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A reliable location model for heterogeneous systems under partial capacity losses☆

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ABSTRACT

Due to the interdependency between multiple infrastructure systems, the performance of a facility may depend on the resources or supplies received from other facilities. However, cross-system interdependence has seldom been studied in the location design context, probably due to the lack of a concise model describing interdependence across heterogeneous systems. This paper proposes a new heterogeneous flow scheme to describe cross-system interdependence. This scheme has two features distinguished from existing models in describing an interdependent facility location problem. First, it is a simple linear model upon which a compact facility location model can be built. Secondly, it relaxes the need to maintain flow conservation between different systems and is suitable in describing heterogeneous systems that take in and output different resources or services. Built on this scheme, this paper proposes a reliable location design model for a nexus of interdependent infrastructure systems. This model aims to locate the optimal facility locations in multiple heterogeneous systems to balance the tradeoff between the facility investment and the expected nexus operation performance. Different from other reliable facility location models, this expected performance captures interdependence among heterogeneous systems due to the resource input-output relationships. The consideration of continuous partial capacity losses complements the reliable location literature that mainly focuses on binary disruptions. Two numerical examples are conducted for investigating features and applications of the proposed model. The results indicate that with a standard off-the-shelf integer programming solver, the proposed model is able to solve optimal facility location design for problem instances of realistic scales to the near-optimum solutions with optimality gap assurance. Sensitivity analyses of key parameters indicate that improving facility capacity and reducing interdependency between systems can mitigate impacts of facility capacity losses and reduce the overall system cost.

1. Introduction

Facility location design aims to select the optimal facility locations to minimize the total cost of an infrastructure system, including the system's one-time construction investment and its long-term operational cost. Traditional facility location models assume that the infrastructure system, once built, will remain functioning all the time. This assumption however ignores possible

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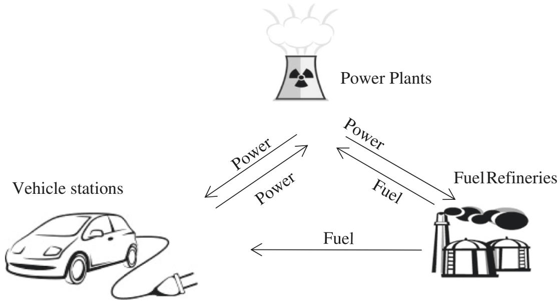


Fig. 1. An example of the interconnected system with three different components.

infrastructure disruptions that have been observed in many real-world infrastructure systems. Infrastructure disruptions may cause sharp increase of operational costs, massive drop of system service quality, and deep deterioration of customer satisfaction. In particular, infrastructure disruptions may be widespread and cause severe consequences during anthropogenic or natural disastrous events, such as the 2002 west-coast port lockout (D'Amico, 2002), the 2003 massive power outage (Schewe, 2004) and 2012 Hurricane Sandy (HSRTF, 2013).

Worse, impacts of infrastructure disruptions may be further exacerbated by interdependency between different infrastructure systems. Such cross-system interdependency is prevailing in modern cities and recently became an emerging interdisciplinary research topic. Operating a facility in one system often needs supplies from other systems. Therefore, supplies are circulated across multiple systems and such supply circulations form complicated connections that make these systems highly interdependent. Due to the heterogeneity of the supplies produced by different systems, their circulations may not follow the flow conservation law. Usually, at certain facilities along a circulation, the upstream supplies are only used as an auxiliary resource to sustain their production capacity, and the generated resources will thus have surplus along this circulation. To take a circulation formed by a power plant and a fuel refinery as an example, the amount of fuel production at the refinery supported by one unit electricity from the plant, if fully transported back to this plant, will generate electricity far more than the original one unit. Therefore, it is not suitable to use traditional conserved flows to model interdependency among heterogeneous systems.

In such an interdependent nexus, once some facilities are disrupted in one system, it will have to cut supply to its downstream facilities in other systems and thus may compromise or disrupt the operations of these downstream facilities. These operational damages may be further propagated along the supply chain to jeopardize other further downstream facilities in more systems and even spill back to the originally disrupted facilities. This essentially forms negative circulations of operational damages and spreads facility disruptions across all these systems. This phenomenon is referred to as cascading failures in the literature (e.g., Buldyrev et al., 2010). One illustrative example is shown in Fig. 1, where an infrastructure nexus consists of three systems — power plants in an electricity grid, fuel refineries, and vehicle stations. For simplicity, this study assumes that a vehicle station has both charging/discharging facilities and fuel facilities, and the vehicles considered in this study are hybrid electric vehicles (HEV) that can be powered by either electricity or fuel. We see that the three types of facilities in this nexus need to support one another in addition to supplying external consumer demand. For example, the power plants provide electricity to vehicle stations and refineries to support their daily operations. Meanwhile, the power plants may need fuels from refineries to generate electricity. Further, the electricity generated by HEVs can be shared with power plants through the V2G (vehicle-to-grid) technology. Note that if one system, let's say the power system, loses some generators due to random disruptions, then it will cut down the electricity supply to refineries and HEVs. As a result, some refineries may not be able to sustain their full production capacity, and the charging demands of HEVs may not be fully met. Next, power plants will receive less fuel and less discharged electricity from HEVs, and thus will have to shut down more generators. Such cascading disruptions are circulated in this way to cause even further damages.

The impacts of cascading disruptions are actually much affected by the location decisions of the constructed facilities. First, if each system builds a large number of facilities with sufficient capacity redundancy, when some facilities are disrupted, the propagated impacts can be absorbed by redundant capacities in the downstream systems, and the downstream facilities may also use the extra redundancy to raise their supplies to the system with original disruptions so as to increase the production of the first system to compensate its capacity due to initial disruptions. Secondly, if facilities are located closer to each other, then activation of redundant supplies after disruptions will incur less associated operational costs (e.g., fuel transportation cost and electricity transmission cost). Therefore, it would be beneficial to consider the impacts of cascading disruptions as early as the location design phase such that the planned infrastructure nexus will be reliable against cross-system cascading disruptions.

Despite abundant developments separately in reliable facility location design (Snyder, 2006) and cascading disruption modeling (Ouyang, 2014), only limited efforts were made to integrating the impacts of interdependent heterogeneous systems into location design (see the Literature Review section). While these efforts have explored novel ways of addressing facility protection and correlated disruptions, modeling of facility disruptions that may occur in interdependent heterogeneous systems has not been investigated in the location design phase. The major challenge to integrating these two classes of fruitful research outcomes is probably due to the lack of a concise model describing interdependence across heterogeneous systems. In order to take advantage of mature developments of mathematical programming, most successful location models are built upon simple system operation mechanisms that can be well described by explicit linear formulations. However, most available cascading disruption models are based on complicated network settings and dynamic disruption propagation mechanisms that can only be examined with iterative numerical simulations but not compact linear formulations. This imposes a significant barrier in addressing cross-system cascading disruptions in location planning.

To overcome this challenge and bridge this methodology gap, this paper proposes a new heterogeneous flow scheme to describe cross-system interdependence. This scheme has two features distinguished from existing models in describing facility interdependency. First, it is a simple linear model upon which a compact facility location model can be built. Secondly, it relaxes the need to maintain flow conservation between different systems and is suitable in describing heterogeneous systems that take in and output different resources or services. Built on this scheme, this paper proposes a reliable location design model for a nexus of interdependent infrastructure systems. This model aims to locate the optimal facility locations in multiple heterogeneous systems to balance the tradeoff between the facility investment and the expected nexus operation performance. Different from other reliable facility location models, this expected performance captures possible cascading impacts as an emergency cost among multiple systems induced by probabilistic facility capacity loses. The emergency cost is derived from getting emergency resources supplied by the emergency source to avoid cascading failures. The consideration of continuous partial capacity losses complements the reliable location literature that mainly focuses on binary disruptions. As an exploratory study, this proposed model proposes a general distribution for the capacity loss of each facility and then only investigates uniform and triangular distributions that are two typical distributions. It also assumes the service relationships for facility-to-facility supply and facility-to-customer supply will remain unchanged once facilities are built, regardless of facility disruptions. Therefore this study does not intend to investigate service reassignments in response to actual disruption scenarios as some other reliable facility location problems (Snyder and Daskin, 2005). Two case studies are conducted for investigating features and applications of the proposed model. The results indicate that with a standard off-the-shelf integer programming solver, the proposed model is able to solve optimal facility location design for problem instances of realistic scales to the near-optimum solutions with optimality gap assurance. Sensitivity analyses of key parameters indicate that improving facility capacity and reducing interdependency between systems can mitigate impacts of facility capacity losses and reduce the overall system cost.

The remainder of this paper is organized as follow. Section 2 reviews the relevant literature and highlights research gaps. Section 3 formulates the reliable facility location problem as a mixed integer programming problem. Section 4 conducts two case studies to test the model performance and draw managerial insights. Section 5 concludes this work and briefly discusses future research directions.

2. Literature review

Facility location design problems have been studied for many years. In the early twentieth century, Weber (1929) initiated a pioneering study about facility locations. Afterwards, many classic facility location design models are developed for solving different facility location problems. In the late twentieth century, Daskin (1995) conducted a comprehensive review on different classic facility location models and introduced a number of key algorithms to solve these models. Later years, a number of studies extended the classic models to various aspects, e.g., transportation network (Melkote and Daskin, 2001), inventory-location and joint inventory-location (Daskin et al., 2002; Shen et al., 2003), transportation-inventory network (Shu et al., 2005), sensor placement (Berry et al., 2006), and remanufacturing network (Lu and Bostel, 2007). In the past decade, probably due to frequent anthropogenic or natural disasters, researchers have paid increasing attention to impacts of possible facility disruptions, so do those in the location design research community. Snyder and Daskin (2005) pointed out that the facility location scheme obtained by the traditional models is likely to be suboptimal even with infrequent disruptions. Cui et al. (2010) proposed both continuous and discrete models to study the reliable uncapacitated UFL problem (RUFL) with site-dependent failure probabilities. Further, researchers have proposed a number of reliable facility location models to address issues including imperfect information (Berman et al., 2009; Yun et al., 2015), joint inventory-location (Chen et al., 2011), emergency service network (An et al., 2013, 2015a), sensor deployment (Li and Ouyang, 2011, 2012), hub-and-spoke design (An et al., 2015b), market competitions (Wang and Ouyang, 2013; Wang et al., 2015), traffic congestion (Ouyang et al., 2015).

Most location design studies share a common feature that the location decision aims to strike the optimal balance between one time facility investment and long-term operational costs. Location decisions are usually captured by binary variables, which raise most location problems to be NP-hard. To alleviate the computational burden and maintain an elegant model structure, the operational are often described in a compact form, e.g., a linear programming formulation oftentimes. Such a model structure is helpful in solution efficiency and revealing managerial insights. It is remarkable that facility operations with independent disruptions (Snyder and Daskin, 2005; Cui et al., 2010), which involves an exponentially number of disruption scenarios, have been successfully formulated into such compact forms involving only a small polynomial number of variables and constraints.

In many real-world observations, especially in severe disasters, disruptions are actually not independent. Rather, they are correlated and may even propagate across chains of facilities to cause cascading failures. Modelling cascading disruptions across multiple systems becomes an emerging research topic in recent years. To just name a few, Buldyrev et al. (2010) studied the catastrophic cascade of failures in interdependent networks. Morris and Barthelemy (2012) studied the key features of interdependent systems. Radicchi (2015) introduced a set of heuristic equation to study the percolation in real interdependent networks. See Ouyang (2014) for a comprehensive review on this topic. These studies adopted numerical simulation and complex network theory to develop innovative models describing disruption propagation dynamics and asymptotical system failure patterns. However, piggybacking location analysis on these models would create overwhelming computational burdens and enormous challenges to model structure analysis. The authors only noticed some limited effort (Zio et al., 2012) in protecting facilities under cascading disruption risks in a single network system.

While a compact form describing general cascading disruptions suitable for location design is yet to be discovered, several plausible efforts have been made in relevant pursuits in the location design context. Li and Ouyang (2010) proposed a continuum approximation approach for large scale reliable facility location problems with several specific correlated probability disruption functions. Liberatore et al. (2012) studied a location protection problem where one facility's disruption may cause capacity losses of other facilities. Li et al. (2013) proposed a support station structure to explicitly represent correlated disruptions to enable compact formulations and efficient solution algorithms for this complex problem. Xie et al. (2015) extended this framework by considering more general correlation patterns and drilling into more theoretical insights. Lu et al. (2015) proposed a distributional robust reliable uncapacitated fixed-charge location (DR-RUFL) model in order to obtain the optimal expected total cost under the worse-case disruption profile. Despite these efforts, so far as our knowledge, general cascading disruptions propagating across three or more layers of facilities among different systems with different resources have not be addressed in the location analysis literature.

It would also worth noting that while most reliable facility location studies consider binary disruptions, partial loss of facility capacity, which may happen more frequently than complete disruptions, receives relatively less attention in the location design context. Hatefi et al. (2015) proposed a fuzzy possibilistic programing model for designing a reliable forward-reverse logistics network with partial and complete facility capacity disruptions. Mohammadi et al. (2016) designed a reliable logistics network based on a hub location problem with two different types of disruptions. Both studies assumed that the facility capacity loss happens at a few discrete levels whereas this study considers the full spectrum of continuous facility capacity loss from 0 to the full capacity.

To address these research gaps, this paper propose a new reliable location model that considers interdependent heterogeneous flows in the nexus of multiple different systems. The emergency source is introduced to avoid cascading failures. Further, we investigate a continuous spectrum of probabilistic facility capacity losses to fit broader facility operational states.

3. Model formulation

3.1. Facility investment

For readers' convenience, the key notation of this study is listed in Table 1.

The studied infrastructure nexus is comprised of a set of systems, denoted by L. Each system $l \in L$ consists of a set of candidate facility locations, denoted by I_l. For the notation convenience, let I denote the set of all candidate locations, i.e., $I = \cup_{l \in L} I_l$, and let l_i denote the type of facility at location i, $\forall\, i \in I$. Constructing a facility at a candidate location $i \in I$ costs a fixed investment of f_i. Define variables $Y := \{y_i\}_{i \in I}$ to denote the facility location decisions: $y_i = 1$ denotes a facility that is constructed at location i and $y_i = 0$ otherwise. This yields the total construction cost as follows:

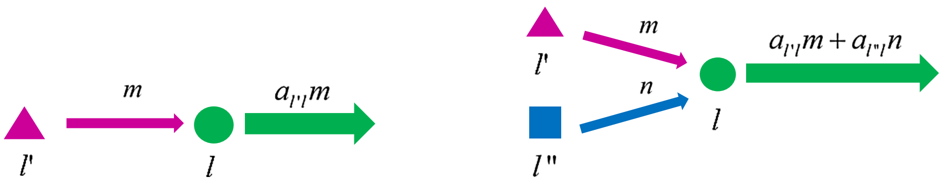
$$C^C := \sum_{i \in I} f_i y_i \tag{1}$$

3.2. Heterogeneous flow scheme

The facility operations in different systems are interdependent based on the following heterogeneous flow scheme. A facility $i \in I_l$, once built, can produce an type-l outbound resource flow no greater than its capacity c_i, which can feed into facilities in the other systems to support their productions. Different from the traditional network flows that require flow conservation, i.e., the total inbound flow to a facility (or node) is always identical to the total outbound flow from the same facility, the study proposes a *heterogeneous flow* scheme that does not necessarily preserve conservation of flows through a facility. We assume that the resource flows can be converted from one system to another according to the following heterogeneous conversion rules as illustrated in Fig. 2.

Table 1
Notation list.

Parameter	Description
a_{ij}	Conversion factor from facility i to j
c_i	Capacity of facility i
d_{ij}	Distance from location i to j
f_i	Fixed cost of facility i
I	Set of all candidate facility locations
L, $l \in L$	Set of systems, indexed by l
q_i	Disruption probability of facility i
U, $u \in U$	Set of customers, indexed by u
β_l	Unit transportation cost for system l
π_i	Unit emergency cost for facility i
μ_{ul}	Demand of customer u for system l
Decision Variable	Description
x_{ij}	Resource flow assignment decisions: x_{ij} denotes the amount of flow sent from facility i to facility or customer j
y_i	Facility location decisions: $y_i = 1$ denotes a facility is constructed at location i and $y_i = 0$ otherwise



(a) Single input flow (b) Multiple input flows

Fig. 2. Heterogeneous flow scheme.

When the flow is sent from a facility in system l' to one facility in system l, the output flow from the type l facility shall be proportional to the input flow sent from the type l' facility. Let conversion factor $a_{l'l}$ denote the units of type l output flow produced with one unit of type l' input flow. Therefore, as illustrated in Fig. 2(a), m units of type l' flow can be converted to $a_{l'l}m$ units of type l flow through a type l facility. Further, the output flows from different input flows through the same facility are additive. As illustrated in Fig. 2(b), m units of type l' flow and n units of type l'' flow shall yield $a_{l'l}m + a_{l''l}n$ units of type n in total through a type l facility. If $a_{l'l} = 0$, it means that the type l' facility has no connection with the type l facility. To make the problem general, we also allow transshipment between the same type facilities. Since this transshipment neither loses nor creates resources, we set $a_{ll} = 1, \forall\, l \in L$.

These interdependent systems collectively serve a number of distributed customers, denoted by U. Besides supporting each other's production, an installed facility can directly supply its respective output resources to a customer. Each customer $u \in U$ consumes at minimum μ_{ul} units of type l. Transportation cost is generated by shipping resource flows between facilities and customers. We assume that β_l denotes the cost when transport one unit of type l per unit distance and d_{ij} denotes the distance from location i to location j. Define variables $X{:} =\{x_{ij}\}_{i\in I, j\in I\cup U}$ to denote the resource flow assignment decisions; i.e., x_{ij} denotes the amount of flow sent from facility i to facility or customer j. Once the resource flows are assigned, the total output flow that facility i needs to provide is obviously formulated by $\sum_{j\in I\cup U} x_{ij}$, which shall be always no greater than facility capacity c_i. With this, we can formulate the total transportation cost as follows:

$$C^T{:} = \sum_{l\in L} \beta_l \sum_{i\in I_l} \sum_{j\in I\cup U} d_{ij}x_{ij} \tag{2}$$

It is worth mentioning that not all relationships of mutual dependency can exist or are feasible in real world. For example, in a very simple infrastructure nexus with two types of resources A and B, consider the case that $a_{AB} = 0.5$, $a_{BA} = 0.2$ as shown in Fig. 3. If 100 units of A is fed to this nexus (through node A), it will reduce to 50 units after passing node B and further reduces to 10 units after circulating back to A. We see that after each circulation, the resources dwindle, and thus this nexus can only consume resources rather than producing them. Actually, to balance to the flow, external flow needs to constantly feed into this nexus. Therefore, this nexus cannot support any customer demand and thus shall not exist in a valid nexus design.

The following analysis will rigorously investigate this issue.

Definition 1. We call flows $\{x_{ij} \geqslant 0\}_{i\in I, j\in I\cup U}$ a *feasible flow solution* if $\sum_{j\in I\cup U} x_{ij} \leqslant c_i y_i, \forall\, i \in I$, $\sum_{j\in I\cup U} x_{ij} \leqslant \sum_{j\in I} a_{l_j l_i} x_{ji}, \forall\, i \in I$ and $\sum_{i\in I_l} x_{ij} \geqslant \mu_{jl}, \forall\, l \in L, j \in U$.

Definition 2. *We call flows* $\{x_{ij} \geqslant 0\}_{i\in I, j\in I\cup U}$ *a flow pattern if* $\sum_{j\in I\cup U} x_{ij} \leqslant \sum_{j\in I} a_{l_j l_i} x_{ji}, \forall\, i \in I$. *Note that a flow pattern may not be a feasible flow solution since it does not need to satisfy the customer demands. If all these inequalities become equalities, we call it a critical flow pattern. For the simplicity of the following presentation, we may omit zero flows in a flow pattern.*

Definition 3. *We call an ordered vector* $P = [i_0, i_1, \cdots, i_{K-1}, i_K]$ *such that* $i_k \in I, \forall\, k \in \mathscr{K}\backslash\{K\}$ *and* $i_K \in U$ *with* $\mathscr{K}{:} =\{0, 1, \cdots, K\}$ *a service chain. The service chain allows the flow a customer receives traceable. If* P *satisfies* $a_{i_{k'} i_0} \prod_{k=0}^{k'-1} a_{i_k i_{k+1}} > 1, \exists\, k' \in \mathscr{K}\backslash\{0, K\}$, *we say* P *is sustainable with respect to facility* $i_{k'}$. *Otherwise, if* $a_{i_{k'} i_0} \prod_{k=0}^{k'-1} a_{i_k i_{k+1}} \leqslant 1, \exists\, k' \in \mathscr{K}\backslash\{0, K\}$, *we say* P *is unsustainable with respect to facility* $i_{k'}$. *Here, for the simplicity of the formulations, we slightly abuse the notation to use* a_{ij} *to denote* $a_{l_i l_j}$.

Definition 4. *For a service chain* $P = [i_0, i_1, \cdots, i_{K-1}, i_K]$, *we call a critical flow pattern* $\{x_{i_k i_{k+1}} > 0, \forall\, k \in \mathscr{K}\backslash\{K\}, x_{i_{k'} i_0} > 0$, *for some* $k' \in \mathscr{K}\backslash\{0, K\}\}$ *a sustainable circulation associated with* P.

Proposition 1. *For a given facility design* $I^* \subset I$, *an optimal flow solution that minimizes the transportation cost can be decomposed into a set*

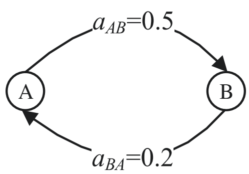


Fig. 3. Illustration of an unfeasible nexus.

of sustainable circulations.

Proof. We propose a decomposition algorithm (Decomp) that decomposes an optimal solution $\mathbf{x}: =\{x_{ij}\}_{i\in I^*, j\in I^*\cup U}$ to a set of sustainable circulations in a finite steps.

Decomp-0. Initialize residual flows $\bar{\mathbf{x}}: =\{\bar{x}_{ij}: =x_{ij}\}_{i\in I^*, j\in I^*\cup U}$ and circulation set $\mathscr{X} = \varnothing$.

Decomp-1. Select any $i_0 \in I$ with positive residual outflows to customers. Select the largest residual outflow from i_0 to a customer, say $i_1 \in U$, denoted by $\bar{x}_{i_0 i_1}$ (a tie can be arbitrarily broken in this algorithm). Initialize $P: =[i_0, i_1]$.

***Decomp*-2.** Denote all nodes that have positive outflows to i_0 by $I_{i_0}^+$. *Select* $j \in I_{i_0}^+$ with the largest $\bar{x}_{ji_0} a_{ji_0}$.

***Decomp*-3.** If $j \notin P$, update $i_{k+1} = i_k$, $\forall\, k \in \mathscr{K}$, $i_0 = j$ and go to Step Decomp-2. Otherwise if $j = i_{k'}$, $\exists\, i_{k'} \in P$, then due to optimality, we know P is sustainable with respect to k'. *If this does not hold, we can must be able to identify a critical flow pattern* $\{\hat{x}_{i_{k'} i_0} \in (0, \bar{x}_{i_{k'} i_0}], \hat{x}_{i_{k-1} i_k} \in (0, \bar{x}_{i_{k-1} i_k}], \forall\, 0 < k \leqslant k' \in \mathscr{K}, \hat{x}_{j i_{k''}} \in (0, \bar{x}_{j i_{k''}}]$, for some $j \in I$, $k'' \leqslant k' \in \mathscr{K}\}$. Then based on Lemma 1 (see the Appendix I), subtracting this critical flow pattern from $\mathbf{x}$ yields a feasible flow solution yet with less transportation cost due to reduced flow amounts. This however contradicts that solution x is optimal.

Since P is sustainable with respect to k', then we can select the maximal sustainable circulation in the following way. First solve

$$\hat{x}_{i_{k'} i_0} = \min\left\{\bar{x}_{i_{k'} i_0},\ \min_{k<k'\in\mathscr{K}} \frac{\bar{x}_{i_k i_{k+1}}}{a_{i_{k'} i_0} \prod_{k''=0}^{k-1} a_{i_{k''} i_{k''+1}}},\ \min_{k'\leqslant k<K\in\mathscr{K}} \frac{\bar{x}_{i_k i_{k+1}}}{(a_{i_{k'} i_0} \prod_{k''=0}^{k'-1} a_{i_{k''} i_{k''+1}} - 1) \prod_{k''=k'}^{k-1} a_{i_{k''} i_{k''+1}}}\right\}$$

Then reclusively solve

$$\hat{x}_{i_0 i_1} = a_{i_{k'} i_0} \hat{x}_{i_{k'} i_0}$$

$$\hat{x}_{i_k i_{k+1}} = a_{i_{k-1} i_k} \hat{x}_{i_{k-1} i_k},\ \forall\, k < k' \in \mathscr{K}$$

$$\hat{x}_{i_{k'} i_{k'+1}} = a_{i_{k'-1} i_{k'}} \hat{x}_{i_{k'-1} i_{k'}} - \hat{x}_{i_{k'} i_0}$$

$$\hat{x}_{i_k i_{k+1}} = a_{i_{k-1} i_k} \hat{x}_{i_{k-1} i_k},\ \forall\, k' < k < K \in \mathscr{K}$$

Denote this sustainable circulation by $\hat{\mathbf{x}}$.

Decomp-4. Add $\hat{\mathbf{x}}$ to $\mathscr{X}$. Update $\bar{x}_{ij} = \bar{x}_{ij} - \hat{x}_{ij}$, $\forall\, i \in I^*, j \in I^* \cup U$. If $\bar{\mathbf{x}}$ contains at least one positive flow going to a customer, go to Step Decomp-1. Otherwise, all customers can be served with circulations in $\mathscr{X}$, now $\bar{\mathbf{x}}$ shall not have any positive flow; otherwise, there will be flows just circulating among facilities without reaching to a customer, which contradicts the optimality condition. Then return $\mathscr{X}$ as the sustainable circulation decomposition of $\mathbf{x}$. This completes the proof.

In order to explain the proposition 1 clearly, we construct a simple example. This nexus includes two types of systems, indexed by A and B. The conversion factors are set as $a_{AB} = 4$ and $a_{BA} = 0.5$. System A has a built facility denoted by A1. System B contains two built facilities denoted by B1 and B2, respectively. The demands of two customers C1 and C2 are satisfied by this nexus. The optimal

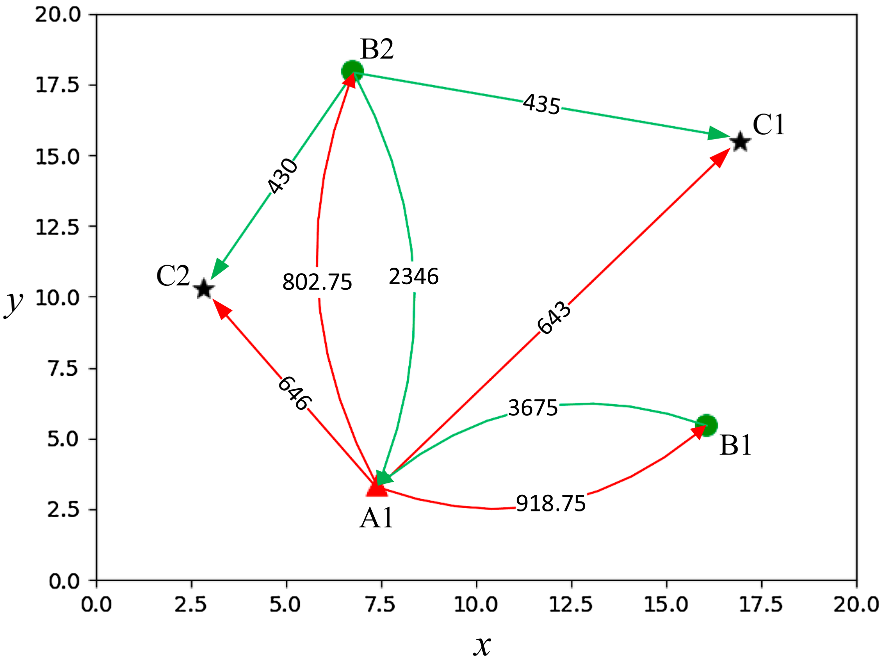


Fig. 4. Optimal solution to a simple nexus.

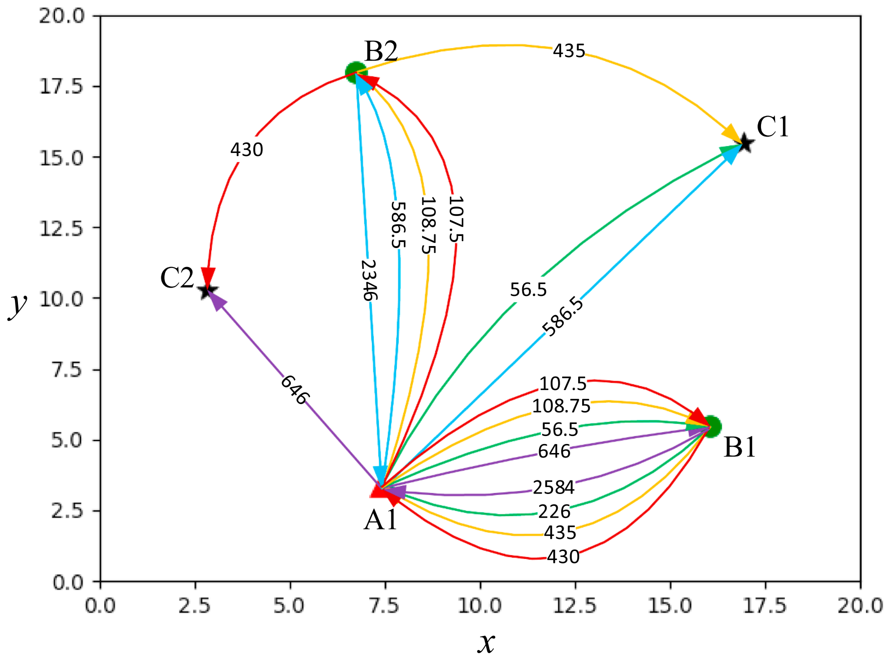


Fig. 5. Set of sustainable circulations to the optimal solution.

solution is shown in Fig. 4. The numbers in directed lines denote the flow assignments between locations. According to the decomposition algorithm, we can decompose the optimal solution into a set of sustainable circulations.

Before the first step, we initialize residual flows $\bar{\mathbf{x}}$: $=\{\bar{x}_{ij}: =x_{ij}\}_{i\in I^*, j\in I^*\cup U}$ and circulation set $\mathscr{X} = \varnothing$.

In the first step, we select facility A1 as i_0. Because the residual outflow from i_0 to customer C2 is the largest, we select customer C2 as i_1. Initialize P: =[A1,C2].

In the second step, we select facility B1 that provides the largest residual outflow to facility A1. Because facility B1 is not in P, we add it to the front of P and repeat this step. Then, we select facility A1 that provides the largest residual outflow to facility B1. Because facility A1 is in P, we stop this step and obtain P = [B1,A1,C2], which is sustainable with respect to A1.

In the third step, we calculate the outflows by the previous formulations for all facilities in P and obtain this sustainable circulation $\hat{\mathbf{x}}$, which is shown as purple lines in Fig. 5.

In the fourth step, we add $\hat{\mathbf{x}}$ to $\mathscr{X}$. Update $\bar{\mathbf{x}} = \bar{\mathbf{x}} - \hat{\mathbf{x}}$. Because $\bar{\mathbf{x}}$ contains positive flows going to customers, we repeat these steps and obtain the other sets of sustainable circulations that are shown in Fig. 5 with different colored lines. Each line denotes a sustainable circulation. At last, no positive flows go to customers in $\bar{\mathbf{x}}$ and the decomposition algorithm is finished.

According to the proposition 1, only sustainable facility circulations are deemed as feasible in a practical nexus. In a non-trivial nexus design, if one facility is not in any sustainable circulation, this facility actually cannot bring in any resource surplus to the system and thus shall not be used. Further, if the nexus does not include a sustainable circulation, this nexus cannot maintain sustainable resource flows to satisfy customer demands. Therefore, a non-trivial nexus design with interdependent systems has a feasible solution (the locations and the flows) only if there exists at least one sustainable circulation for each facility type.

3.3. Probabilistic capacity loss

However, even if a facility has sufficient capacity, it may not always produce the total assigned output flow due to possible facility disruptions. A facility, once built, may be disrupted from time-to-time due to uncertain external hazards. This study considers partial capacity losses that happen frequently in real world (Hatefi et al., 2015; Mohammadi et al., 2016). We assume that each facility i has a random capacity loss r_i that follows a probability density function $p(r_i)$ over the full range $[0, c_i]$ at first. When $r_i = 0$, facility i is completely functioning, the probability is equal to $(1 - q_i)$ where q_i is the probability that facility i is not completely functional. Otherwise, the probability density function $p(r_i)$ is a general function $f(r_i)$ across $r_i \in (0, c_i]$. Therefore, the corresponding probability density function is:

$$p(r_i) = \begin{cases} f(r_i),\ 0 < r_i \leqslant c_i \\ (1 - q_i)\delta(r_i),\ r_i = 0 \\ 0,\ r_i < 0 \text{ or } r_i > c_i \end{cases} \tag{3}$$

where $\delta(r_i)$ is the Dirac delta function (Hassani, 2008). We assume that the output flow is assigned to facility i before any disruption happens. When disruption r_i is realized, facility i can only output a flow no greater than residual capacity $c_i - r_i$. If this residual

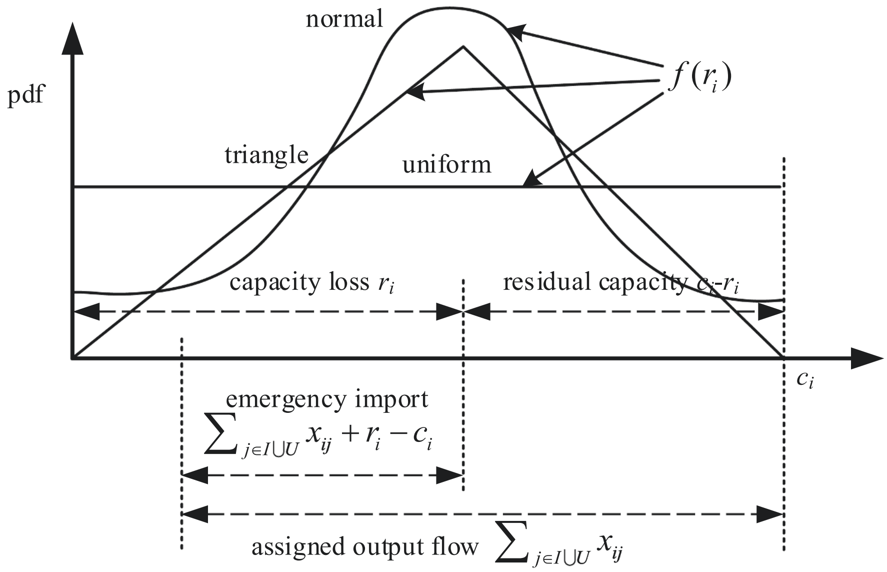


Fig. 6. Probabilistic continuous capacity loss.

capacity is less than the assigned output flow $\sum_{j\in I\cup U} x_{ij}$, then the difference $\sum_{j\in I\cup U} x_{ij} + r_i - c_i$ has to be made up by an expensive import from a perfectly reliable emergency source. Let π_i denote the cost rate for facility i to import one unit resource from the emergency source. This indicates that the emergency cost associated with this facility is $\pi_i\left(\sum_{j\in I\cup U} x_{ij} + r_i - c_i\right)$, which only happens when output flow $\sum_{j\in I\cup U} x_{ij}$ exceeds residual capacity $c_i - r_i$.

With these definitions, the expected emergency/penalty cost for can be expressed as:

$$C^P: = \sum_{i\in I}\int_{c_i-\sum_{j\in I\cup U} x_{ij}}^{c_i} \pi_i\left[\sum_{j\in I\cup U} x_{ij} + r_i - c_i\right]^{+} p(r_i)dr_i = \sum_{i\in I}\pi_i\left(q_i\sum_{j\in I\cup U} x_{ij} - \int_{c_i-\sum_{j\in I\cup U} x_{ij}}^{c_i} F(r_i)dr_i\right) \tag{4}$$

where the operator $[\cdot]^{+}: =\max\{\cdot,0\}$, $F(r_i): = \int_0^{r_i} f(r_i)dr_i$.

The function (4) is a general function and can be converted into several forms according to different probability density functions $f(r_i)$. Function $f(r_i)$ can be assumed to follow various distributions, such as the uniform distribution, triangular distribution and normal distribution, as illustrated in Fig. 6. We can imagine that the complexity of function (4) is determined by the distribution that $f(r_i)$ follows.

The first example is that $f(r_i)$ follows an adapted uniform distribution shown as

$$f(r_i) = \frac{q_i}{c_i} \tag{5}$$

It is arguably the simplest distribution of this paper's concern. It is easy to obtain the expected emergency cost function. With this, the expected emergency cost can be rewritten as:

$$C^P = \sum_{i\in I}\frac{\pi_i q_i}{2c_i}\left(\sum_{j\in I\cup U} x_{ij}\right)^2 \tag{6}$$

The second example is that $f(r_i)$ follows a triangular distribution, the general form is shown as

$$f(r_i) = \begin{cases} \frac{2q_i r_i}{c_i h}, for\ 0 < r_i < h \\ \frac{2q_i(c_i - r_i)}{c_i(c_i - h)}, for\ h \leqslant r_i \leqslant c_i \end{cases} \tag{7}$$

It is a piecewise function and more complex than the uniform distribution. The parameter d not only determines the shape of function (7), but also dictates the complexity of function (4). In order to reduce the complexity of function (4), we assume $d = 0$ to convert the piecewise function into a special adapted triangular distribution shown as

$$f(r_i) = \frac{2q_i(c_i - r_i)}{c_i^2} \tag{8}$$

The function (8) is a decreasing function that reflects a common phenomenon in facility operations; i.e., small capacity losses are often of high probabilities and large capacity losses of low probabilities. Therefore, we can get the emergency cost with an adapted triangular distribution shown as

$$C^P = \sum_{i\in I} \frac{\pi_i q_i}{3c_i^2}\left(\sum_{j\in I\cup U} x_{ij}\right)^3 \tag{9}$$

Another example is that $f(r_i)$ follows the normal distribution shown as

$$f(r_i) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(r_i-\mu)^2}{2\sigma^2}\right) \tag{10}$$

It is more complex than the triangular distribution. According to functions (6) and (9), we derive that the emergency cost is in the form of $\left(\sum_{j\in I\cup U} x_{ij}\right)^4$, which is hard to express in the commercial mathematics planning solver. Therefore, we will not elaboration on the model with the normal distribution in this study. Instead, we just focus on functions (6) and (9) in the next section to discuss features and applications of our proposed model.

3.4. Model formulation

The optimization problem is to determine where to site facilities among the candidate locations in all systems and how to assign flows between these facilities to minimize the total cost, including construction cost C^C(Eq. (1)), transportation cost C^T(Eq. (2)), and expected emergency cost C^P(Eq. (4)). Thus, the targeted location design problem can be described as a non-linear integer programming model as:

$$\min_{X,Y} \sum_{i\in I} f_i y_i + \sum_{l\in L}\beta_l \sum_{i\in I_l}\sum_{j\in I\cup U} d_{ij}x_{ij} + \sum_{i\in I}\pi_i\left(q_i\sum_{j\in I\cup U}x_{ij} - \int_{c_i-\sum_{j\in I\cup U}x_{ij}}^{c_i} F(r_i)dr_i\right) \tag{11}$$

Subject to

$$\sum_{i\in I_l} x_{iu} \geqslant \mu_{ul},\ \forall\ l\in L,\ u\in U \tag{12}$$

$$\sum_{j\in I\cup U} x_{ij} \leqslant \sum_{j\in I} a_{l_j l_i} x_{ji},\ \forall\ i\in I \tag{13}$$

$$\sum_{j\in I\cup U} x_{ij} \leqslant c_i y_i,\ \forall\ i\in I \tag{14}$$

$$y_i\in\{0,1\},\ \forall\ i\in I \tag{15}$$

$$x_{ij}\geqslant 0,\ \forall\ i\in I, j\in I\cup U \tag{16}$$

Constraints (12) require the end customer demands are met for resources from all types of systems. Constraints (13) are key constraints in modeling interdependency between different types of facilities. They indicate that facility i should obtain enough input from other facilities to generate enough output for other facilities and end customers. Constraints (14) mean that the combined transported output away from facility i for other facilities and end customers should be less than the expected capacity of facility i. Constraints (15) and (16) are binary and non-negative constraints for the decision variables, respectively.

In the proposed model, we can fix the values of the decision variables to test whether an instance has a feasible solution. As we known, if the feasible solution exists, the largest total output must be more than the total demand in the worst case. Therefore, we set all the candidate facilities built and no facility disrupted, namely $y_i = 1,\ \forall\ i\in I$ and $q_i = 0,\ \forall\ i\in I$. With this setting, the proposed model converts into a simple facility location model. If it yields a feasible solution by solving this simple model, we know that this instance has a feasible solution. Also, as Lemma 2 (see the Appendix I) states, if the instance has a feasible solution, after all facilities are built, we shall be able to find at least one sustainable circulation connected to each demand. This test process will screen out infeasible instances for the following numerical analysis.

4. Numerical examples

This section composes a number of numerical examples to test the computational performance, illustrate the model applications and draw managerial insights. This proposed model is a mixed-integer quadratic programming problem and can be solved by commercial programming solvers. A state-of-the-art mixed-integer programming solver, Gurobi (http://www.gurobi.com/), is adopted to solve these numerical examples. This solver can not only provides a solution but also indicates the optimality gap for this solution. If the optimality gap is 0, then this solution is the optimal solution. Otherwise, as long as this gap is small, this solution is deemed as a near optimum solution that yields an overall performance close to the true optimum. Because Gurobi cannot solve a cubical programming problem, we use the SCIP solver (http://scip.zib.de/) to solve the examples with facility disruptions followed the triangular distribution. Section 4.1 investigates a hypothetical numerical example with relatively simple settings to test how

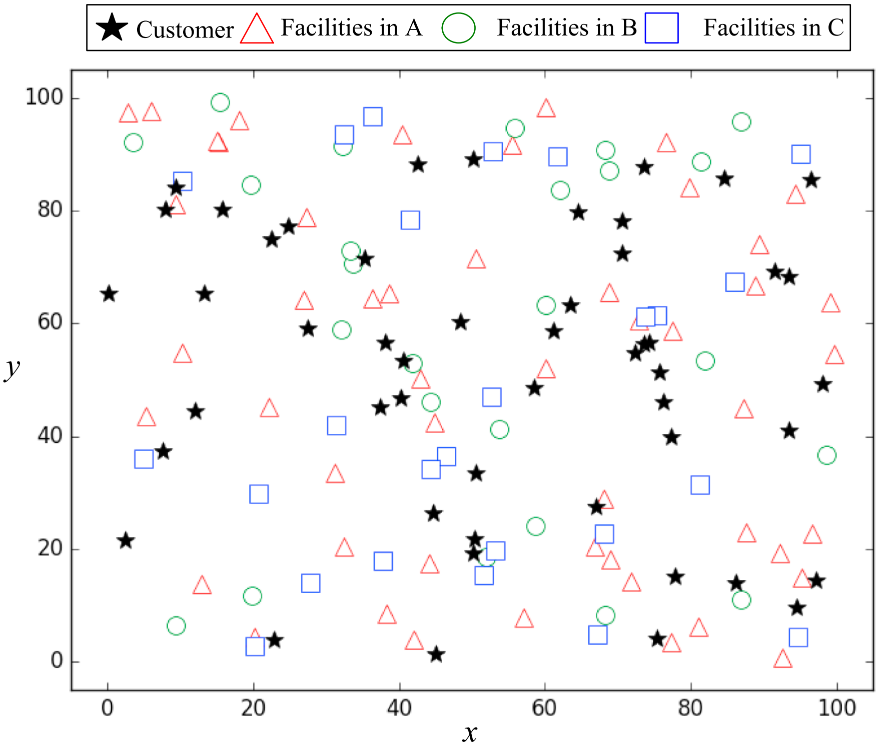


Fig. 7. All customers and candidate facility locations in the planning area.

parameters impacts cost components and facility distribution. Section 4.2 applies the proposed model to a case study based on real world data from the cities in the Louisiana State, US. This case study shows how the model can be applied to relevant facility location design problems in real world. All problem instances are solved on a PC with 3.6 GHz CPU and 16 GB RAM.

4.1. Hypothetical example

This example constructs a nexus with three types of systems, indexed by A, B and C, respectively. Systems A, B and C have $0.5\ |I|$, $0.25\ |I|$ and $0.25\ |I|$candidate facility locations, respectively. The default value of candidate location size $|I|$ is set to 100, while we may vary $|I|$ to construct problem instances of different sizes. These candidate facility locations are randomly distributed in a $[0,\ 100] \times [0,\ 100]$ area. The candidate facility locations are shown in Fig. 7 for the default instance. In this figure, the hollow shapes denote the candidate facilities. The red triangles denote the candidate facilities in system A, the green circles denote those in system B, and the blue squares denote those in system C. The fixed facility costs in systems A, B and C are set uniformly distributed between 2000 and 4000, between 8000 and 10000, and between 12,000 and 15000, respectively. The default facility capacities in systems A, B and C are set uniformly distributed between 800 and 1000, between 8000 and 10000, and between 3000 and 5000, respectively. All facility disruption probabilities q_i are set to 0.1. The unit emergency costs for resources A, B and C are set to 3, 4 and 5, respectively. The unit transportation costs for resources A, B and C are set to 0.2, 0.3 and 0.4, respectively. The default conversion factor values are shown in Table 2. A number of $|U|$ customers are randomly distributed in this $[0,\ 100] \times [0,\ 100]$ space. Customer size $|U|$ is set to 50 in the default instance and these customers are shown as the black stars in Fig. 7. The customer demands in systems A, B and C are set randomly between 50 and 150, between 400 and 500, and between 200 and 300, respectively. These parameter values are set as the benchmark values for the parts of sensitivity analysis. In order to vary the default parameter values (conversion factor and facility capacity), a conversion factor scalar a is introduced to multiply the default conversion factors between facilities and a capacity scalar

Table 2
Conversion factors between different systems.

From	To		
	A	B	C
A	1	0.9	0
B	10	1	6
C	5	0	1

Table 3
Performance comparison for different disruption patterns.

	Uniform distribution	Triangular distribution
Constructions cost	122,472	119,762
Transportation cost	145538.06	149117.30
Emergency cost	3553.47	1968.60
Total cost	271563.53	270847.90
Facility number	23	22
Solution time (s)	26 s	6289 s
Gap (%)	0.0037%	1.68%

c to multiply the default facility capacity in different systems. We set $a = 1$ and $c = 1$ in the default instance.

To discuss the performance of different capacity loss distribution (uniform and triangular), we conduct two simple instances with $|I| = 60$ and $|U| = 20$. The other parameters are set as the default values. Table 3 shows the results of comparison between the uniform and triangular distributions. We see that the cost components of the two distributions are very close. The relative difference is approximately 0.2%. However, the solution time of the triangular distribution instance is much longer than that of the uniform distribution instance. The gap of the triangular distribution instance is also greater than that of the uniform distribution instance. Therefore, for simplification, we will only investigate the uniform distribution in the following analysis.

In order to verify Proposition 1, we use the results of instances with the uniform distribution in Table 3. Through the proposed decomposition algorithm, we obtain the set of sustainable circulations shown in Table 4. It confirms that an optimal nexus flow design must be composed of sustainable circulations. Ideally, no residual flow exists among the facilities when the decomposition algorithm is finished. But due to the computational accuracy in the process of decomposition algorithm, we find a slight numerical error that residual flows with small values (less than 1) between facilities may exist. These results verify the proposition 1 although it has slight numerical errors. On the other hand, the decomposition algorithm also can be used to improve the current solutions by removing the residual flows in the end of decomposition algorithm.

Table 5 shows the computational performance of our proposed model in different instances. We set a solution time limit of 1000 s. We test 9 instances with $|I|$ to 200 and $|U|$ up to 120. Note that some instances have “INF” gaps, which means these instances do not have feasible solutions due to over stringent parameter settings. Some instances may not have a feasible solution when the customer number $|U|$ is too large in relation to $|I|$; i.e., there are not enough facilities to satisfy customer demand. In this table, we can see that all instances can be solved by Gurobi in a short solution time, and all feasible instances have small optimality gaps (less than 2%). Note that further increase of the solution time limit will further reduce the optimality gap. Overall, these tests indicate that the computational performance of our proposed model when solved by Gurobi is satisfactory for problem instances of realistic sizes.

Fig. 8 shows the optimal facility locations and the flows between facilities and customers with different conversion factor scales and different capacity scales. In these figures, the solid shapes denote the constructed facilities and the line color indicates the line’s start point. According to the conversion factors, there are two sustainable facility circulations in the hypothetical example: *A-B-C-A* and *A-B-A*. In the left figures (Fig. 8(a), (c) and (e)), we can see that these two circulations exist simultaneously in the intermediate flows. Note that interestingly, each facility in system B serves as a hub for a cluster of intermediate flows and facilities, while these clusters are often linked up by the facilities in system A. Comparing Fig. 8(a) with Fig. 8(c), we can see that a larger conversion factor scalar reduces the number of clusters. It means that the conversion factor influences the connection degree between facilities: high

Table 4
Set of sustainable circulations in a simple instance.

(P,k')	([1,38,54,3,6],54)	([54,7,38,29,17],38)	([6,44,6],44)	([29,38,54,16],38)
	([44,6,6],6)	([54,7,38,29,10],38)	([19,44,1],44)	([13,38,54,2],38)
	([4,45,10,20],45)	([54,1,38,29,19],38)	([6,44,19],44)	([1,38,54,1],54)
	([4,45,10,5],45)	([58,30,37,8],37)	([19,44,19],44)	([29,38,54,13],38)
	([30,37,58,11,7],58)	([58,2,37,12],37)	([4,45,9],45)	([13,38,54,11],38)
	([54,1,38,13,11],38)	([6,44,16,37,15],44)	([27,45,5],45)	([1,38,54,17],54)
	([54,1,38,13,16],38)	([58,11,37,4],37)	([21,45,20],45)	([29,38,54,17],38)
	([6,44,16,19],44)	([19,44,16,37,18],44)	([10,45,10],45)	([13,38,54,17],38)
	([6,44,19,1],44)	([58,2,37,7],37)	([4,45,14],45)	([30,37,58,18],58)
	([58,30,37,20,18],37)	([58,11,37,18],37)	([21,45,14],45)	([11,37,58,12],58)
	([58,30,37,20,15],37)	([58,30,37,19],37)	([27,45,14],45)	([30,37,58,15],58)
	([58,30,37,20,4],37)	([58,11,37,19],37)	([27,45,46,20],45)	([11,37,58,19],58)
	([58,2,37,20,8],37)	([54,1,38,16],38)	([27,45,46,5],45)	([30,37,58,8],58)
	([58,30,37,20,12],37)	([54,7,38,13],38)	([27,45,46,14],45)	([11,37,58,7],58)
	([58,2,37,20,19],37)	([54,3,38,3],38)	([27,45,46,10],45)	([30,37,58,7],37)
	([4,45,21,14],45)	([13,38,11],38)	([10,45,46,9],45)	([30,37,58,4],37)
	([4,45,27,9],45)	([54,7,38,17],38)	([27,45,46,4],45)	([30,37,58,8],37)
	([54,1,38,29,2],38)	([54,3,38,2],38)	([10,45,46,4],45)	
	([54,1,38,29,13],38)	([54,1,38,2],38)	([29,38,54,6],38)	
	([54,1,38,29,3],38)	([54,1,38,17],38)	([1,38,54,3],54)	

Table 5
Computational performance in different instances.

#	$\|I\|$	$\|U\|$	Solution time (s)	Gap (%)
1	60	20	26	0.0037
2	60	50	0.35	INF
3	60	120	1.15	INF
4	100	20	100	0.0077
5	100	50	124	0.0064
6	100	120	2.73	INF
7	200	20	1000	0.3459
8	200	50	1000	0.2859
9	200	120	1000	1.8328

conversion factor values mean low interdependency. Comparing Fig. 8(a) with Fig. 8(e), we can see that enlarging the capacity scalar also reduces the number of clusters. As we know, a large facility capacity means that a facility can output more resources. Therefore, a facility can obtain sufficient resources from fewer other facilities. Correspondingly, the number of clusters is reduced. In the right figures (Fig. 8(b), (d) and (f)), we find that a few facilities in system A do not serve any customers. These facilities in system A just play a connection role between facilities in system B and facilities in system C in order to offset the capacity loss of the other facilities in system A. Comparing Fig. 8(b) with Fig. 8(d), we can see that the number of connection facilities is reduced with increasing the conversion factor scalar. Comparing Fig. 8(b) with Fig. 8(f), we also find that the capacity scalar can reduce the number of connection facilities. Therefore, we can project that the number of connection facilities may approach zero with further increase of facility capacity or conversion factor.

Now, we conduct a sensitivity analysis to some key parameters, such as the conversion factor, disruption probability and capacity.

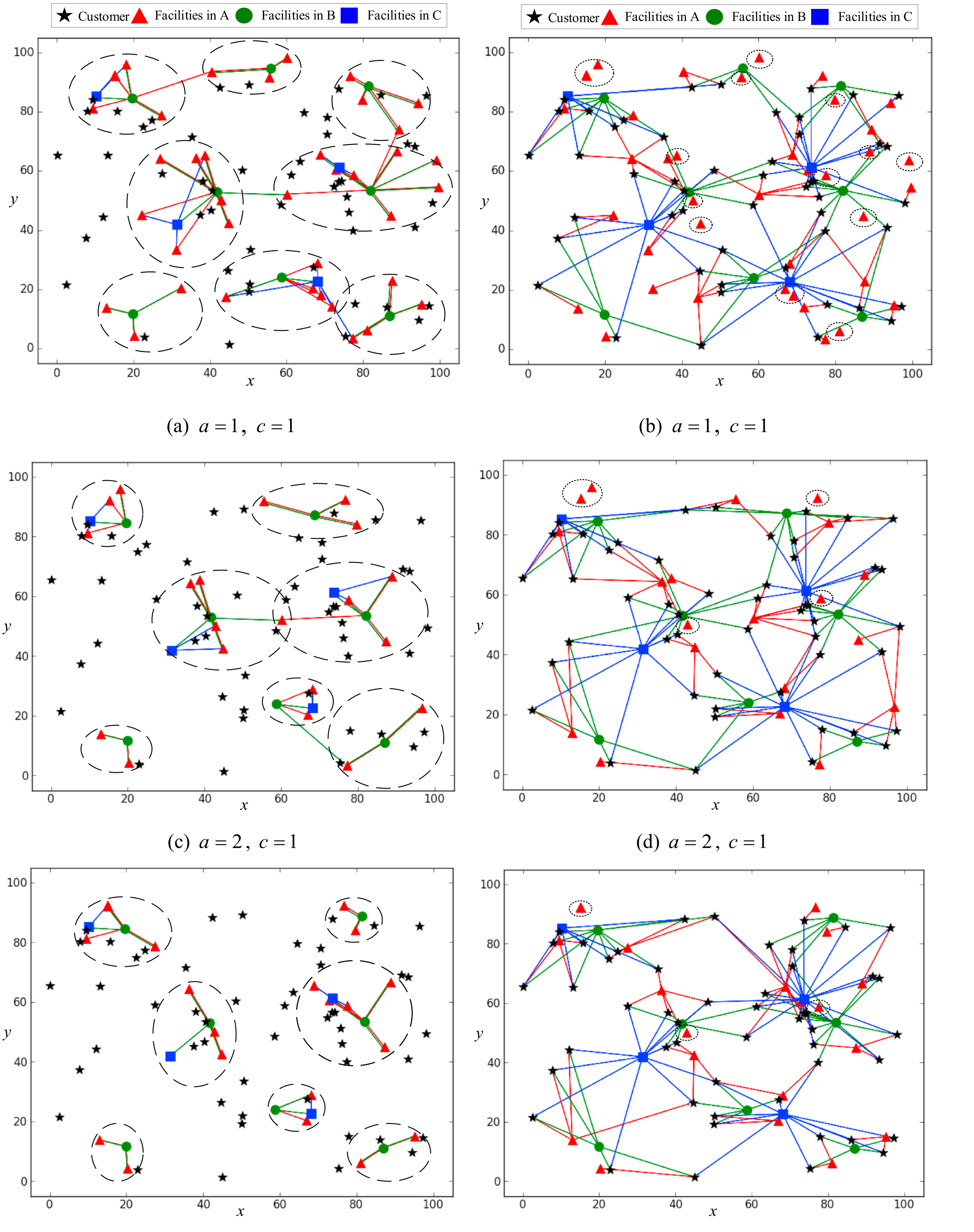
First, we analyze the conversion factor which reflects the production efficiency of a facility. Fig. 9 shows how the conversion factor scalar influences the total cost and the facility number. In this figure, we can see that the total cost and its three component costs are decreasing significantly at first with the increase of the conversion factor scalar. Then the decreasing trend becomes flattening out as the conversion factor scalar grows large. The change of the facility number also has the same trend with the increase of the conversion factor scalar. This trend indicates that improving the conversion factor can reduce the constructed facility number and the total cost, but the marginal effect decreases as the conversion factor grows. Note that as the conversion factor approaches infinity, a facility can produce its resource almost without much upstream resource input, and thus the problem reduces to one where facilities operate independently. The flat tails of Fig. 9 actually correspond to this asymptotic independent case. From these results we see that the conversion factor plays a key role in improving the nexus performance.

The disruption probability is another key parameter in our model. It reflects the risk with which the facilities must be faced. Let C_1, C_2 andC_4 denote the total costs for $a = 1$, 2 and 4, respectively. Fig. 10 shows how the disruption probability influences the total cost with different conversion factor scales in the nexus. As the disruption probability increases, the total cost C_1 is increasing gradually. Although C_2 and C_4 are not shown directly in this figure, we can speculate that these costs have the same trend as C_1. In this figure, we see that the difference $(C_1 - C_2)$ is increasing with the increase of disruption probability. It means that the increase rate of the total cost when $a = 1$ is larger than that when $a = 2$. A large conversion factor makes the nexus have a high resistance to facility disruptions. Therefore, when the conversion factor scalar is large, the effect of disruption probability on the nexus will drop off and the total cost will increase slowly. From Fig. 8, we know that large conversion factor scalar indicates low interdependency between facilities. That also means that a nexus with high interdependency between facilities is weak when it faces on the risk of facility disruptions.

Fig. 11 shows the sensitivity analysis results with respect to capacity scalar c. Again, C_1, C_2 andC_4 denote the total costs for $a = 1$, 2 and 4, respectively. It is obvious that the total cost decreases sharply with the increase of c at first and the decrease range is small when the capacity scalar c is large. This trend can be concluded from the third term of objective function (11). This tend indicates the utility of enlarging capacity is decreasing with the capacity improvement. In this figure, we see that the difference $(C_1 - C_2)$ is decreasing with the increase of capacity scalar. It means that the total cost when $a = 1$ is closed to that when $a = 2$. Therefore, the effect of enlarging capacity on the performance of this nexus will weaken when the conversion factor is large. Compared this figure with Fig. 9, we can see that the decrease degree of total cost in Fig. 9 is larger than that in Fig. 11. Therefore, the conversion factor is more important than the capacity on improving the nexus performance.

4.2. Case study with real world data

This section illustrates how to apply the proposed model to real-world problems by investigating a case study with real world data. The three major energy systems in this case study are refineries, power plants and vehicle stations. The fuel produced by refineries can be used to generate electricity at power stations. The fuel can also be used by HEVs at vehicle stations for transportation. Power plants generate electricity and will be consumed by refineries for production and by HEVs for transportation. HEVs are also capable of discharging electricity into power stations if needed. A link from a vehicle station to a power plant is considered as a simple transshipment link with a conversion factor of 1 since they produce the same type of resource. Note that there are multiple other systems in real world other than the three considered systems in this case study. In order to model the location problems of this paper, other systems are considered as already being built and their roles can be equivalently represented as fixed "local demand" in



(a) $a = 1, \; c = 1$ (b) $a = 1, \; c = 1$

(c) $a = 2, \; c = 1$ (d) $a = 2, \; c = 1$

(e) $a = 1, \; c = 2$ (f) $a = 1, \; c = 2$

Fig. 8. Optimal facility locations and flows with different a and c.

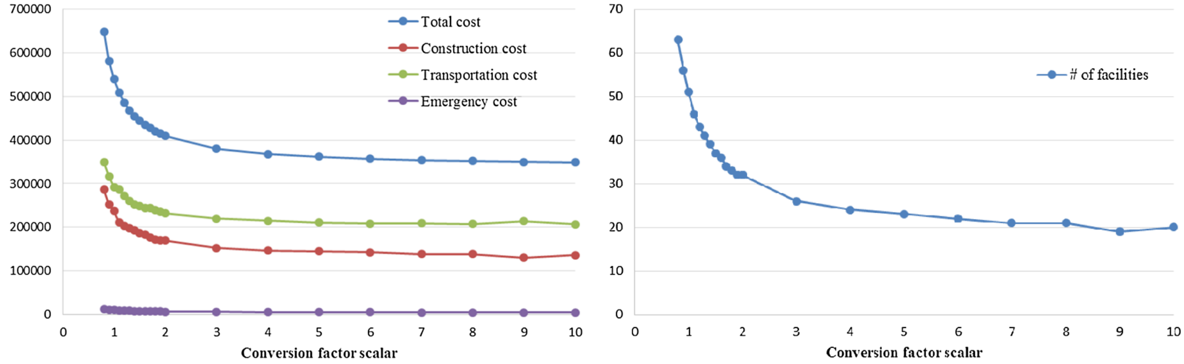


Fig. 9. Sensitivity analyses of conversion factor scalar.

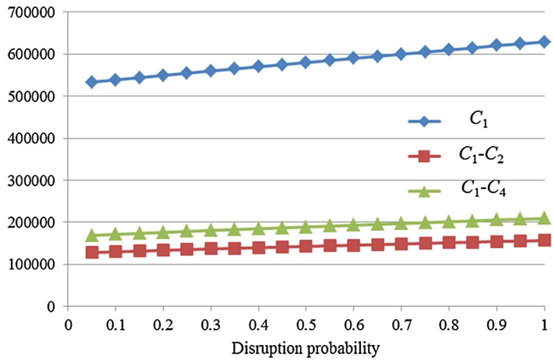


Fig. 10. Sensitivity analyses of disruption probability.

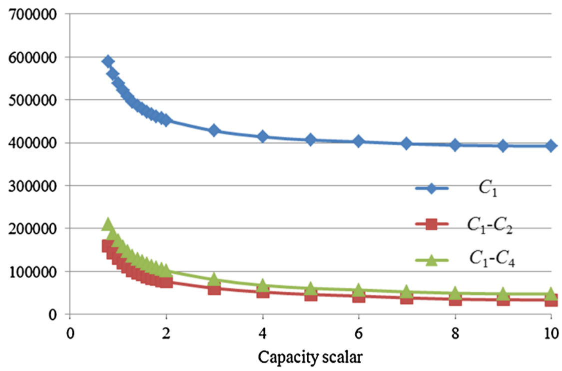


Fig. 11. Sensitivity analyses of capacity scalar.

this problem.

In the studied problem, we are given a set of pre-selected candidate locations for refineries, power plants, and potential counties/cities for vehicle stations, as well as attributes at these locations for various facilities, such as local demand and fixed costs. The model will decide the most cost-effective location design to minimize the total system cost caused by random facility disruptions.

The region of interest in this case study is Louisiana State. There are 111 pre-selected locations for three types of facilities. 18 of them are potential refineries, 29 of them are power plants and the remaining are pilot county locations for V2G demonstration program. Types 1, 2 and 3 indicate power plants, vehicle stations and fuel refineries, respectively. For type 1, the facility attributions are obtained from the eGRID2012 file[1]. The total annual net generation is treated as the facility capacity. We select the maximum capacity facility from each county whose primary fuel is gas, oil or biomass. We assume that the fixed cost of each facility is proportional to the facility capacity. For type 2, each facility is located in the each parish seat[2]. The corresponding facility capacity

[1] https://www.epa.gov/energy/emissions-generation-resource-integrated-database-egrid.
[2] https://en.wikipedia.org/wiki/Louisiana_statistical_areas.

and fixed cost are set proportional to the population of the parish. For types 1 and 2, the unit emergency cost and the unit transportation cost are set as 300 and 0.01, respectively. For type 3, the facility attributions are obtained from the Wikipedia website.[3] We assume that the facility fixed cost is 25,000,000, the unit emergency cost is 237.5 and the unit transportation cost is 1.25. For customers, the locations are the same as the type 2 facilities in the parish seats. The customer demand for each type resource is set proportional to the consumption of Louisiana State[4] and the population of each parish. All the parameters used in this paper for facilities and customers are shown in Appendix II. We also assume the following conversion rate in Table 6. The conversion factors from 1 to 3 and 3 to 1 are calculated according to the relationship between the consumption and generation.[5] The conversion factor from 3 to 2 is also set according to that the 6 kW hours is equal to a gallon of gasoline for HEVs.[6]

We solve this case study by our proposed model. The solution time is 141 sec and the optimality gap is 0.0092%. It indicates that the solver Gurobi has a good performance on our proposed model. Fig. 12 shows the population density of Louisiana State and the optimal facility location scheme. In this figure, we can see that most of optimal facility locations, especially the power plants and refineries, are concentrated around the area of a high population density, which to some extent reflects large demand. Building facilities in this area can satisfy these demands with relatively low unit cost. These results indicate that our model can solve the realistic problems and get a reasonable optimal facility location scheme.

Fig. 13 shows the intermediate flows between the facilities in different systems and the out flows to the customers. In Fig. 13(a), we can see that the intermediate flows exist in all different types of facilities and they form a number of sustainable facility circulations. In Fig. 13(b), all facilities provide resources to the customers and some customer demands are satisfied by two or more facilities similar to Fig. 8.

Fig. 14 shows how the facility location scheme changes with different q_i and α. And Table 7 shows the estimated cost of the real world synthetic example with different q_i and α values. In this figure, the sizes of markers are proportional the corresponding facility capacities. The proportional rates are different in different type systems. Comparing Fig. 14(a) with Fig. 14(b) (or Fig. 14(c) with Fig. 14(b)), we find that more facilities are constructed and a larger number of facilities with lower capacities are installed at a higher disruption risk. While more facilities mean a higher construction cost, they reduce the transportation cost and increase system reliability, which yields a good balance of the cost components in the disruption scenario despite the increase of the total cost. We also see that facilities are built in the area with high population density as explained in Fig. 12. Comparing Fig. 14(a) and Fig. 14(c), we find that similarly, a larger number of power plants with low capacities are constructed when α is higher. However the total cost is reduced as α increases. The same phenomenon is also observed when comparing Fig. 14(b) with Fig. 14(d). This indicates that a higher α (possibly as results from conversion technology innovations) leads to a leaner and more economic nexus with higher reliability.

Now, we study the effects of fortifying facilities on the facility layout. According to the results of Fig. 14(b), we set $q_1 = q_{32} = q_{99} = 0$ since they have the largest capacities among the built facilities in each type and are thus deemed as critical facilities. The other parameters are all identical to those in Fig. 14(b). Fig. 15 compares the facility layouts without and with facility fortification and Table 8 shows the corresponding estimated costs. We see that after fortifying the facilities with largest capacities, several facilities with higher capacities are now replaced by facilities with lower capacities, particularly for vehicle stations. This means that a large amount of demands are satisfied by the fortified facilities and the rest can be satisfied by facilities with lower capacities instead. Form Table 8, we see that fortifying the critical facilities reduces the construction cost and increases the number of built facilities. Further, the decrease of emergency cost also indicates improvement of system reliability. The transportation cost however increases, which may be because that the numbers of power plants and refineries decrease. In general, the total cost reduces due to facility fortification. Therefore, facility fortification, if feasible and not of too high cost, is a good approach to reduce system cost and improve system reliability.

Finally, we study the effects of disruption risk magnitudes on the facility layout. Because the area along the Mississippi River may face on the natural disaster, such as the flood in 2016, we set the disruption probabilities of facilities located in the lower-right-corner area (less than 30.9°N and more than −91.6°W) as 0.3. The disruption probabilities of other facilities are set as 0.1. The other parameters are all identical to those in Fig. 14(a). Fig. 16 compares the facility layouts with different disruption probabilities. From these figures, we can see that facilities are more likely to be built in the area with lower disruption probabilities, particularly for vehicle stations, to improve the resilience of the nexus. However, a number of facilities still are built in the lower-right-corner area, e.g. power plants and refineries. That is because building these facilities in this area can obtain more benefits although they may bear higher disruption risk that will cause the loss. In general, higher disruption risk will let the facilities away from this area. Nevertheless, if the loss derived from disrupted facilities is less than the benefits from built facilities, the facilities will still be built in the area with higher disruption probabilities.

5. Conclusions

This paper investigates the network location design problem for a nexus of interdependent heterogeneous infrastructure systems. This model considers probabilistic facility disruptions with a continuous capacity loss. Such disruptions can cause cascading failures

[3] https://en.wikipedia.org/wiki/List_of_oil_refineries#Louisiana.
[4] http://www.eia.gov/state/data.cfm?sid=LA#ConsumptionExpenditures.
[5] http://www.eia.gov/totalenergy/data/annual/index.php#electricity.
[6] http://gatewayev.org/how-much-electricity-is-used-refine-a-gallon-of-gasoline.

Table 6
Conversion factor between three facility types.

	To		
From	1	2	3
1	1	1	19.11
2	1	1	0
3	4.12	1.85	1

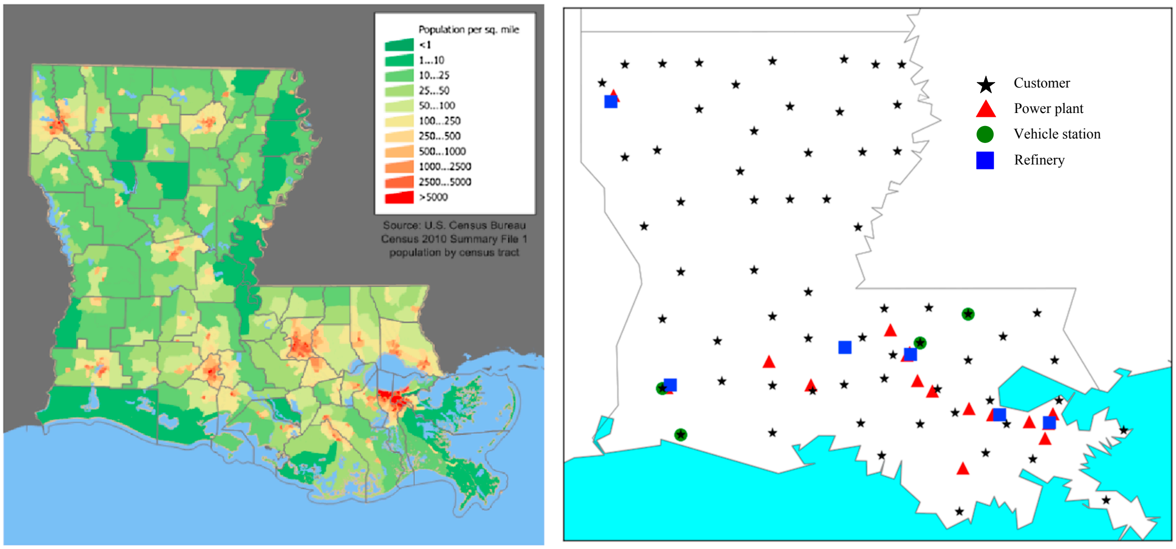
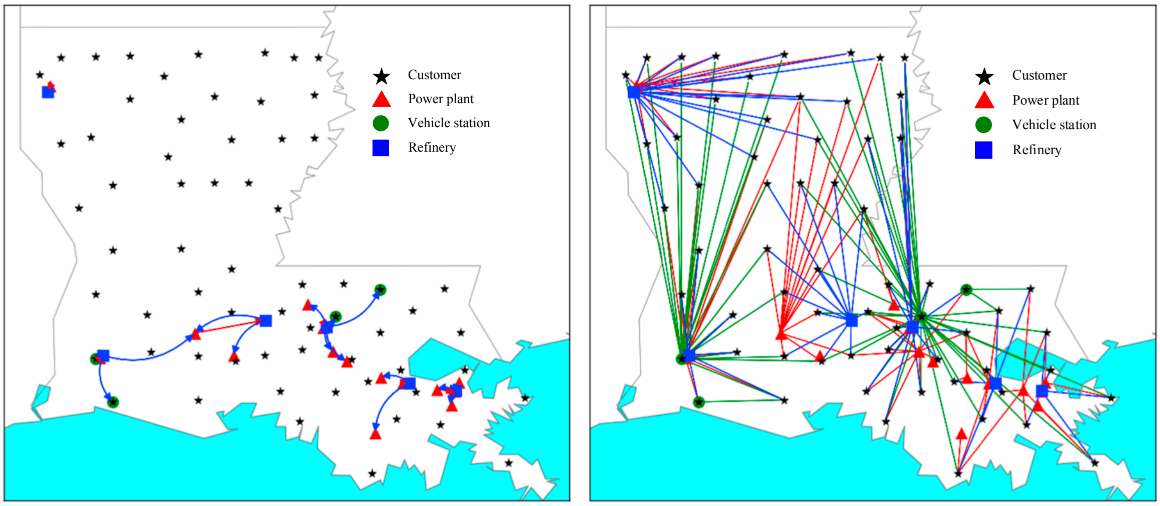


Fig. 12. Facility location scheme of real world example with $q = 0.1$.



(a) Intermediate flows between facilities (b) Output flows from facilities to customers

Fig. 13. Flows between facilities and customers.

due to interdependencies among different systems. We proposed a novel heterogeneous flow scheme that describes interdependencies between heterogeneous systems producing different resources in a parsimonious form. This allows the development of a compact integer programming model for solving the optimal facility location design for an infrastructure nexus. Numerical experiments are conducted to test the proposed model and to draw managerial insights. We find that with this formulation, relevant problem instances of realistic sizes can be efficiently solved with off-the-shelf solvers. The analysis results show that when the conversion factors are larger, the nexus includes fewer clusters, indicating less intersystem interdependency. When cross-system interdependency is higher,

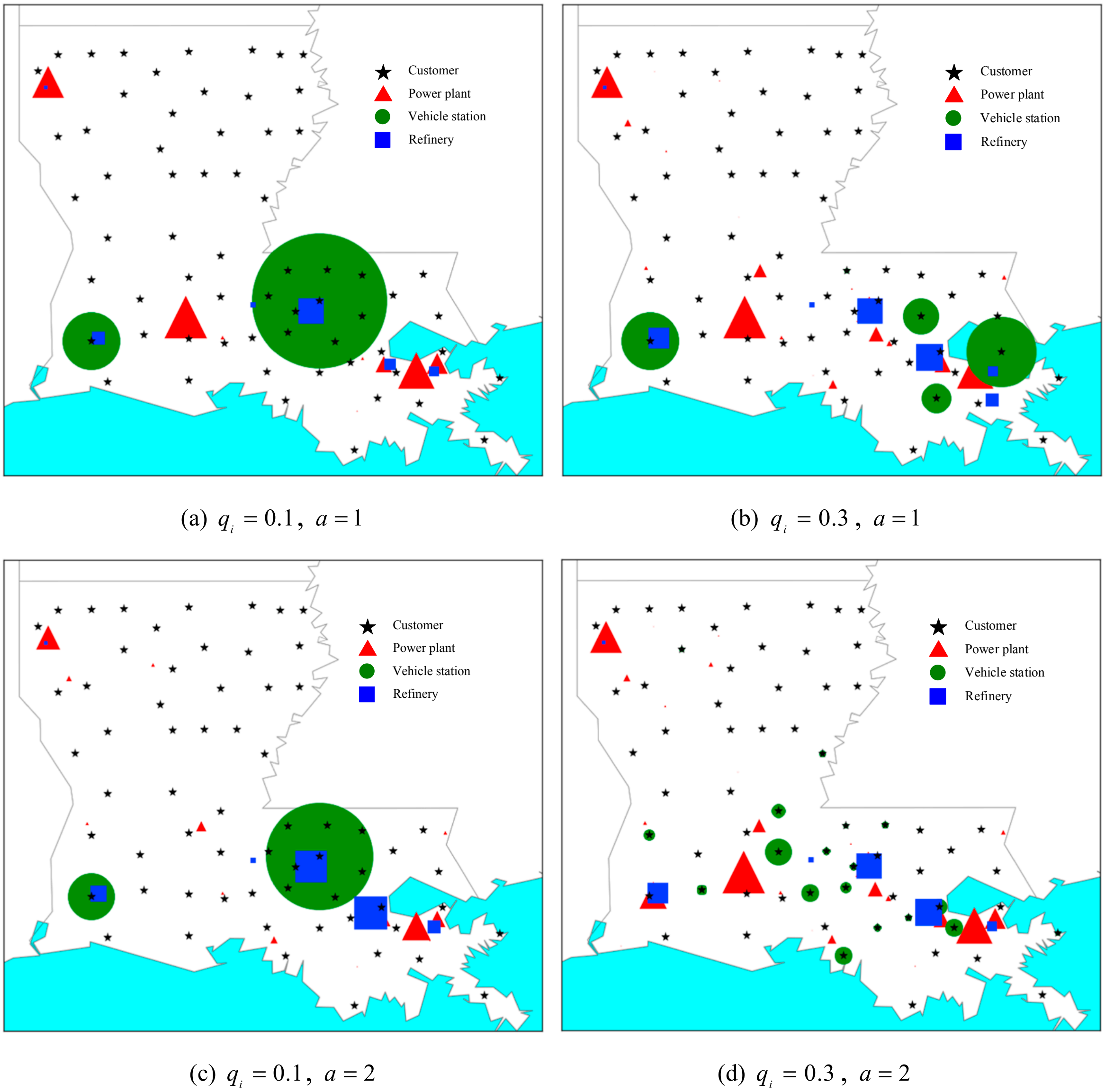


Fig. 14. Facility location schemes of the real world synthetic example with different q_i and a values.

Table 7
Estimated cost of the real world synthetic example with different q_i and a values.

q_i	a	Construction cost	Transportation cost	Emergency cost	Total cost	Facility number (Type 1,2,3)
0.1	1	7.61E8	4.40E8	4.45E8	1.65E9	(16, 4, 6)
0.3	1	8.69E8	4.75E8	1.13E9	2.48E9	(25, 5, 7)
0.1	2	7.53E8	3.78E8	4.13E8	1.54E9	(21, 4, 6)
0.3	2	8.54E8	4.10E8	1.04E9	2.30E9	(28, 21, 6)

the nexus becomes more vulnerable to disruptions and more facility investment is needed to offset the system vulnerability. We also find that the conversion factor is more efficient than the capacity in improving the performance of the nexus. The case study indicates that our proposed model can be applied to location design for real-world interdependent systems.

This study is a pioneering attempt to incorporate cascading disruptions in reliable location design. It can be extended in a number of directions. For example, we should consider other interdependency structures different from the studied additive linear heterogeneous flow scheme. We can also consider more general disruption patterns that include both parametric and non-parametric

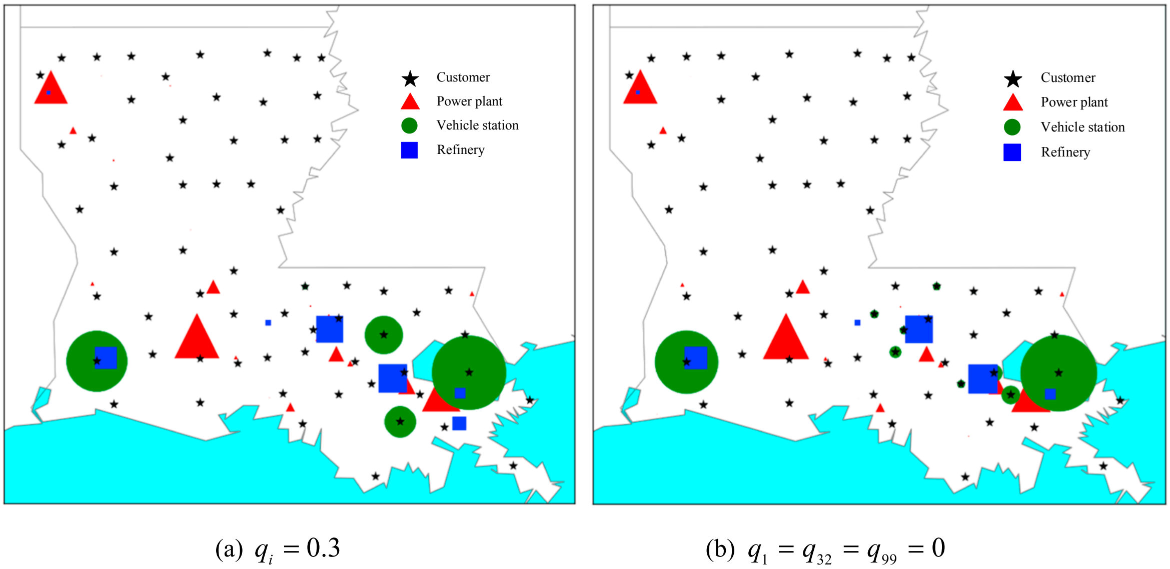


Fig. 15. Facility layouts of the real world synthetic example without and with fortification facilities.

Table 8
Estimated cost of the real world synthetic example without and with fortification facilities.

Disruption probability	Construction cost	Transportation cost	Emergency cost	Total cost	Facility number (Type 1,2,3)
$q_i = 0.3$	8.69E8	4.75E8	1.13E9	2.48E9	(25, 5, 7)
$q_1 = q_{32} = q_{99} = 0$	8.41E8	5.08E8	9.16E8	2.26E9	(24, 10, 6)

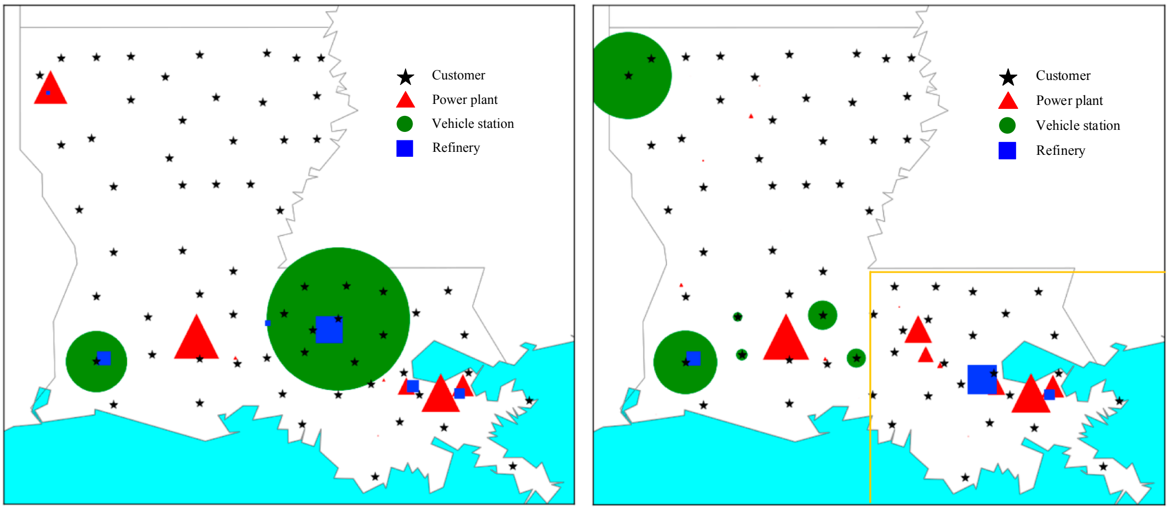


Fig. 16. Facility layouts of the real world synthetic example with different disruption risk magnitudes.

distributions to be calibrated from real world data. In addition to location planning, real time operations and controls of such interdependent systems against possible disruption risks and their propagation are worth investigation. Further, demand uncertainties may be incorporated into this framework, e.g., through stochastic programming.

Acknowledgment

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Appendix I

Lemma 1. *For a feasible flow solution* $\{x_{ij} \geqslant 0\}_{i\in I, j\in I\cup U}$ *and a critical flow pattern* $\{x'_{ij} \geqslant 0\}_{i,j\in I}$ (*where flows can only exist between facilities but not from facilities to customers*), *if* $\{x''_{ij}: = x_{ij} - x'_{ij} \geqslant 0\}_{i\in I, j\in I\cup U}$, *then* $\{x''_{ij}\}_{i\in I, j\in I\cup U}$ *is also a feasible flow solution.*

Proof. Because $\{x_{ij} \geqslant 0\}_{i\in I, j\in I\cup U}$ is a feasible solution and $\{x'_{ij} \geqslant 0\}_{i,j\in I}$ is a critical flow pattern, we obtain $\sum_{j\in I\cup U} x_{ij} \leqslant \sum_{j\in I} a_{l_j l_i} x_{ji}$, $\forall\, i \in I$, $\sum_{j\in I} x'_{ij} = \sum_{j\in I} a_{l_j l_i} x'_{ji}$, $\forall\, i \in I$ and $\{x'_{ij} = 0\}_{\forall\, i\in I, j\in U}$. Thus, for each $i \in I$, we know

$$\begin{aligned}\sum_{j\in I\cup U} x''_{ij} &= \sum_{j\in I\cup U} (x_{ij} - x'_{ij}) = \sum_{j\in I\cup U} x_{ij} - \sum_{j\in I\cup U} x'_{ij}\\ &= \sum_{j\in I\cup U} x_{ij} - \sum_{j\in I} x'_{ij} - \sum_{j\in U} x'_{ij}\\ &= \sum_{j\in I\cup U} x_{ij} - \sum_{j\in I} a_{l_j l_i} x'_{ji}\\ &\leqslant \sum_{j\in I} a_{l_j l_i} x_{ji} - \sum_{j\in I} a_{l_j l_i} x'_{ji}\\ &= \sum_{j\in I} a_{l_j l_i}(x_{ji} - x'_{ji})\\ &= \sum_{j\in I} a_{l_j l_i} x''_{ji}\end{aligned}$$

These inequalities mean that $\sum_{j\in I\cup U} x''_{ij} \leqslant \sum_{j\in I} a_{l_j l_i} x''_{ji}$, $\forall\, i \in I$. Due to $\{x'_{ij} = 0\}_{\forall\, i\in I, j\in U}$, for each $i \in I$, we obtain $\sum_{j\in U} x''_{ij} = \sum_{j\in U} (x_{ij} - x'_{ij}) = \sum_{j\in U} x_{ij} - \sum_{j\in U} x'_{ij} = \sum_{j\in U} x_{ij}$. These equalities mean that the outputs $\{x''_{ij}\}_{i\in I, j\in U}$ are unchanged. Therefore, $\{x''_{ij}\}_{i\in I, j\in I\cup U}$ is also a feasible flow solution if $\{x''_{ij} \geqslant 0\}_{i\in I, j\in I\cup U}$. This completes the proof.

Lemma 2. *If a service chain* $P = [i_0, i_1, \cdots, i_{K-1}, i_K]$ *has a sustainable circulation* $\{x_{i_k i_{k+1}} > 0,\ \forall\, k \in \mathscr{K},\ x_{i_{k'} i_0} > 0$ *for* some $k' \in \mathscr{K}\backslash\{0, K\}\}$, P *must sustainable with respect to* k'.

Proof. Based on the definition of a sustainable circulation, we obtain $x_{i_{k'-1} i_{k'}} a_{i_{k'-1} i_{k'}} = x_{i_{k'} i_{k'+1}} + x_{i_{k'} i_0} > x_{i_{k'} i_0}$. Further $x_{i_{k'-1} i_{k'}} = a_{i_{k'-2} i_{k'-1}} x_{i_{k'-2} i_{k'-1}} = \cdots = a_{i_{k'} i_0} \prod_{k=0}^{k'-2} a_{i_k i_{k+1}} x_{i_{k'} i_0}$. This yields $a_{i_{k'} i_0} \prod_{k=0}^{k'-1} a_{i_k i_{k+1}} > 1$. This completes the proof.

Appendix II

See Tables 9 and 10.

Table 9
Attributions of candidate facilities.

	Lat (N)	Lon (W)	Type	Fixed cost ($)	Capacity (MWh, ton)	Emergency ($/MWh)	Transportation ($/mile/MWh, $/mile/ton)
1	30.4286	92.4131	1	95,710,060	4,785,503	300	0.01
2	30.1875	90.9840	1	11,161,620	558,081	300	0.01
3	30.8606	93.3756	1	7049299.92	352464.996	300	0.01
4	32.5195	93.7601	1	71,059,640	3,552,982	300	0.01
5	30.2210	93.2992	1	60,915,980	3,045,799	300	0.01
6	29.7614	93.6086	1	218,834	10941.7	300	0.01
7	32.1575	93.5562	1	15522300.04	776115.002	300	0.01
8	30.4922	91.1864	1	58981340.18	2949067.009	300	0.01
9	30.8439	92.2611	1	28,696,840	1,434,842	300	0.01
10	29.9006	91.7333	1	25,000	1250	300	0.01
11	30.2743	91.1164	1	31,175,020	1,558,751	300	0.01
12	32.2752	92.7277	1	8653993.2	432699.66	300	0.01
13	29.9472	90.1458	1	83,574,020	4,178,701	300	0.01
14	30.2380	92.0463	1	6,854,160	342,708	300	0.01
15	32.5256	92.6497	1	900306.4	45015.32	300	0.01
16	31.9033	93.1739	1	2970111.86	148505.593	300	0.01
17	30.0081	89.9372	1	46,922,900	2,346,145	300	0.01
18	32.7061	92.0683	1	33,160,500	1,658,025	300	0.01
19	29.8120	90.0084	1	1,732,300	86,615	300	0.01
20	30.6739	91.3556	1	2,151,800	107,590	300	0.01
21	31.3207	92.4613	1	39,480	1974	300	0.01
22	29.9327	89.9782	1	1028384.7	51419.235	300	0.01
23	30.0051	90.4617	1	37744419.32	1887220.966	300	0.01
24	30.0540	90.6693	1	5,268,500	263,425	300	0.01

(continued on next page)

Table 9 (*continued*)

	Lat (N)	Lon (W)	Type	Fixed cost ($)	Capacity (MWh, ton)	Emergency ($/MWh)	Transportation ($/mile/MWh, $/mile/ton)
25	29.8222	91.5425	1	19,502,900	975,145	300	0.01
26	29.5806	90.7225	1	796,540	39,827	300	0.01
27	30.7811	89.8575	1	8698992.96	434949.648	300	0.01
28	32.6046	93.2944	1	55,500	2775	300	0.01
29	30.4770	91.2110	1	997581.2	49879.06	300	0.01
30	29.64987	90.11207	2	64,704,150	157446.765	300	0.01
31	30.43574	89.92532	2	34,590,750	84170.825	300	0.01
32	30.11269	89.88793	2	33,508,200	81536.62	300	0.01
33	29.92737	90.33719	2	7,914,150	19257.765	300	0.01
34	30.11183	90.48799	2	7,280,550	17716.005	300	0.01
35	29.324	89.47422	2	3,376,800	8216.88	300	0.01
36	29.87979	89.3227	2	2,327,100	5662.61	300	0.01
37	30.80393	90.07468	2	6,712,500	16333.75	300	0.01
38	30.56936	91.09694	2	64,360,950	156611.645	300	0.01
39	30.43401	90.6773	2	17,220,750	41903.825	300	0.01
40	30.20173	90.94385	2	14,600,250	35527.275	300	0.01
41	30.28992	91.40482	2	4,946,100	12035.51	300	0.01
42	30.61022	91.5984	2	3,397,200	8266.52	300	0.01
43	30.47512	91.32764	2	3,369,450	8198.995	300	0.01
44	30.84589	91.02032	2	3,138,300	7636.53	300	0.01
45	30.84231	91.40482	2	2,330,250	5670.275	300	0.01
46	30.80219	90.6773	2	1,613,850	3927.035	300	0.01
47	29.92325	91.09694	2	3,520,800	8567.28	300	0.01
48	30.18951	92.02732	2	36,014,700	87635.77	300	0.01
49	29.93067	91.6077	2	11,115,450	27047.595	300	0.01
50	30.22973	92.38136	2	9,386,550	22840.605	300	0.01
51	29.86014	92.38136	2	8,981,250	21854.375	300	0.01
52	30.23959	91.75388	2	8,075,250	19649.775	300	0.01
53	30.60365	92.06652	2	12,577,200	30604.52	300	0.01
54	29.67851	91.43028	2	7,921,500	19275.65	300	0.01
55	32.61368	93.86553	2	37,967,700	92388.07	300	0.01
56	32.75513	93.66232	2	16,090,500	39153.55	300	0.01
57	32.03106	93.66232	2	3,958,500	9632.35	300	0.01
58	32.76414	93.33889	2	6,195,150	15074.865	300	0.01
59	30.20893	93.33889	2	27,678,600	67351.26	300	0.01
60	29.84344	93.17797	2	1,168,800	2844.08	300	0.01
61	30.26723	92.81783	2	4,712,700	11467.57	300	0.01
62	29.22997	90.75328	2	16,402,200	39912.02	300	0.01
63	29.69523	90.52578	2	14,033,100	34147.21	300	0.01
64	32.42719	92.22367	2	22,388,850	54479.535	300	0.01
65	32.78591	92.38136	2	3,444,600	8381.86	300	0.01
66	32.79622	91.75388	2	4,464,150	10862.765	300	0.01
67	31.14611	92.5396	2	19,530,150	47523.365	300	0.01
68	31.69437	92.5396	2	2,981,850	7255.835	300	0.01
69	30.61945	90.37484	2	16,970,550	41295.005	300	0.01
70	31.132	93.17797	2	7,012,200	17063.02	300	0.01
71	30.76115	93.33889	2	5,269,500	12822.45	300	0.01
72	32.59889	92.69839	2	6,278,550	15277.805	300	0.01
73	32.23998	92.5396	2	2,280,300	5548.73	300	0.01
74	31.68012	93.17797	2	5,807,850	14132.435	300	0.01
75	31.4809	91.63722	2	2,919,000	7102.9	300	0.01
76	32.44016	91.2891	2	1,849,200	4499.72	300	0.01
77	30.97118	92.06652	2	6,399,450	15571.995	300	0.01
78	30.78218	92.38136	2	5,386,650	13107.515	300	0.01
79	30.58875	92.85771	2	3,817,050	9288.155	300	0.01
80	31.48933	93.50035	2	3,590,100	8735.91	300	0.01
81	30.01793	90.79132	2	3,258,150	7928.165	300	0.01
82	32.38873	91.79284	2	3,083,100	7502.21	300	0.01
83	32.07371	91.5984	2	3,068,250	7466.075	300	0.01
84	32.77226	93.01757	2	2,431,500	5916.65	300	0.01
85	31.91961	92.65864	2	2,375,250	5779.775	300	0.01
86	32.41074	93.01757	2	2,275,200	5536.32	300	0.01
87	31.70025	92.22367	2	2,113,950	5143.945	300	0.01
88	32.75458	91.48215	2	1,759,800	4282.18	300	0.01
89	32.06697	92.06652	2	1,592,250	3874.475	300	0.01
90	31.70531	91.90992	2	1,585,050	3856.955	300	0.01
91	32.08429	93.37921	2	1,415,700	3444.87	300	0.01
92	32.75709	91.2506	2	1,304,850	3175.135	300	0.01
93	32.07722	91.2891	2	920,700	2240.37	300	0.01

(*continued on next page*)

Table 9 (*continued*)

	Lat (N)	Lon (W)	Type	Fixed cost ($)	Capacity (MWh, ton)	Emergency ($/MWh)	Transportation ($/mile/MWh, $/mile/ton)
94	29.68167	89.97389	3	25,000,000	11120054.57	237.5	1.25
95	30.48361	91.18056	3	25,000,000	22622783.08	237.5	1.25
96	29.93866	89.97007	3	25,000,000	8508867.667	237.5	1.25
97	30.10769	90.90943	3	25,000,000	10,579,809	237.5	1.25
98	32.79733	93.4149	3	25,000,000	586166.4393	237.5	1.25
99	30.06568	90.59366	3	25,000,000	24266030.01	237.5	1.25
100	30.53903	91.75126	3	25,000,000	3601637.108	237.5	1.25
101	30.13463	93.31824	3	25,000,000	3601637.108	237.5	1.25
102	30.24215	93.25071	3	25,000,000	19259754.43	237.5	1.25
103	30.28615	93.14715	3	25,000,000	6753069.577	237.5	1.25
104	30.24099	93.27144	3	25,000,000	11705320.6	237.5	1.25
105	29.93709	89.94281	3	25,000,000	5627557.981	237.5	1.25
106	30.00159	90.40039	3	25,000,000	10714870.4	237.5	1.25
107	30.47666	91.21086	3	25,000,000	3376534.789	237.5	1.25
108	32.58921	93.5151	3	25,000,000	373669.8499	237.5	1.25
109	32.47037	93.78969	3	25,000,000	2566166.439	237.5	1.25
110	29.98636	90.39246	3	25,000,000	9679399.727	237.5	1.25
111	29.94557	90.3288	3	25,000,000	2025920.873	237.5	1.25

Table 10
Attributions of customers.

	Lat (N)	Lon (W)	Type 1	Type 2	Type 3
1	29.64987	90.11207	2,505,381	22964.83	607362.8
2	30.43574	89.92532	1,339,373	12276.97	324695.3
3	30.11269	89.88793	1,297,456	11892.75	314533.7
4	29.92737	90.33719	306440.3	2808.894	74288.28
5	30.11183	90.48799	281,907	2584.016	68340.83
6	29.324	89.47422	130751.6	1198.496	31697.24
7	29.87979	89.3227	90106.62	825.9355	21843.95
8	30.80393	90.07468	259911.8	2382.404	63008.68
9	30.56936	91.09694	2,492,092	22843.02	604141.3
10	30.43401	90.6773	666797.1	6111.997	161647.2
11	30.20173	90.94385	565329.9	5181.928	137049.1
12	30.28992	91.40482	191515.8	1755.472	46427.89
13	30.61022	91.5984	131541.5	1205.736	31888.73
14	30.47512	91.32764	130,467	1195.887	31628.24
15	30.84589	91.02032	121516.7	1113.847	29458.49
16	30.84231	91.40482	90228.59	827.0535	21873.51
17	30.80219	90.6773	62489.18	572.7885	15148.83
18	29.92325	91.09694	136327.4	1249.604	33048.93
19	30.18951	92.02732	1,394,509	12782.36	338061.6
20	29.93067	91.6077	430396.5	3945.101	104338.1
21	30.22973	92.38136	363452.5	3331.479	88109.36
22	29.86014	92.38136	347759.1	3187.63	84304.9
23	30.23959	91.75388	312678.2	2866.072	75800.49
24	30.60365	92.06652	486996.3	4463.906	118059.3
25	29.67851	91.43028	306724.9	2811.503	74357.28
26	32.61368	93.86553	1,470,131	13475.52	356,394
27	32.75513	93.66232	623033.2	5710.848	151037.8
28	32.03106	93.66232	153275.4	1404.953	37157.52
29	32.76414	93.33889	239879.7	2198.786	58152.43
30	30.20893	93.33889	1,071,731	9823.703	259812.6
31	29.84344	93.17797	45256.59	414.8311	10971.25
32	30.26723	92.81783	182478.4	1672.634	44237.02
33	29.22997	90.75328	635102.4	5821.477	153963.6
34	29.69523	90.52578	543369.5	4980.635	131725.4
35	32.42719	92.22367	866908.9	7946.262	210158.9
36	32.78591	92.38136	133376.9	1222.559	32333.66
37	32.79622	91.75388	172854.4	1584.418	41903.94
38	31.14611	92.5396	756218.4	6931.651	183,325
39	31.69437	92.5396	115458.9	1058.32	27989.93
40	30.61945	90.37484	657109.3	6023.196	159298.6
41	31.132	93.17797	271516.3	2488.774	65821.89

(*continued on next page*)

Table 10 (*continued*)

	Lat (N)	Lon (W)	Type 1	Type 2	Type 3
42	30.76115	93.33889	204,038	1870.254	49463.57
43	32.59889	92.69839	243,109	2228.386	58935.29
44	32.23998	92.5396	88294.5	809.3252	21404.65
45	31.68012	93.17797	224883.2	2061.325	54516.94
46	31.4809	91.63722	113025.3	1036.013	27399.97
47	32.44016	91.2891	71602.07	656.319	17358.01
48	30.97118	92.06652	247790.3	2271.296	60070.15
49	30.78218	92.38136	208574.1	1911.833	50563.23
50	30.58875	92.85771	147798.3	1354.749	35829.76
51	31.48933	93.50035	139010.7	1274.2	33699.43
52	30.01793	90.79132	126157.4	1156.384	30583.5
53	32.38873	91.79284	119379.4	1094.255	28940.34
54	32.07371	91.5984	118804.4	1088.985	28800.95
55	32.77226	93.01757	94149.05	862.9892	22823.92
56	31.91961	92.65864	91971.02	843.0249	22295.92
57	32.41074	93.01757	88097.03	807.5151	21356.77
58	31.70025	92.22367	81853.34	750.2842	19843.16
59	32.75458	91.48215	68140.45	624.5891	16518.83
60	32.06697	92.06652	61652.82	565.1222	14946.08
61	31.70531	91.90992	61374.03	562.5667	14878.5
62	32.08429	93.37921	54816.7	502.461	13288.85
63	32.75709	91.2506	50524.53	463.118	12248.32
64	32.07722	91.2891	35650.02	326.7753	8642.397

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