we make in this work. Section III presents the main contributions and provides further insights and interpretations of the results. The proof of the outer-bounds are presented in Section IV, and the achievability region is derived in Section V. Finally, Section VI concludes the paper.

## II. PROBLEM FORMULATION

To quantify the impact of available random receiver side-information on network capacity, we consider the erasure interference channel (EIC) of Figure 1. In this network, two single-antenna transmitters,  $\text{Tx}_1$  and  $\text{Tx}_2$ , wish to transmit two independent messages,  $W_1$  and  $W_2$ , to their corresponding single-antenna receiving terminals,  $\text{Rx}_1$  and  $\text{Rx}_2$ , respectively, over  $n$  channel uses.

**Channel model:** The channel gain from transmitter  $\text{Tx}_j$  to receiver  $\text{Rx}_i$  at time  $t$  is denoted by  $G_{ij}[t]$ ,  $i, j \in \{1, 2\}$ . The channel gains are either 0 or 1 (i.e.  $G_{ij}[t] \in \{0, 1\}$ ), and they are distributed as Bernoulli  $(1 - \delta)$  random variables. The channels are assumed to be distributed independently across time but at each time, the channel may only be in one of the four states shown in Figure 2–Topology A, B, C, and D, with respective probabilities:

$$p_A = (1 - \delta)^2, \quad p_B = p_C = \delta(1 - \delta), \quad p_D = \delta^2. \quad (1)$$

There are multiple reasons for choosing this specific channel distribution as in general with four binary links, a total of 16

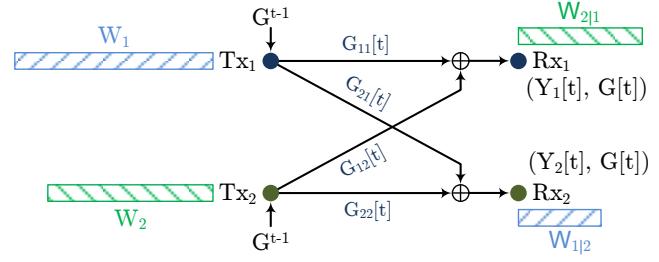


Fig. 1. The two-user interference channel with altering topology, random local cache at the receivers, and delayed channel knowledge.

channel realizations would be possible as considered in [7]. However, including all cases would make tracking the status of the previously transmitted signals more complicated; while the four channel realizations of Figure 2 maintain the key technical challenges and simplify the analysis. Second, the channel realizations of Figure 2 have an interesting motivation from two-unicast networks with a group of relays and the end-to-end network could be captured with these realizations [16], [31]–[33]. Finally, we note that correlation across users can be induced with antenna designs [34], [35].

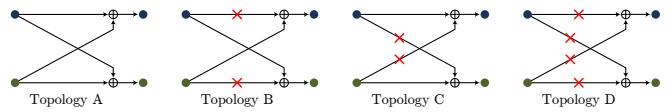


Fig. 2. For the inner-bounds, we assume the channel can only fall into one of four configurations at any given time.

**Input and output signals:** At each time instant  $t$ , the transmit signal of  $\text{Tx}_j$  is denoted by  $X_j[t] \in \{0, 1\}$ , and the received signal at  $\text{Rx}_i$  is given by

$$Y_i[t] = G_{ii}[t]X_i[t] \oplus G_{i\bar{i}}[t]X_{\bar{i}}[t], \quad i = 1, 2, \quad \bar{i} \triangleq 3 - i, \quad (2)$$

where all algebraic operations are in  $\mathbb{F}_2$ . We note that one could assign a continuous channel gain beyond the binary coefficient and also assume additive noise at the receivers, however, this will not change the fundamental of this problem as was the case in [15], [16]. Further, the results can be easily extended to the case where signals are in  $\mathbb{F}_q$  and a correction factor of  $\log_2 q$  will be added to the inner and outer bounds.

**Remark 1.** *Each point-to-point link is an erasure channel, but instead of representing the output by a symbol in  $\{0, e, 1\}$ , we use a channel gain in the binary field. When the link is equal to 1 (i.e. the link is on), the binary output equals to the input, and when the link is equal to 0 (i.e. the link is off), the output is deterministically zero.*

**Channel state information:** We define the channel state information (CSI) at time  $t$  to be the quadruple

$$G[t] \triangleq (G_{11}[t], G_{12}[t], G_{21}[t], G_{22}[t]), \quad (3)$$

and for natural number  $k$ , we set

$$G^k \triangleq (G[1], G[2], \dots, G[k])^\top, \quad (4)$$

where  $G[t]$  is defined in (3), and  $(\cdot)^\top$  denotes the transpose operation. Finally, we set

$$\begin{aligned} G_{ii}^t X_i^t \oplus G_{i\bar{i}}^t X_{\bar{i}}^t &\triangleq \\ [G_{ii}[1]X_i[1] \oplus G_{i\bar{i}}[1]X_{\bar{i}}[1], \dots, G_{ii}[t]X_i[t] \oplus G_{i\bar{i}}[t]X_{\bar{i}}[t]]^\top. \end{aligned} \quad (5)$$

**Messages:** Each message,  $W_i$ , contains  $m_i$  data packets, and we denote the packets for  $\text{Rx}_1$  with  $\vec{a} = (a_1, a_2, \dots, a_m)$ , and the packets for  $\text{Rx}_2$  with  $\vec{b} = (b_1, b_2, \dots, b_m)$ . Here, we note each packet is a collection of encoded bits, however, for simplicity and without loss of generality, we assume each packet is in the binary field, and we refer to them as bits. As mentioned earlier, if we assume the packets are in  $\mathbb{F}_q$  instead, all that would be needed is a correction factor of  $\log_2 q$  in the inner and outer bounds.

**Available CSI at the Transmitters:** In this work, we consider the delayed CSIT model in which at time  $t$ , each transmitter has the knowledge of the channel state information up to the previous time instant (i.e.  $G^{t-1}$ ) as depicted in Figure 1, and the distribution from which the channel gains are drawn,  $t = 1, 2, \dots, n$ .

**Available CSI at the Receivers:** At time instant  $t$ ,  $\text{Rx}_i$  has the its local channel state information up to time  $t$  (i.e.  $G_{ii}^t$  and  $G_{i\bar{i}}^t$ ), see Figure 1, and the distribution from which the channel gains are drawn. Each receiver then broadcasts its local CSI which becomes available to all other nodes with unit delay. To make notations simpler, and since receivers only decode the messages at the end of the communication block, we assume both receivers have instantaneous knowledge of the entire CSI. We note that each channel gain in the intermittent (erasure) model captures the success or the failure in delivering a large number of bits in the forward channel, and thus, the feedback overhead is negligible. This also explains why the feedback channel is used to share CSI rather than information about the received signals.

**Random receiver cache:** We assume a random fraction  $(1 - \epsilon)$  of the bits intended for receiver  $\text{Rx}_{\bar{i}}$  are available at  $\text{Rx}_i$ ,  $i = 3 - \bar{i}$ , and we denote this side information with  $W_{\bar{i}|i}$  as in Figure 1. In particular, we assume that each packet intended for  $\text{Rx}_{\bar{i}}$  becomes available to  $\text{Rx}_i$  according to a Bernoulli  $(1 - \epsilon)$  process distributed independently from all other processes and the messages, and that

$$H(W_{\bar{i}|i}) = (1 - \epsilon) H(W_{\bar{i}}), \quad i = 1, 2. \quad (6)$$

**Transmitter's knowledge of side-information:** We assume the transmitters know exactly what fraction of their own messages is available to the unintended receiver.

**Encoding:** The constraint imposed at the encoding function  $f_{i,t}(\cdot)$  at time index  $t$  is given by:

$$X_i[t] = f_{i,t}(W_i, G^{t-1}), \quad (7)$$

however, to highlight the transmitters' knowledge of the available side-information at the unintended receiver, we use the following notation:

$$X_i[t] = f_{i,t}(W_i, W_{i|\bar{i}}, G^{t-1}), \quad (8)$$

where we implicitly assume the knowledge of  $\delta$ , and  $\epsilon$  is available to each transmitter as side-information.

**Decoding:** Each receiver  $\text{Rx}_i$ ,  $i = 1, 2$ , uses a decoding function  $\varphi_{i,n}(Y_i^n, G^n, W_{\bar{i}|i})$  to get an estimate  $\widehat{W}_i$  of  $W_i$ . An error occurs whenever  $\widehat{W}_i \neq W_i$ . The average probability of error is given by

$$\lambda_{i,n} = \mathbb{E}[P(\widehat{W}_i \neq W_i)], \quad (9)$$

where the expectation is taken with respect to the random choice of the transmitted messages.

**Capacity region:** We say that a rate pair  $(R_1, R_2)$  is achievable, if there exist block encoders at the transmitters, and block decoders at the receivers, such that  $\lambda_{i,n}$  goes to zero as the block length  $n$  goes to infinity. The capacity region is the closure of the set of the achievable rate pairs and is denoted by  $\mathcal{C}$ .

### III. MAIN RESULTS

In this section, we present the main contributions of this paper and provide interpretations of the results.

#### A. Outer-bounds

The following theorem establishes a new set of outer-bounds on the capacity region of the two-user interference channel with altering topology and random receiver cache.

**Theorem 1** (Outer-bounds). *For the two-user erasure interference channel with delayed CSIT and random receiver side-information as described in Section II, we have*

$$\mathcal{C} \subseteq \mathcal{C}^{\text{out}} \equiv \begin{cases} 0 \leq R_i \leq (1 - \delta), & i = 1, 2, \\ 0 \leq \frac{\epsilon}{1 + \delta} R_i + R_{\bar{i}} \leq (1 - \delta^2), & i = 1, 2. \end{cases} \quad (10)$$

Proof of Theorem 1 is presented in Section IV, and a generalization to a broader set of channel parameters is provided in the extended version of this work [36].

**Remark 2.** *From (10), we conclude that when  $\epsilon \leq \delta(1 + \delta)$ , the region is simply described by  $0 \leq R_i \leq (1 - \delta)$ . We note that this latter expression describes the capacity of two parallel non-interfering point-to-point erasure channels. To put into perspective, when  $\delta = 1/2$  and  $\epsilon \leq 3/4$  (i.e. only  $1/4$  of each message is available to the unintended user), the outer-bound matches that of two non-interfering erasure channels.*

Figure 3 depicts the parallel (non-interfering) sum-rate bound of  $2(1 - \delta)$  as well as the sum-rate outer-bound. For convenience, the x-axis represents  $(1 - \delta)$ , which the probability of each link being active. As noted in Remark 2 (and also observed in [7] for the no side-information scenario), depending on the value of  $\epsilon$  and  $\delta$ , the sum-rate might be dominated by either of these bounds. For instance, for  $\epsilon = 1/2$ ,

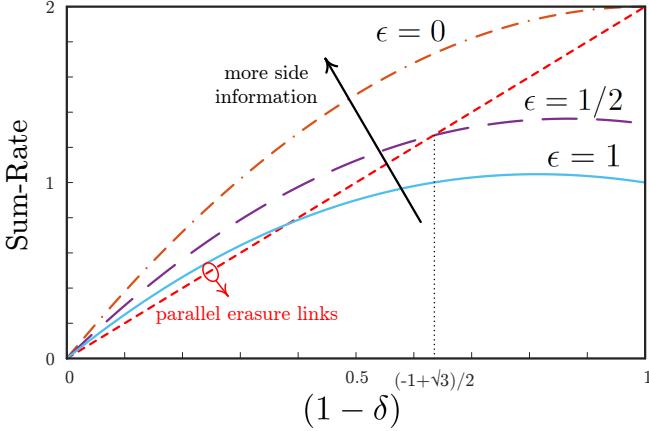


Fig. 3. The outer-bounds of (10) for  $\epsilon \in \{1, 1/2, 0\}$  and the non-interfering sum-rate bound as a red dashed line. Note that for convenience, the x-axis represents  $(1 - \delta)$ , which the probability of each link being active.

when  $(1 - \delta) \leq (-1 + \sqrt{3})/2$ , the parallel erasure bounds are dominant and when  $(1 - \delta) \geq (-1 + \sqrt{3})/2$ , the other bounds in (10) are dominant. We also note that  $\epsilon = 0$  corresponds to the scenario in which the entire message of each user is available to the unintended user, and thus,  $2(1 - \delta)$  is easily achievable. On the other end,  $\epsilon = 1$  is the scenario with no side-information, and the results recover the region in [7].

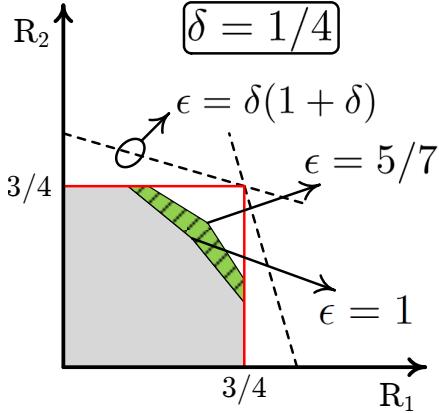


Fig. 4. The outer-bound region for  $\delta = 1/4$  and different values of  $\epsilon$ .

Figure 4 depicts the outer-bound region for  $\delta = 1/4$  and different values of  $\epsilon$ . The grey shaded region is the baseline with no side-information ( $\epsilon = 1$ ), and the hashed green region is the gain when  $\epsilon = 5/7$ . As we show in Theorem 2 and under the specified conditions, for  $5/7 \leq \epsilon \leq 1$ , we can achieve the outer-bounds and thus, the capacity region is characterized. We further note that for  $\epsilon \leq \delta(1 + \delta) = 5/16$ , the outer-bound region is simply expressed by  $R_i \leq (1 - \delta) = 3/4$ .

#### B. Achievable rates

The following theorem establishes the conditions under which the outer-bounds of Theorem 1 are achievable.

**Theorem 2** (Achievability Conditions). *For the two-user erasure interference channel with delayed CSIT and random receiver side-information as described in Section II, we have*

- 1) **Sum-Capacity:** *The maximum sum-rate outer-bound of Theorem 1 is achievable when*

$$\epsilon \geq \frac{1}{1 + \frac{(1-2\delta)^+}{1+\delta}}. \quad (11)$$

- 2) **Capacity Region:** *The entire outer-bound region of Theorem 1 is achievable when*

$$\epsilon \geq \max \left\{ \frac{1}{1 + \frac{(1-2\delta)^+}{1+\delta}}, \frac{\delta(1 + \delta)}{(1 - \delta)} \right\}. \quad (12)$$

First, we note that for  $\delta > 1/2$ , the condition expressed in (11) implies  $\epsilon = 1$ , *i.e.* no side-information at the receivers, which is covered in [7]. Thus, we focus on  $\delta \leq 1/2$ , and (11) becomes:

$$\epsilon \geq \frac{1 + \delta}{2 - \delta}, \quad \text{for } \delta \leq \frac{1}{2}. \quad (13)$$

We further note that (13) also implies that  $\epsilon \geq \delta(1 + \delta)$ , and based on the outer-bounds expressed in (10) and Remark 2, at the maximum sum-rate point, we have

$$R_i = \frac{(1 + \delta)(1 - \delta^2)}{1 + \delta + \epsilon}. \quad (14)$$

Finally, if  $\delta \leq (+3 - \sqrt{5})/2 \approx 0.382$  and the condition in (11) is satisfied, then (12) also holds. In other words, for  $\delta \leq (+3 - \sqrt{5})/2$  and  $\epsilon$  satisfying (11), the capacity region is characterized.

Proof of Theorem 2 is presented in the extended version of this work [36], but in Section V, we summarize the key ideas behind the achievability strategy.

#### IV. PROOF OF THEOREM 1: DERIVING THE OUTER-BOUNDS

In this section, we derive the outer-bounds of Theorem 1. The derivation of the outer-bounds on individual rates is straightforward and omitted. We derive the following bound, and the derivation of the other bound follows from symmetry:

$$\epsilon/(1 + \delta)R_1 + R_2 \leq (1 - \delta^2). \quad (15)$$

Suppose rate-type  $(R_1, R_2)$  is achievable. We enhance receiver  $\text{Rx}_1$  by providing the entire  $W_2$  to it, as opposed to  $W_{2|1}$ , and we note that this cannot reduce the rates. Then, for  $\beta = \epsilon/(1 + \delta)$ , we have

$$\begin{aligned} n(\beta R_1 + R_2) &= \beta H(W_1) + H(W_2) \\ &\stackrel{(a)}{=} \beta \underbrace{H(W_1|W_2, G^n) + H(W_2|W_{1|2}, G^n)}_{\text{Enhanced Rx}_1} \\ &\stackrel{(\text{Fano})}{\leq} \beta I(W_1; Y_1^n|W_2, G^n) + I(W_2; Y_2^n|W_{1|2}, G^n) + n\xi_n \\ &= \beta H(Y_1^n|W_2, G^n) - \beta \underbrace{H(Y_1^n|W_1, W_2, G^n)}_{= 0} \\ &\quad + H(Y_2^n|W_{1|2}, G^n) - H(Y_2^n|W_{1|2}, W_2, G^n) + n\xi_n \\ &\stackrel{(b)}{\leq} H(Y_2^n|W_{1|2}, G^n) + 2n\xi_n \stackrel{(c)}{\leq} n(1 - \delta^2) + 2\xi_n, \end{aligned} \quad (16)$$

where  $\xi_n \rightarrow 0$  as  $n \rightarrow \infty$ ; (a) follows from the independence of the messages and the channels, and captures the enhancement of receiver  $\text{Rx}_1$ ; (b) follows from Theorem 3 below; (c) is true since the entropy of a binary random variable is at most one, and the receiver is not in erasure a fraction  $(1 - \delta^2)$  of the communication time. Dividing both sides by  $n$  and let  $n \rightarrow \infty$ , we get (15).

**Theorem 3.** *For the two-user erasure interference channel with delayed CSIT and random receiver side-information as described in Section II, and  $\beta = \epsilon/(1 + \delta)$ , we have*

$$H(Y_2^n|W_{1|2}, W_2, G^n) + n\xi_n \geq \beta H(Y_1^n|W_2, G^n), \quad (17)$$

where  $\xi_n \rightarrow 0$  as  $n \rightarrow \infty$ .

Proof of Theorem 3 is provided in the extended version of this work [36]. This theorem captures the baseline entropy that becomes available *regardless* of the transmission strategy to the unintended user under the assumptions of the problem. We note that for specific strategies, we might be able to provide a tighter bound. As an example, for a transmitter that ignores channel feedback  $\beta = \epsilon$  would suffice. However, to prove the converse, the bound must hold for any encoding strategy.

## V. THEOREM 2: KEY IDEAS AND MOTIVATING EXAMPLE

In this section, we provide a summary of the techniques as well as an example. The complete proof is deferred to [36].

**Key ideas:** The power of wireless is in multicast. More specifically, gains are magnified if we can satisfy multiple users simultaneously. This simple and intuitive idea is behind most wireless communication algorithms. To see how this idea can be used in interference channels with altering topology, delayed CSIT, and random receiver cache, we first explain the network coding opportunities, and then, present an example to explain the overall achievability strategy.

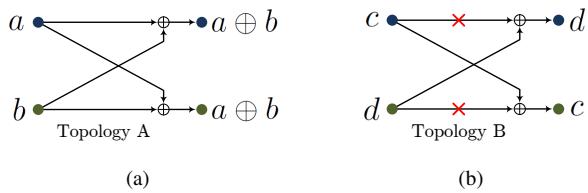


Fig. 5. The transmitted bits in these examples can be combined for efficient multicast retransmission.

Suppose at some time instant  $t_1$ , network topology  $A$  is realized, meaning that all wireless links are active as in Figure 5(a). If bits  $a$  and  $b$  were transmitted from  $\text{Tx}_1$  and  $\text{Tx}_2$ , respectively, then each receiver obtains a linear combination of these bits. It seems that providing only  $a$  or  $b$  to both receivers would be the optimal solution. However, we show there are other opportunities that can improve the network throughput. Suppose  $c$  is part of  $W_{1|2}$  (intended for  $\text{Rx}_1$  but in  $\text{Rx}_2$ 's cache) and  $d$  is part of  $W_{2|1}$ . This scenario could have also happened if at the time of transmitting  $c$  and  $d$ , topology  $B$  was realized as in Figure 5(b). This time, it seems  $c$  and  $d$  could be sent to

their respective receivers through two non-interfering point-to-point erasure links as they are apriori known to the unintended receiver. Interestingly, we can come up with a more efficient solution:  $\text{Tx}_1$  should deliver  $a \oplus c$  and  $\text{Tx}_2$  should deliver  $b \oplus d$  to *both* receivers. This way,  $\text{Rx}_1$  will end up with  $a \oplus b, d, a \oplus c$ , and  $b \oplus d$ , from which, it can recover  $a$  and  $c$ . A similar story goes for  $\text{Rx}_2$ . In summary, we achieved multicasting gains by mixing the signals available locally to each transmitter rather than retransmitting individual ones.

**Motivating example:** To keep the description short and convey the main points, for the motivating example, we focus on the maximum sum-rate point. We further use expected values of random variables as opposed to a more careful analysis involving concentration theorems and defer such analysis to the next subsection where we present the complete proof. We further choose an example where  $\epsilon$  satisfies (11) of Theorem 2 with equality, which further shortens the description of achievability. In particular, we assume

$$\delta = \frac{1}{5}, \epsilon = \frac{2}{3}. \quad (18)$$

This scenario corresponds to strong point-to-point erasure links (success rate of  $4/5$ ) and when each receiver has apriori access to  $1/3$  of the message of the other user. For these parameters, the maximum sum-rate point using (10) is:

$$(R_1, R_2) \approx (0.62, 0.62). \quad (19)$$

We start with  $m$  bits for each receiver where  $1/3m$  of the bits for each receiver is apriori known to the unintended receiver. Each transmitter separates its bits into two groups, the first are those known to the unintended receiver, called the side-information bits, and the second would be the complement of the first group. Each transmitter keeps sending out one bit from the second group until the channel realization learned through the feedback channel is *not* topology  $D$ . This process on average takes

$$\frac{\epsilon m}{1 - \delta^2}. \quad (20)$$

After this initial phase, each bit falls into three categories based on the topology that was realized during its transmission (topology  $A$ ,  $B$ , or  $C$ ). Those in topology  $C$  are already delivered and no further action is needed. For those in topologies  $A$  and  $B$ , we can retransmit the combination of them as discussed above. Further, for the choice of  $\delta = 1/5$ , there will be more bits associated with topology  $A$  than  $B$ . We take advantage of this and mix the remaining bits of topology  $A$  with the side-information bits at each transmitter, *i.e.* those known apriori to the unintended receiver through the random cache. In this example,  $\delta$  and  $\epsilon$  were carefully chosen such that the number of bits in topology  $A$  was exactly equal to those in topology  $B$  and the random cache. The combined (XORed) bits can be delivered at the multiple-access channel (MAC) capacity formed at each receiver equal to  $(1 - \delta^2)$ . In summary, for this particular example, the total communication time is given

by

$$t_{\text{total}} = \underbrace{\frac{\epsilon m}{1 - \delta^2}}_{\text{initial phase}} + \underbrace{\frac{\epsilon(1 - \delta)^2 m}{(1 - \delta^2)^2}}_{\text{multicasting XORed bits}} \approx 1.62, \quad (21)$$

which combined with the fact that each transmitter had  $m$  bits to deliver, immediately implies the desired sum-rate.

## VI. CONCLUSION

In this paper, we studied the benefit of having random receiver cache in interference channels with altering topology and delayed feedback. We provided a new set of outer-bounds based on a key theorem that quantifies the baseline entropy available to each receiver under the specific assumptions of the problem. We showed that these bounds are tight under certain conditions, thus, characterizing the capacity region in such cases. The next steps include investigating whether non-linear coding may improve the inner-bounds or the outer-bounds need improvement when the capacity remains open. Further, it would be interesting to understand the implications of random receiver cache on latency and age of information in interference channels with altering topology.

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