Information Preferences of Individual Agents in Linear-Quadratic-Gaussian Network Games

Furkan Sezer, *Student Member, IEEE*, Ceyhun Eksin, *Member, IEEE*

***Abstract*— We consider linear-quadratic-Gaussian (LQG) network games in which agents have quadratic payoffs that depend on their individual and neighbors' actions, and an unknown payoff-relevant state. An information designer determines the fidelity of information revealed to the agents about the payoff state to maximize the social welfare. Prior results show that full information disclosure is optimal under certain assumptions on the payoffs, i.e., it is beneficial for the average individual. In this paper, we provide conditions for general network structures based on the strength of the dependence of payoffs on neighbors' actions, i.e., competition, under which a rational agent is expected to benefit, i.e., receive higher payoffs, from full information disclosure. We find that all agents benefit from information disclosure for the star network structure when the game is homogeneous. We also identify that the central agent benefits more than a peripheral agent from full information disclosure unless the competition is strong and the number of peripheral agents is small enough. Despite the fact that all agents expect to benefit from information disclosure ex-ante, a central agent can be worse-off from information disclosure in many realizations of the payoff state under strong competition, indicating that a risk-averse central agent can prefer uninformative signals ex-ante.**

***Index Terms*— Information design, welfare maximization, network games**

I. Introduction

In an incomplete information network game, multiple players compete to maximize their individual payoffs that depend on the action of the player, the neighboring players' actions, and on unknown states. Incomplete information games are employed to model traffic flow in communication or transportation networks [1], [2], power allocation of users in wireless networks with unknown channel gains [3], oligopoly price competition [4], and coordination of autonomous teams [5], [6]. The information design problem refers to the determination of the information fidelity of the signals given to the players about the payoff state so that the induced actions of players maximize a system level objective.

An information designer is an entity that is more informed about the realized payoff state than the players. Information designer selects an optimal probability distribution of signals based on the state realization with respect to its objective (see Fig. 1). Various entities, e.g., a system operator overseeing the spectrum allocation, a market-maker, an independent system operator in the power grid, or the federal reserve, can be considered as an information designer. Information designers may define different objectives such as maximizing social welfare [7], minimizing misinformation [8], or maximizing auction revenue [9]. In control systems, information design is used for robust sensor design [10], deception/privacy modelling [11] and queues with heterogeneous users [12]. In the absence of a (real) designer, an information design problem can quantify the sensitivity of a system level objective to the information available to the players [13], [14].

Authors are with the Department of Industrial and Systems Engineering, Texas A&M University, College Station, TX 77843 USA (e-mails: furkan.sezer@tamu.edu, eksinc@tamu.edu). This work was supported by NSF CCF-2008855.

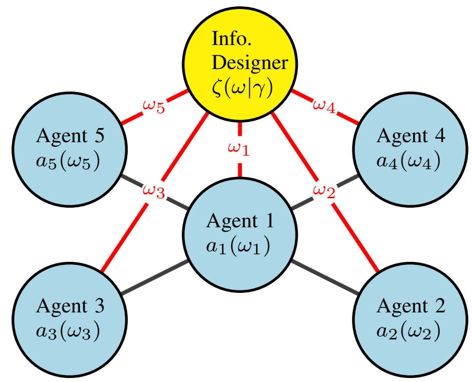


Fig. 1. Agents play a network game with individual payoffs that depend on their neighbors' actions and an unknown payoff state γ. An information designer sends a signal $\boldsymbol{w}_i$ drawn from information structure $\zeta(\boldsymbol{\omega}|\gamma)$ to each agent i. Agent i takes an equilibrium action $\boldsymbol{a}_i$ based on the received signal $\boldsymbol{\omega}_i$ to maximize its expected utility.

In this paper, we consider social welfare maximization via information design in linear-quadratic-Gaussian (LQG) network games (Fig. 1). Social welfare is defined as the aggregate utility of the players. In an LQG game, the players have quadratic payoff functions, and the state and the signals (types) come from a Gaussian distribution [15]. Under certain assumptions for the quadratic payoff coefficients, the rational behavior, defined as the Bayesian Nash equilibrium (BNE), in LQG games is unique. In [7], we show that full information disclosure is the optimal solution to social welfare maximization under public information structures and/or common payoff states (see Theorem 1).

While full information disclosure may be optimal from the system perspective, here we analyze the effect of such information disclosure policy on the payoffs of individual agents and its dependence on the centrality of the agents. We identify sufficient conditions for the individual preference of informative signals based on the payoff coefficients prior to realization of the state (ex-ante) in Theorem 2. We leverage this result, and identify that both central and peripheral agents in a star network structure prefer information disclosure ex-ante for homogeneous LQG games (Proposition 1). In computing the benefit of information disclosure to individual agents, we find that a peripheral agent can benefit more than the central agent under full information disclosure if competition is strong and number of agents is small (Proposition 2). In sum, the incentives of the agents and the system designer are in congruence ex-ante given the conditions considered.

We find that joint incentives of individual agents and the system designer can cease to exist ex-post, i.e., after the realization of the payoff state. In contrast to Proposition 1, the central agent prefers no information disclosure ex-post if realization of the payoff state is lower than expected. In the context of Bertrand competition among firms in networked markets, these results imply that central firms may not benefit from information disclosure when the competition among firms is strong. Ex-post analysis is not useful because agents do not observe the realized payoff state when taking actions, but they observe signals generated by the information designer based on the realized state. Still, the ex-post analysis of incentives imply that a risk averse central agent can prefer uninformative signals ex-ante. These results extend prior knowledge on the information design problem [14], [16] by providing a characterization of the benefit of informative signals on players' payoffs and its dependence on centrality of the players in network games with incomplete information.

II. Information Design in LQG Network Games

A. Information Design in Incomplete Information Games

An incomplete information game G involves a set of $n \in \mathbb{N}^+$ players belonging to the set $\mathcal{N} := \{1, ..., n\}$, each of which selects actions $a_i \in A_i$ to maximize the expectation of its individual payoff function $u_i(a, \gamma)$ where $a \equiv (a_i)_{i\in\mathcal{N}} \in A$ and $\boldsymbol{\gamma} \equiv (\gamma_i)_{i\in\mathcal{N}} \in \Gamma$ correspond to an action profile and an unknown payoff state, respectively. Agents form expectations about their payoffs based on their beliefs/types $\omega_i \in \Omega$ about the state. We represent the incomplete information game by the tuple $G := \{\mathcal{N}, A, \{u_i\}_{i\in\mathcal{N}}, \{\omega_i\}_{i\in\mathcal{N}}\}$.

A strategy of player i in an incomplete information game maps each possible value of its type $\omega_i \in \Omega$ to an action, i.e., $s_i : \Omega \to A_i$. A strategy profile $s = (s_i)_{i\in\mathcal{N}}$ is a BNE with respect to an information structure (distribution function) ζ, if it satisfies the following inequality

$$E_\zeta[u_i(s_i(\omega_i), s_{-i}, \boldsymbol{\gamma})|\omega_i] \geq E_\zeta[u_i(a_i', s_{-i}, \boldsymbol{\gamma})|\omega_i], \quad (1)$$

for all $a_i' \in A_i, \omega_i \in \Omega, i \in \mathcal{N}$ where $s_{-i} = (s_j(\omega_j))_{j\neq i}$ denotes the equilibrium strategy of all the other players, and E_ζ is the expectation operator with respect to the distribution function ζ and a prior distribution on the payoff state.

An information designer aims to optimize the expected value of its objective function $f(s, \boldsymbol{\gamma})$, e.g., social welfare, by deciding on an information structure ζ belonging to the feasible space of probability distributions Z, i.e.,

$$\max_{\zeta\in Z} E_\zeta[f(s, \boldsymbol{\gamma})] \quad (2)$$

where s is a BNE strategy profile (1) for the game G under ζ. The timeline for the information design problem is as follows—also see Fig. 1.

1) Designer selects $\zeta \in Z$ and notifies players.
2) Payoff state $\boldsymbol{\gamma}$ is realized.
3) Players observe signals ω drawn from $\zeta(\omega|\boldsymbol{\gamma})$.
4) Players act according to BNE given ζ.

The information design problem in (2) is intractable in the general case. Here, we focus on LQG network games which yield to a tractable semi-definite program formulation of the information design problem for objectives $f(\cdot)$ that are quadratic in strategies and state [16].

B. Linear-Quadratic-Gaussian (LQG) Network Games

An LQG game corresponds to an incomplete information game with quadratic payoff functions and Gaussian information structures. In an LQG game, each player $i \in \mathcal{N}$ decides on his action $a_i \in \mathbb{R}$ according to a payoff function of the form given below,

$$u_i(a, \boldsymbol{\gamma}) = -H_{ii}a_i^2 - 2\sum_{j\neq i} H_{ij}a_i a_j + 2\gamma_i a_i + d_i(a_{-i}, \boldsymbol{\gamma}) \quad (3)$$

where $a \equiv (a_i)_{i\in N} \in \mathbb{R}^n$ and $\boldsymbol{\gamma} \equiv (\gamma_i)_{i\in\mathcal{N}} \in \mathbb{R}^n$. The term $d_i(a_{-i}, \boldsymbol{\gamma})$ is an arbitrary function of the opponents' actions $a_{-i} \equiv (a_j)_{j\neq i}$ and payoff state $\boldsymbol{\gamma}$. We assume the utility function is strictly concave, i.e., $H_{ii} > 0$ for all $i \in \mathcal{N}$. We collect the coefficients of the utility function in a matrix $H = [H_{ij}]_{n\times n}$. Payoff state $\boldsymbol{\gamma} \in \mathbb{R}^n$ follows a Gaussian distribution, i.e., $\boldsymbol{\gamma} \sim \psi(\boldsymbol{\mu}, \Sigma)$ where ψ is a multivariate normal probability distribution with mean $\boldsymbol{\mu} \in \mathbb{R}^n$ and covariance matrix $\Sigma \in \mathbb{R}^{n\times n}$. Each player $i \in \mathcal{N}$ receives a private signal $\omega_i \in \Omega_i \equiv \mathbb{R}^{m_i}$ for some $m_i \in \mathbb{N}^+$. We define the information structure of the game $\zeta(\omega|\boldsymbol{\gamma})$ as the conditional distribution of $\omega \equiv (\omega_i)_{i\in N}$ given $\boldsymbol{\gamma}$. We assume the joint distribution over the random variables $(\omega, \boldsymbol{\gamma})$ is Gaussian; thus, ζ is a Gaussian distribution. For a positive definite matrix H, there is a unique BNE strategy that is linear in private signals. Moreover, we can obtain the BNE strategy by solving a set of linear equations [15].

We consider network games where the nodes are the players $\mathcal{N}$, and edges $\mathcal{E}$ determine the payoff dependencies, i.e., if $(i, j) \notin \mathcal{E}$ then $H_{ij} = 0$, otherwise $H_{ij} \in \mathbb{R}$ for $(i, j) \in \mathcal{E}$. Next, we provide an example.

Example 1 (Bertrand Competition in Networked Markets): Firms determine the price for their goods (a_i) facing a marginal cost of production (γ_i). Firms compete over markets that are connected [17]. The demand is a function of the price of all the firms, $q_i = \vartheta - \varpi a_i + \varrho \sum_{j\neq i} a_j$ with positive constants ϑ, ϖ and ϱ. Firm i's profit is its revenue $q_i a_i$ minus the cost of production $\gamma_i q_i$,

$$u_i(a, \gamma) = q_i a_i - \gamma_i q_i \quad (4)$$

Nodes of networks correspond to a firm in Bertrand competition. If two nodes share an edge, they compete over the same market. For a star network, the central node can be a multinational firm competing with local competitors (peripheral nodes).

Remark 1: Prior studies in network games with quadratic payoffs focus on computation and characterization of equilibria, and analyze the changes to the equilibria or social welfare when network topology is modified via adding/removing links or nodes [18]–[22]. In contrast, this paper considers the effects of information design on individual payoffs when the design objective is to maximize social welfare.

III. Social Welfare Maximization via Information Design

Social welfare is the sum of agents' (quadratic) utilities:

$$f(a,\boldsymbol{\gamma}) = \sum_{i=1}^{n} u_i(a,\boldsymbol{\gamma}) \tag{5}$$

$$= \sum_{i=1}^{n}(-H_{ii}a_i^2 - 2\sum_{j\neq i} H_{ij}a_i a_j + 2\gamma_i a_i + d_i(a_{-i},\boldsymbol{\gamma})). \tag{6}$$

Given quadratic utilities and Gaussian information structure, the information design problem (2) is transformed to the maximization of a linear function of a positive semi-definite covariance matrix ($X = \text{cov}(a,\boldsymbol{\gamma})$) subject to linear constraints stemming from the BNE condition in (1). That is, the information design problem is a semi-definite program (SDP)—see [7], [16] for an explicit formulation of the SDP.

Using this SDP formulation, it is shown in [7], [16] that *full information disclosure*, i.e., signals that reveal the payoff state, is an optimal strategy for the information designer under certain special scenarios.

Theorem 1 (Proposition 7-8, [7]): Full information disclosure is the optimal solution to (2) for social welfare maximization objective in (5), if either of the following conditions hold:

(a) H is positive definite and information designer reveals a single (public) signal.

(b) H is symmetric and there is a common payoff state, i.e. $\gamma_i = \gamma_j, \forall i,j \in \mathcal{N}$.

See [7] for the proof and details. We interpret the results in Theorem 1 for network games where $H_{ij} = 0$ if $(i,j) \notin \mathcal{E}$. Theorem 1(a) implies that if H is diagonally dominant, then full information disclosure is optimal for public signal structures. In the context of Bertrand competition, this result implies that if firms receive the same signal on the cost of their production, it is preferable to reveal the realized cost of production. According to Theorem 1(b), full information disclosure is optimal given a common payoff state and symmetric H. A common payoff state corresponds to a common marginal cost for firms in Bertrand competition. This result implies that each firm should receive a fully informed signal on the marginal cost to maximize the social welfare. Next, we analyze the ex-ante information structure preferences of individual agents based on their position in the network.

IV. Ex-ante Information Structure Preferences of Agents based on Network Position

When there is a common payoff state γ, i.e., $\gamma_i = \gamma$, for $i \in \mathcal{N}$ and public signals $\omega_i = \bar{\omega}$ for $i \in \mathcal{N}$, individual equilibrium actions under full and no information disclosure are given by $a_i = \gamma[H^{-1}\mathbf{1}]_i$ and $a_i = \mu[H^{-1}\mathbf{1}]_i$, respectively for $i \in \mathcal{N}$ where $\mathbf{1} \in \mathbb{R}^n$ is a vector of ones and $[\cdot]_i$ represents the ith element of a vector—see Appendix for the derivation. In this section, we treat the actions as random variables where we assume $\gamma \sim \psi(\mu,\sigma^2)$ and $\mu \sim \psi(\mu_0,\sigma_0^2)$.

Theorem 2: Consider a LQG network game with common payoff state γ and public information structures. Define

$$V_i(H) := [H^{-1}\mathbf{1}]_i\Big(2 - H_{ii}[H^{-1}\mathbf{1}]_i - 2\sum_{j\neq i} H_{ij}[H^{-1}\mathbf{1}]_j\Big). \tag{7}$$

If $V_i(H) > 0$, then full information disclosure is preferable by agent $i \in \mathcal{N}$ over no information disclosure.

Proof: If agent i's expected utility given full information disclosure is larger than its expected utility at no information disclosure, then full information disclosure is preferable. We start with computing agent i's expected utility under full information disclosure by plugging in the equilibrium action profile $a = \gamma H^{-1}\mathbf{1}$ (see Lemma 1) into (3):

$$E[u_i(a,\gamma)] = E[\gamma^2][H^{-1}\mathbf{1}]_i\Big(2 - H_{ii}[H^{-1}\mathbf{1}]_i - 2\sum_{j\neq i} H_{ij}[H^{-1}\mathbf{1}]_j\Big) + E[d_i(a_{-i},\gamma)] \tag{8}$$

Next, we plug in the equilibrium action profile for no information disclosure $a = \mu H^{-1}\mathbf{1}$ (see Lemma 1) into (3):

$$E[u_i(a,\gamma)] = [H^{-1}\mathbf{1}]_i\Big(E[\mu^2](-H_{ii}[H^{-1}\mathbf{1}]_i - 2\sum_{j\neq i} H_{ij}[H^{-1}\mathbf{1}]_j) + 2E[\gamma\mu]\Big) + E[d_i(a_{-i},\gamma)] \tag{9}$$

We subtract (9) from (8) to obtain the difference between expected utilities under full information and no information:

$$E[\Delta u_i(a,\gamma)] = [H^{-1}\mathbf{1}]_i\Big(E[\gamma^2-\mu^2](-H_{ii}[H^{-1}\mathbf{1}]_i - 2\sum_{j\neq i} H_{ij}[H^{-1}\mathbf{1}]_j) + 2E[\gamma^2-\gamma\mu]\Big) \tag{10}$$

$$= \sigma^2[H^{-1}\mathbf{1}]_i\Big(2 - H_{ii}[H^{-1}\mathbf{1}]_i - 2\sum_{j\neq i} H_{ij}[H^{-1}\mathbf{1}]_j\Big). \tag{11}$$

To get the second equality, we use $E[\mu^2] - \mu_0^2 = \sigma_0^2$, $E[\gamma^2] = \sigma^2 + \sigma_0^2 + \mu_0^2$ and $E[\gamma\mu] = \sigma_0^2 + \mu_0^2$ given that $\gamma \sim \psi(\mu,\sigma^2)$ and $\mu \sim \psi(\mu_0,\sigma_0^2)$. If $E[\Delta u_i(a,\gamma)] > 0$, full information is preferred. The condition $E[\Delta u_i(a,\gamma)] > 0$ is equivalent to $V_i(H) > 0$ by the fact that $\sigma^2 > 0$. ■

A. Information Structure Preferences under Star Network

We showcase Theorem 2 by applying to LQG games over star networks. A star network is comprised of a central agent ($i = 1$) and $n-1$ peripheral agents ($j \in \mathcal{N}\setminus\{1\}$). We derive conditions for information structure preferences of both the central and peripheral agents in homogeneous games.

Definition 1 (Homogeneous LQG games): A LQG network game with a payoff coefficients matrix where $H_{ii} = 1$ and $H_{ij} = \beta$, for $(i, j) \in \mathcal{E}$, and $\beta \in \mathbb{R}$ is homogeneous.

Proposition 1: If the LQG game is homogeneous and $(n-1)|\beta| < 1$, then full information disclosure is preferred over no information disclosure by both the central and peripheral agents in the star network.

Proof: We can compute $[H^{-1}\mathbf{1}]_i$ in close form for star networks,

$$[H^{-1}\mathbf{1}]_i = \frac{|\mathcal{N}_i|\beta - 1}{(n-1)\beta^2 - 1} \quad \text{for} \quad i \in \mathcal{N}, \tag{12}$$

where $\mathcal{N}_i : \{j : (i, j) \in \mathcal{E}\}$ denotes the neighbors of agent i, and $|\mathcal{N}_i|$ denotes its cardinality. We check the condition $V_i(H) > 0$ for the central agent, say $i = 1$, by substituting in (12), $|\mathcal{N}_1| = n - 1$ and $|\mathcal{N}_j| = 1$ for $j \in \mathcal{N} \setminus \{1\}$,

$$\frac{(n-1)\beta - 1}{(n-1)\beta^2 - 1}\left(2 - \frac{(n-1)\beta - 1 + 2(n-1)\beta(\beta - 1)}{(n-1)\beta^2 - 1}\right) > 0. \tag{13}$$

We simplify (13) to get $((n-1)\beta-1)((n-1)\beta-3) > 0$. Given that $(n-1)\beta < 1$, the inequality is always true. Thus, full information disclosure is always preferable to no information disclosure by the central agent.

Now we consider peripheral agents $j \in \mathcal{N} \setminus \{1\}$. We check the condition $V_i(H) > 0$ for a peripheral agent by substituting in (12), $|\mathcal{N}_1| = n - 1$, and $|\mathcal{N}_j| = 1$ for $j \in \mathcal{N} \setminus \{1\}$:

$$\frac{\beta - 1}{(n-1)\beta^2 - 1}\left(2 - \frac{\beta(\beta - 1) + 2((n-1)\beta - 1)}{(n-1)\beta^2 - 1}\right) > 0. \tag{14}$$

(14) simplifies to $(\beta - 1)^2 > 0$ which is always satisfied. This means $E[\Delta u_i(a, \gamma)]$ is always positive. Therefore, full information disclosure is always preferable over no information disclosure by the peripheral agents. ■

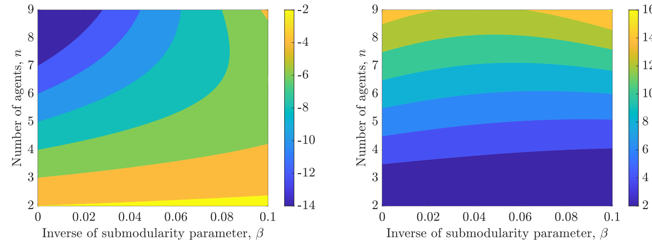
This result shows that all agents regardless of their position in the star network are expected to benefit from information disclosure. We analyze the change in the value of information as a function of competition and number of players in homogeneous games in Fig. 2. We note that for homogeneous games $V_i(H) = V_i(\beta, n)$, and $V_i(\beta, n)$ is given by (13) and (14), respectively for central and peripheral agents. We observe that $V_i(\beta, n)$ is a decreasing function for the central agent while it is an increasing function for a peripheral agent with respect to β. Also, $\frac{\partial V_i(\beta,n)}{\partial \beta}$ decreases further as β decreases or the number of agents increases for the central agent. In contrast, $\frac{\partial V_i(\beta,n)}{\partial \beta}$ is not affected much by a change in the value of β for a peripheral agent.

Next, we identify the region for β where the expected benefit of the information disclosure to a peripheral agent is higher than that of the central agent.

Proposition 2: If the LQG game is homogeneous and

$$\frac{2(n-1) - \sqrt{\nu(n)}}{n(n-2)} < \beta < \frac{2(n-1) + \sqrt{\nu(n)}}{n(n-2)} \tag{15}$$

where $\nu(n) = n^2 - 2n + 4$, then the gain of a peripheral agent from information disclosure is higher than the gain of the central agent. For β values outside interval (15), the gain of the central agent is higher than that of a peripheral agent.



(a) Central agent (b) Peripheral agent

Fig. 2. Contour plot of $\frac{\partial V_i(\beta,n)}{\partial \beta}$ for central (a) and peripheral (b) agents under homogeneous payoff matrix $\boldsymbol{H}$ where $\boldsymbol{H_{ii}} = \mathbf{1}$ and $\boldsymbol{H_{ij}} = \boldsymbol{\beta}$, for $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{\mathcal{E}}$, and $\boldsymbol{\beta} \in \mathbb{R}$. $\frac{\partial V_i(\beta,n)}{\partial \beta} < \mathbf{0}$ for the central agent and $\frac{\partial V_i(\beta,n)}{\partial \beta} > \mathbf{0}$ for a peripheral agent.

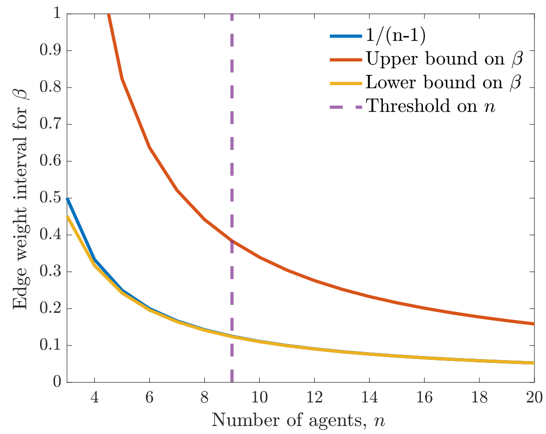


Fig. 3. We plot (15) for number of agents from 3 to 20. We also plot positive definiteness condition we impose on $\boldsymbol{\beta}$, i.e., $(\boldsymbol{n} - \mathbf{1})\boldsymbol{\beta} < \mathbf{1}$. Indeed, the positive definiteness line $(\mathbf{1}/(\boldsymbol{n} - \mathbf{1}))$ crosses below the lower bound in (15) at $\boldsymbol{n} > \mathbf{9}$, indicating that the central agent benefits more than a peripheral agent from information disclosure.

Proof: We consider the difference between $E[\Delta u_1(a, \gamma)]$ in (13), i.e., central agent's benefit from information disclosure, and $E[\Delta u_j(a, \gamma)]$ in (14) for $j \in \mathcal{N} \setminus \{1\}$, i.e., a peripheral agent's benefit, to get

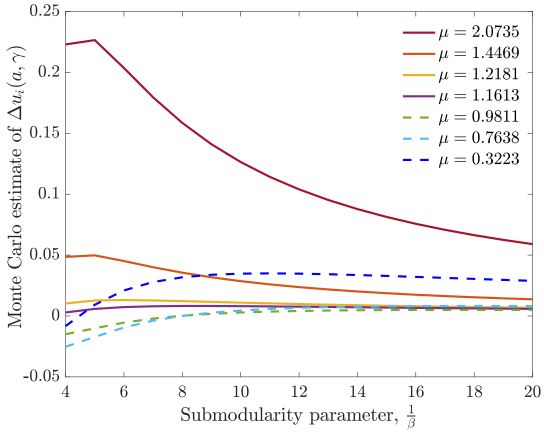
$$E[\Delta u_1(a, \gamma)] - E[\Delta u_j(a, \gamma)] = \frac{((n-1)\beta - 1)((n-1)\beta - 3) - (\beta - 1)^2}{((n-1)\beta^2 - 1)^2} > 0. \tag{16}$$

We remove the positive valued denominator, and simplify the numerator to get

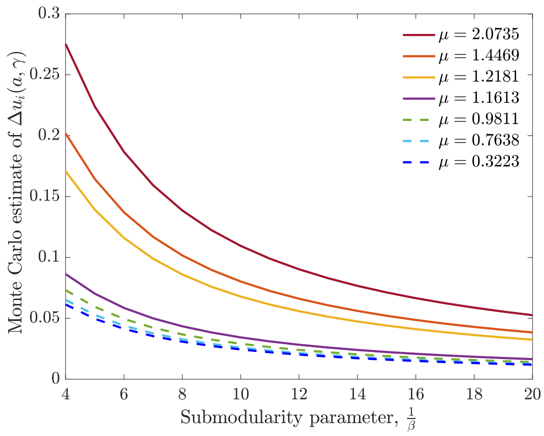
$$n(n-2)\beta^2 - 4(n-1)\beta + 3 > 0. \tag{17}$$

Solving quadratic inequality (17) indicates that when β is in the range given in (15), $E[\Delta u_1(a, \gamma)] - E[\Delta u_j(a, \gamma)] < 0$. Thus, a peripheral agent benefits more than the central agent from full information disclosure. The second part of the result follows from the fact that we have $E[\Delta u_1(a, \gamma)] - E[\Delta u_j(a, \gamma)] > 0$ for β values outside the interval (15). ■

In Fig. 3, we plot the upper and lower bound values in (15) as a function of n. We observe the bounds get closer as n increases. When we contrast these bounds with the bound for positive-definiteness, i.e., $\beta < 1/(n-1)$, we observe that the upper bound is not realized for any β value. For $n > 9$, the



(a) Central agent



(b) Peripheral agent

Fig. 4. Ex-post information preference estimates of central and peripheral agents in submodular games on a star network with $n = 4$. Lines show seven realized μ values generated from $\mu \sim \psi(\mu_0 = 1, 0.3^2)$. Dashed lines indicate $\mu < \mu_0$. Solid lines indicate $\mu > \mu_0$. For each μ and β value, 1000 γ values are generated from $\psi(\mu, 0.1^2)$. We estimate $\Delta u_i(a,\gamma)$ by averaging the values over γ realizations. For large β values, full information disclosure may not be preferred by the central agent when $\mu < \mu_0$.

positive definiteness condition implies that the lower bound cannot be exceeded. Thus, the central agent always benefits more than a peripheral agent for $n > 9$.

V. Ex-post Information Structure Preferences

Depending upon the realizations of μ and γ, agents may prefer no information disclosure ex-post. We can say an agent prefers full information disclosure over no information disclosure if its change in the utility function from information disclosure $\Delta u_i(a,\gamma) > 0$, upon realization of μ and γ. We express $\Delta u_i(a,\gamma)$ as follows by removing the expectation operator in (10),

$$\Delta u_i(a,\gamma) = (\gamma-\mu)[H^{-1}\mathbf{1}]_i \Big((\gamma+\mu)(-H_{ii}[H^{-1}\mathbf{1}]_i - 2\sum_{j\neq i} H_{ij}[H^{-1}\mathbf{1}]_j) + 2\gamma \Big). \tag{18}$$

We estimate (18) numerically via Monte Carlo simulation for homogeneous submodular ($\beta < 0$) and supermodular ($\beta > 0$) games. In submodular games, agents' actions are strategic substitutes, i.e., when agent j increases its action agent i's incentive to increase its action decreases ($\frac{\partial^2 u_i}{\partial a_i \partial a_j} < 0$). In supermodular games, agents' actions complement each other, i.e., when agent j increases its action, agent i's incentive to increase its action increases ($\frac{\partial^2 u_i}{\partial a_i \partial a_j} > 0$)—see [21, Section 3]. The Bertrand competition with payoffs in (4) is an example of a supermodular game.

We compute $\Delta u_i(a,\gamma)$ for submodular and supermodular games in Figs. 4 and 5, respectively. In particular, we generate μ values from $\psi(\mu_0 = 1, 0.3^2)$, and γ values from $\psi(\mu, 0.1^2)$ where ψ denotes the normal distribution. We estimate $\Delta u_i(a,\gamma)$ for every combination of β and μ value by averaging over realizations of γ.

In both types of games, the average change in utility function over realizations of μ is positive indicating that information disclosure is preferable and confirming Proposition 1. The value of information decreases on average for both central and peripheral agents in both types of games as submodularity parameter $\frac{1}{|\beta|}$ increases. This is reasonable because the dependence of the payoffs on others' actions reduces as $|\beta|$ decreases. In both of the games, central agent prefers no information disclosure ex-post when realized μ is less than μ_0 and the absolute value of submodularity parameter is low (Figs. 4(a) and 5(a)). Otherwise, the central agent prefers full information disclosure ex-post. This indicates a risk-averse central agent may prefer no information disclosure ex-ante. For instance, a multinational company in a Bertrand competition with local firms may prefer that information remains hidden when the production costs are high and competition is stiff. In contrast, a peripheral agent always prefers full information disclosure regardless of the realized μ values (Figs. 4(b) and 5(b)).

VI. Conclusion

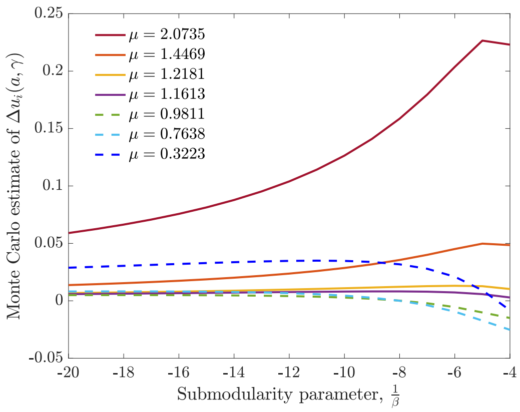
We considered whether the incentives of agents in a network game align with the information designer's objective to maximize social welfare or not. We provide a condition on the benefit of full information disclosure on an individual agent for general networks. Given only prior information about the payoff state, agents in a star network preferred full information disclosure. Unless the competition is strong and the number of agents is small, the central agent benefited more than a peripheral agent from full information disclosure. A rise in strategic interaction results in a lower value of information for central agent. However, ex-post incentive estimates showed that a risk-averse central agent can prefer no information disclosure ex-ante, while the peripheral agents continue to benefit from information disclosure.

References

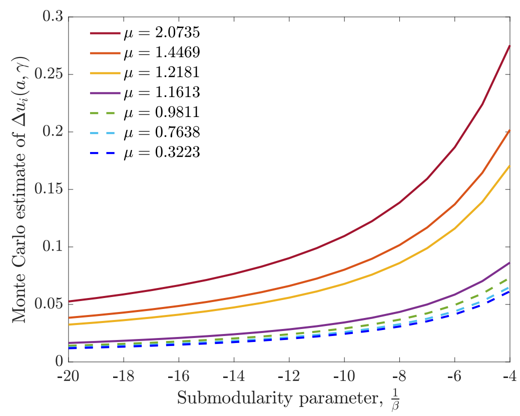
[1] P. N. Brown and J. R. Marden, "Studies on robust social influence mechanisms: Incentives for efficient network routing in uncertain settings," *IEEE Control Systems Magazine*, vol. 37, no. 1, pp. 98–115, 2017.

[2] M. Wu, S. Amin, and A. E. Ozdaglar, "Value of information in bayesian routing games," *Operations Research*, vol. 69, no. 1, pp. 148–163, 2021.

[3] G. Bacci and M. Luise, "A pre-bayesian game for cdma power control during network association," *IEEE Journal of Selected Topics in Signal Processing*, vol. 6, no. 2, pp. 76–88, 2011.



(a) Central agent



(b) Peripheral agent

Fig. 5. Ex-post information preference estimates of central and peripheral agents in supermodular games on a star network with $\boldsymbol{n = 4}$. Lines show seven realized $\boldsymbol{\mu}$ values generated from $\boldsymbol{\mu \sim \psi(\mu_0 = 1, 0.3^2)}$. Dashed lines indicate $\boldsymbol{\mu < \mu_0}$. Solid lines indicate $\boldsymbol{\mu > \mu_0}$. For each $\boldsymbol{\mu}$ and $\boldsymbol{\beta}$ value, $\boldsymbol{1000}$ $\boldsymbol{\gamma}$ values are generated from $\boldsymbol{\psi(\mu, 0.1^2)}$. We estimate $\boldsymbol{\Delta u_i(a, \gamma)}$ by averaging the values over $\boldsymbol{\gamma}$ realizations. For large $|\boldsymbol{\beta}|$ values, full information disclosure is not preferred by the central agent when $\boldsymbol{\mu < \mu_0}$.

[4] P. Molavi, C. Eksin, A. Ribeiro, and A. Jadbabaie, "Learning to coordinate in social networks," *Operations Research*, vol. 64, no. 3, pp. 605–621, 2016.

[5] C. Eksin, P. Molavi, A. Ribeiro, and A. Jadbabaie, "Learning in network games with incomplete information: Asymptotic analysis and tractable implementation of rational behavior," *IEEE Signal Processing Magazine*, vol. 30, no. 3, pp. 30–42, 2013.

[6] C. Eksin, P. Molavi, A. Ribeiro, and A. Jadbabaie, "Bayesian quadratic network game filters," *IEEE transactions on signal processing*, vol. 62, no. 9, pp. 2250–2264, 2014.

[7] F. Sezer, H. Khazaei, and C. Eksin, "Social welfare maximization and conformism via information design in linear-quadratic-gaussian games," *arXiv 2102.13047*, 2021.

[8] O. Candogan and K. Drakopoulos, "Optimal signaling of content accuracy: Engagement vs. misinformation," *Operations Research*, vol. 68, no. 2, pp. 497–515, 2020.

[9] Y. Emek, M. Feldman, I. Gamzu, R. PaesLeme, and M. Tennenholtz, "Signaling schemes for revenue maximization," *ACM Trans. Econ. Comput.*, vol. 2, jun 2014.

[10] M. O. Sayin and T. Basar, "Persuasion-based robust sensor design against attackers with unknown control objectives," *IEEE Transactions on Automatic Control*, vol. 66, no. 10, p. 4589–4603, 2021.

[11] M. O. Sayin and T. Basar, "Bayesian persuasion with state-dependent quadratic cost measures," *IEEE Transactions on Automatic Control*, vol. 67, no. 3, p. 1241–1252, 2022.

[12] N. Heydaribeni and A. Anastasopoulos, "Joint information and mechanism design for queues with heterogeneous users," *2021 60th IEEE Conference on Decision and Control (CDC)*, 2021.

[13] D. Bergemann and S. Morris, "Bayes correlated equilibrium and the comparison of information structures in games," *Theoretical Economics*, vol. 11, no. 2, pp. 487–522, 2016.

[14] D. Bergemann and S. Morris, "Information design: A unified perspective," *Journal of Economic Literature*, vol. 57, pp. 44–95, March 2019.

[15] R. Radner, "Team decision problems," *The Annals of Mathematical Statistics*, vol. 33, no. 3, pp. 857–881, 1962.

[16] T. Ui, "LQG Information Design," Working Papers on Central Bank Communication 018, University of Tokyo, Graduate School of Economics, Mar. 2020.

[17] K. Bimpikis, S. Ehsani, and R. İlkılıç, "Cournot competition in networked markets," *Management Science*, vol. 65, no. 6, pp. 2467–2481, 2019.

[18] C. Ballester, A. Calvó-Armengol, and Y. Zenou, "Who's who in networks. wanted: The key player," *Econometrica*, vol. 74, pp. 1403 – 1417, 09 2006.

[19] J. de Martí and Y. Zenou, "Network games with incomplete information," *Journal of Mathematical Economics*, vol. 61, pp. 221–240, 2015.

[20] A. Galeotti, S. Goyal, M. O. Jackson, F. Vega-Redondo, and L. Yariv, "Network games," *The review of economic studies*, vol. 77, no. 1, pp. 218–244, 2010.

[21] M. O. Jackson and Y. Zenou, "Games on networks," in *Handbook of game theory with economic applications*, vol. 4, pp. 95–163, Elsevier, 2015.

[22] F. Parise and A. Ozdaglar, "Analysis and interventions in large network games," *Annual Review of Control, Robotics, and Autonomous Systems*, vol. 4, 2020.

Appendix

The expectation of the payoff state γ given two Gaussian signals (prior μ and public signal $\bar{\omega}$) as follows

$$E[\gamma|\omega_i = \bar{\omega}] = (1 - \xi_i)\mu + \xi_i\bar{\omega} \tag{19}$$

where $\xi_i(\nu) = \frac{var(\gamma)}{var(\gamma)+var(\bar{\omega})}$, and ν is the covariance matrix of the distribution $\zeta(\omega|\gamma)$.

Lemma 1: Bayesian Nash equilibrium of LQG network game given public signals $\bar{\omega}$ and common payoff state γ can be represented by the following function

$$a_i^*(\bar{\omega}) = E[\gamma|\bar{\omega}][H^{-1}\mathbf{1}]_i \qquad \forall i \in \mathcal{N}, \tag{20}$$

where $\mathbf{1} \in \mathbb{R}^n$ is a vector of ones, and $[\cdot]_i$ indicates the ith element of a vector.

Proof: First order condition of the expectation of the utility function in (3) with respect to a_i yields

$$\frac{\partial E[u_i|\{\omega_i = \bar{\omega}\}]}{\partial a_i} = -H_{ii}a_i^*(\bar{\omega}) - \sum_{i \neq j} H_{ij}E[a_j^*|\{\omega_i = \bar{\omega}\}] + E[\gamma|\{\omega_i = \bar{\omega}\}] = 0, \forall i \in \mathcal{N} \tag{21}$$

We incorporate (19) into (21):

$$H_{ii}a_i^*(\bar{\omega}) = -\sum_{i \neq j} H_{ij}E[a_j^*|\bar{\omega}] + (1 - \xi_i)\mu + \xi_i\bar{\omega} = 0, \forall i \in \mathcal{N} \tag{22}$$

We assume agent $i \in \mathcal{N}$'s strategy is linear in its information $a_i^*(\bar{\omega}) = \alpha_{i1}\bar{\omega} + \alpha_{i2}\mu$ with coefficients α_{i1} and α_{i2}. We substitute linear actions in (22) to get

$$H_{ii}(\alpha_{i1}\bar{\omega} + \alpha_{i2}\mu) = -\sum_{i \neq j} H_{ij}(\alpha_{j1}\bar{\omega} + \alpha_{j2}\mu) + (1 - \xi_i)\mu + \xi_i\bar{\omega} = 0, \forall i \in \mathcal{N} \tag{23}$$

We solve for the action coefficients $\boldsymbol{\alpha}_1 = [\alpha_{11}, \ldots, \alpha_{n1}] \in \mathbb{R}^n$ and $\boldsymbol{\alpha}_2 = [\alpha_{12}, \ldots, \alpha_{n2}] \in \mathbb{R}^n$: $\boldsymbol{\alpha}_1 = \mathbf{1} - \boldsymbol{\alpha}_2 = H^{-1}\xi$ where $\xi = [\xi_1, \ldots, \xi_n]$ and ξ_i is as in (19). Thus, $a^*(\bar{\omega}) = H^{-1}\xi\mathbf{1}\bar{\omega} + (I - H^{-1})\xi\mathbf{1}\mu$ where I is the identity matrix. (20) follows from rearranging terms in a^* and using (19). ∎