TUNING FREQUENCY STABILITY IN MICROMECHANICAL RESONATORS WITH PARAMETRIC PUMPING

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ABSTRACT

In this work we experimentally investigate the influence of parametric amplification and parametric suppression on the frequency stability of micromechanical resonators. We isolate the influence of phase slope tuning from changes in the vibrational amplitude and find that parametric suppression improves the frequency stability in the thermal noise regime by over threefold, while parametric amplification degrades the frequency stability by nearly a factor of two.

KEYWORDS

frequency stability, quality factor, effective quality factor, parametric pumping, microcantilever

INTRODUCTION

Frequency stability is a key figure of merit that affects the signal-to-noise ratio (SNR) of resonant sensors and timing references. In microelectromechanical (MEM) resonators, frequency stability is known to depend on nonlinearity [1, 2], environmental noise [3], and resonator parameters such as the quality factor (Q) [4, 5]. A variety of techniques such as parametric amplification and thermal-piezoresistive pumping have been studied for tuning the effective quality factor (Q_{eff}) of MEM resonators [6, 7], and are known to affect the thermomechanical noise of a resonator [8], but the exact role they play in improving sensor and oscillator performance is still under investigation. These techniques have typically been used to construct oscillators [9] or improve the sensitivity of amplitude-modulated sensors by increasing the vibration amplitude [10, 11], but recently there is interest in using effective quality factor tuning mechanisms to improve the performance of frequency-shift sensors. When placed in a phase-locked loop, frequency-shift sensors transduce signals that induce a shift in the resonant frequency of the device to a change in the loop phase via the resonator's phase-frequency relationship. In this operational mode, a larger phase slope results in more phase change for a given input signal. Effective quality factor tuning mechanisms have been used to improve the performance of resonant sensors using this principle [12, 13], but the effect on a resonator's fundamental frequency stability under the influence of Q_{eff} tuning mechanisms is under investigation [14].

In this work we experimentally investigate the influence of parametric amplification and suppression on frequency stability of a resonator while controlling for the influence of vibrational amplitude. Increasing a resonator's intrinsic quality factor reduces the linewidth of the amplitude response of the device transfer function, steepens the slope of the phase response of the device transfer function, and results in reduced phase noise [15]. However, because phase-dependant Q_{eff} tuning methods like parametric pumping have the opposite effect on the phase slope as they do on the linewidth of the resonator, Q_{eff} suppression increases the magnitude of the phase slope at resonance. When additive white noise dominates, the frequency stability, σ_a, of a resonant sensor over integration time, τ, is given by [4]:

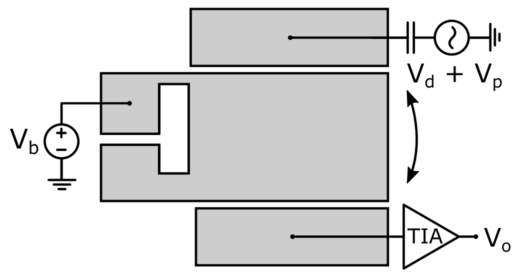

Figure 1: The setup for measuring the open loop response of the cantilevered beam resonator under test. The motion of the resonant beam, held at a bias voltage, V_b, induces a current in the sensing electrode which is transduced into a measurable voltage by a transimpedance amplifier (TIA). We apply a drive voltage, V_d, at a frequency near the resonant frequency of the cantilever, and a parametric pump voltage, V_p, at twice the frequency of the drive voltage.

$$\sigma_a(\tau) \approx \left(\omega_0 \left|\frac{\partial\phi}{\partial\omega}\right|_{\omega_0}\right)^{-1} \sqrt{\frac{V_n^2}{V_s^2}} \sqrt{\frac{1}{2\pi\tau}}, \qquad (1)$$

where $\left|\frac{\partial\phi}{\partial\omega}\right|_{\omega_0}$ is the phase slope at resonance, ω_0 is the resonant frequency, V_n is the white noise floor, and V_s is the signal amplitude. Effective quality factor tuning mechanisms modify the transfer function of the resonator, which changes the observed thermomechanical noise, but in most practical cases the noise floor of the system is dominated by another source, such as noise in the readout method or an amplifier. In the case where the Q_{eff} tuning method does not modify the noise of floor of the combined system and the amplitude of the device remains constant, Eq. (1) demonstrates that increasing the phase slope at resonance improves frequency stability. Counterintuitively, this results in a scenario where parametric suppression improves the frequency stability of the resonator and parametric amplification, which reduces the resonator's linewidth, degrades frequency stability.

MODEL AND EXPERIMENTAL SETUP

We study a single-crystal silicon cantilevered resonator with a lumped mass of $8.53\,\mu$g, fabricated within a wafer-scale encapsulation process [16] and capacitively driven and

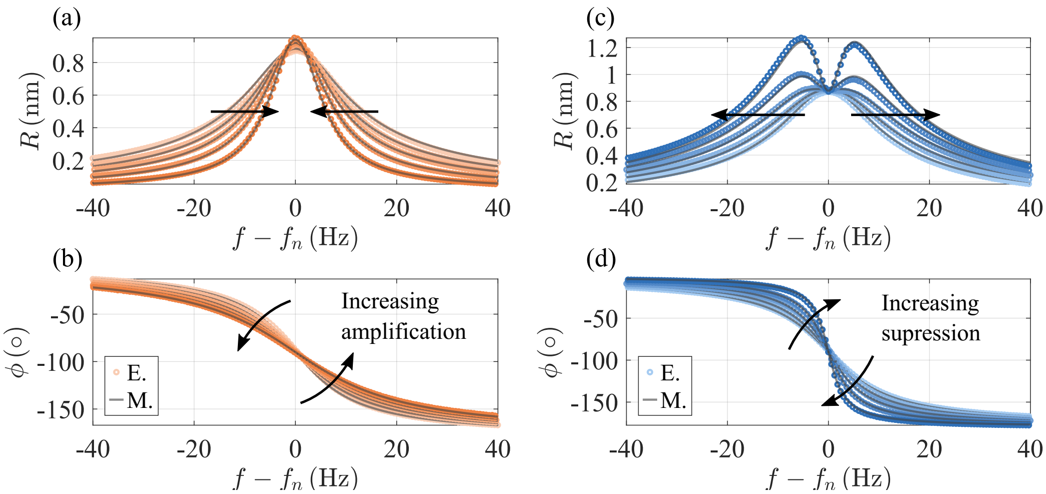

Figure 2: The measured (a,c) amplitude and (b,d) phase of a driven mechanical mode subject to parametric pumping (open circles) plotted against the model from Eq. (5) (grey lines) for the case of (a,b) parametric amplification pump amplitudes of 0:45:180 mV (light orange to dark orange) and (c,d) parametric suppression pump amplitudes of 0:45:180 mV (light blue to dark blue). For the amplification case as the parametric pump strength increases, the slope of the phase response becomes shallower, while in the parametric suppression case the opposite effect occurs. A thermomechanical calibration is used to calibrate the displacement to units of meters.

sensed across a $1.0\,\mu$m gap. Using a high gain readout amplifier suitable for transducing small signals, we first measure the thermomechanical motion of the device. For subsequent measurements we use a low-gain, high dynamic range readout which has a noise floor dominated by a constant white noise source. We calibrate the voltage amplitude at the output of the capacitive readout to units of displacement, and measure the gap size between the device and capacitively coupled electrodes using the measured thermomechanical motion of the device [17, 18] and the known gain of the two readouts. We measure the device's amplitude-frequency and phase-frequency response under the presence of direct forcing and a parametric pump using the experimental setup depicted in Figure 1. For each level of parametric pump, the level of direct drive is adjusted to achieve a constant amplitude at the resonant frequency for fair comparison of frequency fluctuations. We model the system by considering a simple harmonic oscillator with natural frequency ω_0 and quality factor Q under the presence of a mass-normalized direct drive force (f) and parametric pump with strength (λ) and phase (ψ) given by:

$$\ddot{x} + \frac{\omega_0}{Q}\dot{x} + \omega_0^2 x + \lambda\cos(2\omega t)\,x = f\cos(\omega t + \psi)\,. \quad (2)$$

To derive the system response, we first decompose the displacement into two slowly varying quadratures, $a(t)$ and $b(t)$:

$$x(t) = a(t)\cos(\omega t) + b(t)\sin(\omega t), \quad (3)$$

which must satisfy the equation of constraint:

$$0 = \dot{a}(t)\cos(\omega t) + \dot{b}(t)\sin(\omega t). \quad (4)$$

Via the method of averaging [19] we obtain the slow time equations for the two quadratures, $a(t)$ and $b(t)$, accurate to the first order:

$$\begin{aligned} \frac{\partial a}{\partial t} &= -\frac{\omega b}{2} - \frac{\omega_0 a}{2Q} + \frac{\omega_0^2 b}{2\omega} - \frac{\lambda b}{4\omega} + \frac{f\sin(\psi)}{2\omega}, \\ \frac{\partial b}{\partial t} &= \frac{\omega a}{2} - \frac{\omega_0 b}{2Q} - \frac{\omega_0^2 a}{2\omega} - \frac{\lambda a}{4\omega} + \frac{f\cos(\psi)}{2\omega}. \end{aligned} \quad (5)$$

These equations govern the slow time dynamics, and constant values of a and b correspond to a periodic response. Setting the expressions for the derivatives of the two slowly varying quadratures to zero and solving for a and b gives closed form solutions for the device response which can be expressed in terms of amplitude ($R = \sqrt{a^2 + b^2}$), and phase ($\phi = -\arctan(b/a)$). Maximum parametric amplification occurs when the relative phase, ψ, is $-\pi/4$ and maximum parametric suppression occurs when the relative pump phase is $\pi/4$. Fitting to those expressions allows us to extract the device quality factor and frequency, and independently confirm the capacitive gap size. The phase slope of the response at resonance can be derived from the model as:

$$\left|\frac{\partial\phi}{\partial\omega}\right|_{\omega_0} = \frac{4\,\omega_0}{2\omega_0^2/Q \pm \lambda} \quad (6)$$

where $\pm\lambda$ corresponds to the maximum amplification or suppression cases, respectively. Finally, for each combination of drive and pump we sample the amplitude and phase of the resonator at resonance for an extended period. Using the measured device phase response, we calculate the implied frequency fluctuations from the measured phase fluctuations. We then calculate the Allan deviation of frequency stability as a function of integration time for the resonator during the sampling period [20].

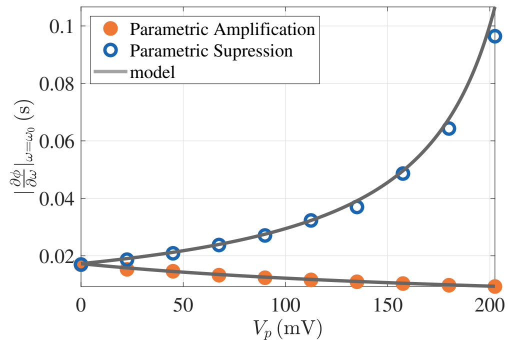

Figure 3: The measured phase slope of the resonator response at resonance as a function of parametric pump voltage for both parametric amplification (orange filled circles) and parametric suppression (blue open circles) plotted against the model (grey lines).

RESULTS AND DISCUSSION

Figure 2 presents open-loop sweeps of the resonator with the pump phase chosen to achieve maximum parametric amplification ($\psi = -\pi/4$), and maximum parametric suppression ($\psi = \pi/4$), demonstrating the effect of parametric pumping on the resonator's phase slope. Fitting to the model given by solving Eq. (5) allows us to measure the natural freqeuncy, $\omega_0/2\pi = 515\,\text{kHz}$, and quality factor, $Q = 2.8\,\text{k}$, for the device and verify that the measured gap size and applied pump and drive voltages corresponds to the expected value of λ and f in each case. In the parametric amplification case as the pump strength increases, the linewidth of the resonator decreases whereas in the parametric suppression case as the pump strength increases, the linewidth of the resonator increases. In both cases, the amplitude of the device at resonance is kept constant constant by tuning the magnitude of direct forcing.

The magnitude of phase slope of the device at resonance for each level of parametric pumping is plotted in Figure 3 for both the amplification and suppression cases against the model in Eq. (6). For the parametric amplification case the magnitude of phase slope of the device decreases, or becomes shallower, as the pump strength increases and in the parametric suppression case the opposite occurs. This behavior is opposite from what would be expected if the linewidth of the resonator was being modified by changing its quality factor, as increasing the Q of a resonator decreases its linewidth and increases the magnitude of the phase slope at resonance. Modifying the effective quality factor of a resonator with parametric pumping counterintuitively has the opposite effect on the amplitude response as it does on the phase response.

Figure 4 shows the Allan deviation frequency, measured in open loop, of the device operating at resonance for each of the cases plotted in Figure 3. The Allan deviation in the white noise regime, for $\tau < 10^{-2}\,\text{s}$, increases when parametric amplification is applied but decreases when parametric suppression is applied. The Allan deviation in the drift regime, for $\tau > 1\,\text{s}$, is unaffected by parametric pumping. This suggests that the steeper phase slope of the resonator in the case of parametric suppression reduces phase to frequency noise conversion, mediated by the phase slope of the resonator, but does not influence mechanisms that induce drift. When strong parametric suppression is applied, unexplained frequency fluctuations limit the Allan deviation and appear as a "floor" in the Allan deviation [21]. The Allan deviation in the thermal regime, plotted in Figure 5 shows that parametric suppression reduces the Allan deviation in the thermal regime by more than threefold, while parametric amplification almost doubles Allan deviation.

We employed parametric pumping to modify the effective quality factor of the resonator under study, and ex-

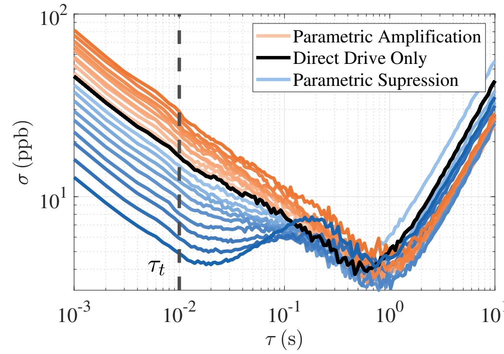

Figure 4: The Allan deviation of frequency stability of a mechanical mode measured in open loop subject to direct drive and parametric pump for, direct drive only (black line), increasing parametric enhancement pump strength (light orange to dark orange), and increasing parametric suppression pump strength (light blue to dark blue) for the pump amplitudes plotted in Figure 3. The direct forcing was tuned to maintain a constant device amplitude. The Allan deviation in the thermal noise regime decreases in the case of parametric suppression and increases in the case of parametric amplification.

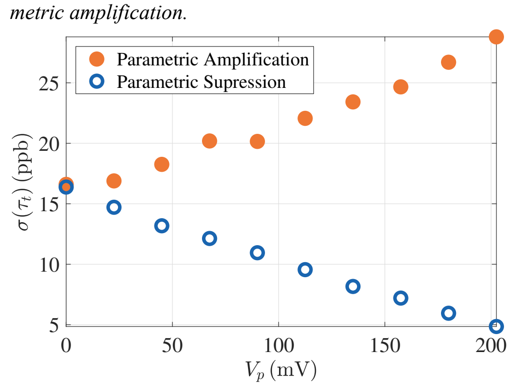

Figure 5: The Allan deviation of frequency at $\tau_t = 10^{-2}$ s, in the thermal noise regime, for constant device amplitude and varying levels of parametric pump for both parametric amplification (orange filled circles) and parametric suppression (blue open circles).

perimentally demonstrated that the phase dynamics of the resonator, not the linewidth modification, dominate the frequency stability of the system in practical cases where the system noise floor is constant. Using parametric suppression to increase the magnitude of the resonator phase slope, we demonstrate a threefold improvement in resonator frequency stability.

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