

The fractal interpolation applied on brownian motion particles trajectories reconstruction

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The particles in condensed matter physics are almost characterized by Brownian motion. This phenomenon is the basis for a very important understanding of the particles motion in condensed matter. For our previous research, there is already applied and confirmed the complex fractal correction which includes influence of parameters from grains and pores surface and also effects based on particles' Brownian motion. As a chaotic structure of these motions, we have very complex research results regarding the particles' trajectories in three-dimension (3D). In our research paper, we applied fractal interpolation within the idea to reconstruct the above mentioned trajectories in two dimensions at this stage. Because of the very complex fractional mathematics on Brownian motion, we found and developed much simpler and effective mathematization. The starting point is within linear interpolation. In our previous research, we presented very original line fractalization based on tensor product. But, in this paper, we applied and successfully confirmed that by fractal interpolation (Akimo polynomial method) that is possible to reconstruct the chaotical trajectories lines structures by several fractalized intervals and involved intervals. This novelty is very important because of the much more effective procedure that we can reconstruct and in that way control the particles' trajectories. This is very important for further advanced research in microelectronics, especially inter-granular micro capacitors.

Keywords: Linear interpolation; fractal interpolation; Brownian motion; particles; microstructure.

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1. Introduction

Mathematical methods, like the basic linear interpolation or fractal interpolation method, are very useful for the reconstruction of various particles' trajectories, such as electrons, molecules, bacteria, and viruses. All of these particles share the same motion pattern, designated as Brownian motion, a stochastic, unpredictable motion, which has a fractal nature. The main idea of our research is to apply fractal interpolation on particles' Brownian motion, to obtain their two-dimension (2D) reconstructed trajectories, which we can use in our further examination.

By applying the recursive fractal interpolation method, we can divide and refract a straight line until it fractally becomes a smooth line. It is represented as a series of broken lines, within fractal nature self-similarities.

It is of great importance to determine electrons' trajectories on the ceramic materials' grain boundaries because it has a great effect on dielectric properties. Also, knowledge of bacterial and viruses' motility behavior is one of the essential points in disease prediction and prevention. Thus, it is evident that different particle Brownian motion trajectories' reconstruction is necessary for numerous fields, and

therefore, implementation of these mathematical methods provides new possibilities and perspectives.

The basis for this research is fractal lines' tensor product,¹ and now by this extended mathematical fractal interval approach, it is possible to define and reconstruct bio and condensed matter systems' particles Brownian motion.

1.1. Basic linear interpolation method

If we start our analysis, using two given points $A_1(x_{A_1}, y_{A_1})$ and $B_1(x_{B_1}, y_{B_1})$, we can determine an equation of a line l_1 , through those two points,

$$l_1 : y - y_{A_1} = \frac{y_{B_1} - y_{A_1}}{x_{B_1} - x_{A_1}}(x - x_{A_1}), \quad (1)$$

where $k_1 = \frac{y_{B_1} - y_{A_1}}{x_{B_1} - x_{A_1}}$ is direction coefficient of this line.

Based on condition of orthogonality of two lines l and n

$$l_1 \perp n_1 \Leftrightarrow k_1 \cdot k_2 = -1 \Leftrightarrow k_2 = -\frac{1}{k_1}. \quad (2)$$

We have direction of a line n coefficient, orthogonal to the line l equal to

$$k_2 = \frac{x_{B_1} - x_{A_1}}{y_{A_1} - y_{B_1}}. \quad (3)$$

We can calculate middle point of this line $S(x_s, y_s)$, where

$$x_s = \frac{x_{A_1} + x_{B_1}}{2}, \quad y_s = \frac{y_{A_1} + y_{B_1}}{2}. \quad (4)$$

Now, we can obtain an equation of line n , orthogonal to the line l

$$n_1 : y - y_s = \frac{x_{B_1} - x_{A_1}}{y_{A_1} - y_{B_1}}(x - x_s), \quad (5)$$

i.e.,

$$n_1 : y = \frac{x_{B_1} - x_{A_1}}{y_{A_1} - y_{B_1}}x + \left(y_s - \frac{x_{B_1} - x_{A_1}}{y_{A_1} - y_{B_1}}x_s\right). \quad (6)$$

Now we can choose some arbitrary point $C(x_c, y_c)$, $C \neq S$, on a line n , and we can obtain equations of two lines p_1 and q_1

$$p_1 : y - y_{A_1} = \frac{y_c - y_{A_1}}{x_c - x_{A_1}}(x - x_{A_1}), \quad (7)$$

$$q_1 : y - y_{B_1} = \frac{y_c - y_{B_1}}{x_c - x_{B_1}}(x - x_{B_1}). \quad (8)$$

All explained are shown in Fig. 1.

If we put $A_2 = A_1$ and $B_2 = C$, we can go into the next iteration, in order to refract line A_2B_2 using new point C

$$l_2 : y - y_{A_2} = \frac{y_{B_2} - y_{A_2}}{x_{B_2} - x_{A_2}}(x - x_{A_2}), \quad (9)$$

where $k_1 = \frac{y_{B_2} - y_{A_2}}{x_{B_2} - x_{A_2}}$ is direction coefficient of this line.

Based on condition of orthogonality of two lines l and n

$$l_2 \perp n_2 \Leftrightarrow k_1 \cdot k_2 = -1 \Leftrightarrow k_2 = -\frac{1}{k_1}. \quad (10)$$

We have coefficient of direction of a line n , orthogonal to the line l equal to

$$k_2 = \frac{x_{B_2} - x_{A_2}}{y_{A_2} - y_{B_2}}. \quad (11)$$

We can calculate middle point of this line $S(x_s, y_s)$, where

$$x_s = \frac{x_{A_2} + x_{B_2}}{2}, \quad y_s = \frac{y_{A_2} + y_{B_2}}{2}. \quad (12)$$

Now, we can obtain an equation of line n , orthogonal to a line l

$$n_2 : y - y_s = \frac{x_{B_2} - x_{A_2}}{y_{A_2} - y_{B_2}}(x - x_s), \quad (13)$$

i.e.,

$$n_2 : y = \frac{x_{B_2} - x_{A_2}}{y_{A_2} - y_{B_2}}x + \left(y_s - \frac{x_{B_2} - x_{A_2}}{y_{A_2} - y_{B_2}}x_s\right). \quad (14)$$

Now we can choose some arbitrary point $C(x_c, y_c)$, $C \neq S$, on a line n , and we can obtain equations of two lines p_2 and q_2

$$p_2 : y - y_{A_2} = \frac{y_c - y_{A_2}}{x_c - x_{A_2}}(x - x_{A_2}), \quad (15)$$

$$q_1 : y - y_{B_2} = \frac{y_c - y_{B_2}}{x_c - x_{B_2}}(x - x_{B_2}). \quad (16)$$

This situation is shown in Fig. 2.

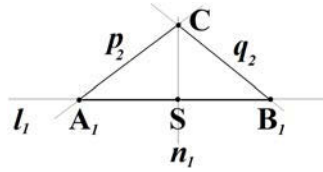


Fig. 1. S is in the middle of the line A_1B_1 , i.e., $A_1S : SB_1 = 1:1$.

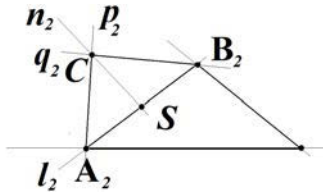


Fig. 2. S is in the middle of the line A_2B_2 , i.e., $A_2S : SB_2 = 1:1$.

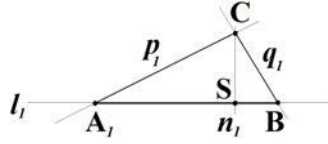


Fig. 3. S is somewhere on the line A_1B_1 , where $A_1S : SB_1 = \lambda$.

In similar manner we can divide and refract new line A_2C , then A_3C , then A_4C , and so on. If we want to divide line in some other proportion $A_1S : SB_1 = \lambda$, we can calculate appropriate point $S(x_s, y_s)$, on this line

$$x_s = \frac{x_{A_1} + \lambda x_{B_1}}{1 + \lambda}, \quad y_s = \frac{y_{A_1} + \lambda y_{B_1}}{1 + \lambda}. \quad (17)$$

Now, we can obtain an equation of line n , orthogonal to a line l

$$n_1 : y - y_s = \frac{x_{B_1} - x_{A_1}}{y_{A_1} - y_{B_1}}(x - x_s), \quad (18)$$

i.e.,

$$n_1 : y = \frac{x_{B_1} - x_{A_1}}{y_{A_1} - y_{B_1}}x + \left(y_s - \frac{x_{B_1} - x_{A_1}}{y_{A_1} - y_{B_1}}x_s\right). \quad (19)$$

Now we can choose some arbitrary point $C(x_c, y_c)$, $C \neq S$, on a line n , and we can obtain equations of two lines p_1 and q_1

$$p_1 : y - y_{A_1} = \frac{y_c - y_{A_1}}{x_c - x_{A_1}}(x - x_{A_1}), \quad (20)$$

$$q_1 : y - y_{B_1} = \frac{y_c - y_{B_1}}{x_c - x_{B_1}}(x - x_{B_1}). \quad (21)$$

All explained are shown in Fig. 3 and so on.

1.2. Fractal interpolation method

We will now construct a fractal interpolant through the points (x_i, y_i) , $i = 1, \dots, N + 1$ determined by the position of the point C in the iterations. We have used the algorithm for obtaining the fractal interpolant developed in our previous papers.²⁻⁴ Over points (x_i, y_i) , $i = 1, \dots, N + 1$, we have defined IFS with affine transformations

$$w_i \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_i & 0 \\ c_i & d_i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e_i \\ f_i \end{bmatrix}, \quad i = 1, \dots, N, \quad (22)$$

and constraints

$$w_i \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_{i-1} \\ y_{i-1} \end{bmatrix} \quad \text{and} \quad w_i \begin{bmatrix} x_N \\ y_N \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix}, \quad i = 1, \dots, N. \quad (23)$$

As before, we chose parameter d_i for the free parameter. When $|d_i| \leq 1, i = 1, \dots, N$, all affine transformations w_i are contractions, and the obtained IFS attractor is a graph of a continuous function passing through interpolation points.

In the case when the ordinates x_i of the interpolation points are not in ascending order, direct application of IFS, defined by (1) and (2), does not give an interpolation curve that passes through the interpolation points. In that case, it is necessary to apply a reversible transformation

$$T(x_i, y_i) = (u_i, v_i), \quad i = 1, \dots, N + 1, \quad (24)$$

where

$$u_i = x_0 + \sum_{j=1}^i (|x_j - x_{j-1}| + p) = u_{i-1} + (|x_i - x_{i-1}|), \quad (25)$$

$$v_i = y_i.$$

In the last step, we have mapped the obtained attractor in (u, v) -plane by inverse transformation

$$T'(u', v') = (x', y'), \quad (26)$$

where

$$x' = x_{i-1} + (x_i - x_{i-1}) \frac{u' - u_{i-1}}{u_i - u_{i-1}}, \quad u' \in [u_{i-1}, u_i], \quad (27)$$

$$y' = v',$$

back into the (x, y) -plane.

2. Experimental Part

The above mentioned mathematical methods could be applied on various subjects, and in our research, we implemented them on particles' motion prediction and reconstruction.

In our experimental work we applied the Heywang inter-granular capacity model⁵ in order to analyze and characterize electron motion on the grain boundaries and between the grains.⁶ Particles' collision, while crossing from one side of the boundary to another, influences their trajectories leading to Brownian fractal motion during sintering. By applying fractal interpolation method on random and chaotic electrons' paths, we are able to transform them into orderly forms.⁷⁻⁹

The theoretical experiment¹⁰ implied predicting the particles' motion on the grain boundaries, which principally manifests as a possible collision, with the influence of the grain boundary fractality on the particles' trajectories. We observed one electron cluster that crossed the grain boundary and another electron cluster on the other side of the grain boundary, that holds the previous one, and could participate in a collision. Next, the experiment was reduced to two electrons, which are representative of the mentioned clusters. Due to the infinitesimal nature of the trajectory

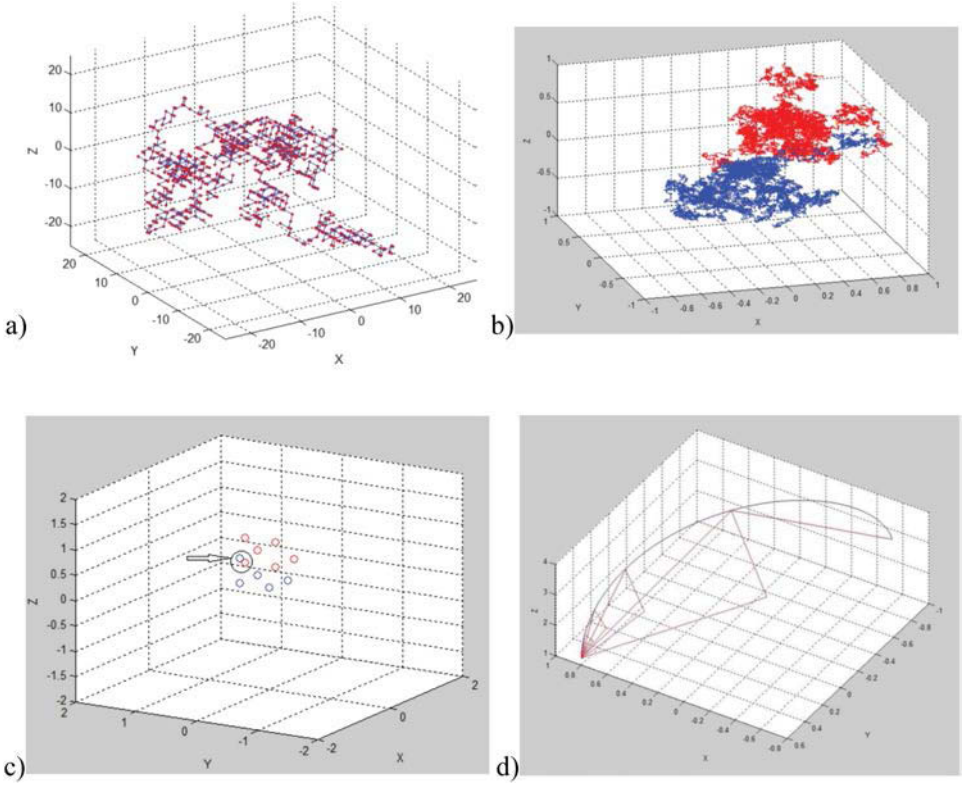


Fig. 4. (Color online) Electrons Brownian motion and collisions on the grain boundary: (a) the trajectory of one electron; (b) collisions of two electron clouds; (c) collision of two electrons and (d) the curve line between two collision points.

geometry between two collision points, we can create the curve line between these two points [Fig. 4(d)], which represents the segment of the electron's trajectory, as the beginning of fractal interpolation.

In Fig. 4, the theoretical experiment results are presented, showing the motion of one electron (a), collision of two electrons (c), and electron clouds with possible collisions on the grain boundary (b), as well as the curve line between two collision points (d).

Considering the fractal self-similarity nature of these particles' trajectories, we introduced previously described fractal analysis methods, and after mathematical transformations, we reconstructed electrons' trajectories. That represents the goal of our research, as well as the possibility to determine and predict particles trajectories.

The porosity of ceramic material influences its microelectronic properties, thus fractal analytical method, based on the application of fractal Cantor set and fractal curvature tensor product, provides improved grains and pores surfaces characterization and reconstruction (Fig. 5), as we reported in our previous papers.¹

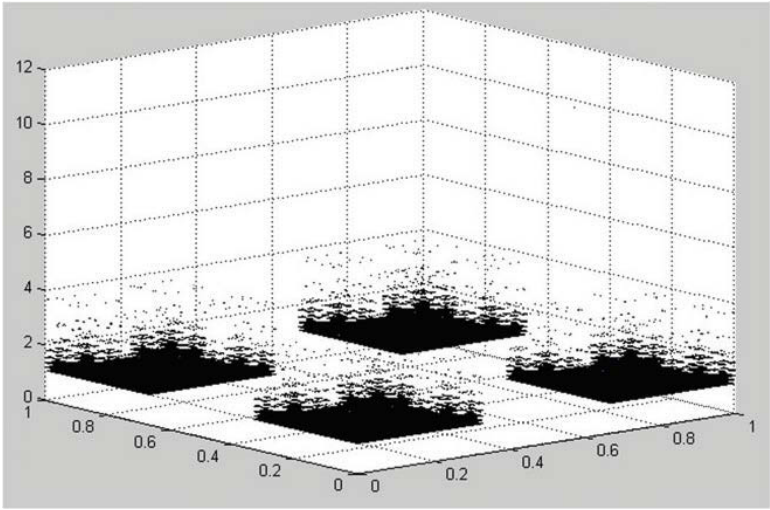


Fig. 5. Infinitesimal division of Cantor 2D dust.

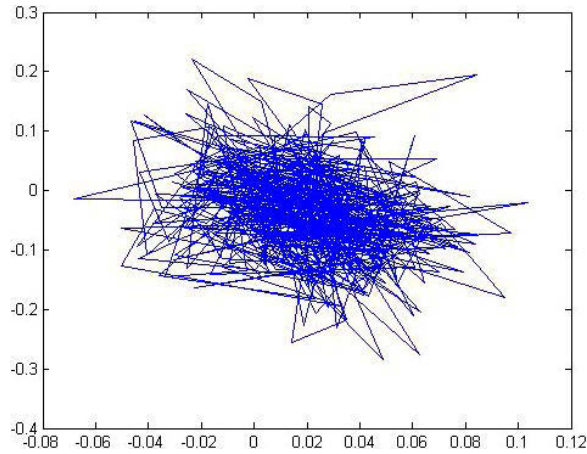


Fig. 6. (Color online) 2D bacterial trajectories.

We also performed some experiments regarding bacterial and viruses' motion, in the frame of Brownian motion fractal nature similarities, and applied fractal interpolation method on the obtained results. The complex chaotic microorganisms' motion pattern,² consisting of multiple broken lines, is presented in Fig. 6.

After applying fractal analysis on bacterial motion diagrams, we established fractal interpolated 2D trajectories,² shown in Fig. 7.

Regarding viruses' Brownian motion, we developed the experimental procedure for determination of Coronavirus trajectories,³ which are also irregular and

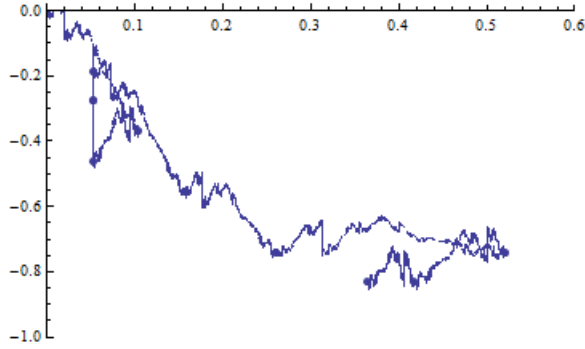


Fig. 7. (Color online) Fractal interpolation applied on bacterial motion.

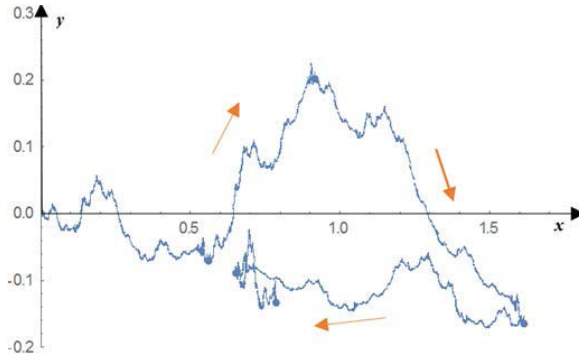


Fig. 8. (Color online) Coronavirus Brownian motion trajectories.

unpredictable. Based on the obtained viruses' position coordinates, and by applying fractal analysis, we created 2D trajectory diagrams, presented in Fig. 8.

In this experimental part we selected and presented results related to real and theoretical experiments based on bacteria, Coronavirus, and particles like electrons in condensed matter, which are confirming variety of the Brownian motion character. In all of these, we successfully introduced fractal interval mathematics methods to reconstruct trajectories and to establish control over the different dynamic structures' movements. The important aspect of trajectories' control and reconstruction is based on the idea that we have confirmed biophysical similarities and biomimetics symmetries between non-alive condensed matter structures and the alive matter in biophysical systems. These two parts correspond to each other and even slowly approach each other, as well. This is very important for creating original novelties in higher-level microelectronic integrations.¹¹ Also, this is important for the next step advancement in hybrid microelectronic solutions in alive and non-alive integrations which opens new frontiers in the field of artificial intelligence, as well. This research phenomenon is also very exciting in space research of alive matter investigation.

By these experimental results, we confirmed the statement of the particles' Brownian motion within the total matter processes. The particles like electrons and similar others behave in their different morphology structures with no differences in phenomena within alive and non-alive matters. So, this is an important joint and integrative parameter in total nature and Universe.

3. Results and Discussion

Now, in the next step of our research, we applied and analyzed fractal interpolation in the process of reconstruction and prediction of the trajectories' structures within the Brownian motion particles' nature. The importance of this procedure lies in the possibility that we can, in advance, prognosticate and design the shapes when they are reconstructed. So, this is of great importance for controlling and somehow directing the motion of the particles. In this phase, we present results in 2D.

The fractal interpolant in Fig. 9 is obtained by the procedure explained in Secs. 1.1 and 1.2. In each interval, we have chosen vertical scaling factor d as a random number from the interval $(-0.06, 0)$. The interpolation points are marked in red color.

The vertical scaling factor d in each interval is here, in Fig. 10, a random number from the interval $(-0.02, 0.02)$.

By comparing fractal interpolant in Figs. 9 and 10. We can see that interpolant in Fig. 10 is smoother than interpolant in Fig. 9 because vertical scaling factor d in Fig. 10 is obtained as a random number from the narrow interval $(-0.02, 0.02)$ in each interval, comparing with the scaling factor d , in Fig. 9, where a scaling factor is a random number from the wider interval $(-0.06, 0)$. Further reduction of the

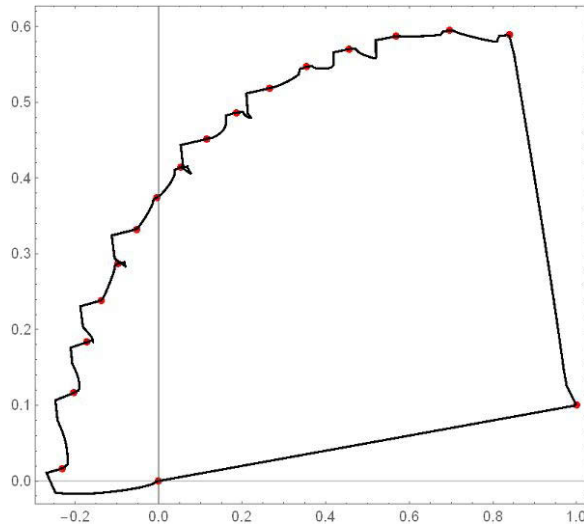


Fig. 9. (Color online) Fractal interpolant obtained with $d \in (-0.06, 0)$.

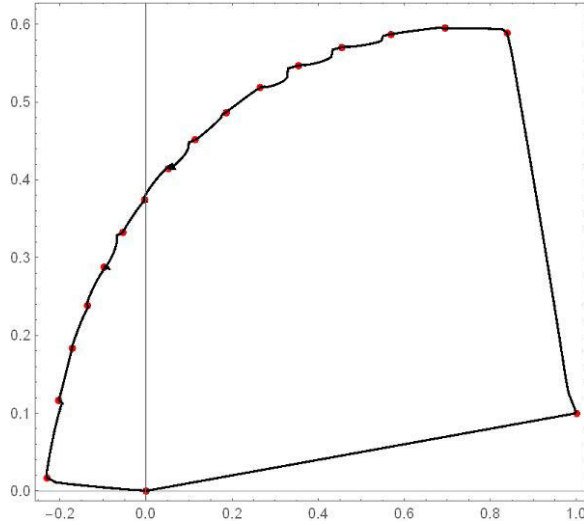


Fig. 10. (Color online) Fractal interpolant obtained with $d \in (-0.02, 0.02)$.

range from which the values of the vertical scaling factor d are taken would lead to obtaining an ever smoother graph of the obtained interpolation function. In the boundary case, if d has a value of 0 at all intervals, we would get straight lines connecting the interpolation nodes.

We expressed the precise data of fractal interpolation application-dependent on scaling factor size. We have got the different structures of fractal interpolation approximations to any type of chaotical curvature by infinitesimal asymptotic approach with the broken line, so we completed the important tool for particles' motion designing.

4. Outlook

We would like to continue our research by extension on 3D Brownian motion structures.

5. Conclusion

In a few of our research papers, we developed different ideas regarding particles' Brownian motion and also for microorganisms. We analyzed several aspects of Brownian motion as a joint property within a biophysical system and also as the important characteristic of hybrid microelectronics advanced devices. As one of the three parameters (influence of grain and pore surface and Brownian motion of particles) the Brownian motion particles' phenomena is very important to characterize the particles in the relation, so that is a joint parameter within the biophysical systems. In that sense, it is very important to develop the methods and tools for the reconstruction of the Brownian trajectories. Ones what we got as reconstruction

opens possibilities for predicting, designing, and prognosing the particles dynamic. We open quite new perspectives for projecting the trajectories in advance and so we can establish control over particles' chaotic motion. The goal is not to transform the disorder of the points and curvatures' structures in clear order. In that case, does not exist "potential difference" if everything becomes order. So, the intermediate step is to come from chaos to control the chaos with possibilities to reconstruct and predict the trajectories of morphologies and structures. From this point of view fractal interpolation, as quite a new application, opens the perspectives that we can easily reconstruct and predict Brownian motion particles' curvatures. Our presented research in this paper demonstrates the most effective and simple mathematical-physical method for the Brownian motion trajectories' reconstruction and prediction, as well. The presented results are very important as a tool for particles' motion designing by predicting goals especially for different microelectronics and hybrid microelectronics designs where alive and non-alive matter are incorporated.

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