Transportation Research Part E 95 (2016) 143–163

Contents lists available at ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

Reliable design of an integrated supply chain with expedited shipments under disruption risks

Jianxun Cui [a], Meng Zhao [a], Xiaopeng Li [b,*], Mohsen Parsafard [b], Shi An [a,*]

[a] School of Transportation Science and Engineering, Harbin Institute of Technology, Harbin 150090, China
[b] Department of Civil and Environmental Engineering, University of South Florida, FL 33620, USA

ARTICLE INFO

Article history:
Received 25 November 2015
Received in revised form 2 September 2016
Accepted 21 September 2016
Available online 11 October 2016

Keywords:
Supply chain design
Transportation disruptions
Expedited shipment
Inventory management
Lagrangian relaxation

ABSTRACT

This paper studied the design of a two-echelon supply chain where a set of suppliers serve a set of terminals that receive uncertain customer demands. In particular, we considered probabilistic transportation disruptions that may halt product supply from certain suppliers. We formulated this problem into an integer nonlinear program to determine the optimal system design that minimizes the expected total cost. A customized solution algorithm based on Lagrangian relaxation was developed to efficiently solve this model. Several numerical examples were conducted to test the proposed model and draw managerial insights into how the key parameters affect the optimal system design.

1. Introduction

Supply chain operations are susceptible to various uncertainties such as supplier disruptions, transportation disruptions or delays, and customer demand fluctuations. As evidenced in recent catastrophic events (e.g., West Coast Lockdown (Gibson Brian et al., 2015), Sichuan Earthquake (Chan, 2008), Fukushima nuclear leak (Holt et al., 2012), and Hurricane Sandy (Blake et al., 2013)), supply chain facilities are vulnerable to various natural and anthropogenic disruption risks such as floods, earthquakes, power outages, and labor actions. Such disruptions, once happening, can choke the supply of corresponding commodities (or services) at the very source. Even if the commodities are successfully sent out from the supply facilities, they may experience extensive transportation delays, especially when they are shipped with slow transportation modes (e.g., waterways and railroads (Tseng et al., 2005; Ouyang and Li, 2010)). Such transportation delay may cause depletion of downstream stocks and loss of customer demand, particularly when customer demand is stochastic and fluctuating. To ensure customer service levels, one way is to hold a high inventory of commodities at the downstream terminals (or retailer stores), which however incurs excessive inventory holding cost. Or expedited transportation can be used to largely reduce the delivery time to avoid accumulation of unmet demand, which however may dramatically increase transportation cost. Li (2013) showed that a better way would be wisely combining inventory management and expedited transportation such that neither a high inventory nor frequent expedited services are needed. This series of uncertainties throughout these interdependent planning and operational stages, if not properly managed, may seriously damage system performance and deteriorate customer satisfaction. An integrated design methodology is needed to plan an efficient and reliable supply chain system that not only smartly balances cost tradeoffs over space and time but also robustly hedges against the unexpected uncertainties from supply, inventory, and demand.

* Corresponding authors.
E-mail addresses: xiaopengli@usf.edu (X. Li), anshi@hit.edu.cn (S. An).

http://dx.doi.org/10.1016/j.tre.2016.09.009
1366-5545/

There have been a number of studies addressing different facets of supply chain design. On the supplier location side, one recently intensively investigated topic is reliable supplier location design. Studies on this topic basically aim to increase the expected performance of a supply chain system across various supplier disruption scenarios by adding proper redundancy to the location design. On the operational side, freight lead time uncertainties and customer demand fluctuations have been well recognized as major challenges to inventory management and customer service quality. A recent study by Li (2013) proposed an integrated methodological framework that incorporated both planning and daily operations under a stochastic setting. This framework also enhanced the reliability of the supply chain system by taking the expedited transportation service into account, where a set of terminals ordered products from a set of suppliers based on the uncertain demands and inventory levels. The mathematical model proposed in this paper mainly minimized the fixed investment involving the cost of setting up service relationships with selected suppliers, the inventory holding cost at terminals, and the operational cost involving regular and expedited shipment cost from suppliers to terminals. It provided an integrated planning paradigm that balanced all the involved decision components and yielded a more reliable logistic system.

This paper aims to bridge this research gap by proposing an integrated supply chain system design model that simultaneously determines supplier location, multi-modal transportation configuration, and inventory management decisions all together under both transportation disruption risks and operational uncertainties. This model considers a two-echelon supply chain system where a set of downstream terminals order products from a subset of candidate upstream suppliers per arriving customer demands. Each shipment from a supplier to a terminal can be delivered via either a regular transportation mode that is cheap yet has a long and uncertain lead time or an expedited transportation mode that is much more expensive yet assures timely delivery. The adoption of expedited services also affects a terminal's inventory position and the corresponding inventory holding cost. Note that the transportation disruptions mentioned in our study refer to the disasters that disrupt the regular shipment service of suppliers. Then the terminals that used to be served by this supplier have to divert to other suppliers or completely lose the service. To assure the regular service reliability, a terminal may be assigned to a sequence of suppliers such that if regular services of some of them are disrupted, the terminal can resort to the remaining according to the assignment priorities. The system design of this problem is very challenging. Not to mention the inherited NP-hardness of a location problem, the system has to face an extremely large number of possible disruption scenarios of regular shipment service that are exponential to the number of the suppliers. Further, the nested uncertainties from transportation delays and customer arrivals will complicate this problem even more. With our efforts, a compact polynomial-size mathematical programming model is proposed that integrates all these decisions components, including supplier location selections, supplier assignments to terminals, expedited transportation activation rules and inventory holding positions, so as to minimize the expected system cost from both location planning and operations under various uncertainties. The compact structure of this model formulation allows the development of an efficient Lagrangian relaxation algorithm that can efficiently solve this problem to a near-optimum solution. Numerical examples show that the proposed model can yield a supply chain system design that minimizes the impacts from probabilistic disruptions and also leverages expedited shipments and inventory management to balance tradeoffs between transportation and inventory costs.

The rest of this paper is organized as follows. Section 2 reviews related literature. Section 3 formulates the design of the studied supply chain system into an integer nonlinear programing model. Section 4 develops a customized solution algorithm based on Lagrangian relaxation. Section 5 conducts numerical studies and discusses the experiment results. Section 6 concludes this paper and briefly discusses future research directions.

2. Literature review

Studies on facility location can be traced back to about a century ago (Weber, 1929). Earlier location models focused on the single tradeoff between one-time facility investment and day-to-day transportation cost (see Daskin (1995) and Drezner (1995) for a review on these developments). These fundamental models were later extended in a number of directions that largely enriched the contents of facility location models. Spatially, the fundamental two-layer supply structures were extended to multi-layer (or multi-echelon) topologies (Şahin and Süral, 2007). Temporally, single-period stationary operations were generalized to multi-period dynamic operations (Melo et al., 2006). The system service was extended from a single commodity to multiple commodities that share the supply chain infrastructure (Klose and Drexl, 2005). Direct transportation was extended to less-than-truck-load operations that involve vehicle routing decisions (Laporte, 1987; Salhi and Petch, 2007). Most of these models assume that all components of the supply chain system behave deterministically and their actions are fully predictable.

In reality, however, uncertainties exist almost ubiquitously throughout all components in a supply chain. Studies in 1980s (Daskin, 1982, 1983; ReVelle and Hogan, 1989; Batta et al., 1989) pointed out the need for facility redundancy under stochastic demand. Later studies (Lee et al., 1997; Ouyang and Daganzo, 2006; Ouyang and Li, 2010) further recognized that demand uncertainties cause serious challenges to inventory management when transportation takes long and uncertain lead times. Ouyang and Li (2010) analyzed the bullwhip effect for general supply chains system that consists of general network topology, general linear ordering policies and various customer demands. To test the existence of the bullwhip effect, a robust formulation was proposed without the knowledge of demand process. Moreover, another formulation that characterized order streams with certain customer demand processes was also presented, which strengthened findings in Ouyang and Daganzo (2006). These formulations offered logistics planners a series of robust ordering policies and operational strategies to mitigate the bullwhip effect of uncertain demands. To address this problem, facility location design had been integrated into inventory management to balance the tradeoff between spatial inventory distribution and transportation (Daskin et al., 2002; Shen et al., 2003; Shu et al., 2005; Shen and Qi, 2007; Snyder et al., 2007; Qi et al., 2010; Chen et al., 2011). Typically, to solve the stochastic transportation-inventory network design problem, Shen et al. (2003) built a nonlinear set-covering integer-programming model that formulated the distribution centers location and retailers allocation problem. Moreover, since involving a nonlinear part in the retailers allocation terms, the set-covering model was restructured and solved as part of the column generation algorithm. Chen et al. (2011) proposed a reliable joint inventory-location problem to simultaneously optimize facility locations, customer allocations, and inventory management decisions under facility disruption risks. Then the problem was formulated as a nonlinear mixed-integer model that incorporates inventory costs and a more general customer assignment scheme into the reliable facility location design framework. To solve the model efficiently, a Lagrangian Relaxation based customized algorithm was developed to obtain a near optimum solution. Peng et al. (2014) proposed a system dynamics model to describe the disaster relief supply chain by simulating the uncertainties associated with predicting post-seismic road network and delayed information. To evaluate the impact of the environmental factors and the effect of the response decisions, they also defined and tested supplies' replenishment solutions combined with three inventory planning strategies and four forecasting methods under different lead time uncertainties. After analyzing the simulation results, a decision tree was formulated to assist the decision-makers to choose the stocking strategies based on quantified risks after a disaster strikes.

In addition to investigate inventory management and transportation, using faster transportation can alleviate the need for keeping high inventory, and thus expedited shipments can be adopted in the supply chain system to improve the overall system performance (Taghaboni-Dutta, 2003; Huggins and Olsen, 2003; Caggiano et al., 2006; Zhou and Chao, 2010; Li, 2013; Qi and Lee, 2014). Li (2013) proposed a compact modeling framework that integrated location planning, inventory management, and expedited shipment configuration to mitigate the impacts from uncertain transportation and stochastic customer demands on the long-term supply chain performance. By studying the problem structure, a series of mathematical models were proposed to minimize the system cost. To solve the nonlinear terms that formulated the total transportation cost for both regular and expedited shipments, a customized solution approach based on Lagrangian relaxation was designed to efficiently solve a near-optimum solution.

Another major source of uncertainties in supply chain operations is unexpected facility disruptions, which was however largely overlooked in the facility location design literature in the last century. However, the catastrophic disasters in the recent years resumed the recognition of the need for siting redundant facilities to hedge against disruption risks, and a number of modeling methods have been introduced for reliable location design under independent disruption risks (Snyder and Daskin, 2005; Cui et al., 2010; Qi et al., 2010; Chen et al., 2011; Li and Ouyang, 2011, 2012; Bai et al., 2015; Shishebori et al., 2014) and correlated (Li and Ouyang, 2010; Berman and Krass, 2011; Liberatore et al., 2012; Li et al., 2013a; Lu et al., 2015). For example, Li et al. (2013a) proposed a compact modeling framework that transformed complex failure correlation patterns into an explicit supporting station structure subjecting to independent and identically distributed disruptions. Then the problem was further formulated into a compact integer linear program to optimize the network location design under the correlated facility failure risks. The formulation was created to minimize the total expected system cost including the fixed cost of constructing the supporting stations and service facilities and the day-to-day operational cost. To study the grain processing/storage facility location problem, An and Ouyang (2016) proposed a bi-level robust optimization model that maximize the profile of food company and minimize the post-harvest loss with the competition among multiple non-cooperative farmers under yield uncertainty and market equilibrium. Then Karush–Kuhn–Tucker (KKT) conditions were derived to reformulate the bi-level problems into equivalent single-level problems (with complementarity constraints) that are solved by customized Lagrangian relaxation approaches. At last the proposed model and solution approach were applied to case studies for Illinois and Brazil. Moreover, multiple sourcing for supply chain design under disruption risks is another method to handle facility disruptions in a series of studies (Tomlin, 2006; Yu et al., 2009; Chen et al., 2011; An et al., 2014; Shahabi et al., 2014; Sawik, 2015; Xie et al., 2015; Hasani and Khosrojerdi, 2016). For example, papers such as Chen et al. (2011), Shahabi et al. (2014) and Xie et al. (2015) focus on the reliable supply chain design under facility disruption risks. However, freight lead time uncertainties and customer demand fluctuations are not considered in this study. Sawik (2015) and Hasani and Khosrojerdi (2016) both aim to design a robust network under uncertainties and correlated disruptions. A series of methods are proposed in these papers to mitigate the risk of supply chain unreliability. Nevertheless, no location problem is involved in their studies.

This study aims to incorporate transportation disruptions of regular shipments in the integrated supply chain system design framework proposed by Li (2013). Note after considering transportation disruptions, the problem becomes of much higher complexity. For a network that consists of n suppliers, there are 2^n disruption scenarios if we take probabilistic transportation disruptions into consideration, which makes it rather difficult to design a compact model with simple adaptions from the one proposed by Li (2013). Therefore, substantial modeling efforts are needed to develop a comprehensive yet computationally-tractable model to describe this problem. With our efforts, an integer nonlinear programming of only a polynomial size is formulated, which completely describes this problem originally with an exponential number of disruption scenarios. Besides, it is also assumed that the expedited shipment service is not only fast, expensive but also absolutely immune to disruptions, and can be provided by an arbitrary candidate supplier.

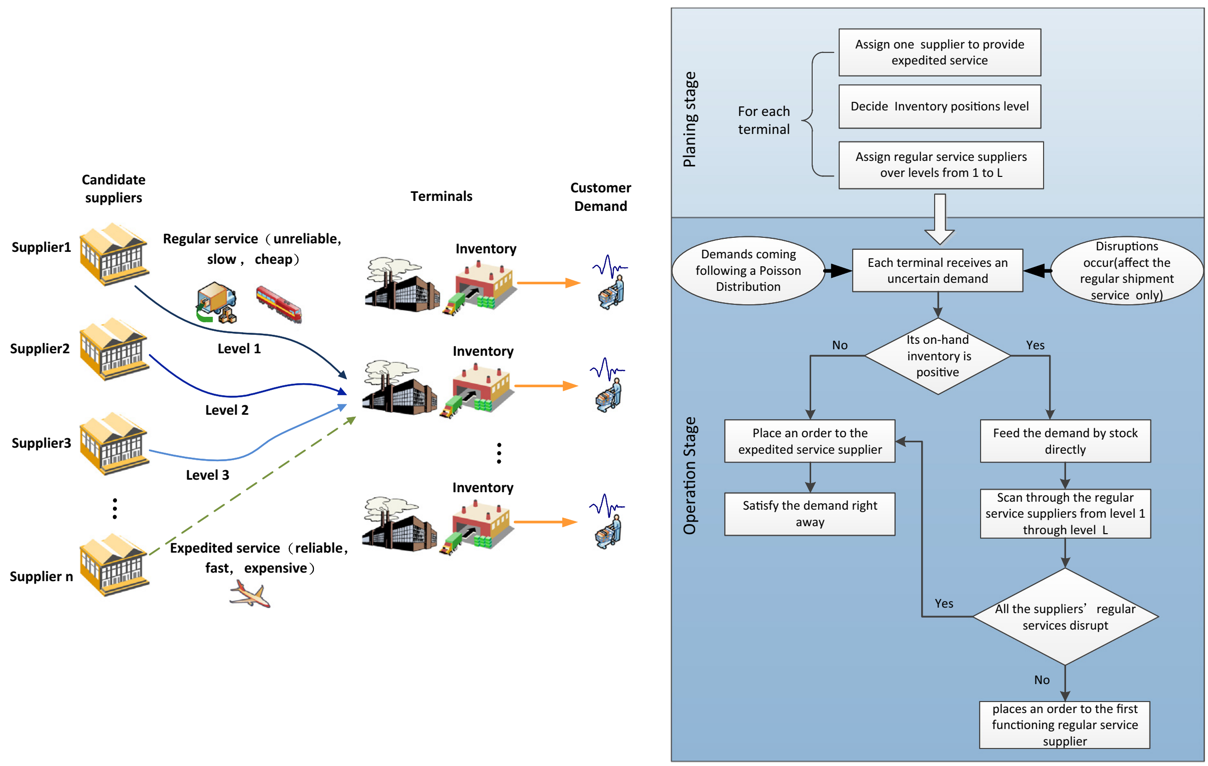
3. Model formulation

For the convenience of the readers, the mathematical notation is summarized in Table 1.

Fig. 1(a) is the structure of studied supply chain system that includes a set of terminals denoted by **J** and a set of candidate suppliers denoted by **I**, while Each terminal $j \in \mathbf{J}$ receives discrete demand for a certain commodity from a fixed pool of customers over time. In the planning stage as shown in Fig. 1(b), each terminal selects an expedited service supplier and a series of regular service suppliers from level 1 to level L, then sets a proper inventory position accordingly. we assume that at each terminal j, demand units arrive randomly with an expected rate of d_j. To feed the arriving demand, we assume that each

Table 1
Notation list.

d_j	Demand rate at the terminal j
e_{ij}	Unit expedited shipment cost from supplier i to terminal j
f_i	Fixed cost to install supplier i
h_j	Unit inventory holding cost at terminal j
q	Disruption probability for the regular service
r_{ij}	Unit regular shipment cost from supplier i to terminal j
t_{ij}	Expected regular shipment lead time from supplier i to terminal j
L	Maximum assignment level
$P_{ij}(S_j)$	Stock-out probability at terminal j with base stock S_j and regular supplier i
S_j	Base-stock position at terminal j
$\bar{S}_j$	Maximum allowable base-stock position at terminal j
X_i	Whether supplier i is installed for service
Y_{ijl}	Whether supplier i provides regular service to terminal j at assignment level l
Z_{ij}	Whether supplier i provides expedited service to terminal j
$\mathbf{I}$	Set of candidate suppliers, indexed by i
$\mathbf{J}$	Set of terminals, indexed by j
$\mathbf{L} = \{1, 2, \ldots, L\}$	Set of assignment levels, indexed by l
$\mathbf{S}_j = \{0, 1, 2, \ldots, \bar{S}_j\}$	Set of candidate base-stock positions, indexed by S_j



(a) (b)

Fig. 1. Structure (a) and flowchart (b) of the studied supply chain system.

terminal j initially keeps a base-stock position $S_j \in \mathbf{S}_j := \{0, 1, 2, \ldots, \bar{S}_j\}$ where $\bar{S}_j$ is a given capacity of the inventory at j, and the cost of holding one unit base stock is $h_j \geqslant 0$. This yields the system inventory cost as follows

$$C^{\mathrm{H}} := \sum_{j \in \mathbf{J}} h_j S_j \tag{1}$$

In the operational stage, we assume that a specific disruption scenario of regular shipment services of all suppliers realizes. The supply chain system proposed in our study follows the Kanban policy. In this policy, whenever receiving a demand unit, terminal j will first check its on-hand inventory and take one unit from this inventory, if any, to feed this demand unit. Meanwhile in order to maintain the base-stock inventory position, terminal j places a unit order right away to a supplier from **I**. Since this study focuses on the supply chain design and daily operations, we assume that the production of all suppliers are constant. Therefore, the product cost is also constant and independent of the optimization decisions, and thus there is no need to formulate the product cost in the proposed optimization model. We assume that disruptions of the corresponding regular transportation services are independent and identically distributed with an identical probability q. To mitigate the impact from uncertain disruptions, a terminal is assigned to $L > 1$ suppliers at different priority levels for regular shipments. Every time, this terminal scans through these assigned suppliers from level 1 through level L and places the order to the first functioning supplier. For the notation convenience, we define level set $\mathbf{L} := \{1, 2, \ldots, L\}$. In this way, the probability for a terminal to be served by its level-l supplier is $(1-q)q^{l-1}, \forall l \in \mathbf{L}$. The assignments are specified by binary variables $\mathbf{Y} = \{Y_{ijl}\}_{i \in I, j \in J, l \in \mathbf{L}}$ such that $Y_{ijl} = 1$ if supplier i is assigned to terminal j at level l or $Y_{ijl} = 0$ otherwise. Let r_{ij} denote the cost to ship a unit commodity from supplier i to terminal j. We assume that each order is served by regular shipment, then the total expected regular shipment cost is

$$C^{\mathrm{R}} := \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{l \in \mathbf{L}} d_j r_{ij} (1-q) q^{l-1} Y_{ijl} \tag{2}$$

We assume that the studied supply chain system has to maintain very high service quality such that customer demand has to be served right after the arrival. So once the on-hand inventory is empty, an expedited order will be placed to the corresponding supplier. The expedited shipment takes place right away with a negligible transportation time, and thus this outstanding demand can be met without much delay as well. In this case, no regular order is needed since it is too slow to supply the demand on time. In our study, we assume that the disruption only affects the regular shipment service and the expedited service is a fast and absolutely reliable transportation mode (i.e., without any disruption). Compared with the regular service, the expedited service yet costs a much higher unit shipment cost, denoted by e_{ij} from supplier i to terminal j, i.e., $e_{ij} \gg r_{i'j}, \forall i' \in \mathbf{I}$. In order to quantify the expected expedited transportation cost, we will first quantify the probability for a terminal to activate the expedited service. As illustrated in Li (2013), this system can be equivalently formulated as a lost-sales system with Poisson-distributed demand and stochastic but independent lead times, proposed by Karush (1957), by reinterpreting the lost sales as the demand fed by the expedited service. Then conditioning on that supplier i is the active regular service provider to terminal j, then the probability for terminal j to use the expedited service can be represented as a function of initial inventory S_j:

$$P_{ij}(S_j) = \frac{(d_j t_{ij})^{S_j} / S_j!}{\sum_{s=0}^{S_j} (d_j t_{ij})^s / s!}. \tag{3}$$

Note that once terminal j places an expedited order from supplier i', then the original regular order to supplier i is equally replaced to an expedited one, which will apparently increase the total shipment cost.

According to Eq. (3), the percentage of terminal j calling for the expedited shipment service from supplier j is denoted by $P_{ij}(S_j)$ with inventory position S_j. Then the total expected percentage of terminal j calling for the expedited shipment service is:

$$P^{\mathrm{T}} = \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{l \in \mathbf{L}} P_{ij}(S_j)(1-q) q^{l-1} Y_{ijl} \tag{4}$$

We define variables $\mathbf{Z} = \{Z_{ij}\}_{i \in I, j \in J}$ to denote the expedited service assignments such that $Z_{ij} = 1$ if terminal j's expedited service provider is supplier i or $Z_{ij} = 0$ otherwise. Then we formulate the total expected expedited shipment cost as:

$$C^{\mathrm{EX}} = \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{l \in \mathbf{L}} \sum_{i' \in \mathbf{I}} e_{i'j} Z_{i'j} P_{ij}(S_j)(1-q) q^{l-1} Y_{ijl} \tag{5}$$

Then the total expected additional cost due to expedited shipments (or the marginal expedited cost) can be formulated as:

$$C^{\mathrm{M}} = \sum_{i \in \mathbf{I}} \sum_{j \in \mathbf{J}} \sum_{l \in \mathbf{L}} \sum_{i' \in \mathbf{I}} d_j (e_{i'j} Z_{i'j} - r_{ij}) P_{ij}(S_j)(1-q) q^{l-1} Y_{ijl} \tag{6}$$

Note that in our study, each terminal is assigned to one and only one expedited supplier, i.e., $\sum_{i \in \mathbf{I}} Z_{ij} = 1$. Then C^M can be rewritten as follows,

$$C^{M} = \sum_{i\in\mathbf{I}}\sum_{j\in\mathbf{J}}\sum_{l\in\mathbf{L}}\sum_{i'\in\mathbf{I}} d_j(e_{i'j} - r_{ij})P_{ij}(S_j)(1-q)q^{l-1}Z_{i'j}Y_{ijl} \tag{7}$$

Remark. Note that since a longer shipment time is usually associated with a higher shipment cost for the same mode of transportation, we assume that $r_{ij} \geqslant r_{i'j} \Longleftrightarrow t_{ij} \geqslant t_{i'j}, \forall i \neq i' \in I, j \in J$.

Another risk that the regular service is that all its suppliers may be disrupted simultaneously at probability q^L. If this happens, the regular service to this terminal becomes inactive, and we assume that it is now only served by emergency shipments from the previously assigned expedited supplier. The emergency cost structure stays the same as the previously defined expedited cost structure since they come from the same sources. Thus the expected system emergency cost is formulated as

$$C^{E} := \sum_{j\in\mathbf{J}}\sum_{i\in\mathbf{I}} d_j e_{ij} q^L Z_{ij}. \tag{8}$$

Finally, in this supply chain system, if candidate supplier i is used by one or more terminals for either regular or expedited service, a fixed installation cost f_i (prorated per unit time) is incurred. Define binary variables $\mathbf{X} = \{X_i\}_{i\in\mathbf{I}}$ to denote the supplier location decisions such that $X_i = 1$ if candidate supplier i is installed or $X_i = 0$ otherwise. This results in the system fixed installation cost as follows,

$$C^{F} := \sum_{i\in\mathbf{I}} f_i X_i. \tag{9}$$

The system design includes integrated decisions of supplier location $\{X_i\}$, regular service assignments $\{Y_{ijl}\}$, expedited service assignments $\{Z_{ij}\}$, and initial inventory positions $\{S_j\}$ that collectively minimize the total system cost composed of (1), (2), (7), (8) and (9). Note that these cost components shall generally exhibit the following tradeoffs. Increasing supplier installations shall raise one-time fixed cost (9) but reduce day-to-day operational costs(2), (7) and (8). The higher inventory positions $\{S_j\}$ we set, which though increase inventory cost (1), the less frequent expedited shipments are needed according to probability function (3), and thus the less extra expedited transportation cost (7) is consumed. In order to quantitatively solve the detailed system design, the follow integer programming model is formulated.

$$\min\ C := \sum_{i\in\mathbf{I}} f_i X_i + \sum_{j\in\mathbf{J}} h_j S_j + \sum_{j\in\mathbf{J}} d_j \left\{ \sum_{i\in\mathbf{I}}\sum_{l\in\mathbf{L}} \left[r_{ij} + \sum_{i'\in\mathbf{I}} (e_{i'j} - r_{ij}) Z_{i'j} P_{ij}(S_j) \right] (1-q) q^{l-1} Y_{ijl} + q^L \sum_{i\in\mathbf{I}} e_{ij} Z_{ij} \right\}, \tag{10}$$

$$\text{s.t.} \quad \sum_{l\in\mathbf{L}} Y_{ijl} - X_i \leqslant 0, \quad \forall i \in \mathbf{I}, j \in \mathbf{J}, \tag{11}$$

$$Z_{ij} - X_i \leqslant 0, \quad \forall i \in \mathbf{I}, j \in \mathbf{J}, \tag{12}$$

$$\sum_{i\in\mathbf{I}} Y_{ijl} = 1, \quad \forall j \in \mathbf{J}, l \in \mathbf{L}, \tag{13}$$

$$\sum_{i\in\mathbf{I}} Z_{ij} = 1, \quad \forall j \in \mathbf{J}, \tag{14}$$

$$S_j \in \mathbf{S}_j, \quad \forall j \in \mathbf{J}, \tag{15}$$

$$Y_{ijl} \in \{0,1\}, \quad \forall i \in \mathbf{I}, j \in \mathbf{J}, l \in \mathbf{L}, \tag{16}$$

$$Z_{ij} \in \{0,1\}, \quad \forall i \in \mathbf{I}, j \in \mathbf{J}, \tag{17}$$

$$X_i \in \{0,1\}, \quad \forall i \in \mathbf{I}. \tag{18}$$

Objective (10) aims to minimize the summation of all cost components (2), (7), (8) and (9) across the entire system. Constraints (11) and (12) indicate that a supplier need to be installed first prior to its usage. Constraint (11) also ensures that if one of the suppliers is selected to provide the regular service to a terminal, it can only serve this terminal at one assignment level. Constraint (13) requires that one terminal has one and only one regular supplier at each level. Constraint (14) postulates that each terminal is assigned to one and only one expedited supplier. Constraints (15)–(18) are the corresponding integer and variable constraints for all variables.

4. Solution approach

Note that problem (10)–(18) is not only a complex nonlinear integer problem, but also apparently NP-hard since the basic uncapacitated facility location problem is its special case. Therefore, off-the-shelf solvers are not appropriate to solve exact optimum to a moderate-sized instance of this problem. Furthermore, though a series of homologous methods based on Lagrangian relaxation (LR) algorithm has been proposed in some researches, e.g., Chen et al. (2011), Li (2013) and Yun et al. (2015), the disruptions of regular shipment service considered in our study causes the toughest point to determine the multi-level suppliers for each terminal. To tackle this challenge, we propose a customized solution approach, which can solve a near-optimum solution to this problem very efficiently. Section 4.1 proposes a LR algorithm adapted from Li (2013) to obtain a lower bound to the optimal value of objective (10). Basically, by adapting LR algorithm, we decompose the original problem into a series of sub-problems that can be easily solved by simple enumeration. This relax solution is however likely infeasible yet yielding a lower bound to the objective. Section 0 proposes a heuristic algorithm to generate an upper bound to the optimal objective (10), which is proved to provide an optimal feasible solution transformed from the relaxed solution.

Finally, Section 4.3 adopts a sub-gradient search algorithm to obtain a near optimality by iteratively updating the upper and lower bound to narrow the gap between them until within an applicable tolerance. Then we obtain a near-optimum solution that is not less than the true optimum within the optimality gap. Better yet, final feasible solution is the true optimum if the gap is reduced to zero.

4.1. Lagrangian relaxation

By relaxing constraints (11) and (12), we add the product of the-left hand side of these constraints with Lagrangian multipliers $\boldsymbol{\lambda} := \{\lambda_{ij} \geqslant 0\}_{i\in\mathbf{I},j\in\mathbf{J}}$ and $\boldsymbol{\mu} := \{\mu_{ij} \geqslant 0\}_{i\in\mathbf{I},j\in\mathbf{J}}$ into objective (10). To improve the relaxed problem solution, the following constraints

$$\sum_{l\in\mathbf{L}} Y_{ijl} \leqslant 1, \quad \forall i \in \mathbf{I}, \quad j \in \mathbf{J}, \tag{19}$$

is also added, which are redundant to constraints (11). Then we obtain the following relax problem

$$\Delta(\boldsymbol{\lambda},\boldsymbol{\mu}) := \min_{\mathbf{X},\mathbf{Y},\mathbf{Z},\mathbf{S}} \sum_{i\in\mathbf{I}} \left[f_i - \sum_{j\in\mathbf{J}} (\lambda_{ij} + \mu_{ij}) \right] X_i + \sum_{j\in\mathbf{J}} \left\{ \sum_{i\in\mathbf{I}} \sum_{l\in\mathbf{L}} \left(\sum_{i'\in\mathbf{I}} \alpha_{ii'jl} Z_{i'j} + \beta_{ijl} \right) Y_{ijl} + h_j S_j \right\}, \tag{20}$$

subject to (12)–(19), where

$$\alpha_{ii'jl} = d_j \left[(e_{i'j} - r_{ij}) P_{ij}(S_j)(1-q)q^{l-1} + \frac{e_{i'j} q^L}{L} \right] + \frac{\mu_{i'j}}{L}, \tag{21}$$

and,

$$\beta_{ijl} = d_j r_{ij} (1-q) q^{l-1} + \lambda_{ij}. \tag{22}$$

Note that the variables $\mathbf{X}$ are separated from others in the relaxed problem, which decomposes the relaxed problem into two sets of sub-problems. The first set only includes one sub-problem associated with variables $\mathbf{X}$:

$$\Gamma(\boldsymbol{\lambda},\boldsymbol{\mu}) = \min_{\mathbf{X}} \sum_{i\in\mathbf{I}} \left[f_i - \sum_{j\in\mathbf{J}} (\lambda_{ij} + \mu_{ij}) \right] X_i, \tag{23}$$

subject to binary constraint (18). Sub-problem (23) could be simply solved by setting $X_i = 1$ if $f_i - \sum_{j\in\mathbf{J}}(\lambda_{ij} + \mu_{ij}) \leqslant 0$ or $X_i = 0$ otherwise, which only takes a time complexity of $O(|\mathbf{I}||\mathbf{J}|)$. The second set contains $|\mathbf{J}|$ single terminal sub-problems as follows:

$$\Phi_j(\boldsymbol{\lambda},\boldsymbol{\mu}) := \min_{\{Y_{ijl}, Z_{ij}\}_{i\in\mathbf{I},l\in\mathbf{L}}, S_j} \sum_{i\in\mathbf{I}} \sum_{l\in\mathbf{L}} \left(\sum_{i'\in\mathbf{I}} \alpha_{ii'jl} Z_{i'j} + \beta_{ijl} \right) Y_{ijl} + h_j S_j, \quad \forall j \in \mathbf{J}, \tag{24}$$

subject to (13)–(17), (19) (where $\alpha_{ii'jl}$ and β_{ijl} are the same as those in (21) and (22)). We reformulate sub-problem (24) as a combinatorial problem to facilitate the solution algorithm. Define set $\mathbf{K} = \{(i_1, i_2, \ldots, i_L) | i_1 \neq i_2 \neq \cdots \neq i_L \in \mathbf{I}\}$, where each $(i_1, i_2, \ldots, i_L)$ specifies a strategy to assign the regular suppliers to terminal j at all L levels sequentially; i.e., supplier i_l is assigned to terminal j at level $l, \forall l \in \mathbf{L}$. For short we denote vector $(i_1, i_2, \ldots, i_L)$ with alias k. Then sub-problems (24) can be rewritten as:

$$\Phi_j(\boldsymbol{\lambda},\boldsymbol{\mu}) := \min_{i'\in\mathbf{I},k\in\mathbf{K},S_j\in\mathbf{S}_j} C_{ki'j}(S_j) := A_{ki'j}(S_j) + h_j S_j + B_{ki'j}, \quad \forall j \in \mathbf{J}, \tag{25}$$

where

$$A_{ki'j} = d_j \sum_{l\in\mathbf{L}} [(e_{i'j} - r_{i_l j})(1-q)q^{l-1} P_{i_l j}(S_j)], \tag{26}$$

$$B_{ki'j} = \sum_{l\in\mathbf{L}} [d_j r_{ij} (1-q) q^{l-1} + \lambda_{ij}] + \frac{d_j e_{i'j} q^L + \mu_{i'j}}{L}. \tag{27}$$

For given k and i', $\min_{S_j\in\mathbf{S}_j} C_{ki'j}(S_j)$ can be solved with a bisection search method (BS) described in Appendix A. With this, problem (20) can be solved by a customized enumeration algorithm (EA) that does an exhaustive search through $k \in \mathbf{K}, i' \in \mathbf{I}$ for every $j \in \mathbf{J}$, as follows:

Step EA1: For each terminal $j \in \mathbf{J}$, we iterate through $(k, i') \in (\mathbf{K}, \mathbf{I})$ that specifies terminal j's assignment strategy of both regular and expedited suppliers, and call the BS algorithm to solve $S_j^* := \arg\min_{S_j\in\mathbf{S}} C_{ki'j}(S_j)$.

Step EA2: Find $(k^* = (i_1^*, i_2^*, \ldots, i_L^*), i'^*) := \text{argmin}_{k \in \mathbf{K}, i' \in \mathbf{I}} C_{ki'j}(S_j^*)$;
Step EA3: Return the optimal assignment strategy (k^*, i'^*) and inventory position S_j^*;
Step EA4: Repeat EA1-3 for every supplier $j \in \mathbf{J}$ to get the optimal solution to $\mathbf{X}, \mathbf{Y}$ and $\mathbf{Z}$ as follows:

$$Y_{ijl} = \begin{cases} 1 & \text{if } i = i_l^*; \\ 0 & \text{otherwise,} \end{cases} \quad Z_{ij} = \begin{cases} 1 & \text{if } i = i'^*; \\ 0 & \text{otherwise,} \end{cases} \quad X_i = \max_{j \in \mathbf{J}, l \in \mathbf{L}} \{Y_{ijl}, Z_{ij}\}, \quad \forall j \in \mathbf{J}, i \in \mathbf{I}, l \in \mathbf{L}. \tag{28}$$

Note that in the worst case, the time complexity of the EA algorithm for solving sub-problems (24) is $O(|\mathbf{J}||\mathbf{I}|^{L+1} \ln(\overline{S}_j))$. By solving sub-problem (23) and (24), we obtain the object value of relaxed problem (20) for one set of given $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$, i.e.,

$$\Delta(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \Gamma(\boldsymbol{\lambda}, \boldsymbol{\mu}) + \sum_{j \in \mathbf{J}} \Phi_j(\boldsymbol{\lambda}, \boldsymbol{\mu}), \tag{29}$$

According to the duality property of Lagrangian relaxation (Geoffrion, 1974), the object value obtained from problem (29) also serves as the lower bound for the optimal value of the original problem. Besides, we note that the time complexity of the relaxed problem (29) is dominated by sub-problem (24) since the time complexity of sub-problem (24) is much higher than that of sub-problem (23).

4.2. Feasible solutions

If the solution obtained by solving problem (29) is feasible to original problem, then solution is also optimal to the primal. However, for most large-scale instances, the feasible solution needs to be constructed based on the relaxed solution. Many previous methods (Li and Ouyang (2012), Li (2013), Yun et al. (2015)) can be used to construct a near-optimal objective value. One intuitive solution is to adjust the $\mathbf{X}$ values while keeping $\mathbf{Y}, \mathbf{Z}, \mathbf{S}$ values as:

$$X_i = \max_{j \in \mathbf{J}, l \in \mathbf{L}} \{Y_{ijl}, Z_{ij}\}, \quad \forall i \in \mathbf{I}, \tag{30}$$

which could be solved in relatively short time of $O(|\mathbf{I}||\mathbf{J}|L)$. However, it may not always obtain a good feasible solution, since the scattered $\mathbf{Y}, \mathbf{Z}$ values likely yield an excessive number of suppliers, leading to an unnecessarily high total cost. Conversely, a better solution is to fix $\mathbf{X}$ while adjusting the other variables accordingly. So we define $\bar{\mathbf{I}} := \{i | X_i = 1, \forall i \in \mathbf{I}\}$ as the set of installed suppliers in the relaxed solution. Then by setting $\lambda_{ij} = \mu_{ij} = 0$ and replacing $\mathbf{I}$ with $\bar{\mathbf{I}}$, sub-problems (24) can be rewritten as (31)–(34):

$$\bar{\Phi}_j(\boldsymbol{\lambda}, \boldsymbol{\mu}) := \min_{\{Y_{ijl}, Z_{ij}\}_{i \in \bar{\mathbf{I}}, l \in \mathbf{L}}, \mathbf{S}_j} \sum_{i \in \bar{\mathbf{I}}} \sum_{l \in \mathbf{L}} \left(\sum_{i' \in \bar{\mathbf{I}}} \bar{\alpha}_{ii'jl} Z_{i'j} + \bar{\beta}_{ijl} \right) Y_{ijl} + h_j S_j, \quad \forall j \in \mathbf{J}, \tag{31}$$

$$\text{Subject to } \sum_{l \in \mathbf{L}} Y_{ijl} \leqslant 1, \quad \forall i \in \bar{\mathbf{I}}, j \in \mathbf{J}, \tag{32}$$

$$\sum_{i \in \bar{\mathbf{I}}} Y_{ijl} = 1, \quad \forall j \in \mathbf{J}, l \in \mathbf{L}, \tag{33}$$

$$\sum_{i \in \bar{\mathbf{I}}} Z_{ij} = 1, \quad \forall j \in \mathbf{J}, \tag{34}$$

where

$$\bar{\alpha}_{ii'jl} = d_j \left[(e_{i'j} - r_{ij}) P_{ij}(S_j)(1-q)q^{l-1} + \frac{e_{i'j} q^L}{L} \right], \tag{35}$$

$$\bar{\beta}_{ijl} = d_j r_{ij} (1-q) q^{l-1}. \tag{36}$$

Then other feasible variable values can be obtained similarly to the transformation from (24), (25). We also define set $\bar{\mathbf{K}} = \{(i_1, i_2, \ldots, i_L) | i_1 \neq i_2 \neq \cdots \neq i_L \in \bar{\mathbf{I}}\}$ as all the strategies to assign regular suppliers from $\bar{\mathbf{I}}$ to terminal j at all L levels, and we also use alias k to represent vector $(i_1, i_2, \ldots, i_L)$ for short. Then the transformed sub-problems are formulated as

$$\bar{\Phi}_j(\boldsymbol{\lambda}, \boldsymbol{\mu}) := \min_{i' \in \bar{\mathbf{I}}, k \in \bar{\mathbf{K}}, \mathbf{S_j} \in \mathbf{S_j}} \bar{C}_{ki'j}(S_j) := \bar{A}_{kj} + \bar{B}_{ki'j} + h_j S_j, \quad \forall j \in \mathbf{J} \tag{37}$$

where

$$\bar{A}_{kj} = d_j \sum_{l \in \mathbf{L}} r_{i_l j} (1-q) q^{l-1} (1 - P_{i_l j}(S_j)), \tag{38}$$

$$\bar{B}_{ki'j} = e_{i'j} \left(\frac{d_j q^L}{L} + d_j \sum_{l \in \mathbf{L}} (1-q) q^{l-1} P_{i_l j}(S_j) \right). \tag{39}$$

The exact optimal solution to each sub-problem (31)–(34) with any $j \in \mathbf{J}$ can be solved as follows. First denote $\bar{i}_j'^* := \text{argmin}_{i' \in \bar{\mathbf{I}}} e_{i'j}$. Then we denote with vector $\bar{k}_j^* = \left(\bar{i}_1^*, \bar{i}_2^*, \ldots, \bar{i}_L^*\right) \in \bar{\mathbf{K}}$ the first L regular service suppliers sorted by the shipment cost to terminal j, i.e., $r_{\bar{i}^{j*}_l j} \leqslant r_{\bar{i}^{j*}_m j} \leqslant r_{ij},\ \forall l < m \in \mathbf{L}, i \notin \bar{k}^*_j$. Finally, define $\bar{S}^*_j := \min_{S_j \in \mathbf{S}_j} \bar{C}_{\bar{k}^* \bar{i}'^* j}(S_j)$, which can be again efficiently solved with the BS algorithm in Appendix A. The following proposition proves that $\left(\bar{i}'^*_j, \bar{k}^*_j, \bar{S}^*_j\right)$ is the optimal solution to sub-problem (31)–(34) with respect to terminal j.

Proposition 1.

$$\left(\bar{i}'^*_j, \bar{k}^*_j, \bar{S}^*_j\right) = \min_{i' \in \bar{\mathbf{I}}, k \in \bar{\mathbf{K}}, S_j \in \mathbf{S}_j} \bar{C}_{ki'j}(S_j), \quad \forall j \in \mathbf{J}.$$

Proof. First, it can be seen from the structure of sub-problem (31)–(34) that as i' varies while the other variables are fixed, $\bar{C}_{ki'j}(S_j)$ increases with $e_{i'j}$. Therefore the optimal solution to i' is $\bar{i}'^*_j$. Let $\hat{S}_j$ denote the optimal value of S_j, then the optimal solution to k is $\hat{k}_j := (\hat{i}^j_1, \hat{i}^j_2, \ldots \hat{i}^j_L) := \min_{k \in \bar{\mathbf{K}}} \bar{C}_{k\bar{i}'^*_j j}(\hat{S}_j)$. We will prove $\hat{k}_j = \bar{k}^*_j$ by contradiction. If there exists $l \in \mathbf{L}$ such that $r_{\hat{i}^j_l j} > r_{\hat{i}^j_{l+1} j}$. We construct a new feasible solution $\tilde{k}_j := (\hat{i}^j_1, \ldots, \hat{i}^j_{l+1}, \hat{i}^j_l, \ldots \hat{i}^j_L)$ by swapping the levels of $\hat{i}^j_l$ and $\hat{i}^j_{l+1}$ in $\hat{k}_j$, and then we compare the difference between the two costs with respect to $\hat{k}_j$ and $\tilde{k}_j$, respectively,

$$\begin{aligned}\bar{C}_{\hat{k}_j \bar{i}'^*_j j}(\hat{S}_j) - \bar{C}_{\tilde{k}_j \bar{i}'^*_j j}(\hat{S}_j) &= (1-q)^2 q^{l-1} d_j \left[(r_{\hat{i}^j_l j} - r_{\hat{i}^j_{l+1} j}) + e_{\bar{i}'^*_j j}(P_{\hat{i}^j_l j}(\hat{S}_j) - P_{\hat{i}^j_l j}(\hat{S}_j)) - (r_{\hat{i}^j_l j} P_{\hat{i}^j_l j}(\hat{S}_j) - r_{\hat{i}^j_{l+1} j} P_{\hat{i}^j_l j}(\hat{S}_j)) \right] \\ &> (1-q)^2 q^{l-1} d_j \left[(r_{\hat{i}^j_l j} - r_{\hat{i}^j_{l+1} j}) + (e_{\bar{i}'^*_j j} - r_{\hat{i}^j_l j})(P_{\hat{i}^j_l j}(\hat{S}_j) - P_{\hat{i}^j_l j}(\hat{S}_j)) \right].\end{aligned}$$

Note that $r_{\hat{i}^j_l j} - r_{\hat{i}^j_{l+1} j} > 0$, and $P_{\hat{i}^j_l j}(\hat{S}_j) - P_{\hat{i}^j_l j}(\hat{S}_j) > 0$ due to the assumption that $r_{ij} \geqslant r_{i'j} \Longleftrightarrow t_{ij} \geqslant t_{i'j}, \forall i \neq i' \in \mathbf{I}, j \in \mathbf{J}$. Then we obtain $\bar{C}_{\hat{k}_j \bar{i}'^*_j j}(\hat{S}_j) - \bar{C}_{\tilde{k}_j \bar{i}'^*_j j}(\hat{S}_j) \geqslant 0$, which is contradictive to the premise that $\hat{k}_j$ is the optimal solution. Therefore, we prove $\hat{i}^j_1 \leqslant \hat{i}^j_2 \leqslant \cdots \leqslant \hat{i}^j_L$. If there exists a $i \in \bar{\mathbf{I}} \setminus \hat{k}_j$ and some $l \in \mathbf{L}$ having $r_{\hat{i}^j_l j} > r_{ij}$, replacing $\hat{i}^j_l$ with i in $\hat{k}_j$ will further reduce cost $\bar{C}_{\hat{k}_j \bar{i}'^*_j j}(\hat{S}_j)$ with a similar argument, which is a contradiction, too. This proves that $\hat{k}_j = \bar{k}^*_j$.

Finally, $\hat{S}_j = \bar{S}^*_j$ obviously holds since $\hat{k}_j = \bar{k}^*_j$. This completes the proof. □

By solving problem (31)–(34) for all $j \in \mathbf{J}$, a feasible solution to the primal problem can be obtained as follows:

$$Y_{ijl} = \begin{cases} 1 & \text{if } i = \bar{i}^{j*}_l; \\ 0 & \text{otherwise,} \end{cases} \quad Z_{ij} = \begin{cases} 1 & \text{if } i = \bar{i}'^*_j; \\ 0 & \text{otherwise,} \end{cases} \quad S_j = \bar{S}^*_j, \quad \forall j \in \mathbf{J}, i \in \mathbf{I}, l \in \mathbf{L}. \tag{40}$$

This algorithm is fast (only taking a solution time of $O(|\mathbf{J}| \max\{|\mathbf{I}| \ln(|\mathbf{I}|), L \ln(\bar{S}_j)\})$). By plugging these feasible solution values into primal objective function (10), we obtain an upper bound to the optimal objective value as well.

4.3. Updating Lagrangian multipliers

If the upper bound objective obtained from (19) is equal to the lower bound derived in (11), then it is also the optimal objective to the original problem and the corresponding solution is the optimal solution. Otherwise, multipliers $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ need to be updated iteratively to reduce this gap, which is similar to the process in Chen et al. (2011), Li (2013) and Yun et al. (2015) described in Appendix B.

5. Numerical example

In this section, a series of numerical examples are presented to test the proposed model and provide useful managerial insights based on the datasets provided in Daskin (1995), i.e., a 49-site network involving 48 continental state capital cities and Washington D.C, an 88-sit network involving the previous 49 sits and the other 50 largest cities of US, and a 150-site network consisting of the 150 largest US cities. The numerical algorithms are coded with MATLAB and implemented on a server running two Xeon E5-2640 processors clocked at 2.60 GHz with 8 cores and 16 GB RAM. The LR parameters are set as $\tau = 1, \bar{\tau} = 10^{-3}, K = 5, \theta = 1.005$, and $\bar{K} = 60$. We assume that each site has both a candidate supplier and a terminal, and the parameters are generated as follows. We set $h_j = h$, $r_{ij} = c_r \delta_{ij}$, and $t_{ij} = c_t \delta_{ij}, \forall i \in \mathbf{I}, j \in \mathbf{J}$, where h, c_r and c_t are constant coefficients and δ_{ij} is the great-circle distance between sites i and j. Note that since defined as parameter, the specific expressions of r_{ij} and t_{ij} have no impact on the generality of the proposed model, i.e., r_{ij} and t_{ij} can be adapted to arbitrary values based on application needs. Each e_{ij} is set to be an independent realization of a uniformly distributed random variable in interval $[1, 1 + c_e] \cdot \max a_{i'' \in \mathbf{I}} r_{i'j}$, where $c_e \geqslant 0$ is a constant scalar. In addition, we assume that each f_i is the product of the corresponding city population and a scalar c_f, and each d_j is the product of the corresponding state population and a scalar c_d.

Table 2 shows the sensitivity analysis results of L, where it can be easily seen that when $L > 3$, the total system cost varies slightly with the growth of L, yet the solution time and gap increases greatly. Hence we set $L = 3$ for all the cases based on the result of the sensitivity analysis.

In order to test the performance of the LR algorithm proposed in Section 4, we compare it with the Branch-And-Reduce Optimization Navigator (BARON, coded in GAMS) and the results are listed in Table 3. BARON is the branch-and-bound type enhanced solver for the global solution of nonlinear and mixed-integer nonlinear problems. It can be seen that both the algorithms take a longer solution time and yield a larger gap as q increase. However, BARON take a much longer solution time than LR, while the optimal gap obtained by using LR is no more than 0.5%, which is obviously lower than that by using BARON (16.48–21.10%). So the conclusion drawn from these experiment results is that the performance of customized solution based on LR algorithm is significantly better than that of off-the-shelf solvers like BARON. Besides, the applicability to realistic size problems of the proposed model and solution method can be seen from the results of 88-site and150-site network cases, whose 27 instances are shown as Tables 6 and 7 in Appendix B.

Besides, we also implemented the LR algorithm into a branch-and-bound framework to try to further reduce the optimality gap. However, we find that the branch-and-bound algorithm does not improve the algorithm performance, and thus we opt out the branch-and-bound approach in the proposed algorithm. The detail is documented in Appendix C.

Table 4 summarizes the results of 27 instances on the 49-site network by varying q, c_e and h, where we set other parameters $c_r = 0.01, c_f = 0.02, c_d = 10^{-5},$ and $c_t = 10^{-4}$. The optimal gap between the final feasible objective value and the best relaxed objective is denoted by G, the solution time is denoted by T. The optimal system total inventory and the optimal number of selected suppliers are denoted by S and N, respectively. Moreover, define

$$P^{\mathrm{E}} := \frac{\sum_{i\in \mathbf{I}}\sum_{j\in \mathbf{J}}\sum_{l\in \mathbf{L}} d_j(1-q)q^{l-1}P_{ij}(S_j)Y^*_{ijl}}{\sum_{j\in \mathbf{J}} d_j}$$

as the percentage of demand served by the expedited shipments, where Y^*_{ijl} is the best solution to Y_{ijl}. Inventory cost, regular shipment cost, marginal expedited shipment, emergency cost, supplier set-up cost, and total system cost are denoted by C^{H}, C^{R}, C^{M}, C^{E}, C^{F}, and C, respectively, as defined in Eqs. (1), (2), (7)–(9).

From Table 4 we can briefly see how the variation of q, c_e and h affect the optimal result. When q increases, $C^{\mathrm{F}}, C^{\mathrm{M}}, P^{\mathrm{E}}$, C and P_E significantly increase, and C^{R} slightly increases, while C^{H} and S increase first then drop. It indicates that all the cost components will increase at first, leading to a sharp growth of the total cost. Nevertheless, when q keeps increasing, regular service is more unreliable and keeping a higher inventory is no longer an appealing solution. Instead, a higher percentage of expedited shipments and more suppliers are needed to keep the service quality while shorten the shipping distance to offset the increasing expedited shipment cost. Furthermore, we also notice that the growth of the h rapidly brings up $C^{\mathrm{F}}, C^{\mathrm{M}}, P^{\mathrm{E}}, C^{\mathrm{H}}$ and C, while C^{R} and S keep decreasing. It's probably because as h increase, the inventory position will be reduced to offset the growth of inventory cost, leading to a higher frequency of calling for the expedited shipment and less regular shipment orders. Hence more supplier will also be selected to counteract the growth of expedited shipment cost.

Fig. 2 shows four sets of more detailed sensitivity results, where we can see all the cost components, the inventory position S and the expedited service percentage change over key parameters q, h, c_r and c_e. The default parameters are set as $q = 0.1, h = 100, c_r = 0.01, c_e = 1, c_f = 0.02, c_d = 10^{-5},$ and $c_t = 10^{-4}$, and only one parameter value varies in each experiment. In Fig. 2(a), as q grows from 0 to 1, C^{F} and C^{M} generally increase, while C^{H} increases slightly first and then drops, and C^{R} is originally stationary and then drops. Also, the total cost C has a sharp increase from around 30,000 to 80,000, then

Table 2
The sensitivity analysis of $\boldsymbol{L}$. ($h = 100, c_r = 0.01, c_e = 1, c_f = 0.02, c_d = 10^{-5}, c_t = 10^{-4},$ and $q = 0.1$).

#	L	T (sec)	G (%)	C
1	1	385	0.01	87310.7
2	2	862	0.03	63149.2
3	3	3154	0.18	49883.5
4	4	79,250	1.83	48027.9
5	5	685,072	5.42	46951.7
6	6	9,035,286	12.74	42167.5

Table 3
Cooperation of different solution methods.

#	q	Solution time (sec)		Gap (%)	
		LR	BARON	LR	BARON
1	0.1	804	10,794	0.18	16.48
2	0.3	851	13,147	0.22	18.63
3	0.5	906	22,180	0.41	19.91
4	0.7	937	36,749	0.47	21.10

Table 4
Numerical results for the 49-site network.

#	q	h	c_e	T	G (%)	N	C^F	S	C^H	C^R	C^M	P^E (%)	C
1	0.1	10	1	760	0.18	7	8864	374	3740	8982.4	1292.0	0.76	22878.4
2	0.1	10	2	781	0.17	7	8864	406	4060	9058.1	915.6	0.51	22897.7
3	0.1	10	10	792	0.21	8	9831	413	4130	9103.4	857.2	0.34	23921.6
4	0.1	20	1	786	0.18	9	11,716	367	7340	8703.3	2698.5	1.52	30457.8
5	0.1	20	2	803	0.19	9	11,716	379	7580	8784.9	2490.7	0.66	30571.6
6	0.1	20	10	815	0.21	10	14,216	387	7740	8875.4	1677.9	0.41	32509.3
7	0.1	100	1	804	0.18	12	2,1498	156	15,600	7479.2	5242.7	3.82	49819.9
8	0.1	100	2	815	0.16	13	23,898	163	16,300	7582.6	3791.5	2.47	51572.1
9	0.1	100	10	813	0.17	13	23,898	178	17,800	8409.4	2936.2	1.60	53043.6
10	0.3	10	1	791	0.18	14	24,658	381	3810	9015.6	4831.1	7.93	42314.7
11	0.3	10	2	817	0.18	14	24,658	394	3940	9193.2	4669.0	7.18	42460.2
12	0.3	10	10	811	0.2	15	25,177	419	4190	9286.3	4590.7	6.72	43244.0
13	0.3	20	1	816	0.21	15	25,177	372	7440	8906.7	7628.5	8.08	49152.2
14	0.3	20	2	827	0.22	15	25,177	387	7740	9010.3	7347.8	7.49	49275.1
15	0.3	20	10	835	0.22	16	27,690	404	8080	9170.1	6176.4	6.80	51116.5
16	0.3	100	1	851	0.22	16	27,690	204	20,400	7932.3	13357.0	14.92	69379.3
17	0.3	100	2	859	0.23	16	27,690	218	21,800	8083.7	11982.0	13.70	69555.7
18	0.3	100	10	864	0.24	17	28,013	230	23,000	8916.3	9751.0	11.90	69680.3
19	0.5	10	1	885	0.36	18	29,615	315	3150	9454.1	4911.3	10.61	47130.4
20	0.5	10	2	885	0.37	18	29,615	347	3470	9533.4	4709.7	9.85	47328.1
21	0.5	10	10	886	0.37	19	30,355	368	3680	12091.2	4616.8	8.31	50743.0
22	0.5	20	1	886	0.39	19	30,355	304	6080	9372.0	7901.8	11.02	53708.8
23	0.5	20	2	887	0.39	20	31,162	323	6460	9407.9	7664.1	10.33	54694.0
24	0.5	20	10	887	0.41	20	31,162	348	6960	11327.5	6462.4	9.07	55911.9
25	0.5	100	1	906	0.41	21	32,591	193	19,300	8419.9	19478.0	20.60	79788.9
26	0.5	100	2	910	0.41	22	33,127	221	22,100	8673.0	17621.8	18.68	81521.8
27	0.5	100	10	920	0.42	24	35,073	235	23,500	10191.3	16157.0	17.27	84921.3

followed by a constant and slower increasing rate as q becomes lager. Fig. 2(b) shows that P^E rapidly increases with the growth of q, while S rises at first and then drops slowly. It's probably because when q increases, the regular service from upstream suppliers become increasingly unreliable, and thus the probability of accessing backup suppliers and expedited services grows. Then more suppliers are selected and higher inventory positions are needed to offset the growth of the shipment costs. Furthermore, as q keeps rising, selecting more suppliers gradually becomes the only cure and higher inventory positions are not as helpful. Meanwhile, expedited shipments gradually take over regular shipments and become the dominating shipment mode. In Fig. 2(c), when h grows from 1 to 1000, C^F, C^H and C^M generally increase, C^R continually drop to almost zero, and C increases strictly first followed by a slower growth. Fig. 2(d) shows that the increase of h rapidly brings down S to a slowing down trend in the tail, while P^E generally increase. This implies that installing more suppliers does not help much when h is large, while using more expedited shipments seems more effective in offsetting the inventory cost growth. We can see in Fig. 2(e) and (f) that both C and C^F increase with the growth of c_r from 0.005 to 0.2, while P^E and C^M keep decreasing to almost zero. C^H and C^R grow slowly at first and then drop, which seems to be consistent with the variation of S in Fig. 2(f). This is probably because as c_r grows, the regular shipment cost increases, and thus a higher inventory is needed to offset the growth of expedited shipment cost. The higher inventory leads to a continuous drop of the expedited shipments and a slight increase of the regular shipment cost initially. Nevertheless, as c_r continues to grow (the shipment cost correspondingly increases), building more suppliers becomes a better solution to offset the shipment cost growth, which finally brings down the total inventory. In Fig. 2(g) and (h), as c_e increases, S grows significantly and P^E drops sharply, but the total cost and all its components do not change too much. This indicates that expedited shipments actually cause little increase in overall cost under the optimal inventory management and transportation configuration strategy, and thus it seems an appealing strategy to combine both regular and expedited shipments to reduce the system cost and increase the system reliability.

Besides, we also tested how the results vary with the magnitudes of supplier installation cost (in terms of c_f), customer demand rates (in terms of c_d) and lead times (in terms of c_t), as shown in Fig. 3. It can be seen in Fig. 3(a) and (b) that, when c_f initially grows, all cost components and the total cost generally increase. As c_f keeps increasing, C^F increases first and then turns down and C^R flattens out. This is probably because that, the growth of supplier installation cost likely decreases the number of suppliers, which consequentially raises the shipment distance, cost and leading time. Nevertheless, when c_f continues to increase, the number of suppliers is so small that increasing the inventory position alone is not enough to keep the service quality and thus using more expedited services seems necessary. Fig. 3(c) shows that the increase of c_d initially raises all cost components except C^R. Then C^F keeps increasing but C^H and C^R decrease slightly with a slowing down trend in the tails. We also see in Fig. 3(d) that S increases quickly initially and then flattens out, while P^E significantly decreases to almost zero. It is probably because that expedited services are more suitable for the cases with low demands when the suppliers are scattered and high inventory positions are unnecessary. Nevertheless, as demands increase, regular shipments will become the main shipment mode instead. Fig. 3(e) and (f) shows that as c_t grows, C^F, C^M and P_E increase while C^R drop, and C^H and S increases at the begin-

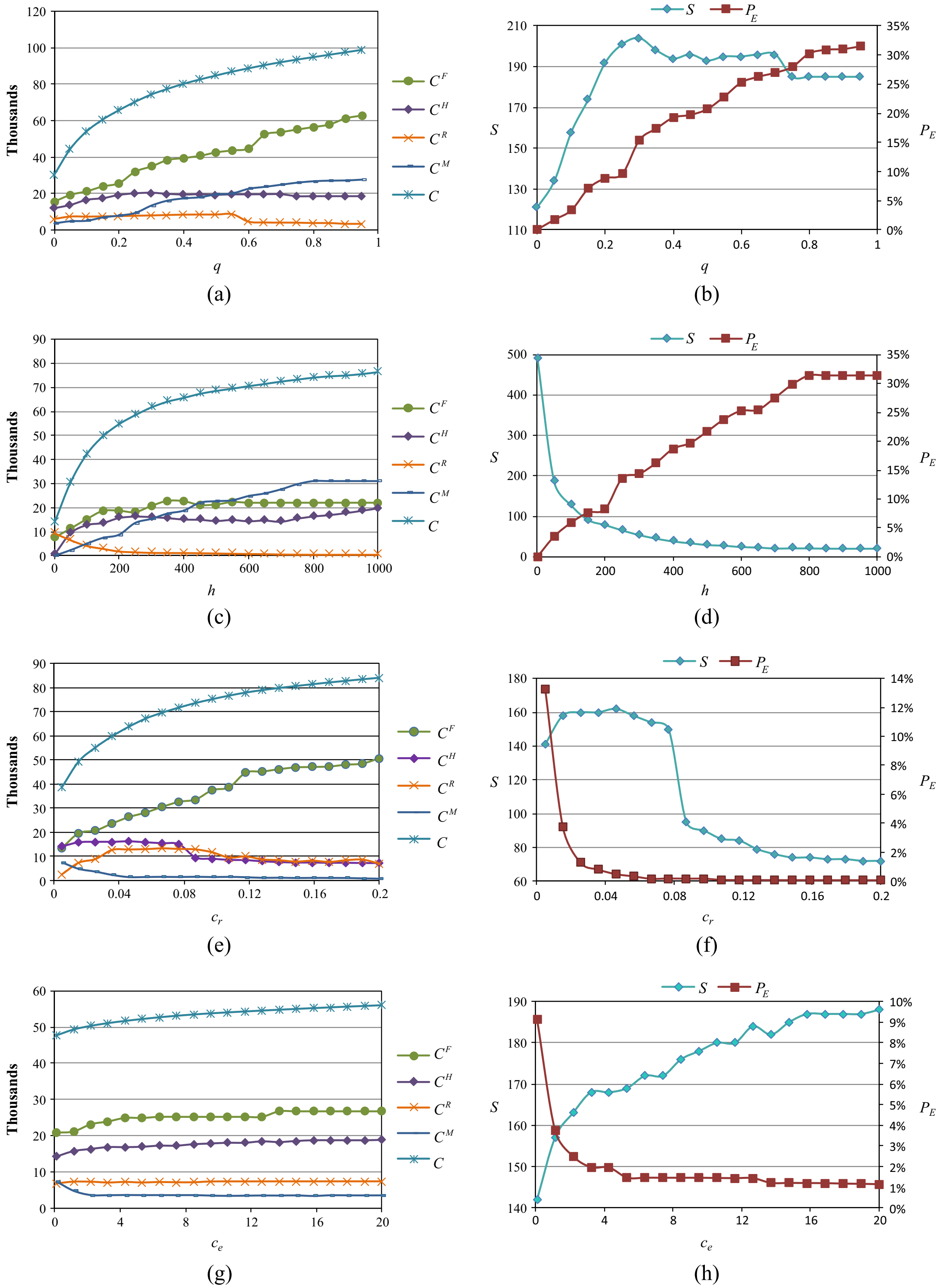


Fig. 2. Sensitivity analysis on parameters $\boldsymbol{q}$ ((a) and (b)), $\boldsymbol{h}$ ((c) and (d)), $\boldsymbol{c_r}$ ((e) and (f)) and $\boldsymbol{c_e}$ ((g) and (h)).

ning then drops. This shows that growth of regular shipment delay will cause the decreasing of inventory positions and consequently increasing the expedited service seems to become a better solution to improve the service quality.

Furthermore, we conducted a series of experiments to verify the interactions among the key parameters in the model. We first set the default parameters as The default parameters were set as $q = 0.1, h = 100, c_r = 0.01,\ c_e = 1, c_f = 0.1,$ $c_d = 2 \times 10^{-5},$ and $c_t = 3 \times 10^{-4}$. Then every time, we varied two of these parameters simultaneously and the corresponding total costs are plotted in Fig. 5(a)–(u) in Appendix E. We see that there are no significant interactions among these parameters.

We also tested how the variations of q affect the optimal suppliers' layouts and terminals' assignments. Again we set $h = 100, c_r = 0.01, c_e = 1, c_f = 0.02, c_d = 2 \times 10^{-5},$ and $c_t = 3 \times 10^{-4}$ and each sub-figure in Fig. 4 shows the optimal layout for a different q value among 0, 0.3 and 0.6. In each sub-figure, the squares denote the selected supplier locations and the circles represent the terminals with their area sizes proportional to the base-stock positions. The arrows show how the selected suppliers are assigned to each terminal with each arrow's width proportional to the percentage of the corresponding

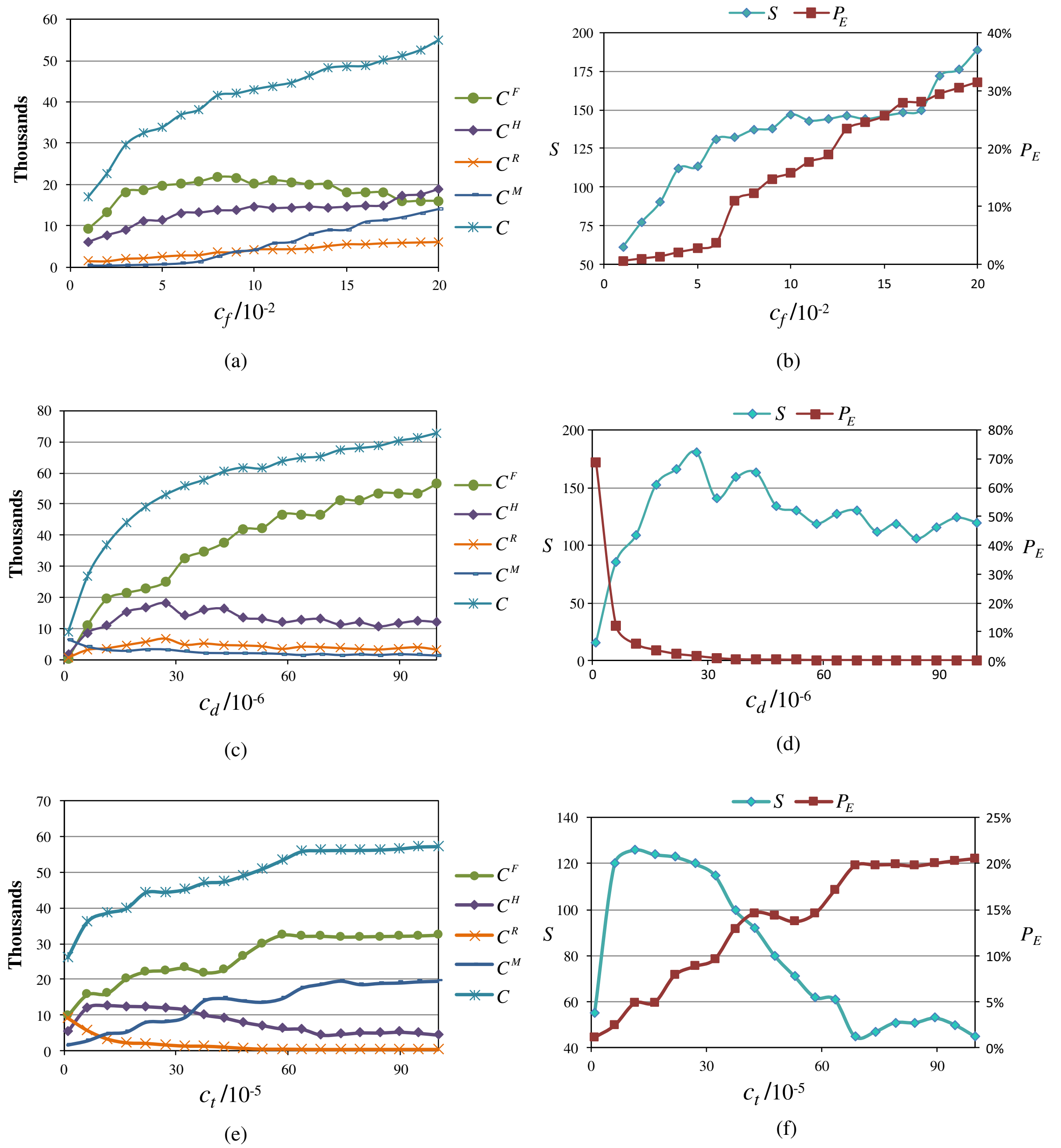
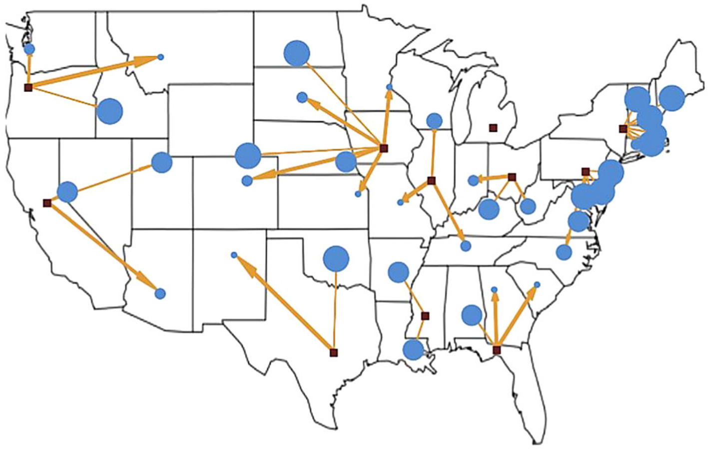


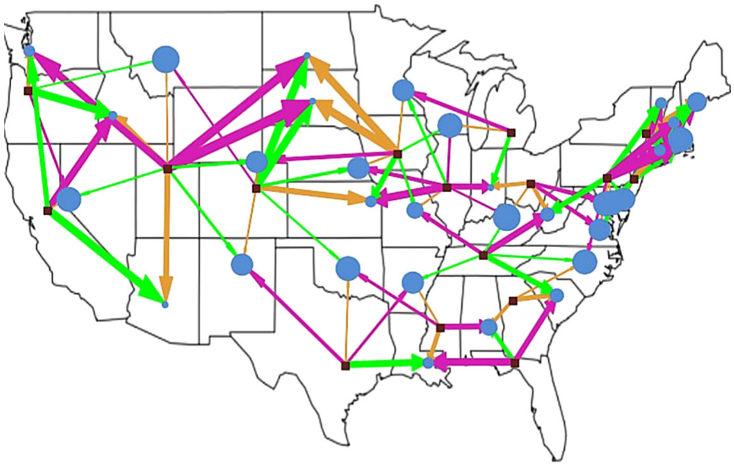
Fig. 3. Sensitivity analysis on parameters $\boldsymbol{c_f}$ ((a) and (b)), $\boldsymbol{c_d}$ ((c) and (d)), and $\boldsymbol{c_t}$ ((e) and (f)).

expedited shipments and different colors denoting different levels, i.e., yellow for the first level, green for the second level and pink for the third level.

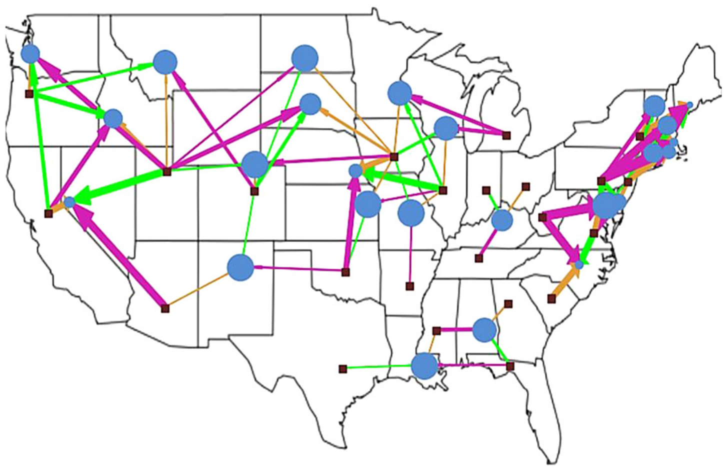
In Fig. 4(a), when the disruption risks are ignored ($q = 0$), the problem would be similar to the integrated model proposed by Li (2013), in which all suppliers are assumed to be reliable and is considered as the benchmark case in our problem. By



(a) $q = 0\%$



(b) $q = 30\%$



(c) $q = 60\%$

Fig. 4. Optimal network layouts under different $\boldsymbol{q}$.

comparing Fig. 4(a) and (b), we note that as failure probability q increases from 0 to 0.3, 5 more supplier installations are selected and more frequent expedited shipments are needed, in particular for the terminals that are far away from their suppliers. This implies that when primary suppliers become unreliable, backup ones are needed and a proper solution is selecting more suppliers to reduce the overall shipment cost. Generally, the expedition percentage increases with the assignment level, and terminals served by farther suppliers hold higher inventory positions.

As q further rises to 0.6, we can see that many more suppliers are installed and the inventory positions of terminals increase generally, but the expedition percentage generally drops. Intuitively, this is because that, as the suppliers become more unreliable, the better solution to offset the growth of shipment cost is to increase the intensity of suppliers distribution and the position of terminals inventory, which will consequently bring down the overall stock-out times and expedition percentage. Also, we can see that 7 more suppliers are installed and most of them (5 suppliers) are located in the northeastern areas with higher population and more terminals. Therefore, under the optimal planning, terminals with more demand may be met first to reduce the shipment costs as much as possible.

6. Conclusion

This paper presented a reliability model that takes the possibility of supplier failures into the design of integrated logistics system involving logistics network planning and long-term operations management. We formulated a nonlinear integer model to investigate this problem. The model aims to optimize the number of suppliers and their distribution, the assignment of regular shipment services at multiple levels and the expedited services between the suppliers and the terminals, and the base-stock inventory positions. Since the proposed model is difficult to solve (nonlinear integer programming problem), a customized solution approach adapted from Li (2013) is created based on LR. This solution approach is able to solve this model efficiently and accurately, as evidenced in a set of numerical experiments. Moreover, according to the results of these experiments, some major observations are drawn as:

Observation 1: The increased expedited shipments can ensure the reliability of the integrated logistics system without an excessive increase of the operating costs.
Observation 2: When the regular shipment services of upstream suppliers become unreliable, expedited shipments will be used more frequently in comparison with regular shipments.
Observation 3: The optimal network layout and the related cost components vary significantly with different disruption probabilities.
Observation 4: When the disruption probability increases, areas with more customer demands tend to have a high priority to receive services, despite the higher transportation costs.

The proposed model can be further improved in several directions. First, this study assumes that the expedited shipment is "non-fallible", which may be not realistic for some applications where the suppliers may suffer serious disasters causing both services failed. Second, it might be worth considering positive lead times even for expedited shipments for some applications where the expedited delivery time is still noticeable. Third, the disruption probabilities of the regular shipment services of all the suppliers are assumed to be the same, which may not be appropriate for all practical problems. Several approaches have been proposed for relaxing this assumption in the recent studies including Li et al. (2013b), Wang and Ouyang (2013) and Lu et al. (2015). Following these developments, the proposed model in this study could be further extended to one with heterogeneous disruption probabilities. Fourth, the transportation costs and travel time between suppliers and terminals are defined as parameters in the proposed model. Further investigation is needed for cases where transportation cost and travel time depend on the system decisions (e.g., they are functions of shipment rates). Finally, this study is set as a two-echelon system where the locations and demand of all the terminals are considered to be in the basic conditions and independent of the network design results. However, in some other applications, a more general structure is needed for terminals distribution planning. Extending the current two-echelon network to a more general structure will be an interesting research topic.

Acknowledgement

This research is supported by in part by the U.S. National Science Foundation through Grants CMMI #1558889, #1541130 and #1638355, the National Center for Intermodal Transportation for Economic Competitiveness (US Tier I University Transportation Center) through Grant # 13091085 and the National Natural Science Foundation of China through Grant #51478151.

Appendix A. Bisecting algorithm to solve (25)

Step BS0: For a given set of $k \in \mathbf{K}, i' \in \mathbf{I}, j \in \mathbf{J}$, initialize two search bounds as $S_L := 0$ and $S_U := \bar{S}_j$, and the difference slope of $C_{ki'j}(S_j)$ defined in Eq. (25) with respect to S_L as:

$$G_L := h_j + d_j \sum_{l \in \mathbf{L}} (e_{i'j} - r_{i_l j})(1-q)q^{l-1}(P_{i_l j}(0) - P_{i_l j}(1)),$$

and that with respect to S_U as:

$$G_U := h_j + d_j \sum_{l \in \mathbf{L}} (e_{i'j} - r_{ij})(1-q)q^{l-1}(P_{ij}(\bar{S}_j) - P_{ij}(\bar{S}_j - 1)).$$

Step BS1: If $G_L, G_U \geqslant 0$, set optimal $S^* := S_L$. Or if $G_L, G_U < 0$, set optimal $S^* := S_U$. Or if $S_U - S_L \leqslant 1$, set $S^* := S_L$ if $C_{ki'j}(S_L) \leqslant C_{ki'j}(S_U)$ or $S^* := S_U$ otherwise. If S^* is found, go to Step BS3.

Step BS2: Set the middle point $S_M := \lfloor (S_L + S_U)/2 \rfloor$. Calculate the slope at S_M as: $G_M := h_j + d_j \sum_{l \in \mathbf{L}} (e_{i'j} - r_{ij})(1-q)q^{l-1}(P_{ij}(0) - P_{ij}(1))$ if $S_M = 0$ or $G_M := h_j + d_j \sum_{l \in \mathbf{L}} (e_{i'j} - r_{ij})(1-q)q^{l-1}(P_{ij}(S_M) - P_{ij}(S_M - 1))$ otherwise. If $S_M > 0$, set $S_U = S_M$ and $G_U = G_M$; otherwise, set $S_L = S_M$ and $G_L = G_M$. Go to Step BS1.

Step BS3: Return S_j^* and $C_{ki'j}(S_j^*)$ as the optimal solution and the optimal objective value to problem (25), respectively.

Appendix B. Subgradient algorithm to update Lagrangian multipliers

Step SG0: Set initial multipliers $\lambda_{ij}^0 = \mu_{ij}^0 = 0, \quad \forall i \in \mathbf{I}, j \in \mathbf{J}$. Set an auxiliary scalar $0 < \tau \leqslant 2$ and an iteration index $k := 0$. Set the best known upper bound objective $C := +\infty$.

Step SG1: Solve relaxed problem $\Delta(\boldsymbol{\lambda}^k, \boldsymbol{\mu}^k)$ with the solution approach proposed in Section 4.1, and $\{X_i^k\}, \{Y_{ij}^k\}, \{Z_{ij}^k\}, \{S_j^k\}$ denote its optimal solution. If the objective value of $\Delta(\boldsymbol{\lambda}^k, \boldsymbol{\mu}^k)$ does not improve in K consecutive iterations (where K is a pre-defined number, e.g., 5), we update $\tau = \tau/\theta$, where h is a contraction ratio greater than it.

Step SG2: Adapt $\{X_i^k\}, \{Y_{ij}^k\}, \{Z_{ij}^k\}, \{S_j^k\}$ to a set of feasible solution with the algorithm described in Section 4.2. Set C equal to this feasible objective if C is greater than it.

Step SG3: Calculate the step size as follows[1]

$$t_k := \frac{\tau(C - \Delta(\boldsymbol{\lambda}^k, \boldsymbol{\mu}^k))}{\sum_{i \in \mathbf{I}, j \in \mathbf{J}} \left(\left(\sum_{l \in \mathbf{L}} Y_{ijl}^k - X_i^k \right)^+ + (Z_{ij}^k - X_i^k)^+ \right)}.$$

Then update multipliers as follows

$$\lambda_{ij}^{k+1} = \left[\lambda_{ij}^k + t_k \left(\sum_{l \in \mathbf{L}} Y_{ijl}^k - X_i^k \right) \right]^+, \mu_{ij}^{k+1} = [\mu_{ij}^k + t_k(Z_{ij}^k - X_i^k)]^+, \quad \forall i \in \mathbf{I}, j \in \mathbf{J}.$$

Step SG4: Terminate this algorithm if (i) optimality gap $\frac{C - \Delta(\boldsymbol{\lambda}^k, \boldsymbol{\mu}^k)}{C} \leqslant \varepsilon$ where ε is a pre-specified error tolerance, (ii) τ is smaller than a minimum value $\bar{\tau}$, or (iii) k exceeds a maximum iteration number $\bar{K}$; return the best feasible solution as the near-optimum solution. Otherwise $k = k + 1$, and go to Step SG1.

Appendix C. Branch-and-bound approach

To further improve the performance of the solution method, we implement the LR algorithm into a branch-and-bound framework. We first obtain the initial solution by running the LR algorithm and then branch on variables X in a depth-first manner. A greedy heuristic is used to determine the branching older: Given the other variables, candidate supplier i is chosen as the next variable to be branched if setting up at i brings the greatest decrease of the objective value (10). Note that the method for obtaining the optimal objective values is referred to Section 4.2. Then two child nodes are obtained where $X_i = 1$ in one node and $X_i = 0$ in the other. At each node, we run the LR algorithm for 100 s (which is less than the running time of the pure LR algorithm without branch-and-bound) to determine the lower and upper bounds. If the lower bound is higher than the best feasible solution so far, then no more branching is needed over this node. If all the candidate suppliers have been branched in the current node and only one feasible solution exists, then the solution shall be returned as both the upper and lower bounds. Note that the final multipliers shall be passed down to its child nodes as their initial multipliers.

To address the performance of the branch-and-bound approach above, we perform 4 instances to compare the results with LR, which are shown in Table 5.

We can see from the experiment results that the branch-and-bound procedure is not efficient for solving the proposed model to a moderate-scale problem. It's probably because it takes a long time to process one iteration of LR algorithm, which causes a relatively small number of iterations for one node. As a result, the gap of the branch-and-bound procedure is no less than that of the LR algorithm alone within the same solution time.

Appendix D. Results of 88-site and 150-site cases

To verify the practical applicability of the proposed model and solution method, we test them with the 88-site and 150-site network cases and the results of 27 representative instances are shown as Tables 6 and 7. The parameters are set the

[1] In the denominator of this formula, we use the absolute value instead of the squared Euclidean norm, because we found it helps improve the solution efficiency.

Table 5
Comparisons of LR and Branch-and-bound.

#	q	Solution time (sec)		Gap (%)	
		LR	Branch-and-bound	LR	Branch-and-bound
1	0.1	1000	1000	0.13	0.14
2	0.3	1000	1000	0.17	0.19
3	0.5	1000	1000	0.41	0.43
4	0.7	1000	1000	0.47	0.50

Table 6
Numerical results for 88-site network.

#	q	h	c_e	T	G (%)	N	C^F	S	C^H	C^R	C^M	P^E (%)	C
1	0.1	10	1	3019	0.62	7	8864	563	5630	12152.6	1630.6	0.62	22878.4
2	0.1	10	2	2745	0.80	7	8864	610	6100	12221.4	1421.1	0.5	22897.7
3	0.1	10	10	2153	0.75	8	9831	659	6590	12418.3	1306.4	0.27	23921.6
4	0.1	20	1	4208	1.05	9	11,716	575	11,500	11606.9	4401.9	1.44	30457.8
5	0.1	20	2	3191	0.61	9	11,716	594	11,880	11752.1	4007.8	0.63	30571.6
6	0.1	20	10	2819	0.33	10	14,216	613	12,260	11827.5	2237.9	0.33	32509.3
7	0.1	100	1	3005	0.90	12	21,498	248	24,800	10257.2	7327.7	3.76	49819.9
8	0.1	100	2	2505	0.83	13	23,898	252	25,200	10378.7	5920.4	2.31	51572.1
9	0.1	100	10	2493	0.95	13	23,898	274	27,400	10427.7	5390.8	1.58	53043.6
10	0.3	10	1	3284	0.65	14	24,658	576	5760	13786.8	8100.3	7.56	42314.7
11	0.3	10	2	3764	0.51	14	24,658	596	5960	13847.5	7839.4	7.04	42460.2
12	0.3	10	10	2422	0.70	15	25,177	636	6360	13,931	7627.9	5.9	43,244
13	0.3	20	1	3351	0.76	15	25,177	583	11,660	13614.4	14330.9	7.78	49152.2
14	0.3	20	2	2511	0.91	15	25,177	592	11,840	13749.7	13467.3	7.11	49275.1
15	0.3	20	10	2497	1.19	16	27,690	637	12,740	13870.1	12512.6	6.23	51116.5
16	0.3	100	1	3924	1.62	16	27,690	314	31,400	12,580	25897.6	14.06	69379.3
17	0.3	100	2	2850	1.83	16	27,690	335	33,500	12665.5	23007.6	13.51	69555.7
18	0.3	100	10	3471	1.92	17	28,013	351	35,100	12804.2	20577.1	11.73	69680.3
19	0.5	10	1	2914	2.04	18	29,615	490	4900	12,360	8249.1	11.65	47130.4
20	0.5	10	2	2833	2.18	18	29,615	539	5390	12484.5	8003.5	11.33	47328.1
21	0.5	10	10	3932	2.63	19	30,355	576	5760	14,820	7912.3	10.31	50,743
22	0.5	20	1	3572	2.80	19	30,355	466	9320	12241.9	14235.5	10.84	53708.8
23	0.5	20	2	2592	2.62	20	31,162	491	9820	12363.7	14033.3	10.05	54,694
24	0.5	20	10	2582	2.73	20	31,162	538	10,760	12466.3	13038.2	8.8	55911.9
25	0.5	100	1	3015	2.87	21	32,591	290	29,000	11660.8	32535.8	20.17	79788.9
26	0.5	100	2	2671	2.92	22	33,127	335	33,500	11752.6	29438.8	18.02	81521.8
27	0.5	100	10	2826	2.89	24	35,073	370	37,000	12278.5	27601.9	16.72	84921.3

Table 7
Numerical results for 150-site network.

#	q	h	c_e	T	G (%)	N	C^F	S	C^H	C^R	C^M	P^E (%)	C
1	0.1	10	1	3600	4.12	12	18,342	725	7250	15058.2	3270.8	0.73	43921.0
2	0.1	10	2	3600	4.00	13	20,065	777	7770	15144.9	2447.1	0.56	45427.0
3	0.1	10	10	3600	4.28	13	20,065	811	8110	15188.3	2171.0	0.37	45534.3
4	0.1	20	1	3600	4.26	14	20,903	751	15,020	14800.4	7032.6	1.57	57756.0
5	0.1	20	2	3600	4.72	14	20,903	767	15,340	14844.6	6898.7	0.72	57986.3
6	0.1	20	10	3600	4.89	15	22,814	791	15,820	14968.3	5369.4	0.38	58971.7
7	0.1	100	1	3600	3.97	18	29,438	317	31,700	13551.1	10611.8	3.79	85300.9
8	0.1	100	2	3600	4.16	19	30,094	327	32,700	13562.8	9928.9	2.68	86285.7
9	0.1	100	10	3600	4.34	20	31,637	349	34,900	13750.1	8039.2	1.61	88326.3
10	0.3	10	1	3600	5.14	21	32,487	750	7500	15077.0	13849.2	8.02	68913.2
11	0.3	10	2	3600	5.03	21	32,487	769	7690	15115.8	13798.9	7.25	69091.7
12	0.3	10	10	3600	5.17	21	32,487	778	7780	15142.4	13749.8	6.96	69159.2
13	0.3	20	1	3600	5.35	21	32,487	733	14,660	14997.3	19557.1	8.83	81701.4
14	0.3	20	2	3600	5.70	22	33,009	751	15,020	15041.0	19653.2	7.45	82723.2
15	0.3	20	10	3600	5.92	22	33,009	779	15,580	15084.8	19344.9	6.77	83018.7
16	0.3	100	1	3600	5.33	22	33,009	564	56,400	14016.0	33685.0	15.81	137110.0
17	0.3	100	2	3600	5.54	22	33,009	580	58,000	14045.0	33648.7	14.14	138702.7
18	0.3	100	10	3600	5.61	23	34,809	595	59,500	14098.9	30402.3	12.38	138810.2
19	0.5	10	1	3600	6.27	27	38,594	636	6360	15522.2	17168.6	11.37	77644.8
20	0.5	10	2	3600	6.50	27	38,594	734	7340	15532.7	18078.8	10.45	79545.5

(continued on next page)

Table 7 (*continued*)

#	q	h	c_e	T	G (%)	N	C^F	S	C^H	C^R	C^M	P^E (%)	C
21	0.5	10	10	3600	6.71	28	39,894	759	7590	15661.9	16542.4	8.94	79688.3
22	0.5	20	1	3600	6.44	30	41,078	618	12,360	15462.6	19686.1	12.10	88586.7
23	0.5	20	2	3600	6.56	31	42,461	654	13,080	15486.3	19768.7	10.97	90796.0
24	0.5	20	10	3600	6.89	31	42,461	680	13,600	15501.3	19317.9	9.38	90880.2
25	0.5	100	1	3600	8.94	38	57,091	575	57,500	14479.0	41186.7	20.79	170256.7
26	0.5	100	2	3600	7.35	38	57,091	603	60,300	14493.2	44379.6	19.42	176263.8
27	0.5	100	10	3600	8.08	39	57,880	625	62,500	14541.4	43348.8	17.74	178270.2

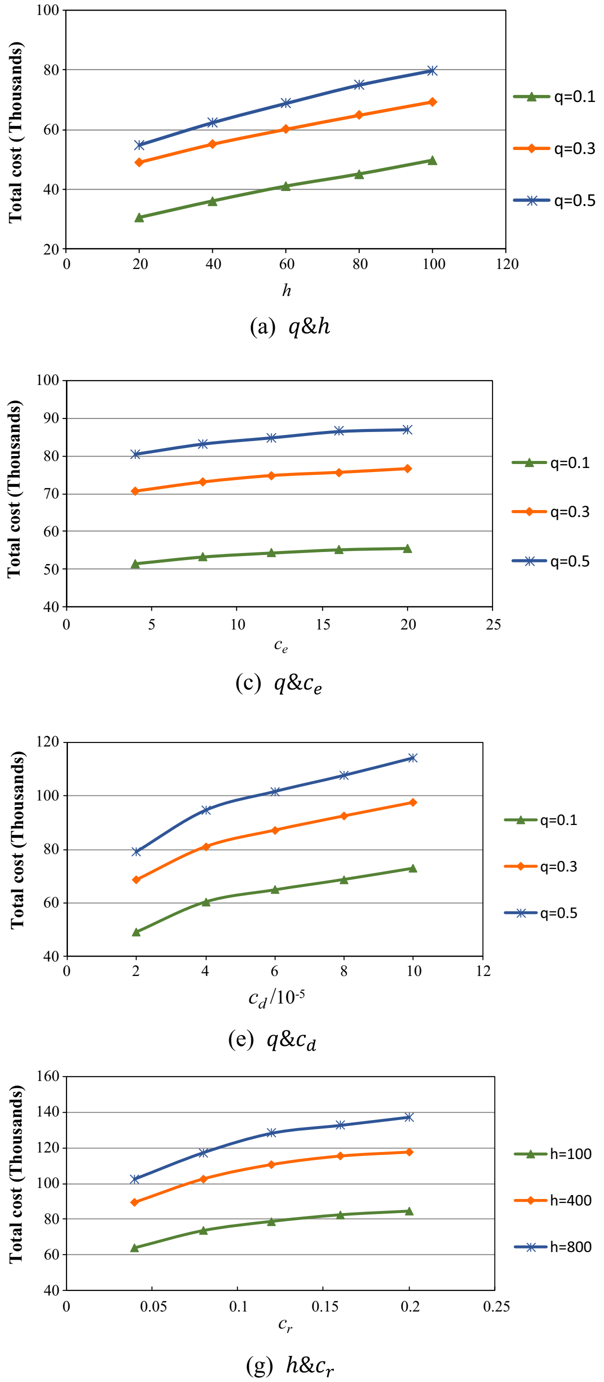
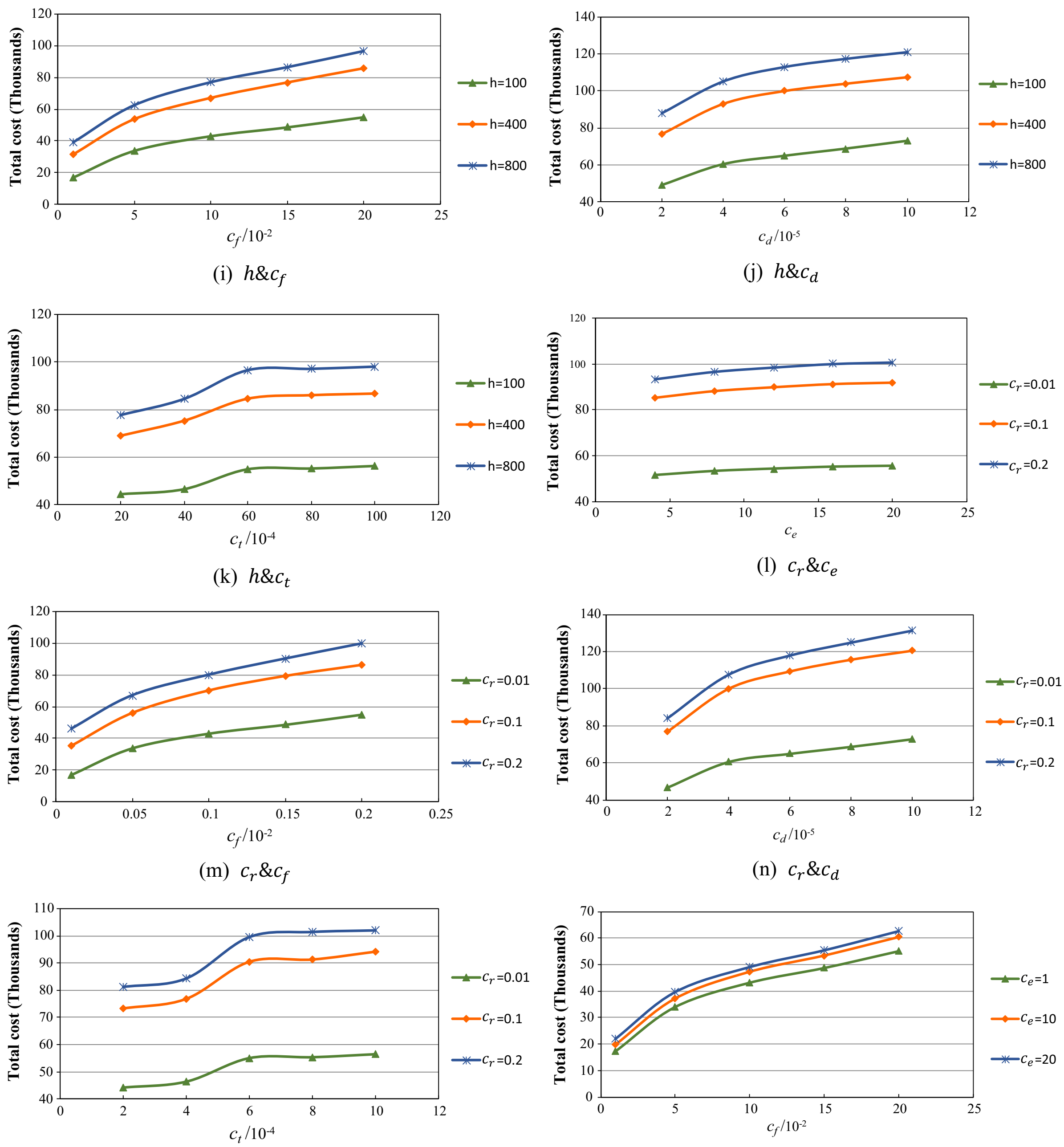


Fig. 5. Interaction analysis among different parameters.

same as those in the 49-site network case except that we set $c_d = 10^{-4}$. We see that each instance can still be solved to near optimum ($G < 10\%$) in an acceptable time despite the much expended network sizes. This indicates that the customized solution method is capable to handling real-word sized problem instances.

Appendix E. Interaction analysis among the key parameters

To test the interactions among the key parameters, we performed a series of experiments and the results are shown as Fig. 5(a)–(u), where we do not see significant interactions among these parameters.



(i) $h\&c_f$ (j) $h\&c_d$

(k) $h\&c_t$ (l) $c_r\&c_e$

(m) $c_r\&c_f$ (n) $c_r\&c_d$

(o) $c_r\&c_t$ (p) $c_e\&c_f$

Fig. 5 (*continued*)

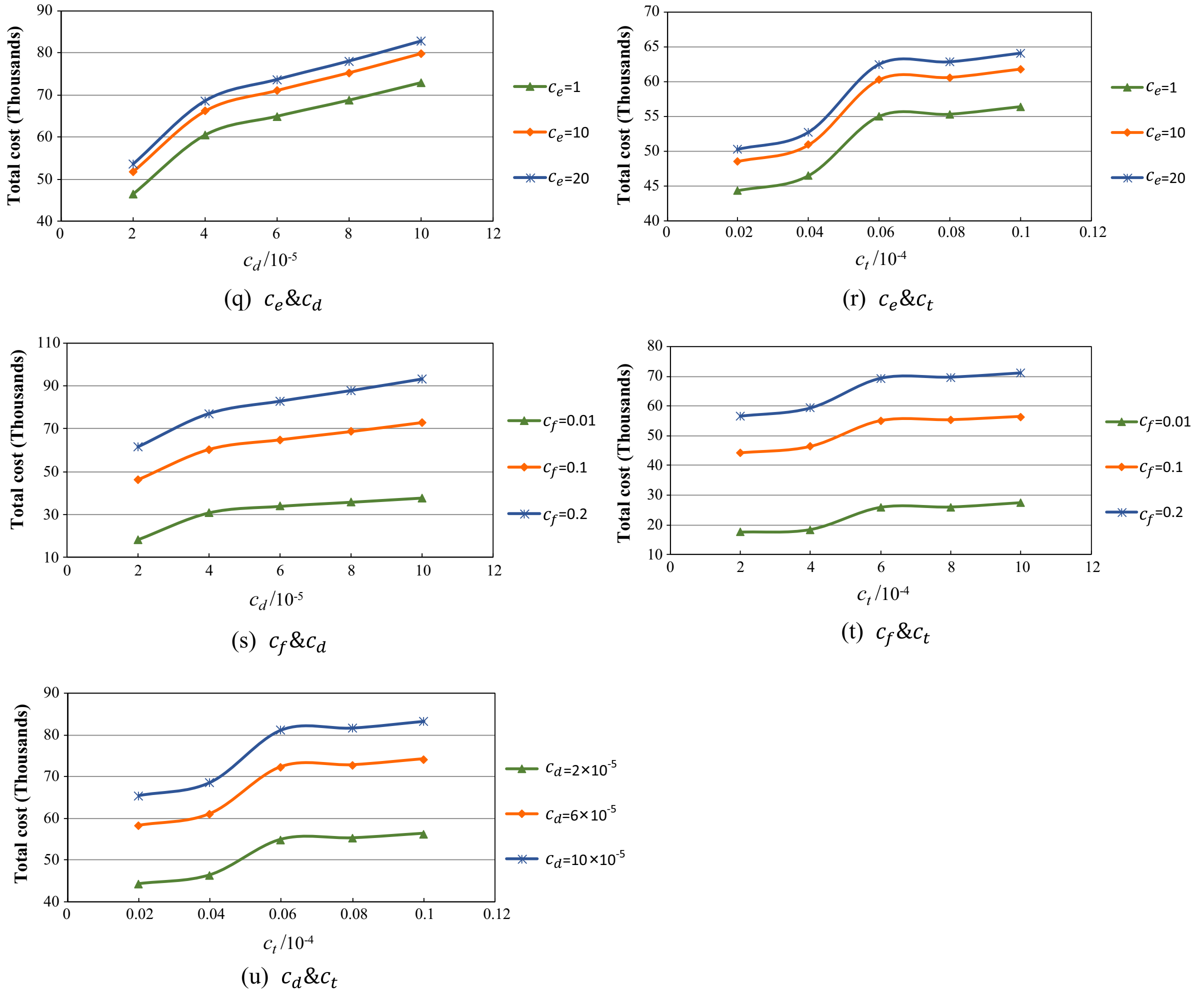


Fig. 5 (*continued*)

References

An, K., Ouyang, Y., 2016. Robust grain supply chain design considering post-harvest loss and harvest timing equilibrium. Transp. Res. Part E: Logist. Transp. Rev. 88, 110–128.

An, Y., Zeng, B., Zhang, Y., Zhao, L., 2014. Reliable p-median facility location problem: two-stage robust models and algorithms. Transp. Res. Part B: Methodol. 64, 54–72.

Bai, Yun, Li, Xiaopeng, Peng, Fan, Wang, Xin, Ouyang, Yanfeng, 2015. Effects of disruption risks on biorefinery location design. Energies 8 (2), 1468–1486.

Batta, Rajan, Dolan, June M., Krishnamurthy, Nirup N., 1989. The maximal expected covering location problem: revisited. Transp. Sci. 23 (4), 277–287.

Berman, Oded, Krass, Dmitry, 2011. On n-facility median problem with facilities subject to failure facing uniform demand. Discrete Appl. Math. 159 (6), 420–432.

Blake, Eric S., Kimberlain, Todd B., Berg, Robert J., Cangialosi, J.P., Beven II, John L., 2013. Tropical cyclone report: hurricane sandy. Natl. Hurricane Center 12, 1–10.

Caggiano, Kathryn E., Muckstadt, John A., Rappold, James A., 2006. Integrated real-time capacity and inventory allocation for reparable service parts in a two-echelon supply system. Manuf. Serv. Oper. Manage. 8 (3), 292–319.

Chan, Emily Y.Y., 2008. The untold stories of the Sichuan earthquake. The Lancet 372 (9636), 359–362.

Chen, Qi, Li, Xiaopeng, Ouyang, Yanfeng, 2011. Joint inventory-location problem under the risk of probabilistic facility disruptions. Transp. Res. Part B: Methodol. 45 (7), 991–1003.

Cui, Tingting, Ouyang, Yanfeng, Shen, Zuo-Jun Max, 2010. Reliable facility location design under the risk of disruptions. Oper. Res. 58 (4-part-1), 998–1011.

Daskin, Mark S., 1982. Application of an expected covering model to emergency medical service system design. Decision Sci. 13 (3), 416–439.

Daskin, M.S., 1983. A maximum expected covering location model: formulation, properties and heuristic solution. Transp. Sci. 17 (1), 48–70.

Daskin, M.S., 1995. Network and Discrete Location: Models, Algorithms and Applications. Wiley, New York, NY. ISBN 0 47101 897.

Daskin, Mark S., Coullard, Collette R., Shen, Zuo-Jun Max, 2002. An inventory-location model: formulation, solution algorithm and computational results. Ann. Oper. Res. 110 (1–4), 83–106.

Drezner, Zvi, 1995. Facility Location: A Survey of Applications and Methods. Springer.

Geoffrion, A.M., 1974. Lagrangean Relaxation for Integer Programming. Springer, Berlin Heidelberg.

Gibson Brian J., Clifford Defee, C., Ishfaq Rafay. 2015. The State of The Retail Supply Chain. <http://www.rila.org/supply/resources/Documents/Fifth%20Annual%20SRSC%20Report.pdf>.

Hasani, A., Khosrojerdi, A., 2016. Robust global supply chain network design under disruption and uncertainty considering resilience strategies: a parallel memetic algorithm for a real-life case study. Transp. Res. Part E: Logist. Transp. Rev. 87, 20–52.

Holt, Mark, Campbell, Richard J., Nikitin, Mary Beth, 2012. Fukushima nuclear disaster. Congr. Res. Serv. <http://digitallibrary.unt.edu/ark:/67531/metadc87170/m1/1/high_res_d/R41694_2012Jan18.pdf>.

Huggins, Eric Logan, Olsen, Tava Lennon, 2003. Supply chain management with guaranteed delivery. Manage. Sci. 49 (9), 1154–1167.

Klose, Andreas, Drexl, Andreas, 2005. Facility location models for distribution system design. Eur. J. Oper. Res. 162 (1), 4–29.

Karush, W., 1957. A queuing model for an inventory problem. Oper. Res. 5 (5), 693703.

Laporte Gilbert. 1987. Location Routing Problems. <http://trid.trb.org/view.aspx?id=1186529>.

Lee, Hau L., Padmanabhan, Venkata, Whang, Seungjin, 1997. The bullwhip effect in supply chains 1. Sloan Manage. Rev. 38 (3), 93–102.

Liberatore, Federico, Scaparra, Maria P., Daskin, Mark S., 2012. Hedging against disruptions with ripple effects in location analysis. Omega 40 (1), 21–30.

Li, Q., Zeng, B., Savachkin, A., 2013a. Reliable facility location design under disruptions. Comput. Oper. Res. 40 (4), 901–909.

Li, Xiaopeng, 2013. An integrated modeling framework for design of logistics networks with expedited shipment services. Transp. Res. Part E: Logist. Transp. Rev. 56, 46–63.

Li, Xiaopeng, Ouyang, Yanfeng, 2010. A continuum approximation approach to reliable facility location design under correlated probabilistic disruptions. Transp. Res. Part B: Methodol. 44 (4), 535–548.

Li, Xiaopeng, Ouyang, Yanfeng, 2011. Reliable sensor deployment for network traffic surveillance. Transp. Res. Part B: Methodol. 45 (1), 218–231.

Li, Xiaopeng, Ouyang, Yanfeng, 2012. Reliable traffic sensor deployment under probabilistic disruptions and generalized surveillance effectiveness measures. Oper. Res. 60 (5), 1183–1198. http://dx.doi.org/10.1287/opre.1120.1082.

Li, Xiaopeng, Ouyang, Yanfeng, Peng, Fan, 2013b. A supporting station model for reliable infrastructure location design under interdependent disruptions. Transp. Res. Part E: Logist. Transp. Rev. 60, 80–93.

Lu, Mengshi, Ran, Lun, Shen, Zuo-Jun Max, 2015. Reliable facility location design under uncertain correlated disruptions. Manuf. Serv. Oper. Manage. 17 (4), 445–455.

Teresa Melo, M., Nickel, Stefan, Saldanha Da Gama, F., 2006. Dynamic multi-commodity capacitated facility location: a mathematical modeling framework for strategic supply chain planning. Comput. Oper. Res. 33 (1), 181–208.

Peng, M., Peng, Y., Chen, H., 2014. Post-seismic supply chain risk management: a system dynamics disruption analysis approach for inventory and logistics planning. Comput. Oper. Res. 42, 14–24.

Ouyang, Yanfeng, Daganzo, Carlos F., 2006. Discretization and validation of the continuum approximation scheme for terminal system design. Transp. Sci. 40 (1), 89–98.

Ouyang, Yanfeng, Li, Xiaopeng, 2010. The bullwhip effect in supply chain networks. Eur. J. Oper. Res. 201 (3), 799–810.

Qi, Lian, Lee, Kangbok, 2014. Supply chain risk mitigations with expedited shipping. Omega <http://www.sciencedirect.com/science/article/pii/S0305048314001492>.

Qi, Lian, Shen, Zuo-Jun Max, Snyder, Lawrence V., 2010. The effect of supply disruptions on supply chain design decisions. Transp. Sci. 44 (2), 274–289.

ReVelle, Charles, Hogan, Kathleen, 1989. The maximum availability location problem. Transp. Sci. 23 (3), 192–200.

Şahin, Güvenç, Süral, Haldun, 2007. A review of hierarchical facility location models. Comput. Oper. Res. 34 (8), 2310–2331.

Salhi, Said, Petch, R.J., 2007. A GA based heuristic for the vehicle routing problem with multiple trips. J. Math. Model. Algorithms 6 (4), 591–613.

Sawik, T., 2015. Integrated supply chain scheduling under multi-level disruptions. IFAC-PapersOnLine 48 (3), 1515–1520.

Shahabi, M., Unnikrishnan, A., Jafari-Shirazi, E., Boyles, S.D., 2014. A three level location-inventory problem with correlated demand. Transp. Res. Part B: Methodol. 69, 1–18.

Shen, Zuo-Jun Max, Coullard, Collette, Daskin, Mark S., 2003. A joint location-inventory model. Transp. Sci. 37 (1), 40–55.

Shen, Zuo-Jun Max, Qi, Lian, 2007. Incorporating inventory and routing costs in strategic location models. Eur. J. Oper. Res. 179 (2), 372–389.

Shishebori, Davood, Snyder, Lawrence V., Jabalameli, Mohammad Saeed, 2014. A reliable budget-constrained fl/nd problem with unreliable facilities. Networks Spatial Econ. 14 (3–4), 549–580.

Shu, Jia, Teo, Chung-Piaw, Shen, Zuo-Jun Max, 2005. Stochastic transportation-inventory network design problem. Oper. Res. 53 (1), 48–60.

Snyder, Lawrence V., Daskin, Mark S., 2005. Reliability models for facility location: the expected failure cost case. Transp. Sci. 39 (3), 400–416.

Snyder, Lawrence V., Daskin, Mark S., Teo, Chung-Piaw, 2007. The stochastic location model with risk pooling. Eur. J. Oper. Res. 179 (3), 1221–1238.

Taghaboni-Dutta, Fataneh, 2003. E-commerce strategies for heightened security needs of supply-chain management. IJEBM 1 (1), 9–14.

Tseng, Yung-yu, Yue, Wen Long, Taylor, Michael A.P., et al, 2005. The role of transportation in logistics chain. Eastern Asia Soc. Transp. Stud. <http://222255.132.18:8085/Portals/0/Docs/51293331-1657.pdf>.

Tomlin, B., 2006. On the value of mitigation and contingency strategies for managing supply chain disruption risks. Manage. Sci. 52 (5), 639–657.

Wang, X., Ouyang, Y., 2013. A continuum approximation approach to competitive facility location design under facility disruption risks. Transp. Res. Part B: Methodol. 50, 90–103.

Weber, Alfred, 1929. Theory of the Location of Industries [Translated by CJ Friedrich From Weber's 1909 Book]. The University of Chicago Press, Chicago.

Xie, S., Li, X., Ouyang, Y., 2015. Decomposition of general facility disruption correlations via augmentation of virtual supporting stations. Transp. Res. Part B: Methodol. 80, 64–81.

Yun, L., Qin, Y., Fan, H., Ji, C., Li, X., Jia, L., 2015. A reliability model for facility location design under imperfect information. Transp. Res. Part B: Methodol. 81, 596–615.

Yu, H., Zeng, A.Z., Zhao, L., 2009. Single or dual sourcing: decision-making in the presence of supply chain disruption risks. Omega 37 (4), 788–800.

Zhou, Sean X., Chao, Xiuli, 2010. Newsvendor bounds and heuristics for serial supply chains with regular and expedited shipping. Naval Res. Logist. 57 (1), 71–87.