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MODELING COVID-19 EPIDEMIC AND ANALYSIS

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ABSTRACT. An epidemic disease caused by coronavirus has spread all over the world with a strong contagion rate. We implement an SIR model to study the evolution of the infected population and the number of infected recovered and dead because of this epidemic in South Carolina consistent with available data. We perform an analysis of the results of the model by varying the parameters and initial conditions, in particular transmission and recovery rates.

We use data covering the period December 1, 2020, to June 1, 2021. The models and results are consistent with the observations. The models developed using data help us understand the recovery rates. The infection and recovery increasing in South Carolina do not show improvement. The number of dead people tends to increase although by small amount.

Models were developed based on the available data. Initially neural networks and machine learning methodology were used to come up with transmission rates. Later, direct calculation and optimal control methodology were used to deduce transmission parameters. For the period December to June there were no available data on recovered populations and we have to determine them as well as transmission and recovery rates based on data of infected populations and dead population using neural networks and optimal control methodologies where transmission, recovery, relapsation immunity and death rates from infection are considered as decision variables.

AMS (MOS) Subject Classification. 34H05, 34D20, 68T07, 92B20

Key Words and Phrases. Optimal control, Reproduction number.

1. INTRODUCTION

The rapid spread of a disease in regions (epidemic) or the global outbreak of a disease (pandemic), can have a detrimental effect on health systems and economical activities locally and globally. Measures to reduce the pandemic spread include curtailing close interactions between using social distancing and face masks and vaccinations. Social distancing has negative economic effects. It is useful to understand the significance of these interventions, ([2], [16], [11],[18]).

Mathematical models have been used in epidemiology for many years, going back to the eighteenth century. Most of the models are compartmental models, with the population divided into classes and with assumptions being made about the rate of transfer from one class to another. Here we consider a Susceptible-Infectious-Recovered (SIR) model to describe the spread of the virus and compute the number of infected and dead individuals. There are models that include exposed and migration. The goal is to compute the number of infected, recovered, and dead individuals on the basis of the number of contacts, probability of disease transmission, incubation period, recovery rate, and fatality rate. The epidemic disease model predicts a peak of infected and dead individuals as a function of time and assumes that births

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and natural deaths are balanced, since we are dealing with a very short period of time. The population members solely decrease due to the disease as dictated by the fatality rate of the disease. The differential equations are solved with a forward Euler scheme, ([8]).

2. MATHEMATICAL MODELS

Mathematical and statistical methods provide essential input for governmental decision making that aims at controlling the outbreak. Statistical methods frequently aim at early detection of disease outbreaks ([16]). Another approach is to develop models that indicate the outbreak dynamics using compartmental models ([16]). In compartmental models we consider a fraction of the population to be susceptible, a fraction to be infected, a fraction that has recovered. In some models exposed group is part of the model. Compartmental models have been used to model HIV epidemic, malaria, and corona virus outbreak,([7],[12],[9] ,[16],[18]). In this paper we consider SIR model. SIR model can be modified in several ways, for example, by including demographics, deceased populations, hidden population, i.e., exposed populations (SEIR). In an accelerating epidemic outbreak contact tracing, the SEIR model needs to be modified to account for it. In the current paper we have two main objectives: (i) to report some new analytical results about SIR model and (ii) to introduce an optimization/neural network approach for the estimation of the parameters of the SIR model from real time series data. The SIR model is formulated in terms of three populations of individuals. The susceptible population, z_1, consists of all individuals susceptible to the infection of concern. The infected population population, z_2, comprises the infected individuals. These persons have the disease and can transmit it to the susceptible individuals. The recovered population, z_3, represents the immune individuals, who cannot become infected and cannot transmit the disease to others.

Another approach we use is neural network approach ([4], [17]).

In this paper we consider an SIR epidemic disease model. The total (initial) population, N, is categorized into three classes, namely, susceptible, $S(t)$, infected-infectious, $I(t)$, and recovered, $R(t)$, where t is the time variable. We consider discrete and continuous models.

The initial value problem we consider is

$$\begin{aligned}
\frac{dz_1}{dt} &= \lambda_{SC} \cdot z_1 - (\mu_{SC})z_1 - u \cdot z_1 z_2(1/N), \\
\frac{dz_2}{dt} &= u \cdot z_1 z_2(1/N) - (v+w)z_2 - (\mu_{SC})z_2 + u \cdot z_2 z_3(1/N), \\
\frac{dz_3}{dt} &= v \cdot z_2 - (\mu_{SC})z_3 - u \cdot z_2 z_3(1/N),
\end{aligned}$$

where $\lambda_{SC} = birthrate$, $\mu_{SC} = natural\ death\ rate$, u=transmission rate, v=recovery rate, w= death rate of infected, N=5149000, susceptible population in SC.

We solve the above system of differential Equations by using MATLAB Euler-scheme. The results are shown below. To determine the necessary parameters, we used data obtained from CDC and optimal control methodology as well as neural network and machine learning tools.

3. DISCRETE MODEL

We use data covering the period December 1, 2020, to June 1, 2021. In this period vaccination has been available although not taken advantage of by a lot of people. In addition, social distancing and face making have been less and less adhered to.

We consider the following discrete model covering the period December 1, 2020, to June 1, 2021. We have data for infected population and dead population for this model. We are going to rely on our model to estimate the recovered populations day by day covering this period. The recovered population for Dec. 1, 2020, is known to be 115152.

$$\begin{aligned} z_1(i+1) &= (1-vc)\cdot\lambda_{SC}\cdot N + z_1(i) - \mu_{SC}\cdot z_1(i) \\ &\quad -(1/(1+exp(-u(i))))z_1(i)z_2(i)(1/N) + (1/(1+exp(-s(i))))z_3(i), \\ z_2(i+1) &= z_2(i) + u(i)z_1(i)z_2(i)/N - (v(i) + 1/(1+exp(-w(i))) + \mu_{SC})z_2(i) \\ &\quad +(1/(1+exp(-r(i))))\cdot z_3(i), \\ z_3(i+1) &= vc\cdot\lambda_{SC}\cdot N + z_3(i) + (1/(1+exp(-v(i))))\cdot z_2(i) - (\mu_{SC} \\ &\quad +1/(1+exp(-r(i))) + 1/(1+exp(-s(i))))\cdot z_3(i), \end{aligned}$$

In this model,
$\lambda_{SC} = .058\, birth\ rate; \mu_{SC} = .0095, natural\ death\ rate$,
vc =.40, $vc\cdot N$ represents proportion of vaccinated people,
N=the susceptible population, 5149000,
transmission rate=1/(1+exp(-u(i))),
recovery rate=1/(1+exp(-v(i))),
relapsation rate= 1/(1+exp(-r(i))),
immunity rate=1/(1+exp(-s(i))),
death rate from infection=1/(1+exp(-w(i))).

Thus, the number of recovered compartment, z_3, increases by $vc\cdot N$, whereas the susceptible compartment z_1 increases by $(1-vc)\cdot\lambda_{SC}\cdot N$. We see the recovery, relapsation, and death rates are numbers between zero and 1. They are known. The optimization model determines what are appropriate. The number of infections arising from an infected individual is then modelled by the number $R_0(i)$ given below. The average basic reproduction number is 1.6133. A sketch of the reproduction number is shown below. We note it is slightly bigger than 1 consistent with the infected-recovered graph shown below.

$$\begin{aligned} A(i) &= (u(i)\cdot z(i,1)/N)/(v(i)+w(i)+\mu_{SC}) \\ R_0(i) &= (A(i) + 1/2\sqrt{A(i)^2 + 4\cdot v(i)\cdot r(i)/((v(i)+w(i)+\mu_{SC})\cdot(\mu_{SC}+r(i)+s(i)))} \end{aligned}$$

We would like to minimize the cost

$$C(i)^2 + D(i)^2 + E(i)^2$$

where

$$\begin{aligned} C(i) &= (z_2(i) - Inf(i)), \\ D(i) &= ((1/(1+exp(-w(i)))) \cdot z_2(i) - Dead(i)), \\ E(i) &= (z_2(i) - z_3(i)). \end{aligned}$$

$$\begin{aligned} \partial z_2/\partial u(i) &= (z_1(i)z_2(i)/N)(-1)exp(-u(i))/(1+exp(-u(i)))^2, \\ \partial z_2/\partial v(i) &= -z_2(i)(-1)exp(-v(i))/(1+exp(-v(i)))^2, \\ \partial z_2/\partial w(i) &= -z_2(i)(-1)exp(-w(i))/(1+exp(-w(i)))^2, \\ \partial z_2/\partial r(i) &= z_3(i)(-1)exp(-r(i))/(1+exp(-r(i)))^2, \\ \partial z_3/\partial r(i) &= -z_3(i)(-1)exp(-r(i))/(1+exp(-r(i)))^2, \\ \partial z_3/\partial s(i) &= -z_3(i)(-1)exp(-s(i))/(1+exp(-s(i)))^2. \end{aligned}$$

To update decision variables set

$$\begin{aligned} au(i) &= 2C(i)\partial z_2/\partial u(i) + 2D(i)(1/(1+exp(-w(i))))\partial z_2/\partial u(i) + 2D(i)\partial z_2/\partial u(i), \\ &\quad +2E(i)\partial z_2/\partial u(i) - 2E(i)\partial z_3/\partial u(i), \\ av(i) &= 2C(i)\partial z_2/\partial v(i) + 2D(i)(1/(1+exp(-w(i))))\partial z_2/\partial v(i) + 2D(i)\partial z_2/\partial v(i), \\ &\quad +2E(i)\partial z_2/\partial v(i) - 2E(i)\partial z_3/\partial v(i), \\ aw(i) &= 2C(i)\partial z_2/\partial w(i) + 2D(i)(1/(1+exp(-w(i))))\partial z_2/\partial w(i) + 2D(i)\partial z_2/\partial w(i), \\ &\quad +2E(i)\partial z_2/\partial w(i) - 2E(i)\partial z_3/\partial w(i) \\ ar(i) &= 2C(i)\partial z_2/\partial r(i) + 2D(i)(1/(1+exp(-w(i))))\partial z_2/\partial r(i) + 2D(i)\partial z_2/\partial r(i) \\ &\quad +2E(i)\partial z_2/\partial r(i) - 2E(i)\partial z_3/\partial r(i), \\ as(i) &= -2E(i)\partial z_3/\partial s(i). \end{aligned}$$

$$\begin{aligned} u(i) &= u(i) - del \cdot au(i), \\ v(i) &= v(i) - del \cdot av(i), \\ w(i) &= w(i) - del \cdot aw(i), \\ r(i) &= r(i) - del \cdot ar(i), \\ s(i) &= s(i) - del \cdot as(i). \end{aligned}$$

$Inf(i)$ is the number of infected people at or on the $i-th$ date after December 1, 2020. The numbers are gotten from CDC. Likewise $Dead(i)$ represents the number of dead people. The quantity $E(i)$ represents the difference between the number of infected people according to our model $z_2(i)$, and infected people, $Inf(i)$, gotten from CDC data. We represent the recovered people by $z_3(i)$.

The following 3 figures represents the number of infected and recovered populations, recovery and contact figures, and reproduction rates that were obtained using the discrete model approach (figure 1).

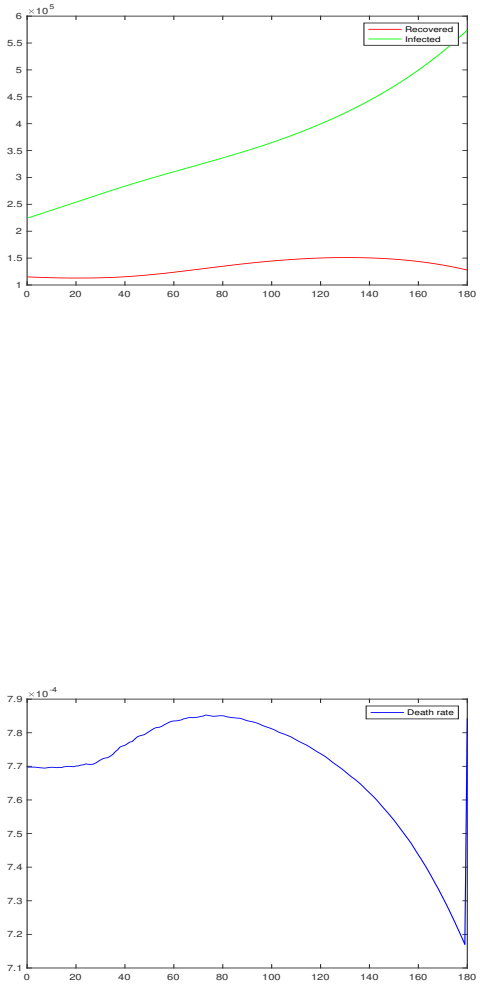


FIGURE 1. Infected and Recovered, Death rate.

4. **Continuous Model-Optimal Control Approach**

Mathematical models are important in analyzing the spread and control of infectious diseases. The model formulation requires carefully designed models with appropriate assumptions, and variables parameters. Mathematical models have been critical in the study of infectious diseases ([8] , [16], [17]). They have been used in studying tuberculosis([15], HIV ([9]), and dengue fever ([1]) models, etc. The aim here is to start with appropriate model and relevant

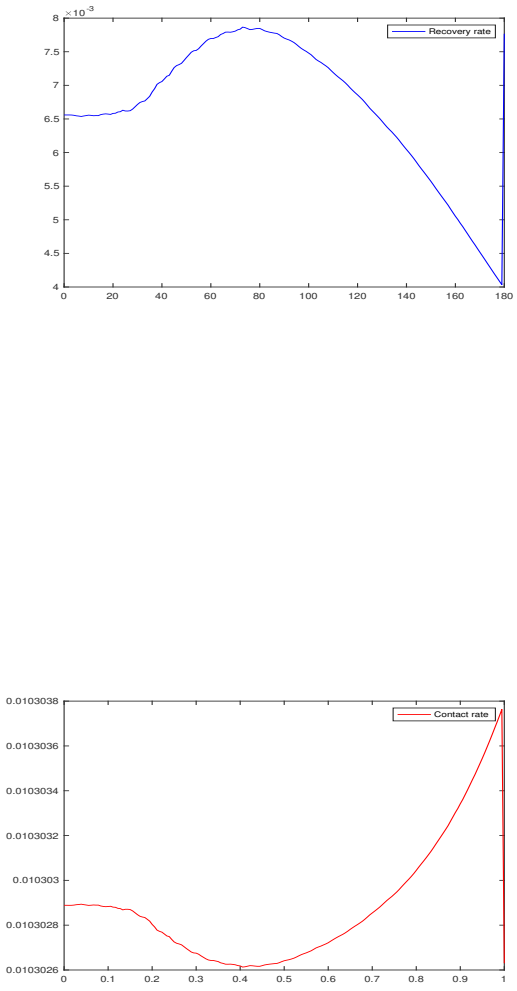


FIGURE 2. Recovery Rates and Contact Rates

parameters to be determined. Among the parameters of importance to be determined are contact rates u, recovery rates v, relapse rates r, infection reproduction rates R_0, death rates w, immunity rates s. We also include the role of vaccination. Although vaccinated people are unlikely to be infected contributing to immunity, there is still a possibility of relapse.

We would like to minimize the cost function

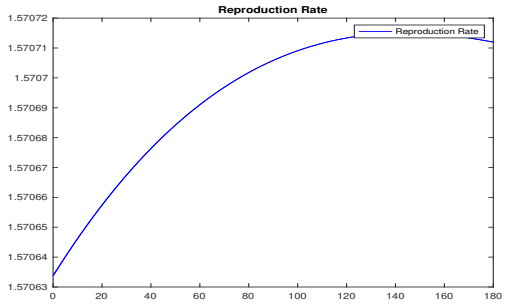


FIGURE 3. Reproduction Number.

$$\int_0^T \{(w(t)z_1(t) - Dead(t))^2 + (v(t)z_2(t) - z_3(t))^2 + (u(t)z_1(t) - z_2(t)^2\}dt$$

Subject to the constraint

$$\begin{aligned}
\frac{dz_1}{dt} &= \lambda_{SC} \cdot N - (\mu_{SC})z_1 - uz_1z_2(1/N) + s \cdot z_3, \\
\frac{dz_2}{dt} &= uz_1z_2(1/N) - (v+w)z_2 - \mu_{SC}z_2 + rz_3, \\
\frac{dz_3}{dt} &= vz_2 - (\mu_{SC})z_3 - rz_2 - sz_3.
\end{aligned}$$

The adjoint equation is

$$\begin{aligned}
dP_1/dt &= 2(uz_1 - z_2)u + (\mu_{SC} + uz_2/N)P_1 - (uz_2/N)P_2, \\
dP_2/dt &= 2(wz_2 - Dead(t))w + 2(vz_2 - z_3)v - 2(uz_1 - z_2) + (uz_1/N)P_1 \\
&\quad -(uz_1/N - v - w - \mu_{SC})P_2 - vP_3, \\
dP_3/dt &= -2(vz_2 - z_3) - sP_1 - rP_2 + (\mu_{SC} + r + s)P_3.
\end{aligned}$$

Next we construct the Hamiltonian.
Set

$$f_0(t) = (w(t)z_1 - Dead(t))^2 + (v(t)z_2 - z_3)^2 + (u(t)z_1 - z_2)^2,$$

Next,

$$\begin{aligned} \partial f_0/\partial u &= 2(uz_1 - z_2)z_1, \\ \partial f_0/\partial v &= 2(v)z_2 - z_3)z_2, \\ \partial f_0/\partial w &= 2(wz_2 - Dead(t))z_2. \end{aligned}$$

$$\begin{aligned} \partial f_1/\partial u &= -z_1z_2/N, \\ \partial f_1/\partial v &= 0, \\ \partial f_1/\partial w &= 0. \end{aligned}$$

$$\begin{aligned} \partial f_2/\partial u &= z_1z_2/N, \\ \partial f_2/\partial v &= -z_2, \\ \partial f_2/\partial w &= -z_2. \end{aligned}$$

$$\begin{aligned} \partial f_3/\partial u &= 0, \\ \partial f_3/\partial v &= z_2, \\ \partial f_3/\partial w &= 0. \end{aligned}$$

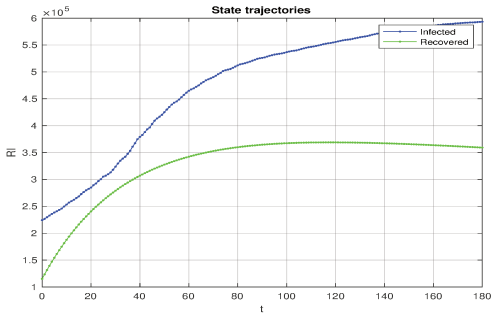
$$\begin{aligned} \partial H/\partial u(t) &= f_0(t)u(t) - P_1\partial f_1/\partial u - P_2\partial f_2/\partial u - P_3\partial f_3/\partial u, \\ \partial H/\partial v(t) &= f_0(t)v(t) - P_1\partial f_1/\partial v - P_2\partial f_2/\partial v - P_3\partial f_3/\partial v, \\ \partial H/\partial w(t) &= f_0(t)w(t) - P_1)\partial f_1/\partial w - P_2\partial f_2/\partial w - P_3\partial f_3/\partial w. \end{aligned}$$

Finally we update our control variables.

$$\begin{aligned} u(t) &= u(t) - randn \cdot del \cdot \partial H/\partial u(t), \\ w(t) &= w(t) - randn \cdot del \cdot \partial H/\partial w(t), \\ v(t) &= v(t) - randn \cdot del \cdot \partial H/\partial v(t). \end{aligned}$$

Again, we use the CDC data of infected population and dead people day by day from December 1, 2020, to June 1, 2021. We use our model to estimate the number of recovered people. The following figure represents the recovered (green) and infected (blue) populations.

We see from both discrete and continuous models is that the number of infected populations increases until mid-April and begins to decrease. The number of recovered populations follows the pattern of recovered populations. The number of recovered people becomes closer to the number of infected populations.



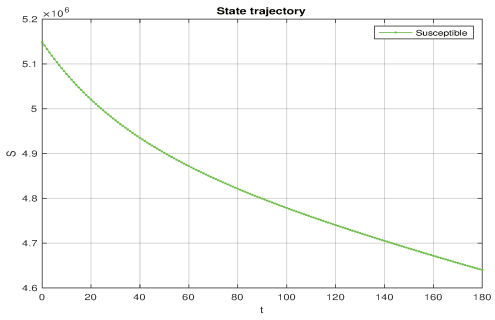
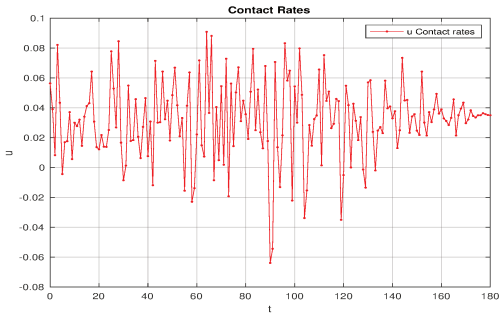


FIGURE 4. *Infected and Recovered States*, Susceptible State.

From the state equation (3.1) we consider

$$\begin{aligned} \frac{dz_2}{dt} &= uz_1z_2(1/N) - (v+w)z_2 - \mu_{SC}z_2 + rz_3, \\ \frac{dz_3}{dt} &= vz_2 - (\mu_{SC})z_3 - rz_2 - sz_3. \end{aligned}$$



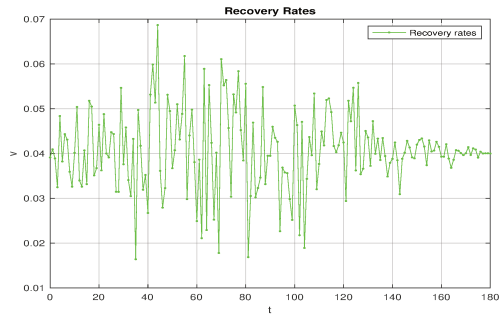


FIGURE 5. Contact Rates, Recovery Rates.

We rewrite this equations as

$$\frac{dz}{dt} = (F + V)z$$

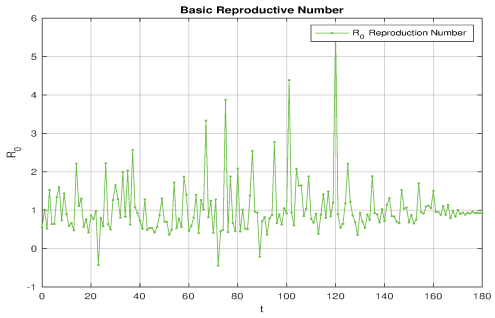


FIGURE 6. Reproduction Number.

where

$$F = \begin{bmatrix} uz_1/N & r \\ v & 0 \end{bmatrix}. \tag{4.1}$$

and

$$V = \begin{bmatrix} -v - w - \mu_{SC} & 0 \\ 0 & -\mu_{SC} - r - s \end{bmatrix}. \tag{4.2}$$

Now,

$$-FV^{-1} = \begin{bmatrix} uz_1/(v + w + \mu_{SC}) & r/(\mu_{SC} + r + s) \\ v/(v + w + \mu_{SC}) & 0 \end{bmatrix}. \tag{4.3}$$

$$\begin{aligned} A(i) &= (u(i) \cdot z(i,1)/N)/(v(i) + w(i) + \mu_{SC}) \\ R_0(i) &= (A(i) + 1/2\sqrt{A(i)^2 + 4 \cdot v(i) \cdot r(i)/((v(i) + w(i) + \mu_{SC}) \cdot (\mu_{SC} + r(i) + s(i)))} \end{aligned}$$

The dominant eigenvalue of $-FV^{-1}$ is R_0 and $R_0(i)$ is 1.0314. A sketch of the reproduction number is shown below. We note it is slightly bigger than 1 consistent with the infected-recovered graph shown below.

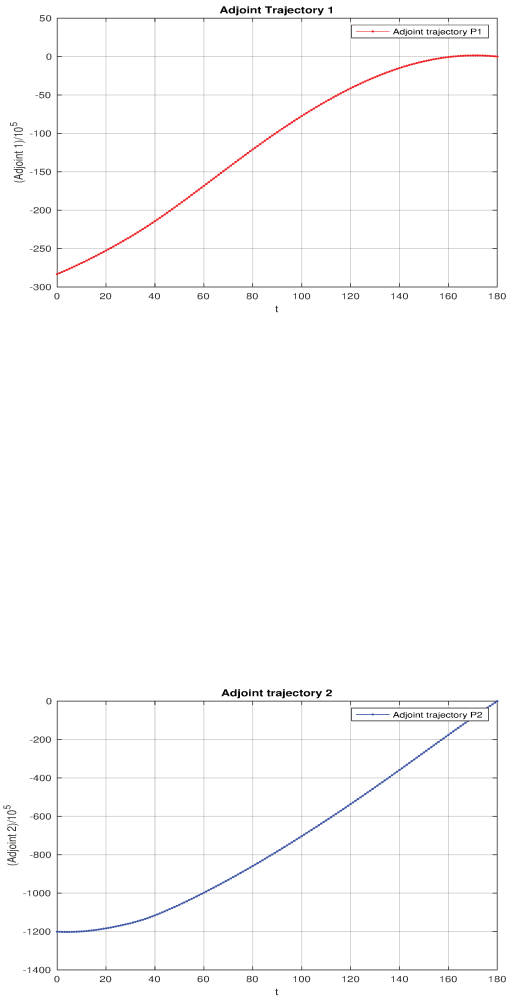


FIGURE 7. AdjointTrajectories 1,2.

5. CONCLUSION

The worldwide spread of corona virus exerts enormous pressure on healthcare systems, societies, and governments. Therefore, predicting the epidemic dynamics is an important problem from a data science and mathematical modeling perspective. The motivation of the current work was to explore the potential of sequential data assimilation to create a regional epidemic model as a forecasting tool. The standard epidemic SIR-type models implement a

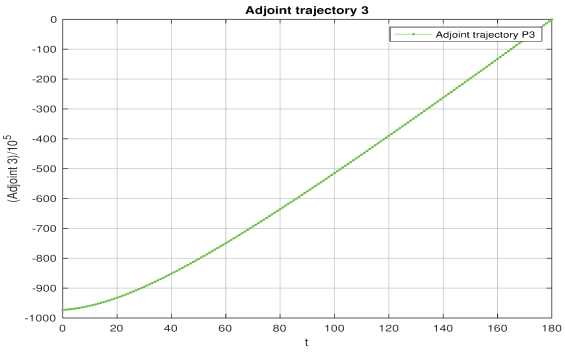


FIGURE 8. AdjointTrajectory 3.

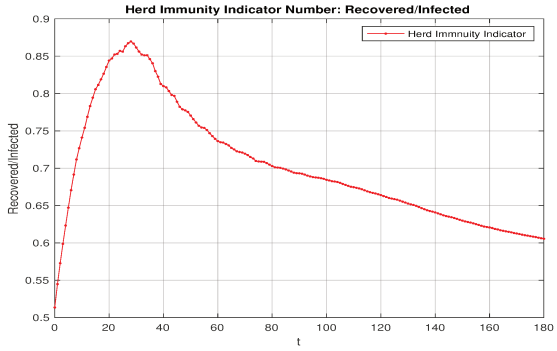


FIGURE 9. Herd Immunity Indicator 3.

compartmental description under the assumption of homogeneous mixing of individuals.

More realistic modeling approaches must account for spatial heterogeneity due to time varying disease onset times, regionally different contact rates, and the time dependence of the contact rates due to the implementation of containment strategies. However, extensive data are not currently available. Thus, we must construct models where control theory, optimization, and neural network methodologies to approximate missing and necessary data. In the work we did relating to data from December 1, 2020, to June 1, 2021, we rely only on available data of infected and dead populations to have some ideas on the transmission, recovery, and relapse rates.

What we see in the last three pictures from the discrete model are a decrease in death rate, high recovery rate, and decreasing infection transmission rate. The basic reproduction rate is consistent with this observation although it trending upward, but less than 1. What we see in the very last picture is like the first picture of the recovered and infected populations. We notice they are similar.

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