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Positioning Students to Explore Math with Technology

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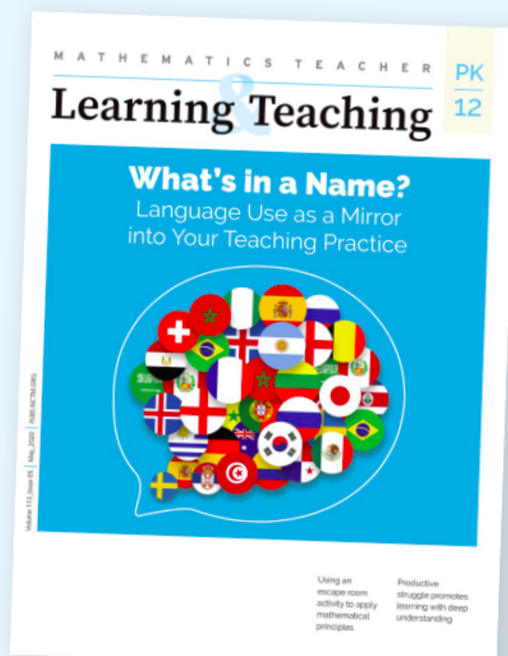

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Positioning Students to Explore Math with Technology

The authors discuss digital equity from the perspective of using math action technologies to position all students as mathematics explorers.

Allison W. McCulloch, Jennifer N. Lovett, Lara K. Dick, and Charity Cayton

When we think about digital equity, often what comes to mind are issues of access to digital technologies. Although that is a divide that must be conquered, once students have access to digital technologies, the next step is to ensure that we wield these technologies in such ways that they help to address the existing inequities in our mathematics classrooms (Suh 2010). We must ask ourselves who gets to use technology tools to engage in meaningful mathematics? Does every student get the opportunity to use technology in ways that position them as mathematically powerful? Or are we using them in ways that perpetuate the inequities we know currently exist in many mathematics classrooms—that many students mostly engage with technology to practice procedures or remediate their mathematical understandings. This means that every student, not just those labeled as honors students, should have the opportunity to engage with high cognitive-demand tasks that used digital mathematical technologies (White, Fernandes, and Civil 2018). It is important that all students get to use digital technologies in ways that support their development as explorers of mathematics—students who think in mathematical ways and feel welcome in mathematical spaces (Su 2020). Our focus in this article is to discuss the ways that digital technologies can be positioned to meet this challenge through providing ways for *each and every student* to (1) enter a mathematical problem, (2) mediate mathematical discussions, and (3) build personal and powerful ways of mathematical thinking. We believe these three characteristics position all students as mathematical explorers when using technology in the classroom.

MATH ACTION TECHNOLOGIES

Many different types of technologies are available for classroom use. They typically fall into two categories: (1) conveyance technologies and (2) math action technologies (Dick and Hollebrands 2011). *Conveyance technologies* are those that are used to convey information. They allow students and teachers to present, communicate, and collaborate with each other. These include presentation software (e.g., PowerPoint®), interactive whiteboards (e.g., Jamboard™), assessment tools (e.g., Kahoot! and clickers), and collaboration tools (e.g., Google Docs™). These technologies are not mathematics specific, and opportunities for mathematical sense making have to come from the users themselves. Yet, when they are used in ways that promote opportunities for students to engage in mathematical sense making and reasoning, they have the potential to profoundly influence learning.

In contrast to conveyance technologies, *math action technologies* can perform mathematical tasks by responding to user actions in mathematically defined ways. These types of technologies offer students opportunities to interact with mathematical objects in ways that are not possible with paper and pencil. Math action technologies include, but are not limited to, virtual manipulatives, graphing technologies, dynamic geometry systems, dynamic data analysis software, spreadsheets, computer algebra systems (CAS), and computer simulations. These technologies have the potential to allow students to focus on reasoning and sense making and explore relationships and patterns in mathematical or statistical behavior by acting on the mathematical objects themselves. This is emphasized in all three of the NCTM *Catalyzing Change* books (2018, 2020a, 2020b) through the discussion of tools and technology; the high school book specifically states:

> Students should have opportunities to use mathematical action technology in all content domains to explore mathematical relationships and deepen their understanding of Essential Concepts, to interpret mathematical representations, and to employ complex manipulations necessary to solve problems. Mastery of skills should not be a prerequisite for using technology in any content area; rather, the focus when using technology should be on developing understanding and interpreting results. (p. 42)

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In fact, research has shown that teachers' strategic use of math action technologies supports student learning (e.g., Gadanidis and Geiger 2010; Nelson, Christopher, and Mims 2009; Suh 2010) and, even more important, can offer students access to mathematics they may otherwise not have opportunities to engage with (e.g., McCulloch 2011). In what follows, we share how carefully positioning students as mathematical explorers through the use of math action technologies in instruction can address issues related to digital equity.

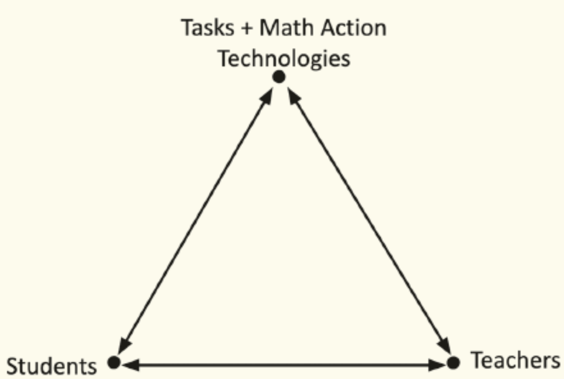
WAYS TO SUPPORT STUDENTS EXPLORING MATHEMATICS WITH TECHNOLOGY

Effective instruction can be thought of as a collection of interactions among students and teachers around the content and tasks, sometimes referred to as the *instructional triangle* (Cohen, Raudenbush, and Ball 2003). When using math action technologies with a task, as either an integral part of the task or as a tool alongside it, we can imagine this same interaction (see figure 1). The interaction among students, teachers, and tasks with math action technologies, when considered carefully, can create spaces in which all students are positioned as explorers of mathematics.

A teacher's use of math action technologies in carefully selected tasks has the potential to support all students in exploring important mathematics, welcoming them to the mathematical space. Specifically, math action technologies can be used to—

- provide a way for all students to enter a mathematical problem;
- mediate mathematical discussions, regardless of familiarity with the language of mathematics or even the dominant language in the classroom; and
- support the development of informal and powerful ways of mathematical thinking.

Fig. 1



The instructional triangle + math action technology can create spaces in which all students are explorers of mathematics (adapted from Cohen, Raudenbush, and Ball 2003).

These ideas are not new. For years, we have known that equitable instructional practices rely on these components. In 2008, in her discussion of equity for English language learners (more recently referred to as emergent bilinguals), Borgioli emphasized Carpenter and Lehrer's (1999) fundamental instructional dimensions for mathematics classrooms. They listed a need for complex tasks that incur meaningful problem solving, the use of tools to represent mathematical ideas, and classroom norms focused on student discourse. When these instructional dimensions were presented in 1999, math action technologies were nothing like we see today. Now that such technologies are readily available to students, carefully considering how we position them in instruction is important.

In fact, as we were writing this article, Desmos released its revised "Guide to Building Great (Digital) Math Activities" (Moynihan, Bejarano, and Meyer 2021), specifically noting their intentional design considerations related to digital equity. They share that their aim is to "help every student experience the kind of mathematics education they deserve" (under "Interrupt Our Biases") by considering the messages about what it means to do mathematics as communicated by the questions that are posed; how to "ensure that every student has access and opportunity to engage"; and how "to invite, celebrate, and develop the brilliance in early student thinking" (under "Interrupt Our Biases," bullets 3 and 4). These ideas mirror the ways we described earlier that math action technologies can support students as explorers of mathematics: by inviting students to play with mathematical objects, to describe what they notice and wonder, and to answer questions that position them as powerful mathematics explorers.

EXAMPLES OF DIVERSE LEARNERS USING MATH ACTION TECHNOLOGIES

In what follows, we present examples of different types of math action technologies and the ways in which they have been used by teachers to position students as mathematical explorers. Each example highlights how the use of the math action technology provided opportunities for students to explore mathematics in the task, to mediate their mathematical discussions, and to support their informal and powerful mathematical thinking. These examples demonstrate how math action technologies can be leveraged to position learners as mathematical explorers and give them opportunities to engage in meaningful mathematics.

Although the three examples that follow are from students in different grade levels, they were specifically chosen to highlight how teachers used math action technologies to position students as mathematical explorers for different types of technologies irrespective of grade level. Each example emphasizes different aspects of student engagement with math action technologies and, therefore, gives information for all K–12 teachers. We first share an example of two high school students in a nonhonors course exploring with a dynamic graphing technology, and then we share an example of an elementary school student exploring a mathematical topic for the first time with a virtual manipulative. Finally, we share an example of two middle school students, one emerging bilingual and one student identified as having a learning disability, exploring with an interactive applet.

Students Exploring with a Dynamic Graphing Technology

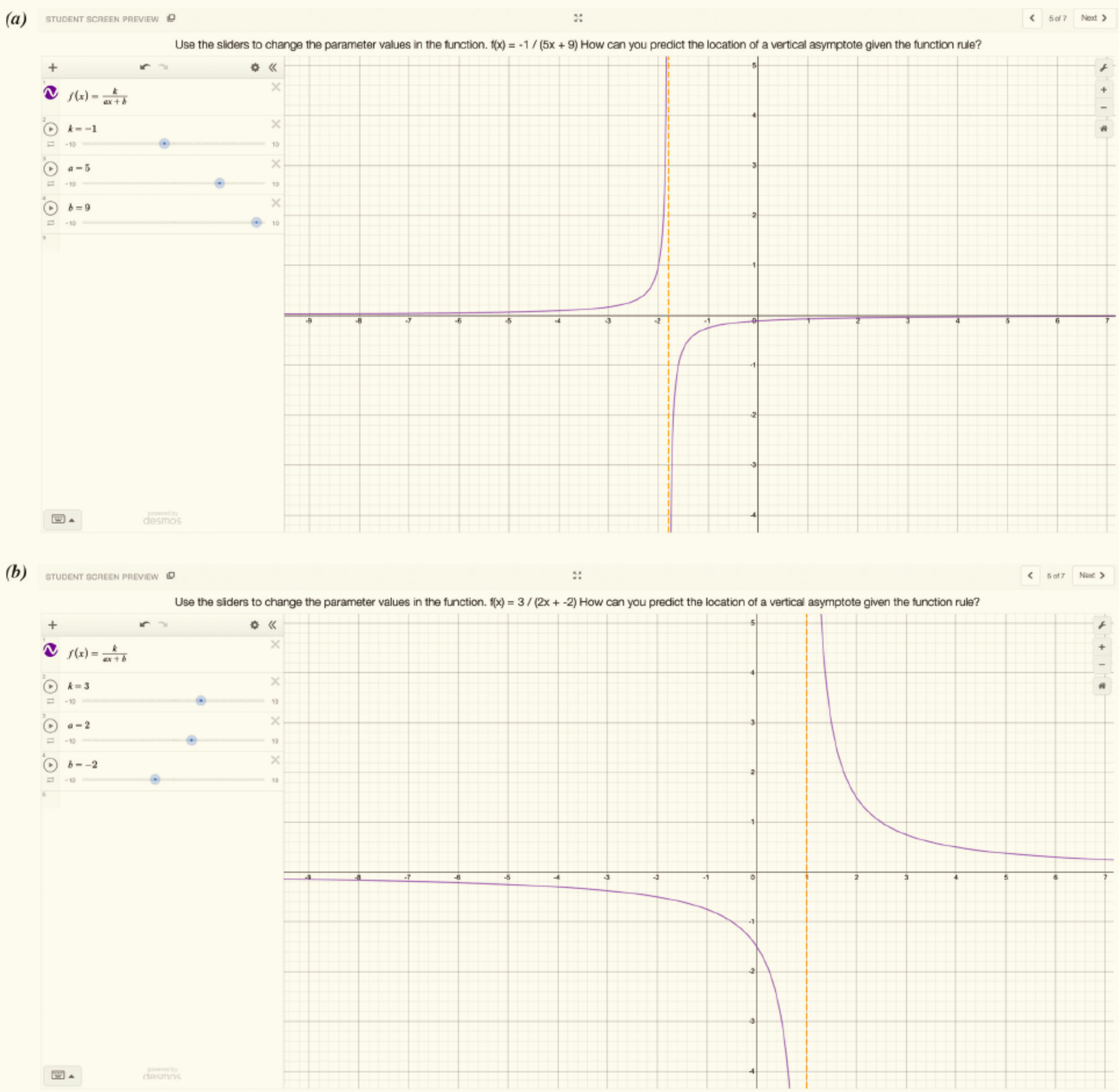
When one mentions digital technologies, often one of the first technologies that comes to mind is the graphing calculator, a dynamic graphing technology. Many free dynamic graphing technologies are available (e.g., Core Math Tools, Desmos, and GeoGebra), and they all allow for students to quickly and accurately create graphs of functions. A common way for students to explore graphs of functions with digital graphing technologies is to use sliders with which they can click and drag to see what happens to a graph as a parameter changes (e.g., Edwards and Özgün-Koca 2010; Paoletti, Monahan, and Visnubhotla 2017; Rodrigues 2006). For example, students can explore linear functions in the form of $y = mx + b$ by dragging values for m and b and examining the way the graph is transformed. Using sliders in this way gives students opportunities to explore the relationship between the structure of a function and its graph, to notice patterns, to make conjectures, and to test those conjectures. In other words, to be mathematical explorers of characteristics of function families (e.g., linear, quadratic, and exponential).

When exploring rational functions, important characteristics to consider include asymptotes. Here, we share an example of the use of sliders in a dynamic graphing technology to explore vertical asymptotes. The Introduction to Vertical Asymptote task was created in Desmos Activity Builder and includes pages with the Desmos Graphing Calculator in the activity. Students are asked to use sliders to explore the parameters of rational functions and their effect on the number of vertical asymptotes a function has and their location. Students explore increasingly more complex rational functions as they move through the activity (see figure 2). (See Dick, McCulloch, and Lovett [2021] for more about this activity.) Ultimately, they are asked to explain to a friend how they know how many vertical asymptotes a rational function may have and where each is located.

In video 1, two students, Eden and McKenzie, are working on this task. Eden and McKenzie are currently enrolled in a nonhonors Integrated Math 3 course. They are familiar with rational functions, having learned about them in the context of direct and indirect variation in prior courses. In the context of indirect variation, the students have heard the term *vertical asymptote*, but their teacher notes that they have not been formally introduced to the concept. In the context of this activity, the first few pages provided graphs of rational functions in which students were asked to describe the domain and range of the function and note anything else they notice. Then, vertical asymptote was defined as "a line with respect to a curve such that the curve approaches that line, but does not touch it even if both the line and the curve are extended infinitely." Once they were provided with a definition, they were sent off to use sliders to explore the relationship between the function and the vertical asymptote. The prompt asked them to explore how they could predict the location of a vertical asymptote given the function rule (see figure 2). Although the students were provided with a definition, they were not given any procedures for determining the number or location of vertical asymptotes. Instead, they were trusted as powerful mathematical explorers to create their own strategies. Before reading any further, watch video 1 in which Eden and McKenzie explore the location of a vertical asymptote.

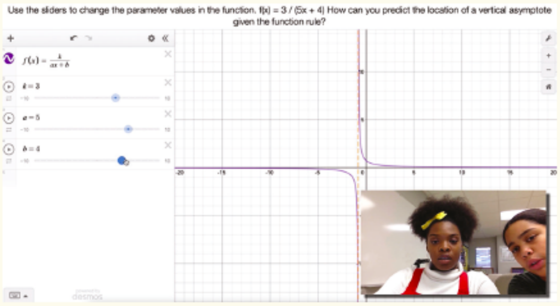
In video 1, we see that Eden and McKenzie immediately entered the task and explored the newly defined concept, vertical asymptote, using the sliders. As the girls worked together, they talked about what they saw as a result of changing the sliders. For example, early in the clip, McKenzie changed b to 5 and said, “Look, if you do it there, it is not going to be exactly on a number.” Both students knew what was meant by “it” and what “it” was that was not going to be “exactly on a number”—what they created with the technology was part of their discourse. In fact, their actions with the technology and the representations they created were important to their discourse through the entire clip. Importantly, the students changed k to be zero, and in doing so they were no longer looking at a rational function. This is important, and intentional, for them; they wanted to be able to focus on only the asymptote. With

Fig. 2



The Introduction to Vertical Asymptote Desmos activity allowed students to explore the relationship between the function and the vertical asymptote using sliders (Dick, McCulloch, and Lovett 2021).

Video 1 Eden and McKenzie Engage with the Introduction to Vertical Asymptotes Desmos Activity



Watch the full video online.

the function out of the way, they continued to explore, and we see they are beginning to build an understanding of how they may determine the location of a vertical asymptote for a rational function of the form $f(x) = k/(ax + b)$. On the basis of their exploration, they conjectured that the vertical asymptote is located at $x = b/a$ and continued to test this conjecture. They recognized that it is not at b/a but rather $(-b)/a$, but they did not seem sure as to why. They explained that it must be one of those "weird flippy thingies that graphs do." The informal understanding that Eden and McKenzie have begun to build is a powerful foundation to which they can now refer. They have clearly noticed the connection between the expression in the denominator of the function and the vertical asymptote, so as their informal strategies are used to build formal procedures, they have this conceptual grounding as a reference.

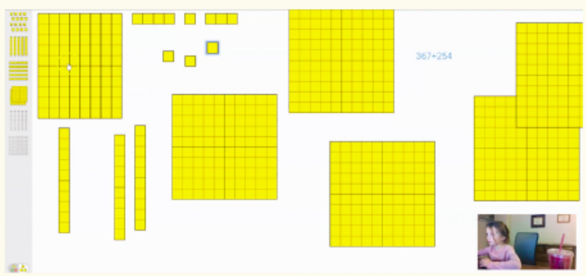
Students Exploring with a Virtual Manipulative

Virtual manipulatives are another type of mathematical action technology that can support students in their exploration of mathematical concepts. Virtual manipulatives are similar to concrete manipulatives, but they allow for students to interact in new ways by dragging, combining, and separating (e.g., Edwards et al. 2012; Moyer, Bolyard, and Spikell 2002). A wide variety of virtual manipulatives are available, ranging from pattern blocks to algebra tiles. Many of these tools, including ten-frames, rekenreks, and base-ten blocks, can be used to support students as they build conceptual understanding of early number and operations. These manipulatives support students in making connections among multiple representations as they develop an understanding of our number system. Although not all schools have all these physical manipulatives, virtual versions are freely available online (e.g., The Math Learning Center). Many of these virtual manipulatives offer affordances not available with the physical counterparts. For example, with physical base-ten blocks, one must exchange 10 tens blocks for a hundreds block or exchange 10 hundreds blocks for a thousands cube. Instead of the action of exchanging blocks, the virtual base-ten blocks allow for the dynamic action of combining or breaking apart. These dynamic actions afforded by the technologies support learners in their exploration and representation of number and operations.

In video 2, we share an example of a second-grade student, Irene, using the free virtual base-ten block app Number Pieces (see figure 3) (Dick, McCulloch, and Lovett 2021). In school, Irene had worked with multiple representations of two-digit numbers using both physical and virtual manipulatives. She had formally added two two-digit numbers, but this was her first experience adding two three-digit numbers. Before reading any further, watch Irene exploring the problem 367 + 254 in video 2.

Even though this was Irene's first experience adding two three-digit numbers and we could see by both her facial expression and mumbling that she thought the numbers "might be a little too hard for me," Irene

Video 2 Irene Uses the Number Pieces Applet to Represent the Addition of Two Three-Digit Numbers



Watch the full video online.

could easily enter the problem. Irene began by simply looking at the place value for each number and dragging appropriate blocks (hundreds, tens, and ones) into the workspace screen to represent their value. As she gathered the necessary hundreds, tens, and ones, she used the technology to communicate her understanding of place value (i.e., "first we need a lot of hundreds," and then we see her count out three hundreds blocks and then two more hundreds blocks). The technology mediated the communication of her strategy as well. She began by counting within each place value, largest to smallest, to see if she could group the numbers. After counting the tens, she said, "We do have ten blocks with us, so that would make an a hundred. . . . Now I need to do glue." The virtual manipulative allowed Irene to combine 10 tens blocks using the "join pieces" button to make a hundreds block.

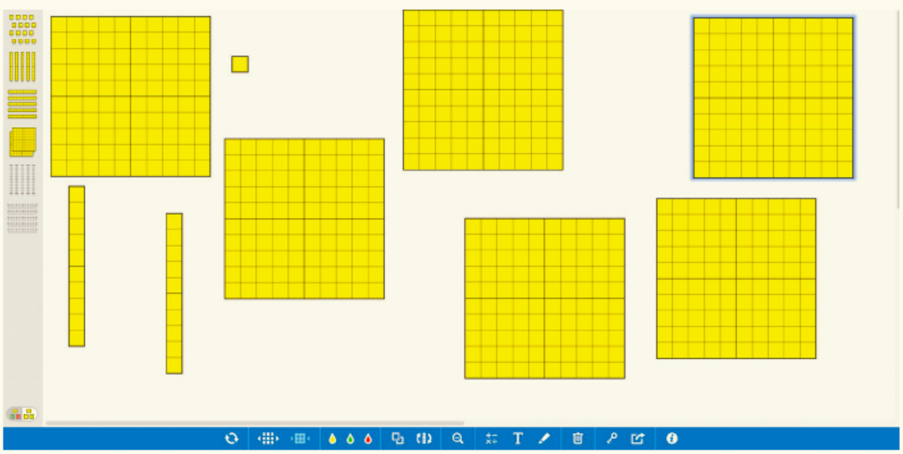
The way the technology works to dynamically combine the blocks gave Irene agency as a mathematical explorer. The technology supported Irene's self-created informal addition strategy in which she first combined the larger place values and then the smaller values; it was not necessary for Irene to be provided with the standard procedure of regrouping smaller place values and carrying over to larger place values to successfully add two three-digit numbers. With the virtual manipulatives, she was able to extend a strategy she had used with two-digit numbers, building on her informal and powerful ways of thinking about both place value and addition. Irene's experience exploring these three-digit numbers with their place values offers her a foundation for procedural understanding of regrouping in third grade. In addition, though Irene completed this task out of school, tasks such as this one can be used in the classroom for individual students or pairs of students to directly model operations with numbers (e.g., Edwards et al. 2012; Moyer, Bolyard, and Spikell 2002).

Students Exploring with an Interactive Applet

Interactive applets are small computer programs that students can interact with to explore mathematical objects. Applets are typically designed to do one thing, making them easy to operate and allowing for students to jump in and explore (Jacques 2013). For example, NCTM's *Illuminations* interactivities include a nice collection of applets, ranging from those that support the development of elementary counting strategies (e.g., the "Grouping and Grazing" applet [NCTM, n.d.]) to those that support the exploration of properties of geometric solids (e.g., the "Geometric Solids" applet [NCTM, n.d.]). Teachers can build their own applets using technologies like GeoGebra.

To help students develop an understanding of the function concept, a common strategy is the use of a function machine applet. A quick internet search will reveal many different function machine applets freely available. Often these applets follow the structure in which the user enters a number (an input), and then the machine applies a "rule" to determine and give the resulting number (the output) to the user. Unlike

Fig. 3



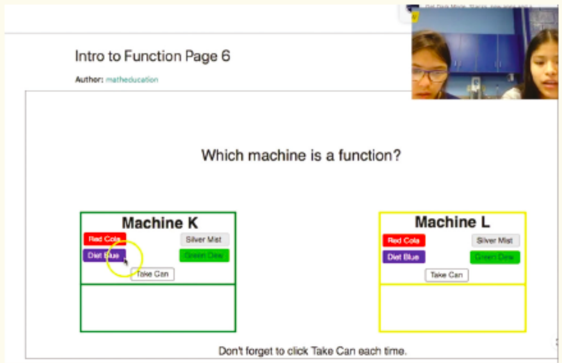
This Number Pieces applet shows Irene's solution to 367 + 254.

typical function machines, the applet in the example that follows was designed to contain no numerical or algebraic expressions, so that students would not focus on specific numerical values or representations. Instead, it was built on the metaphor of a vending machine. By removing the numeric and algebraic representations, it was intended for students to attend to the nature of and relationship between functional input and output, apart from forming a reliance on algebraic procedures without understanding (i.e., without focusing on the vertical line test).

This applet, Introduction to Function, has eight pages (see figure 4). Each vending machine contains four buttons (Red Cola, Diet Blue, Silver Mist, and Green Dew). When a button is pressed, it produces none, one, or more than one of the four different colored cans (red, blue, silver, and green). On the first few pages of the applet, students are provided with two machines: one identified as a function and the other as not a function. They explore by clicking on each button (input) to see the cans (output). They are not told how to do this, but to determine the difference between the two machines, they will need to click each button multiple times and explore the outputs that result. Later, they are shown two machines and asked which is a function and which is not a function. The goal of the task is for students to develop an informal definition of function through their interaction with the applet and with each other. (See Lovett and colleagues (2020) for a full discussion of this task and recommendations of how to build the formal definition with middle school students.)

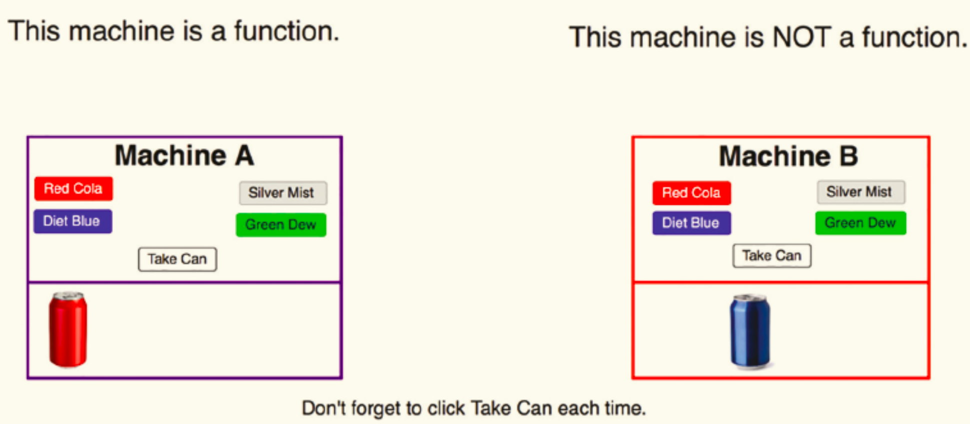
In video 3, you will see seventh graders Luz and Leah working with the Introduction to Function applet. Neither Luz nor Leah had ever been introduced to the mathematical term *function* before. Their class was cotaught by a mathematics teacher and an inclusion support teacher because there were students identified as having a learning disability in mathematics and students who were emergent bilingual. The whole class was working on this task in small groups, with both teachers monitoring their work. One of the teachers spoke only English, so she used Google Translate™ on her phone to

Video 3 Luz and Leah Engage with the Introduction to Function GeoGebra Applet



Watch the full video online.

Fig. 4



This is the first page of the Introduction to Function applet (Lovett et al. 2020).

support her communication with students who chose to speak languages other than English. Before reading any further, watch video 3, which shows Luz and Leah engaging with two vending machines K and L as they are working to determine which is a function and why.

One of our favorite things about this task is that regardless of prior experience, all students have a way to enter the task; they simply click the buttons on the machines and observe the results. At this point in the task, Luz and Leah had made a conjecture about what makes a machine a function or nonfunction and had discovered the importance of trying each button more than once. Leah and Luz clicked each button multiple times. Though they did not say anything, this action alone communicates to us that they understand that one cannot determine if the machine is a function by testing a button only once. In the beginning of video 3, we see the applet and the teacher mediating the discussion between the students. The teacher helped them get started by stating the colors of the cans in both Spanish and English as Luz clicked the buttons on machine K. Noting that when a silver can appeared, the teacher referred to it as "blanco, silver" is important; this is because Luz referred to the silver cans as "blanco" (i.e., white) when she started the activity. The teacher chose to continue to use Luz's description rather than to correct her. As Luz continued to press the buttons and the students examined the output, we see and hear both Luz and Leah's surprise when the Red Cola button was clicked multiple times and different colored sets of two cans resulted each time. On the third click, when yet a different set of two cans appeared, both students said "no." When they moved to test machine L, the teacher spoke less as the students communicated their ideas using both spoken language and the applet. For example, when Luz explained the machine as "solamente el color verde," she used the mouse to circle machine L to communicate which machine she was talking about (i.e., that the output cans are only green).

Throughout video 3, the fact that the technology mediated the girls' discourse with each other and with the teacher is evident. Being able to use the cursor to point at and circle objects while using minimal words to communicate important ideas was vital to their work together. Not only were they communicating with and through the applet, but it is also evident that they were developing powerful informal ideas about the concept of function. For example, at the end of the clip, Leah explained that machine L is a function because "they are all green"; Luz added to that idea by explaining, "Siempre sale el mismo color y no sale revuelto" (Google Translate recognized this for the teacher as "It always comes out the same color, and it's not scrambled"). The students engaged with the task and with each other without the use of formal mathematics language. The way the technology was used in this task positioned them as powerful mathematical explorers. They were not told what a function was but instead were trusted with defining it themselves before returning to a whole-class discussion to formally define function.

Looking across the Examples

Looking across the video clip examples gives insight into ways that the interaction among students, teachers, and tasks with math action technologies can create spaces in which all students are positioned as explorers of mathematics. Regardless of the specific math action technology, the students were using the technologies to investigate, notice patterns, test conjectures, and refine their ideas. In all three examples, we see the different and important ways that the math action technology helped students to enter the problem; how it was used to mediate their mathematical discussion, even when the students and teacher spoke different languages; and how it supported the development of incredibly powerful, informal mathematical ideas.

In each of these examples, a next step would be to make students' thinking visible to the class in a whole-class discussion and use it as the foundation on which to build and connect to more formal mathematical ideas. We encourage teachers to carefully select and sequence (Smith and Stein 2018) students' strategies and representations (e.g., through screenshots, screenshare, or having a student demonstrate their work on the teacher's computer) to facilitate a whole-class discussion that focuses on both student thinking and their use of technology (Dick, McCulloch, and Lovett 2021).

MATH ACTION TECHNOLOGIES THAT SUPPORT EXPLORATION

With an eye toward supporting students as mathematical explorers, here we briefly share some free math action technologies along with suggestions for strategically using them in the classroom (see table 1). Keep in mind that all the technologies listed can be used on computers, tablets, and cell phones. Most importantly, they all produce mathematical responses on the basis of user input, allowing students to observe and explore mathematical ideas as well as make and test conjectures about mathematical structures and relationships.

In addition to the plethora of free math action technologies, there has been a recent emergence of powerful teaching platforms for building digital math activities. Platforms like Desmos Activity Builder and GeoGebra Classroom allow teachers to create a series of activities that provide the teacher with access to students' responses in real time. These activity platforms allow teachers to select a preconstructed activity, to adapt those they find, or to create their own. Although these can be used as solely conveyance technology, they are most powerful when they include math action technologies as well. Whether using an existing activity or creating a new one, it is important to consider the ways in which the activity gives opportunities for students to be mathematics explorers and for teachers to use students' mathematical ideas to drive class discussions as the foundation for building formal mathematical ideas.

Regardless of the specific math action technology, the students were using the technologies to investigate, notice patterns, test conjectures, and refine their ideas.

FINAL THOUGHTS

The context of the pandemic, sending teachers and students into remote spaces, has no doubt pushed us forward in bringing digital technology into the mathematics classroom. We are finally approaching a point in time when access to digital technologies is becoming the norm, especially if we consider the combination of personal handheld devices, Chromebooks™, and laptops. However, the quick move to remote settings required the use of conveyance technologies in lessons and did not allow for time to reflect on the ways in which we were (and were not) including math action technologies.

So, now is a time to stop and reflect on the ways we are using digital technology tools in our instruction. Are they being used in a way that positions students as *mathematical explorers*—as able to think in powerful mathematical ways and as welcome in mathematical spaces? And are we providing each and every student with this opportunity? If not, how might we make choices about the use of math action technologies, so that they are addressing existing inequities rather than perpetuating them?

Table 1 Math Action Technologies That Support Exploration

Type of Mathematical Action Technology	Description	Select Examples
Virtual manipulatives	Virtual manipulatives, created to parallel their physical counterparts, allow for dynamic observation and the creation of flexible visual representations of mathematical ideas. The virtual aspect often affords more flexibility and the ability to save drafts, which are impossible with physical versions.	The Math Learning Center Toy Theater
Interactive applets	Small exploratory computer programs allow students to dynamically interact with objects and observe them in mathematical ways (sometimes referred to as *microworlds*). When carefully selected to align with a mathematical goal, applets give a way for students to quickly and easily explore mathematical relationships.	NCTM Illuminations Interactives Shodor Interactivate Core Math Tools
Dynamic graphing applications	Graphing applications allow students to observe and explore multiple representations of functions and data by generating linked tables, graphs, and symbolic representations.	Desmos Graphing Calculator GeoGebra Core Math Tools
Spreadsheets	Spreadsheet applications quickly display results of repeated calculations and can generate tables of values linked to a variety of graphical representations. Displaying repeated calculations allows for insights into structure and relationships among variables.	Google Sheets GeoGebra Core Math Tools
Dynamic geometry software	Dynamic geometry software allows for exploration of geometric relationships in coordinate, transformational, and synthetic contexts. Students can make and explore conjectures through dragging geometric objects and attending to invariances. Such explorations can offer insight into the existence of relationships and why they hold true, an important step to generating formal proofs.	GeoGebra Desmos Geometry Core Math Tools
CAS	CAS can operate on algebraic statements. This allows for insight into the structure of algebraic functions and expressions and is especially powerful for highlighting patterns of equivalence, such as factoring quadratic equations.	GeoGebra Core Math Tools Wolfram Alpha
Data analysis software	Data analysis tools support the visualization of large data sets with linked representations and tools for simulating observable phenomena. These tools offer opportunities to explore *what if* questions that are invaluable to the study of probability and statistics.	CODAP Core Math Tools

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