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An Adaptive Formation Control Architecture for A Team of Quadrotors with Performance and Safety Constraints

Zhongjun Hu and Xu Jin, *Member, IEEE*

Abstract— **In this work, we propose a novel adaptive formation control architecture for a group of quadrotor systems, under line-of-sight (LOS) distance and relative distance constraints, where the constraint requirements can be both asymmetric and time-varying in nature. Universal barrier functions are adopted in the controller design and analysis, which is a generic framework that can address system with different types of constraints in a unified controller architecture. Furthermore, each quadrotor's mass is unknown, and the system dynamics are subjected to time-varying external disturbance. Through rigorous analysis, an exponential convergence rate can be guaranteed on the distance tracking errors, while the constraints are satisfied during the operation. A simulation example further demonstrates the efficacy of the proposed control framework.**

I. Introduction

The formation control problems of unmanned aerial vehicles (UAVs), especially quadrotors, have received much attention from the research, industrial, and military communities, including notable applications in surveillance [1], [2], search and rescue [3], contour mapping [4], [5], object lifting and transporting [6], [7], just to name a few.

To ensure the precise and safe operations of the quadrotor team, several constraint requirements cannot be ignored and have to be taken into consideration. First, for the *performance constraints*, we need to ensure that the quadrotor team is tracking the desired formation trajectory closely. Failing to meet such constraint requirements would result in undesirable formation performance. Second, for the *safety constraints*, we need to guarantee that the LOS relative distance between any two quadrotors cannot be either too small or too large, which can lead to collisions among quadrotors or loss of communication between an agent and rest of the team.

Few works in the quadrotor or unmanned aerial vehicle formation literature have addressed the above issues regarding the *performance* and *safety constraints*. Some notable exceptions include [8]–[16], which consider mere collision avoidance between UAVs, but ignore the upper constraints of the inter-vehicle distances, and fail to address constraints on the LOS distance tracking errors. A distributed formation control framework for the underactuated quadrotors with the pre-assigned constraints of the position is developed in [17]. The work [18] investigates the attitude synchronization problem for cooperative quadrotors subject to unknown nonlinearities and multiple actuator faults. A formation control algorithm for the leader quadrotors and a finite-time containment control for the follower quadrotors with unknown model dynamics are proposed in [19]. None of these works [17]–[19] can address formation control problems of a team of underactuated quadrotors, with time-varying and asymmetric constraint requirements on the LOS distance and relative inter-quadrotor distance tracking errors.

In this work, we develop a novel adaptive control architecture to address formation control problems for a team of quadrotors. Two types of system constraint requirements, namely *performance constraint* and *safety constraint*, are considered during the operation. For the performance constraint, we address the constraint requirements on the distance tracking error between the real and the desired positions for each quadrotor, so that to ensure the *precise* trajectory tracking and formation keeping. For the *safety constraints*, we consider the constraints on the relative inter-quadrotor distance, so that to ensure the distance between any two quadrotors is neither too large nor too small, hence to ensure the *safe* operations of the quadrotor team. Universal barrier functions are adopted in the controller design and analysis, which is a generic framework that can address system with different types of constraints in a unified controller architecture. Adaptive estimators are employed to deal with the time-varying system uncertainties presented in the system dynamics. Through rigorous analysis, we show that exponential convergence can be guaranteed on the LOS distance and relative distance tracking errors, while all constraint requirements are satisfied during the operation.

The notations used in this work are fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers. Moreover, $(\cdot)^{\mathrm{T}}$ implies the transpose vector, $|\cdot|$ is the absolute value for scalars, and $||\cdot||$ represents the Euclidean norm for vectors and induced norm for matrices. Next, we write $\dot{(\cdot)}$ as the first order time derivative of $(\cdot)$, if $(\cdot)$ is differentiable, and $(\cdot)^{(n)}$ as the n-th order time derivative of $(\cdot)$ for n being a positive integer. Furthermore, C^2 denotes the class of functions that are two-times differentiable with respect to time, with the derivatives being in the class of C^1, which consists of all differentiable functions whose derivative is continuous. Last but not least, the distance between any two points p_1 and p_2 in the three-dimensional space is defined as

$$\mathrm{dist}(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2},$$

where $p_1 = [x_1, y_1, z_1]^{\mathrm{T}}$ and $p_2 = [x_2, y_2, z_2]^{\mathrm{T}}$.

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Zhongjun Hu and Xu Jin are with the Department of Mechanical Engineering, University of Kentucky, Lexington, KY 40506, USA (E-mail: ZhongjunHu@uky.edu; xu.jin@uky.edu).

II. Problem formulation

A. System Dynamics

Consider the following class of multi-vehicle systems with N quadrotors, where, for the ith quadrotor ($i = 1, \cdots, N$), the position and attitude in the inertial reference frame are represented as $p_i(t) = [x_i(t),\ y_i(t),\ z_i(t)]^{\mathrm{T}} \in \mathbb{R}^3$ and $\Theta_i(t) = [\phi_i(t),\ \theta_i(t),\ \psi_i(t)]^{\mathrm{T}} \in \mathbb{R}^3$, respectively. The translational velocities with respect to the inertial reference frame are represented as $v_i(t) = [v_{xi}(t),\ v_{yi}(t),\ v_{zi}(t)]^{\mathrm{T}} \in \mathbb{R}^3$. The kinematics and dynamics for the ith quadrotor ($i = 1, \cdots, N$) are expressed as

$$\dot{p}_i(t) = v_i(t), \tag{1}$$

$$m_i \dot{v}_i(t) = m_i g e_z - F_i(t) R(\Theta_i(t)) e_z + N_{1i}(t), \tag{2}$$

where $p_i(0) = p_{i0} \in \mathbb{R}^3$ and $v_i(0) = v_{i0} \in \mathbb{R}^3$, with p_{i0} and v_{i0} being initial conditions. $m_i \in \mathbb{R}$, $m_i > 0$, $F_i(t) \in \mathbb{R}$, and $N_{1i}(t) \in \mathbb{R}^3$ represent the mass, thrust, and the external disturbance of the ith quadrotor ($i = 1, \cdots, N$). Furthermore, $g \in \mathbb{R}$ is the gravitational acceleration and $e_z = [0,\ 0,\ 1]^{\mathrm{T}} \in \mathbb{R}^3$ is the unit vector. $R(\Theta_i(t)) \in \mathrm{SO}(3)$ is the rotation matrix, with the expression in (3) (see the next page), which translates the translational velocity vector in the body-fixed frame into the rate of change of the position vector in the inertial frame, and $\mathrm{SO}(3) = \{\mathbb{R}^{3\times 3} \mid \mathbb{R}^{\mathrm{T}}\mathbb{R} = I_3\}$ is a set of orthogonal matrices in $\mathbb{R}^{3\times 3}$. It is straightforward to see that

$$||R(\Theta_i(t))|| \leq R_{\max}, \tag{4}$$

with $R_{\max} > 0$ being a known constant.

B. System Performance and Safety Constraints

The coordinate of the reference trajectory for the ith vehicle ($i = 1, \cdots, N$) is denoted by $p_{\mathrm{d}i}(t) \triangleq [x_{\mathrm{d}i}(t),\ y_{\mathrm{d}i}(t),\ z_{\mathrm{d}i}(t)]^{\mathrm{T}} \in \mathbb{R}^3$. Hence the line-of-sight (LOS) distance tracking error for the ith quadrotor $d_{\mathrm{e}i}(t)$ ($i = 1, \cdots, N$), which is the distance between the actual and desired position of the quadrotor, the desired LOS relative distance between ith and jth quadrotors $L_{ij}(t)$ ($i, j = 1, \cdots, N,\ j \neq i$), and the actual LOS relative distance between ith and jth quadrotors $d_{ij}(t)$ ($i, j = 1, \cdots, N,\ j \neq i$), are defined as

$$d_{\mathrm{e}i}(t) = \mathrm{dist}\Big(p_i(t), p_{\mathrm{d}i}(t)\Big), \tag{5}$$

$$L_{ij}(t) = \mathrm{dist}\Big(p_{\mathrm{d}i}(t), p_{\mathrm{d}j}(t)\Big), \tag{6}$$

$$d_{ij}(t) = \mathrm{dist}\Big(p_i(t), p_j(t)\Big). \tag{7}$$

During the formation operation, one *performance constraint* and one *safety constraint* have to be satisfied for each quadrotor in the operation. First, the LOS distance tracking error for the ith quadrotor $d_{\mathrm{e}i}(t)$ ($i = 1, \cdots, N$) has to satisfy the following *performance constraint*

$$d_{\mathrm{e}i}(t) < \Omega_{d\mathrm{H}i}(t), \tag{8}$$

where, for all $t \geq 0$, $\Omega_{d\mathrm{H}i}(t) > 0$ is the user-defined time-varying constraint requirement for the distance tracking error $d_{\mathrm{e}i}(t)$ and is C^2. The constraint requirement (8) means that the trajectory tracking error for the ith quadrotor cannot be too large.

Second, define the LOS relative distance tracking error between the ith and jth quadrotors ($i, j = 1, \cdots, N,\ j \neq i$) as $d_{\mathrm{e}ij}(t) \triangleq d_{ij}(t) - L_{ij}(t)$, which has to meet the following *safety constraint*

$$-\Omega_{\mathrm{L}ij}(t) < d_{\mathrm{e}ij}(t) < \Omega_{\mathrm{H}ij}(t), \tag{9}$$

where, for all $t \geq 0$, $\Omega_{\mathrm{H}ij}(t) > 0$ is the user-defined time-varying upper bound for the distance tracking error $d_{\mathrm{e}ij}(t)$, and $-\Omega_{\mathrm{L}ij}(t) < 0$ is the lower bound, with $L_{ij}(t) > \Omega_{\mathrm{L}ij}(t) > 0$. Both $\Omega_{\mathrm{H}ij}(t)$ and $\Omega_{\mathrm{L}ij}(t)$ are C^2. The constraint requirement (9) means that the inter-quadrotor distance cannot be either too large or too small.

C. Control Objective

The <u>**control objective**</u> for the formation control problem is to design a control framework such that:

1) The LOS distance tracking error $d_{\mathrm{e}i}(t)$ for the ith quadrotor ($i = 1, \cdots, N$) can converge into an arbitrary small neighbourhood of zero;

2) The relative distance $d_{\mathrm{e}ij}(t)$ between the ith and jth ($i, j = 1, \cdots, N,\ j \neq i$) quadrotors can converge into an arbitrarily small neighbourhood of zero;

3) The performance and safety constraint requirements (8) and (9) are satisfied during the operation.

The following assumptions are used to facilitate the discussion and analysis of the main result.

Assumption 2.1: The reference trajectory coordinates for the ith quadrotor ($i = 1, \cdots, N$) $x_{\mathrm{d}i}(t)$, $y_{\mathrm{d}i}(t)$, $z_{\mathrm{d}i}(t)$ are all C^2 with bounded differentiations.

Assumption 2.2: The thrust $F_i(t)$ and disturbance $N_{1i}(t)$ for the ith quadrotor ($i = 1, \cdots, N$) are uniformly bounded with <u>*unknown*</u> bounds.

Assumption 2.3: The mass m_i for the ith quadrotor ($i = 1, \cdots, N$) is <u>*unknown*</u>, but the inverse is assumed to be both upper and lower bounded, such that $\bar{b}_{mi} > \frac{1}{m_i} > \underline{b}_{mi}$, where $\bar{b}_{mi}$ and $\underline{b}_{mi}$ are <u>*unknown*</u> positive constants.

Assumption 2.4 ([20]): Denote the approximation of the time derivative of a continuous function $\dot{\vartheta}(t)$ as $\hat{\dot{\vartheta}}(t)$, where

$$\hat{\dot{\vartheta}}(t) = \frac{\vartheta(t) - \vartheta(t-T)}{T} \tag{10}$$

for some small $T > 0$. Then

$$|\hat{\dot{\vartheta}}(t) - \dot{\vartheta}(t)| \leq \varepsilon_\vartheta \approx o(T). \tag{11}$$

To facilitate the analysis, we present the following lemma from the literature.

Lemma 2.1: For any constant $\varepsilon > 0$ and any variable $z \in \mathbb{R}$, we have $0 \leq |z| - \frac{z^2}{\sqrt{z^2+\varepsilon^2}} < \varepsilon$.

From this point onwards, to simplify the notation, the time and state dependence of the system will be omitted whenever no confusion would arise.

$$R(\Theta_i(t)) = \begin{bmatrix} \cos\theta_i(t)\cos\psi_i(t) & \sin\phi_i(t)\sin\theta_i(t)\cos\psi_i(t) - \cos\phi_i(t)\sin\psi_i(t) & \cos\phi_i(t)\sin\theta_i(t)\cos\psi_i(t) + \sin\phi_i(t)\sin\psi_i(t) \\ \cos\theta_i(t)\sin\psi_i(t) & \sin\phi_i(t)\sin\theta_i(t)\sin\psi_i(t) + \cos\phi_i(t)\cos\psi_i(t) & \cos\phi_i(t)\sin\theta_i(t)\sin\psi_i(t) - \sin\phi_i(t)\cos\psi_i(t) \\ -\sin\theta_i(t) & \sin\phi_i(t)\cos\theta_i(t) & \cos\phi_i(t)\cos\theta_i(t) \end{bmatrix} \quad (3)$$

III. Universal Barrier Function

Regarding the constraint requirements (8) and (9), which are on the LOS distance tracking error $d_{\mathrm{e}i}$ and relative inter-quadrotor distance tracking error $d_{\mathrm{e}ij}$ $(i, j = 1, \cdots, N,\ j \neq i)$, we introduce the transformed error variables for the ith quadrotor $(i = 1, \cdots, N)$ as follows

$$\eta_{\mathrm{e}i} = \frac{\Omega_{d\mathrm{H}i} d_{\mathrm{e}i}}{\Omega_{d\mathrm{H}i} - d_{\mathrm{e}i}}, \quad \eta_{ij} = \frac{\Omega_{\mathrm{H}ij}\Omega_{\mathrm{L}ij} d_{\mathrm{e}ij}}{(\Omega_{\mathrm{H}ij} - d_{\mathrm{e}ij})(\Omega_{\mathrm{L}ij} + d_{\mathrm{e}ij})}. \quad (12)$$

The universal barrier functions used to deal with the constraint requirements (8) and (9) for the ith quadrotor $(i = 1, \cdots, N)$ are then defined as

$$V_{\mathrm{e}i} = \frac{1}{2}\eta_{\mathrm{e}i}^2, \quad V_{ij} = \frac{1}{2}\eta_{ij}^2. \quad (13)$$

Take V_{ij} for an example. It is easy to see that $\eta_{ij} = 0$ if and only if $d_{\mathrm{e}ij} = 0$. Besides, when $d_{\mathrm{e}ij} \to \Omega_{\mathrm{H}ij}$, we have $\eta_{ij} \to +\infty$, hence $V_{ij} \to +\infty$. Alternatively, when $d_{\mathrm{e}ij} \to -\Omega_{\mathrm{L}ij}$, we have $\eta_{ij} \to -\infty$, therefore $V_{ij} \to +\infty$.

Remark 3.1: For the universal barrier function V_{ij}, note that if the constraint functions are symmetric, namely if $\Omega_{\mathrm{H}ij} = \Omega_{\mathrm{L}ij} = \Omega_{ij}$, then the barrier function V_{ij} becomes

$$V_{ij} = \frac{1}{2}\eta_{ij}^2, \quad \eta_{ij} = \frac{\Omega_{ij}^2 d_{\mathrm{e}ij}}{\Omega_{ij}^2 - d_{\mathrm{e}ij}^2}. \quad (14)$$

When there are no constraint requirements on $d_{\mathrm{e}ij}$, which can equivalently be seen as $\Omega_{\mathrm{H}ij} = \Omega_{\mathrm{L}ij} = \Omega_{ij} \to +\infty$, we have

$$\lim_{\Omega_{ij}\to\infty} \eta_{ij} = d_{\mathrm{e}ij}, \quad \lim_{\Omega_{ij}\to\infty} V_{ij} = \frac{1}{2}d_{\mathrm{e}ij}^2, \quad (15)$$

which means systems without output constraint requirements can in fact be regarded as a special case of the generic discussion on asymmetric constraint requirements.

Remark 3.2: In the literature, to deal with asymmetric constraint requirements, the following form of asymmetric barrier Lyapunov function is commonly used [21], [22]

$$V_{\mathrm{b}} = \frac{q(e)}{p}\log\frac{\Omega_{\mathrm{bH}}^p}{\Omega_{\mathrm{bH}}^p - e^p} + \frac{1-q(e)}{p}\log\frac{\Omega_{\mathrm{bL}}^p}{\Omega_{\mathrm{bL}}^p - e^p}, \quad (16)$$

where e is the output tracking error to be constrained, Ω_{bH} and Ω_{bL} are the upper and lower bounds, p is an even number such that $p > n$, with n being the order of the systems, and

$$q(\cdot) = \begin{cases} 1, & \text{if } \cdot > 0; \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

Note that (16) does not have the generic property stated in Remark 3.1. Besides, since $q(\cdot)$ is a discontinuous function, the form (16) requires that the error variable e is raised to the p-th power in order to avoid discontinuity when $e = 0$ for the derivatives of V_{b}, which may put a higher demand than necessary on the control signal when e is large.

IV. Control Design and Analysis

In this section we present the backstepping design procedure that will lead to our controller design and main theorem.

Step 1:

At this step, we consider the position kinematics of the quadrotors. Design the universal barrier function as $V_1 = \sum_{i=1}^N \Big(V_{\mathrm{e}i} + \sum_{j=1, j\neq i}^N V_{ij}\Big)$, and its derivative with respect to time leads to

$$\dot{V}_1 = \sum_{i=1}^N \Big(\eta_{\mathrm{e}i}\dot{\eta}_{\mathrm{e}i} + \sum_{j=1, j\neq i}^N \eta_{ij}\dot{\eta}_{ij}\Big). \quad (18)$$

First we examine the dynamics for $\eta_{\mathrm{e}i}$ $(i = 1, \cdots, N)$. From (12), we have

$$\begin{aligned}\dot{\eta}_{\mathrm{e}i} &= \frac{\partial \eta_{\mathrm{e}i}}{\partial \Omega_{d\mathrm{H}i}}\dot{\Omega}_{d\mathrm{H}i} + \vartheta_{di}\dot{d}_{\mathrm{e}i} \\ &= \Delta_{\mathrm{H}i} + \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(x_i - x_{\mathrm{d}i})\dot{x}_i + \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(y_i - y_{\mathrm{d}i})\dot{y}_i \\ &\quad + \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(z_i - z_{\mathrm{d}i})\dot{z}_i - \xi_i, \end{aligned} \quad (19)$$

where $\Delta_{\mathrm{H}i} \triangleq \frac{\partial \eta_{\mathrm{e}i}}{\partial \Omega_{d\mathrm{H}i}}\dot{\Omega}_{d\mathrm{H}i}$, $\vartheta_{di} \triangleq \frac{\partial \eta_{\mathrm{e}i}}{\partial d_{\mathrm{e}i}} = \frac{\Omega_{d\mathrm{H}i}^2}{(\Omega_{d\mathrm{H}i} - d_{\mathrm{e}i})^2}$, $\xi_i \triangleq \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(x_i - x_{\mathrm{d}i})\dot{x}_{\mathrm{d}i} + \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(y_i - y_{\mathrm{d}i})\dot{y}_{\mathrm{d}i} + \vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(z_i - z_{\mathrm{d}i})\dot{z}_{\mathrm{d}i}$. Hence for $\dot{V}_{\mathrm{e}i}$ $(i = 1, \cdots, N)$ we have

$$\begin{aligned}\dot{V}_{\mathrm{e}i} &= \eta_{\mathrm{e}i}\vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(x_i - x_{\mathrm{d}i})\dot{x}_i + \eta_{\mathrm{e}i}\vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(y_i - y_{\mathrm{d}i})\dot{y}_i \\ &\quad + \eta_{\mathrm{e}i}\vartheta_{di}\frac{1}{d_{\mathrm{e}i}}(z_i - z_{\mathrm{d}i})\dot{z}_i + \eta_{\mathrm{e}i}\Delta_{\mathrm{H}i} - \eta_{\mathrm{e}i}\xi_i. \end{aligned} \quad (20)$$

Similarly, for $\dot{V}_{ij}$ $(i, j = 1, \cdots, N,\ j \neq i)$ we have

$$\begin{aligned}\dot{V}_{ij} &= \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(x_i - x_j)\dot{x}_i + \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(y_i - y_j)\dot{y}_i \\ &\quad + \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(z_i - z_j)\dot{z}_i - \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(x_i - x_j)\dot{x}_j \\ &\quad - \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(y_i - y_j)\dot{y}_j - \eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(z_i - z_j)\dot{z}_j \\ &\quad + \eta_{ij}\Delta_{ij} - \eta_{ij}\xi_{ij}, \end{aligned} \quad (21)$$

where $\Delta_{ij} \triangleq \frac{\partial \eta_{ij}}{\partial \Omega_{\mathrm{H}ij}}\dot{\Omega}_{\mathrm{H}ij} + \frac{\partial \eta_{ij}}{\partial \Omega_{\mathrm{L}ij}}\dot{\Omega}_{\mathrm{L}ij}$, $\vartheta_{ij} \triangleq \frac{\partial \eta_{ij}}{\partial d_{\mathrm{e}ij}} = \frac{\Omega_{\mathrm{H}ij}\Omega_{\mathrm{L}ij}(d_{\mathrm{e}ij}^2 + \Omega_{\mathrm{H}ij}\Omega_{\mathrm{L}ij})}{(\Omega_{\mathrm{H}ij} - d_{\mathrm{e}ij})^2(\Omega_{\mathrm{L}ij} + d_{\mathrm{e}ij})^2}$, $\xi_{ij} \triangleq \vartheta_{ij}\dot{L}_{ij} = \vartheta_{ij}\frac{1}{L_{ij}}(x_{\mathrm{d}i} - x_{\mathrm{d}j})(\dot{x}_{\mathrm{d}i} - \dot{x}_{\mathrm{d}j}) + \vartheta_{ij}\frac{1}{L_{ij}}(y_{\mathrm{d}i} - y_{\mathrm{d}j})(\dot{y}_{\mathrm{d}i} - \dot{y}_{\mathrm{d}j}) + \vartheta_{ij}\frac{1}{L_{ij}}(z_{\mathrm{d}i} - z_{\mathrm{d}j})(\dot{z}_{\mathrm{d}i} - \dot{z}_{\mathrm{d}j})$.

Hence, for $\dot{V}_1$ we have

$$\dot{V}_1 = \sum_{i=1}^N \Big(\eta_{\mathrm{e}i}\Delta_{\mathrm{H}i} - \eta_{\mathrm{e}i}\xi_i + \sum_{j=1, j\neq i}^N (\eta_{ij}\Delta_{ij} - \eta_{ij}\xi_{ij})$$

$$+ E_{xi}\dot{x}_i + E_{yi}\dot{y}_i + E_{zi}\dot{z}_i\Big), \tag{22}$$

where

$$E_{xi} = \eta_{\mathrm{e}i}\vartheta_{\mathrm{d}i}\frac{1}{d_{\mathrm{e}i}}(x_i - x_{\mathrm{d}i}) + \sum_{j=1, j\neq i}^{N} 2\eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(x_i - x_j),$$

$$E_{yi} = \eta_{\mathrm{e}i}\vartheta_{\mathrm{d}i}\frac{1}{d_{\mathrm{e}i}}(y_i - y_{\mathrm{d}i}) + \sum_{j=1, j\neq i}^{N} 2\eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(y_i - y_j),$$

$$E_{zi} = \eta_{\mathrm{e}i}\vartheta_{\mathrm{d}i}\frac{1}{d_{\mathrm{e}i}}(z_i - z_{\mathrm{d}i}) + \sum_{j=1, j\neq i}^{N} 2\eta_{ij}\vartheta_{ij}\frac{1}{d_{ij}}(z_i - z_j).$$

Next, define the fictitious velocity tracking error as $e_{vi} = v_i - \alpha_{vi}$, with the stabilizing function $\alpha_{vi} \in \mathbb{R}^3$ ($i = 1, \cdots, N$) designed as

$$\alpha_{vi} = \frac{E_i}{E_i^{\mathrm{T}} E_i}\Big(-K_{\mathrm{e}i}\eta_{\mathrm{e}i}^2 - \sum_{j=1, j\neq i}^{N} K_{ij}\eta_{ij}^2 - \eta_{\mathrm{e}i}\Delta_{\mathrm{H}i} + \eta_{\mathrm{e}i}\xi_i - \sum_{j=1, j\neq i}^{N}(\eta_{ij}\Delta_{ij} - \eta_{ij}\xi_{ij})\Big), \tag{23}$$

where $E_i = [E_{xi},\ E_{yi},\ E_{zi}]^{\mathrm{T}} \in \mathbb{R}^3$, $K_{\mathrm{e}i} > 0$ and $K_{ij} > 0$ are control gains.

Remark 4.1: In (23), singularity can occur when $||E_i|| = 0$. Since $||E_i|| = 0$ if and only if $E_{xi} = 0$, $E_{yi} = 0$, and $E_{zi} = 0$ at the same time, there are two cases when this can happen. First, $||E_i|| = 0$ when both $d_{\mathrm{e}i} = 0$ and $d_{\mathrm{e}ij} = 0$. In this case, note that all the terms in the bracket on the right-hand-side of (23) are also zero, and we simply have $\alpha_{vi} = 0$. Second, $E_{xi} = 0$, $E_{yi} = 0$, and $E_{zi} = 0$ can happen at the same time when the reference direction vector for tracking is opposite to and same in magnitude with the direction vector for collision avoidance. This is usually referred to as the "deadlock situation" [20] in the literature, which can be resolved by modifying the reference trajectories or the time-varying constraint functions to allow the vehicle to move out of the deadlock. For the rest of the analysis we assume $||E_i|| > 0$ is guaranteed.

Therefore, (22) leads to

$$\dot{V}_1 = \sum_{i=1}^{N}\Big(E_i^{\mathrm{T}} e_{vi} - K_{\mathrm{e}i}\eta_{\mathrm{e}i}^2 - \sum_{j=1, j\neq i}^{N} K_{ij}\eta_{ij}^2\Big). \tag{24}$$

Step 2:
At this step, we consider the translational dynamics of the quadrotors. Design the Lyapunov function candidate at this step as $V_2 = \sum_{i=1}^{N}\frac{1}{2}e_{vi}^{\mathrm{T}} e_{vi}$, and its time derivative gives

$$\dot{V}_2 = \sum_{i=1}^{N} e_{vi}^{\mathrm{T}}\Big(g e_z - \frac{1}{m_i}u_i + \frac{1}{m_i}N_{1i} - \dot{\alpha}_{vi}\Big), \tag{25}$$

where we denote $u_i = F_i R_i e_z$. Now, for the ith quadrotor ($i = 1, \cdots, N$), the control law $u_i \in \mathbb{R}^3$ is designed as

$$u_i = \frac{e_{vi}\bar{u}_i^{\mathrm{T}}\bar{u}_i\hat{\rho}_{mi}^2}{\sqrt{e_{vi}^{\mathrm{T}} e_{vi}\bar{u}_i^{\mathrm{T}}\bar{u}_i\hat{\rho}_{mi}^2 + \varepsilon_i^2}}, \tag{26}$$

$$\bar{u}_i = E_i + g e_z + (K_{vi} + \nu_i)e_{vi} + \hat{\mu}_{mi}\frac{e_{vi}}{\sqrt{e_{vi}^{\mathrm{T}} e_{vi} + \varepsilon_i^2}} - \hat{\dot{\alpha}}_{vi}, \tag{27}$$

where $K_{vi} > 0$ is a control gain, $\nu_i > 0$ and $\varepsilon_i > 0$ are design constants, $\hat{\rho}_{mi}$ is the estimation of the *unknown* constant $\rho_{mi} = \frac{1}{\underline{b}_{mi}}$, and $\hat{\mu}_{mi}$ is the estimation of the *unknown* constant μ_{mi} such that $\Big|\Big|\frac{1}{m_i}N_{1i}\Big|\Big| \leq \mu_{mi}$. Next, we substitute the control design (26) back into (25), which yields

$$-\frac{1}{m_i}e_{vi}^{\mathrm{T}} u_i < \varepsilon_i\underline{b}_{mi} - \underline{b}_{mi}e_{vi}^{\mathrm{T}}\bar{u}_i\tilde{\rho}_{mi} - e_{vi}^{\mathrm{T}}\bar{u}_i, \tag{28}$$

where $\tilde{\rho}_{mi} = \hat{\rho}_{mi} - \rho_{mi}$ ($i = 1, \cdots, N$).

Hence, (24) and (25) lead to

$$\dot{V}_1 + \dot{V}_2 < \sum_{i=1}^{N}\Big(-K_{\mathrm{e}i}\eta_{\mathrm{e}i}^2 - \sum_{j=1, j\neq i}^{N} K_{ij}\eta_{ij}^2 - K_{vi}e_{vi}^{\mathrm{T}} e_{vi} - \underline{b}_{mi}e_{vi}^{\mathrm{T}}\bar{u}_i\tilde{\rho}_{mi} - \tilde{\mu}_{mi}\frac{e_{vi}^{\mathrm{T}} e_{vi}}{\sqrt{e_{vi}^{\mathrm{T}} e_{vi} + \varepsilon_i^2}} + \varepsilon_i(\underline{b}_{mi} + \mu_{mi}) + \frac{1}{\nu_i}\varepsilon_{\alpha_{vi}}^2\Big), \tag{29}$$

where $\tilde{\mu}_{mi} = \hat{\mu}_{mi} - \mu_{mi}$ ($i = 1, \cdots, N$).

Next, design the adaptive laws for the estimators $\hat{\rho}_{mi}$ and $\hat{\mu}_{mi}$ ($i = 1, \cdots, N$) as the following

$$\dot{\hat{\rho}}_{mi} = n_{\rho_{mi}}e_{vi}^{\mathrm{T}}\bar{u}_i - \sigma_{\rho_{mi}}\hat{\rho}_{mi}, \tag{30}$$

$$\dot{\hat{\mu}}_{mi} = n_{\mu_{mi}}\frac{e_{vi}^{\mathrm{T}} e_{vi}}{\sqrt{e_{vi}^{\mathrm{T}} e_{vi} + \varepsilon_i^2}} - \sigma_{\mu_{mi}}\hat{\mu}_{mi}, \tag{31}$$

where $\hat{\rho}_{mi}(0) = 0$ and $\hat{\mu}_{mi}(0) = 0$ are the initial conditions, $n_{\rho_{mi}}$, $n_{\mu_{mi}}$, $\sigma_{\rho_{mi}}$, and $\sigma_{\mu_{mi}}$ ($i = 1, \cdots, N$) are positive design constants. Design the Lyapunov function candidates for the estimators of the quadrotors as $V_{\rho_m} = \sum_{i=1}^{N}\frac{\underline{b}_{mi}}{2n_{\rho_{mi}}}\tilde{\rho}_{mi}^2$, $V_{\mu_m} = \sum_{i=1}^{N}\frac{1}{2n_{\mu_{mi}}}\tilde{\mu}_{mi}^2$. Denote $V_{\mathrm{pos}} = V_1 + V_2 + V_{\rho_m} + V_{\mu_m}$, after some algebraic manipulation, we can arrive at

$$\dot{V}_{\mathrm{pos}} < \sum_{i=1}^{N}\Big(-K_{\mathrm{e}i}\eta_{\mathrm{e}i}^2 - \sum_{j=1, j\neq i}^{N} K_{ij}\eta_{ij}^2 - K_{vi}e_{vi}^{\mathrm{T}} e_{vi} - \frac{\underline{b}_{mi}\sigma_{\mu_{mi}}}{2n_{\rho_{mi}}}\tilde{\rho}_{mi}^2 - \frac{\sigma_{\rho_{mi}}}{2n_{\mu_{mi}}}\tilde{\mu}_{mi}^2 + C_{1i}\Big), \tag{32}$$

where $C_{1i} = \frac{\underline{b}_{mi}\sigma_{\mu_{mi}}\rho_{mi}^2}{2n_{\rho_{mi}}} + \frac{\sigma_{\rho_{mi}}}{2n_{\mu_{mi}}}\mu_{mi}^2 + \varepsilon_i(\underline{b}_{mi} + \mu_{mi}) + \frac{1}{\nu_i}\varepsilon_{\alpha_{vi}}^2$.

Hence, let the overall Lyapunov function be $V = V_{\mathrm{pos}}$, we can get

$$\dot{V} < -\kappa V + \varrho, \tag{33}$$

where $\kappa \triangleq \min_{i,j}(2K_{\mathrm{e}i}, 2K_{ij}, 2K_{vi}, \sigma_{\mu_{mi}}, \sigma_{\rho_{mi}})$, $\varrho \triangleq \sum_{i=1}^{N} C_{1i}$.

The above backstepping design leads to the following theorem.

Theorem 4.1: For the ith quadrotor ($i = 1, \cdots, N$), with the thrust law as (26) and (27), and adaptive laws (30) and (31), the quadrotor formation system described by (1) and (2), under Assumptions 2.1–2.4 has the following properties:

i) The constraint requirements (8) and (9) will not be violated during operation.

ii) The transformed output tracking error $\eta_{\mathrm{e}i}$ and η_{ij} will converge into the sets

$$\left\{x = \eta_{\mathrm{e}i}, \eta_{ij} : x < \varepsilon_\eta, \quad \varepsilon_\eta = \sqrt{\frac{2\varrho}{\kappa}}\right\},$$

and as a result, the output tracking error $d_{\mathrm{e}i}$ and $d_{\mathrm{e}ij}$ will converge to the sets

$$\left\{d_{\mathrm{e}i} : d_{\mathrm{e}i} < \varepsilon_{\chi_{\mathrm{H},i}}\right\}, \tag{34}$$

$$\left\{d_{\mathrm{e}ij} : -\varepsilon_{\iota_{\mathrm{L},i}} < d_{\mathrm{e}ij} < \varepsilon_{\iota_{\mathrm{H},i}}\right\}, \tag{35}$$

where $\varepsilon_{\chi_{\mathrm{H},i}}$ is expressed as

$$\varepsilon_{\chi_{\mathrm{H},i}} = \frac{\varepsilon_\eta \Omega_{d\mathrm{H}i}}{\Omega_{d\mathrm{H}i} + \varepsilon_\eta}, \tag{36}$$

and we have $\varepsilon_{\iota_{\mathrm{L},i}}$ expressed in (37), with $\varepsilon_{\iota_{\mathrm{H},i}}$ expressed in (38) (see the next page), where $\Omega_\mathrm{H} \triangleq \Omega_{\mathrm{H}ij}$ and $\Omega_\mathrm{L} \triangleq \Omega_{\mathrm{L}ij}$, for $i, j = 1, \cdots, N$, $j \neq i$.

Proof: First, from (33), it is clear that the overall Lyapunov function V is bounded, since $V(t) \leq \Big(V(0) - \frac{\varrho}{\kappa}\Big)e^{-\kappa t} + \frac{\varrho}{\kappa}$. The boundedness of V in turn implies boundedness of $\eta_{\mathrm{e}i}$ and η_{ij}. Hence, the constraints requirements (8) and (9) are satisfied during the operation.

Moreover, we have $\lim\sup_{t\to\infty} V = \frac{\varrho}{\kappa}$, hence $\frac{1}{2}\eta_{\mathrm{e}i}^2 \leq \frac{\varrho}{\kappa}$ when $t \to \infty$, therefore $\eta_{\mathrm{e}i}$ will converge to the set $|\eta_{\mathrm{e}i}| < \varepsilon_\eta$. Similar relationship holds for $\eta_{\mathrm{e}ij}$. Furthermore, boundedness of the adaptive estimates $\hat{\rho}_{mi}$, $\hat{\mu}_{mi}$, as well as boundedness of the fictitious error e_{vi} ($i = 1, \cdots, N$), can be concluded from the fact that V is bounded. Next, for $i = 1, \cdots, N$, note that in the range that $d_{\mathrm{e}i} < \Omega_{d\mathrm{H}i}$, $\eta_{\mathrm{e}i}$ is a function in $d_{\mathrm{e}i}$. Hence, the range (8) gives the range for $d_{\mathrm{e}i}$ given as in (34). Besides, within the range of (9), η_{ij} is quadratically related to $d_{\mathrm{e}ij}$. Hence, satisfying the constraints (9) means that the distance tracking errors $d_{\mathrm{e}i}$ and $d_{\mathrm{e}ij}$ will be confined in the ranges defined by (34) and (35). ■

V. Simulation Studies

In this section, a simulation example is carried out with a team of $N = 4$ quadrotors. In this simulation, the model parameters of the quadrotors are $m_i = 2\mathrm{kg}$ and $g = 9.81\mathrm{m/s^2}$, $i = 1, 2, 3, 4$. Note that the units of the position and translational velocity are m and $\mathrm{m/s}$, respectively. The reference signals for the vehicles are given as $p_{\mathrm{d}1} = [2, 2, 5]^\mathrm{T}$, $p_{\mathrm{d}2} = [2, 3, 5]^\mathrm{T}$, $p_{\mathrm{d}3} = [3, 2, 5]^\mathrm{T}$, and $p_{\mathrm{d}4} = [3, 3, 5]^\mathrm{T}$. The constraint functions are selected as $\Omega_{d\mathrm{H}i} = (10.1 - 0.5)e^{-0.24t} + 0.5$, $\Omega_{\mathrm{H}ij} = (7 - 0.1)e^{-0.08t} + 0.1$, and $\Omega_{\mathrm{L}ij} = (2.2 - 0.1)e^{-0.04t} + 0.1$, $i, j = 1, 2, 3, 4, i \neq j$. To implement the adaptive control framework, the design parameters are chosen as $\varepsilon_i = 0.1$, $n_{\rho_{mi}} = 0.29$, $n_{\mu_{mi}} = 0.3$, $\sigma_{\rho_{mi}} = 0.065$, and $\sigma_{\mu_{mi}} = 0.1$, $i = 1, 2, 3, 4$. The control gains are designed as $K_{\mathrm{e}i} = 0.75$, $K_{ij} = 0.6$, and $K_{vi} = 2$, $\nu_i = 0.5$, $i = 1, 2, 3, 4$. The initial positions of the quadrotor team are $[x_1, y_1, z_1]^\mathrm{T} = [0, 0, 0]^\mathrm{T}$, $[x_2, y_2, z_2]^\mathrm{T} = [0, 5, 0]^\mathrm{T}$, $[x_3, y_3, z_3]^\mathrm{T} = [5, 0, 0]^\mathrm{T}$, and $[x_4, y_4, z_4]^\mathrm{T} = [5, 5, 0]^\mathrm{T}$. The initial condition of the translational velocity of each agent is zero. The external disturbance is $N_{1i} = [0.1\sin(0.2t) + 0.005\mathrm{rand}, 0.05\cos(0.15\mathrm{t}) + 0.01\mathrm{rand}, 0.03\cos(0.12\mathrm{t})]^\mathrm{T}$, where $i = 1, 2, 3, 4$, and rand represents the random noise uniformly distributed in the interval $(-1, 1)$. The simulation results are presented in Figs. 1 and 2. The LOS distance tracking errors $d_{\mathrm{e}i}$ under the proposed controller are shown in Fig. 1 with the constraint function $\Omega_{d\mathrm{H}i}$. From this figure, we see that $d_{\mathrm{e}i}$ can converge to a small neighborhood of the origin without violation of the performance constraint $\Omega_{d\mathrm{H}i}$. Fig. 2 gives us the exhibition of the profile of the inter-quadrotor distance tracking errors $d_{\mathrm{e}ij}$ under the proposed controller. It is obvious that the safety constraints are always satisfied during the operation since $d_{\mathrm{e}ij}$ always stayed between the constraint functions $-\Omega_{\mathrm{L}ij}$ and $\Omega_{\mathrm{H}ij}$. Based on the above discussion, we can conclude that the simulation results confirm the theoretic analysis shown in Theorem 4.1.

VI. Conclusion

In this work, we address the formation control problem for a team of quadrotors with two types of constraints, namely the performance constraints and the safety constraints. A new adaptive formation control architecture is proposed. Specifically, we employ the universal barrier functions into the controller design and analysis, to ensure that the constraint requirements on the LOS distance tracking error and relative distance error between two quadrotors are all satisfied during the operation. The universal barrier function approach is also a generic framework that can address system with different types of constraints in a unified controller architecture. Exponential convergence rate can be guaranteed on the LOS distance and relative inter-quadrotor distance tracking errors, while all constraints are satisfied during the operation. Future research includes extension of the analysis to constrained formation control problems for unmanned aerial vehicles with collaborate objectives such as load lifting and transporting.

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$$\varepsilon_{\iota_{\mathrm{H}},i} = \frac{-(\Omega_{\mathrm{H}}\Omega_{\mathrm{L}} - \varepsilon_\eta(\Omega_{\mathrm{H}} - \Omega_{\mathrm{L}})) + \sqrt{\Omega_{\mathrm{H}}^2\Omega_{\mathrm{L}}^2 + \varepsilon_\eta^2(\Omega_{\mathrm{H}} + \Omega_{\mathrm{L}})^2 - 2\varepsilon_\eta\Omega_{\mathrm{H}}\Omega_{\mathrm{L}}(\Omega_{\mathrm{H}} - \Omega_{\mathrm{L}})}}{2\varepsilon_\eta} \tag{37}$$

$$\varepsilon_{\iota_{\mathrm{L}},i} = \frac{(\Omega_{\mathrm{H}}\Omega_{\mathrm{L}} + \varepsilon_\eta(\Omega_{\mathrm{H}} - \Omega_{\mathrm{L}})) - \sqrt{\Omega_{\mathrm{H}}^2\Omega_{\mathrm{L}}^2 + \varepsilon_\eta^2(\Omega_{\mathrm{H}} + \Omega_{\mathrm{L}})^2 + 2\varepsilon_\eta\Omega_{\mathrm{H}}\Omega_{\mathrm{L}}(\Omega_{\mathrm{H}} - \Omega_{\mathrm{L}})}}{2\varepsilon_\eta} \tag{38}$$

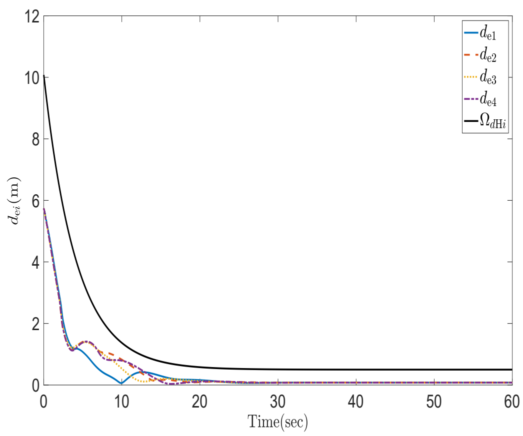

Fig. 1. The profile of the LOS distance tracking errors $d_{\mathrm{e}i}$ with $\Omega_{d\mathrm{H}i}$, $i = 1, 2, 3, 4$.

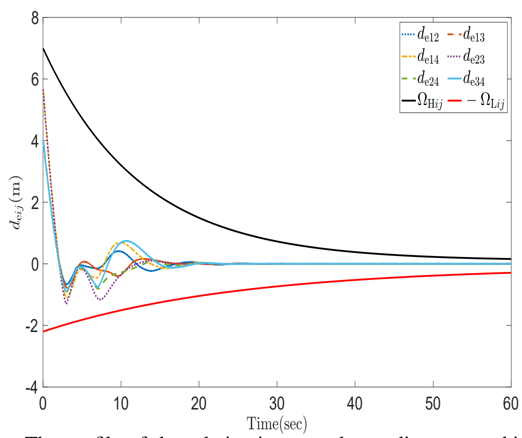

Fig. 2. The profile of the relative inter-quadrotor distance tracking errors $d_{\mathrm{e}ij}$ with $\Omega_{\mathrm{H}ij}$ and $-\Omega_{\mathrm{L}ij}$, $i, j = 1, 2, 3, 4, i \neq j$.

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