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Review

Cosmological particle production: a review

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Abstract

This article will review quantum particle creation in expanding universes. The emphasis will be on the basic physical principles and on selected applications to cosmological models. The needed formalism of quantum field theory in curved spacetime will be summarized, and applied to the example of scalar particle creation in a spatially flat Universe. Estimates for the creation rate will be given and applied to inflationary cosmology models. Analog models which illustrate the same physical principles and may be experimentally realizable are also discussed.

Keywords: particle creation, quantum fields in curved space, quantum effects in the early universe

(Some figures may appear in colour only in the online journal)

1. Introduction and basic concepts

1.1. Preliminary remarks

This review will cover selected aspects of quantum field theory in curved spacetime, with emphasis on quantum particle creation in expanding universes. For broader treatments of quantum field theory in curved spacetime, see, for example, the books by Birrell and Davies [1] and by Parker and Toms [2]. As much as possible, the focus will be on the key physical ideas more than the mathematical formalism, but a certain amount of the latter is essential. I have attempted to give a balanced selection of references, but it is not possible to cite all of the numerous papers which have been written on cosmological particle creation and the related topics covered in this review. I apologize in advance to the authors whose work has not been cited.

1.2. An overview

This review will deal with quantized fields propagating on a classical background spacetime other than Minkowski space. As general relativity describes a gravitational field as a curved four dimensional spacetime, this is a means to describe the interaction of quantum particles with a background gravitational field. One of the key features of quantum field theory in Minkowski spacetime is heavy reliance on Poincare symmetry, which is broken by the background gravitational field. If the gravitational field is spatially inhomogeneous, then space translation symmetry is broken, and linear momentum in no longer conserved. The initial and final momenta of particles scattering from a localized gravitational field need not be equal, as the gravitational field can absorb some momentum, as in the case of a particle scattering from a black hole.

If the background gravitational field is time-dependent, as in the case of an expanding Universe, then time translation symmetry is broken. Now the energy of the quantum particles need not be conserved, and the quantum field may absorb energy from the background. This will play a key role in cosmological particle creation.

The outline of this review is as follows: the formalism of field quantization in a curved spacetime is reviewed in section 1.3, and the mathematical tools needed to describe quantum particle creation will be developed in section 2. This will include Bogolubov transformations, and a discussion of some exactly soluble models, as well as of a perturbative

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approximation method. Some estimates of particle creation rate will be discussed. Section 3 will treat the quantum state of the created particles, and discuss decoherence and entropy production. The implications of quantum particle creation for inflationary cosmology, and for subsequent epochs of the Universe, are discussed in section 4. The backreacion of the created particle on the expansion of the Universe is the topic of section 5. Section 6 discusses some other physical systems which rely upon the same principles as cosmological particle creation, and may be able to be studied in the laboratory. Section 7 summarizes the key topics covered in the review.

1.3. Quantization in curved spacetime

There are four basic ingredients in the construction of a quantum field theory. These are

- The Lagrangian, or equivalently, the equation of motion of the classical theory.
- A quantization procedure, such as canonical quantization or the path integral approach.
- The characterization of the quantum states.
- The physical interpretation of the states and of the observables.

In flat spacetime, Lorentz invariance plays an important role in each of these steps. For example, it is a guide which generally allows us to identify a unique vacuum state for the theory. However, in curved spacetime, we do not have Lorentz symmetry. This is not a crucial problem in the first two steps listed above. The formulation of a classical field theory and its formal quantization may be carried through in an arbitrary spacetime. The real differences between flat space and curved space arise in the latter two steps. In general, there does not exist a unique vacuum state in a curved spacetime. As a result, the concept of particles becomes ambiguous, and the problem of the physical interpretation becomes much more difficult.

The best way to discuss these issues in more detail is in the context of a particular model theory. Let us consider a real, massive scalar field for which the Lagrangian density is

$$\mathcal{L} = -\frac{1}{2}(\partial_\alpha\varphi\partial^\alpha\varphi + m^2\varphi^2 + \xi R\varphi^2). \tag{1.1}$$

(We adopt the $(+++)$ sign conventions of Misner, Thorne, and Wheeler [3]. In particular, the metric signature will be $(-+++)$. Unless otherwise noted, units in which $G = c = \hbar = 1$ are used.) The corresponding wave equation is

$$\Box\varphi - m^2\varphi - \xi R\varphi = 0. \tag{1.2}$$

Here m is the mass, R is the scalar curvature, and ξ is a new coupling constant. There are two popular choices for ξ: minimal coupling ($\xi = 0$) and conformal coupling ($\xi = 1/6$). The former leads to the simplest equation of motion, whereas the latter leads to a theory which is conformally invariant in four dimensions in the massless limit. For our purposes, we need not settle this issue, but rather regard ξ on the same footing as m, as a parameter which specifies our theory. Note that here $\Box$ denotes the generally covariant d'Alembertian operator, $\Box = \nabla_\mu\nabla^\mu$.

A useful concept is that of the *inner product* of a pair of solutions of the generally covariant Klein–Gordon equation, equation (1.2). It is defined by

$$(f_1, f_2) = i\int (f_2^*\overset{\leftrightarrow}{\partial}_\mu f_1)\mathrm{d}\Sigma^\mu = i\int (f_2^*\partial_\mu f_1 - f_1\partial_\mu f_2^*)\mathrm{d}\Sigma^\mu, \tag{1.3}$$

where $\mathrm{d}\Sigma^\mu = \mathrm{d}\Sigma\, n^\mu$, with $\mathrm{d}\Sigma$ being the volume element in a given spacelike hypersurface, and n^μ being the timelike unit vector normal to this hypersurface. The crucial property of the inner product is that it is independent of the choice of hypersurface. That is, if Σ_1 and Σ_2 are two different, non-intersecting hypersurfaces, then

$$(f_1, f_2)_{\Sigma_1} = (f_1, f_2)_{\Sigma_2}. \tag{1.4}$$

The proof of this property is straightforward. We assume that f_1 and f_2 are both solutions of equation (1.2). Furthermore, if the space is such that the hypersurfaces are non-compact, we assume that these functions vanish at spatial infinity. Let V be the four-volume bounded by Σ_1 and Σ_2, and, if necessary, time-like boundaries on which $f_1 = f_2 = 0$. Then we may write

$$\begin{aligned}(f_1, f_2)_{\Sigma_2} - (f_1, f_2)_{\Sigma_1} &= i\oint_{\partial V}(f_2^*\overset{\leftrightarrow}{\partial}_\mu f_1)\mathrm{d}\Sigma^\mu \\ &= i\int_V \nabla_\mu(f_2^*\overset{\leftrightarrow}{\partial}_\mu f_1)\mathrm{d}V,\end{aligned} \tag{1.5}$$

where the last step follows from the four dimensional version of Gauss' law, and $\mathrm{d}V$ is the four dimensional volume element. However, we may write this integrand as

$$\begin{aligned}\nabla_\mu(f_2^*\overset{\leftrightarrow}{\partial}_\mu f_1) = \nabla_\mu(f_2^*\partial_\mu f_1 - f_1\partial_\mu f_2^*) &= f_2^*\Box f_1 - f_1\Box f_2^* \\ &= -f_2^*(m^2 + \xi R)f_1 + f_1(m^2 + \xi R)f_2^* = 0.\end{aligned} \tag{1.6}$$

Thus equation (1.4) is proven.

The quantization of a scalar field in a curved spacetime may be carried out by canonical methods. Choose a foliation of the spacetime into spacelike hypersurfaces. Let Σ be a particular hypersurface with unit normal vector n^μ labeled by a constant value of the time coordinate t. The derivative of φ in the normal direction is $\dot{\varphi} = n^\mu\partial_\mu\varphi$, and the canonical momentum is defined by

$$\pi = \frac{\delta\mathcal{L}}{\delta\dot{\varphi}}. \tag{1.7}$$

We impose the canonical commutation relation

$$[\varphi(\mathbf{x}, t), \pi(\mathbf{x}', t)] = i\delta(\mathbf{x}, \mathbf{x}'), \tag{1.8}$$

where $\delta(\mathbf{x}, \mathbf{x}')$ is a delta function in the hypersurface with the property that

$$\int \delta(\mathbf{x}, \mathbf{x}')\mathrm{d}\Sigma = 1. \tag{1.9}$$

Let $\{f_j\}$ be a complete set of positive norm $[(f_j, f_j) > 0]$ solutions of equation (1.2). Then $\{f_j^*\}$ will be a complete set of negative norm solutions, and $\{f_j, f_j^*\}$ form a complete set

of solutions of the wave equation in terms of which we may expand an arbitrary solution. Write the field operator φ as a sum of annihilation and creation operators:

$$\varphi = \sum_j (a_j f_j + a_j^\dagger f_j^*), \tag{1.10}$$

where $[a_j, a_{j'}^\dagger] = \delta_{j,j'}$ follows from equation (1.8). This expansion defines a vacuum state $|0\rangle$ such that $a_j|0\rangle = 0$. In flat spacetime, we take our positive norm solutions to be positive frequency solutions, $f_j \propto \mathrm{e}^{-\mathrm{i}\omega t}$. Regardless of the Lorentz frame in which t is the time coordinate, this procedure defines the same, unique Minkowski vacuum state.

In curved spacetime, the situation is quite different. There is, in general, no unique choice of the $\{f_j\}$, and hence no unique notion of the vacuum state. This means that we cannot identify what constitutes a state without particle content, and the notion of 'particle' becomes ambiguous. One possible resolution of this difficulty is to choose some quantities other than particle content to label quantum states. Possible choices might include local expectation values [4], such as $\langle\varphi\rangle$, $\langle\varphi^2\rangle$, etc. In the particular case of an asymptotically flat spacetime, we might use the particle content in an asymptotic region. Even this characterization is not unique. However, this non-uniqueness is an essential feature of the theory with physical consequences, namely the phenomenon of particle creation, which we will now discuss.

2. Particle creation by gravitational fields

2.1. Bogolubov transformations

Let us consider a spacetime which is asymptotically flat in the past and in the future, but which is non-flat in the intermediate region. Let $\{f_j\}$ be positive frequency solutions in the past (the 'in-region'), and let $\{F_j\}$ be positive frequency solutions in the future (the 'out-region'). We may choose these sets of solutions to be orthonormal, so that

$$\begin{aligned}(f_j, f_{j'}) &= (F_j, F_{j'}) = \delta_{jj'}\\ (f_j^*, f_{j'}^*) &= (F_j^*, F_{j'}^*) = -\delta_{jj'}\\ (f_j, f_{j'}^*) &= (F_j, F_{j'}^*) = 0.\end{aligned} \tag{2.1}$$

Although these functions are defined by their asymptotic properties in different regions, they are solutions of the wave equation everywhere in the spacetime. We may expand the in-modes in terms of the out-modes:

$$f_j = \sum_k (\alpha_{jk} F_k + \beta_{jk} F_k^*). \tag{2.2}$$

Inserting this expansion into the orthogonality relations, equation (2.1), leads to the conditions

$$\sum_k (\alpha_{jk}\alpha_{j'k}^* - \beta_{jk}\beta_{j'k}^*) = \delta_{jj'}, \tag{2.3}$$

and

$$\sum_k (\alpha_{jk}\beta_{j'k} - \beta_{jk}\alpha_{j'k}) = 0. \tag{2.4}$$

The inverse expansion is

$$F_k = \sum_j (\alpha_{jk}^* f_j - \beta_{jk} f_j^*). \tag{2.5}$$

The field operator, φ, may be expanded in terms of either the $\{f_j\}$ or the $\{F_j\}$:

$$\varphi = \sum_j (a_j f_j + a_j^\dagger f_j^*) = \sum_j (b_j F_j + b_j^\dagger F_j^*). \tag{2.6}$$

The a_j and $a_j^\dagger$ are annihilation and creation operators, respectively, in the in-region, whereas the b_j and $b_j^\dagger$ are the corresponding operators for the out-region. The in-vacuum state is defined by $a_j|0\rangle_{\mathrm{in}} = 0, \forall j$, and describes the situation when no particles are present initially. The out-vacuum state is defined by $b_j|0\rangle_{\mathrm{out}} = 0, \forall j$, and describes the situation when no particles are present at late times. Noting that $a_j = (\varphi, f_j)$ and $b_j = (\varphi, F_j)$, we may expand the two sets of creation and annihilation operator in terms of one another as

$$a_j = \sum_k (\alpha_{jk}^* b_k - \beta_{jk}^* b_k^\dagger), \tag{2.7}$$

or

$$b_k = \sum_j (\alpha_{jk} a_j + \beta_{jk}^* a_j^\dagger). \tag{2.8}$$

This is a Bogolubov transformation, and the α_{jk} and β_{jk} are called the Bogolubov coefficients.

Now we are ready to describe the physical phenomenon of particle creation by a time-dependent gravitational field. Let us assume that no particles were present before the gravitational field is turned on. If the Heisenberg picture is adopted to describe the quantum dynamics, then $|0\rangle_{\mathrm{in}}$ is the state of the system for all time. However, the physical number operator which counts particles in the out-region is $N_k = b_k^\dagger b_k$. Thus the mean number of particles created into mode k is

$$\langle N_k\rangle = {}_{\mathrm{in}}\langle 0|b_k^\dagger b_k|0\rangle_{\mathrm{in}} = \sum_j |\beta_{jk}|^2. \tag{2.9}$$

If any of the β_{jk} coefficients are non-zero, i.e. if any mixing of positive and negative frequency solutions occurs, then particles are created by the gravitational field. The above discussion describes the spontaneous creation of bosons, which will be the main interest in this review, but an analogous procedure may be used to describe fermion creation.

The most straightforward application of the concepts developed above is to particle creation by an expanding Universe. This phenomenon was first hinted at in the work of Schrödinger [5], but was first carefully investigated by Parker [6, 7]. Let us restrict our attention to the case of a spatially flat Robertson–Walker Universe, for which the metric may be written as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}\mathbf{x}^2\right), \tag{2.10}$$

where a is the scale factor. We may use either the comoving time t or the conformal time η, but the solutions of the wave

equation are simpler in terms of the latter. The positive norm solutions of equation (1.2) in this metric may be taken to be

$$f_{\mathbf{k}}(\mathbf{x},\eta) = \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{a(\eta)\sqrt{(2\pi)^3}}\chi_k(\eta), \tag{2.11}$$

where $\chi_k(\eta)$ satisfies

$$\frac{d^2\chi_k}{d\eta^2} + [k^2 + V(\eta)]\chi_k = 0, \tag{2.12}$$

with

$$V(\eta) \equiv a^2(\eta)\left[m^2 + \left(\xi - \frac{1}{6}\right)R(\eta)\right]. \tag{2.13}$$

The norm of $f_{\mathbf{k}}$ being equal to one is equivalent to the Wronskian condition

$$\chi_k\frac{d\chi_k^*}{d\eta} - \chi_k^*\frac{d\chi_k}{d\eta} = i. \tag{2.14}$$

Let us consider the idealized situation in which the Universe is static both in the past and in the future, as illustrated in figure 1. In this case, we have the necessary asymptotically flat regions needed to define in and out vacua. Let

$$a(\eta) \to \begin{cases} a_{\rm i} & \eta \to -\infty \\ a_{\rm f} & \eta \to \infty \end{cases}, \tag{2.15}$$

where $a_{\rm i}$ and $a_{\rm f}$ are constants. Let $\chi_k^{(\rm in)}(\eta)$ be a solution of equation (2.12) which is pure positive frequency in the past:

$$\chi_k^{(\rm in)}(\eta) \sim \frac{e^{-i\omega_{\rm in}\eta}}{\sqrt{2\omega_{\rm in}}}, \quad \eta \to -\infty. \tag{2.16}$$

Similarly, $\chi_k^{(\rm out)}(\eta)$ is pure positive frequency in the future:

$$\chi_k^{(\rm out)}(\eta) \sim \frac{e^{-i\omega_{\rm out}\eta}}{\sqrt{2\omega_{\rm out}}}, \quad \eta \to \infty. \tag{2.17}$$

Here

$$\omega_{\rm in} = \sqrt{k^2 + a_{\rm i}^2 m^2}, \tag{2.18}$$

and

$$\omega_{\rm out} = \sqrt{k^2 + a_{\rm f}^2 m^2}. \tag{2.19}$$

When we include the spatial dependence, the positive frequency modes in the past are given by equation (2.11) with $\chi_k(\eta) = \chi_k^{\rm in}(\eta)$, and those which are positive frequency in the future, $F_{\mathbf{k}}(\mathbf{x},\eta)$, are given by the same relation, but with $\chi_k(\eta) = \chi_k^{\rm out}(\eta)$.

Note that although these modes are defined by their asymptotic forms in the past or future, both sets are valid solutions for all η, and hence one set may be expanded in terms of the other. Let

$$\chi_k^{(\rm in)}(\eta) = \frac{a_{\rm i}}{a_{\rm f}}[\alpha_k\chi_k^{(\rm out)}(\eta) + \beta_k\chi_k^{(\rm out)}(\eta)^*] \tag{2.20}$$

for some constants α_k and β_k. This leads to the relation

$$f_{\mathbf{k}}(\mathbf{x},\eta) = \alpha_k F_{\mathbf{k}}(\mathbf{x},\eta) + \beta_k F_{\mathbf{k}}^*(\mathbf{x},\eta). \tag{2.21}$$

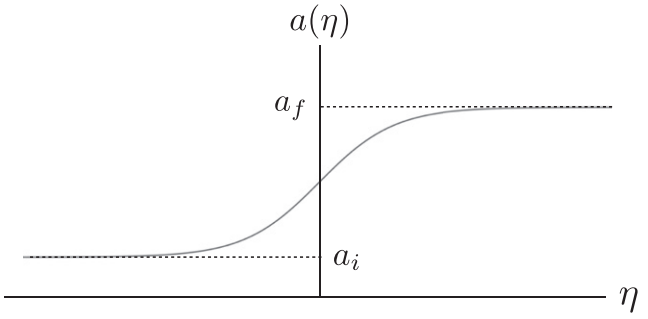

Figure 1. Here the scale factor $a(\eta)$ for an asymptotically bounded expansion of the universe is illustrated, in the past, the in-region, spacetime is flat and $a = a_{\rm i}$. The universe then expands for a finite interval of conformal time, η. In the future, the out-region, spacetime is again flat and $a = a_{\rm f}$. Often we set $a_{\rm f} = 1$ for convenience.

This is just equation (2.2), where the Bogolubov coefficients are given by $\alpha_{\mathbf{k}\mathbf{k}'} = \alpha_k\delta_{\mathbf{k}\mathbf{k}'}$ and $\beta_{\mathbf{k}\mathbf{k}'} = \beta_k\delta_{\mathbf{k},-\mathbf{k}'}$. The condition equation (2.3) becomes

$$|\alpha_k|^2 - |\beta_k|^2 = 1, \tag{2.22}$$

and the mean number of particles created in a given mode, from equation (2.9), is now $\langle N_k\rangle = |\beta_k|^2$. The simple forms of the Bogolubov coefficients in a spatially flat Universe follow from the spatial translation symmetry of the metric, equation (2.10). This means that three-momentum is conserved, even though energy is no longer conserved on a time-dependent background. Indeed, it is the lack of energy conservation which allows particle creation from the vacuum in an expanding Universe. It also allows other processes which are forbidden in flat spacetime, including new decay channels for unstable particles [8, 9].

If we quantize the field φ using periodic boundary condition in a box with coordinate volume $\mathcal{V}$, then the wave number $\mathbf{k}$ takes discrete values, and in the limit of large $\mathcal{V}$,

$$\sum_{\mathbf{k}} \to \frac{\mathcal{V}}{(2\pi)^3}\int d^3k, \tag{2.23}$$

just as in flat spacetime. However, the physical volume of the box in the out-region is $a_{\rm f}^3\mathcal{V}$, so the total number of created particle per unit volume becomes

$$n = \frac{1}{(2\pi a_{\rm f})^3}\int d^3k|\beta_k|^2. \tag{2.24}$$

The density of created particle per unit volume and per unit wave number interval is

$$\frac{dn}{dk} = \frac{1}{2\pi^2 a_{\rm f}^3}k^2|\beta_k|^2. \tag{2.25}$$

The energy of a particle in the final region is

$$E_{\rm out} = \frac{\omega_{\rm out}}{a_{\rm f}} = \sqrt{\left(\frac{k}{a_{\rm f}}\right)^2 + m^2}. \tag{2.26}$$

Note that in $\hbar = 1$ units, k has the units of momentum, and $k/a_{\rm f}$ is the physical momentum of the particle as measured by

an observer at rest in the cosmological frame in the final region. The energy density of created particles is

$$u = \frac{1}{(2\pi a_{\rm f})^3}\int {\rm d}^3k E_{\rm out}|\beta_k|^2. \tag{2.27}$$

In the massless limit, $m = 0$, this becomes

$$u_0 = \frac{1}{(2\pi)^3 a_{\rm f}^4}\int {\rm d}^3k k|\beta_k|^2. \tag{2.28}$$

The extra factor of $a_{\rm f}$ in the denominator of equation (2.28), as compared to equation (2.24), arises from the redshifting of the energy of massless particles by the cosmological expansion.

Note that conformally invariant massless particles are not created by cosmological expansion in a conformally flat spacetime, such as that of equation (2.10), as was first noted by Parker [6]. We can see this in the case of the scalar field; if we set $m = 0$ and $\xi = 1/6$ in equation (2.13), then $V = 0$ and the solutions of equation (2.12) become proportional to $e^{\pm ik\eta}$. This means that a positive frequency solution in the past remains a positive frequency solution, so that $\beta_k = 0$ for all modes and no particles are created. Thus particle creation requires a nonzero mass or non-conformal coupling to the curvature, $\xi \neq 1/6$, or both. For most choices of the scale factor, $a(\eta)$, it can be difficult to find exact solutions to equation (2.12). However, there are a few exactly soluble models which we will now consider.

2.2. Specific models

2.2.1. The Bernard and Duncan model: effects of nonzero mass. One exact solution which illustrates the effects of nonzero mass was given by Bernard and Duncan [10], and is also reviewed in section 3.4 of [1]. This model was originally formulated in a two-dimensional spacetime, but can be easily extended to a four-dimensional model, which will be done here. Consider a Universe with the metric equation (2.10), where

$$a^2(\eta) = \frac{1}{2}[1 + a_{\rm i}^2 + (1 - a_{\rm i}^2)\tanh(\rho\eta)]\,. \tag{2.29}$$

This describes a Universe which expands from an initial scale factor of $a_{\rm i} < 1$ to a final scale factor of $a_{\rm f} = 1$, as illustrated in figure 1. For the case of conformal coupling, $\xi = 1/6$, equation (2.12) becomes

$$\frac{{\rm d}^2\chi_k}{{\rm d}\eta^2} + \left\{k^2 + \frac{1}{2}m^2\left[1 + a_{\rm i}^2 + (1 - a_{\rm i}^2)\tanh(\rho\eta)\right]\right\}\chi_k = 0. \tag{2.30}$$

This equation may be solved in terms of hypergeometric functions. The solution which is pure positive frequency in the past, and has the asymptotic form given in equation (2.16), is

$$\chi_k^{\rm in}(\eta) = \frac{1}{\sqrt{2\omega_{\rm in}}}\exp\left\{-i\omega_+\eta - i\frac{\omega_-}{\rho}\ln\left[2\,\cosh(\rho\eta)\right)\right\} \times F\left(1 + \frac{i\omega_-}{\rho}, \frac{i\omega_-}{\rho}; 1 - \frac{i\omega_{\rm in}}{\rho}; \frac{1}{2}[1 + \tanh(\rho\eta)]\right), \tag{2.31}$$

where $\omega_\pm = (\omega_{\rm out} \pm \omega_{\rm in})/2$. Here F, sometimes denoted by ${}_2F_1$, is a hypergeometric function. The corresponding solution which is pure positive frequency in the future and has the asymptotic form in equation (2.17) is

$$\chi_k^{\rm out}(\eta) = \frac{1}{\sqrt{2\omega_{\rm out}}}\exp\left\{-i\omega_+\eta - i\frac{\omega_-}{\rho}\ln\left[2\,\cosh(\rho\eta)\right)\right\} \times F\left(1 + \frac{i\omega_-}{\rho}, \frac{i\omega_-}{\rho}; 1 + \frac{i\omega_{\rm out}}{\rho}; \frac{1}{2}[1 - \tanh(\rho\eta)]\right). \tag{2.32}$$

The two solutions $\chi_k^{\rm in}(\eta)$ and $\chi_k^{\rm out}(\eta)$ are related by the linear transformation properties of the hypergeometric function, which leads to a relation of the form of equation (2.20). Here

$$\alpha_k = \frac{1}{a_{\rm i}}\sqrt{\frac{\omega_{\rm out}}{\omega_{\rm in}}}\frac{\Gamma(1 - i\omega_{\rm in}/\rho)\Gamma(-i\omega_{\rm out}/\rho)}{\Gamma(1 - i\omega_+/\rho)\Gamma(-i\omega_+/\rho)}, \tag{2.33}$$

and

$$\beta_k = \frac{1}{a_{\rm i}}\sqrt{\frac{\omega_{\rm out}}{\omega_{\rm in}}}\frac{\Gamma(1 - i\omega_{\rm in}/\rho)\Gamma(i\omega_{\rm out}/\rho)}{\Gamma(1 + i\omega_-/\rho)\Gamma(i\omega_-/\rho)}. \tag{2.34}$$

This leads to

$$|\beta_k|^2 = \frac{\sinh^2(\pi\omega_-/\rho)}{a_{\rm i}^2\,\sinh(\pi\omega_{\rm out}/\rho)\,\sinh(\pi\omega_{\rm in}/\rho)}. \tag{2.35}$$

These particles each have energy $\omega_{\rm out} = \sqrt{k^2 + m^2}$ in the out-region.

In the limit of small mass, so $m \ll ka_{\rm i} < k$, we have

$$\begin{aligned}\omega_{\rm out} &\approx k + \frac{m^2}{2k^2} + \cdots,\\ \omega_{\rm in} &\approx k + \frac{a_{\rm i}^2 m^2}{2k^2} + \cdots,\\ \omega_- &\approx \frac{m^2}{4k}(1 - a_{\rm i}^2),\end{aligned} \tag{2.36}$$

so

$$|\beta_k|^2 \approx \frac{\pi^2(1 - a_{\rm i}^2)m^4}{16k^2a_{\rm i}^2\rho^2\,\sinh^2(\pi k/\rho)}. \tag{2.37}$$

Thus the number of created particles vanishes as m^4 when $m \to 0$. If we also assume that $k \gtrsim \rho$, then

$$|\beta_k|^2 \approx \frac{\pi^2(1 - a_{\rm i}^2)^2m^4}{4k^2a_{\rm i}^2\,\rho^2}\,{\rm e}^{-2\pi k/\rho}. \tag{2.38}$$

In the limit of large mass, so $m \gg k > a_{\rm i}k$, equation (2.35) becomes

$$|\beta_k|^2 \approx \frac{1}{a_{\rm i}^2}\,{\rm e}^{-2\pi m a_{\rm i}/\rho}. \tag{2.39}$$

This limit describes particles which are created at rest in the cosmological frame, but with an exponentially suppressed probability.

A somewhat different limit may be explored by assuming that ρ becomes small with k, m, and $a_{\rm i}$ fixed, so that $\omega_{\rm in}$, $\omega_{\rm out}$,

and ω_-, are all large compared to ρ. This is the limit of slow expansion, where we find

$$|\beta_k|^2 \approx \frac{1}{a_{\rm i}^2}\, {\rm e}^{-2\pi\omega_{\rm in}/\rho}. \tag{2.40}$$

If we use equation (2.18), and assume that $k \lesssim ma_{\rm i}$, then we again obtain equation (2.39). However, if we now assume that $k \gtrsim ma_{\rm i}$, then we find

$$|\beta_k|^2 \approx \frac{1}{a_{\rm i}^2}\, {\rm e}^{-2\pi k/\rho}. \tag{2.41}$$

This result contains the same exponential factor as does equation (2.38), although with a different prefactor, arising from assuming somewhat different limits. Both forms show that the creation of very energetic particles is exponentially suppressed.

Note that in the two-dimensional version of this model described in [1, 10], the factors of $a_{\rm i}$ in the final results, such as equation (2.35), do not appear. This is because in two spacetime dimensions, the factor of $1/a(\eta)$ which appears in four dimensional solutions of the Klein–Gordon equation, e.g. equation (2.11), does not appear. In four spacetime dimensions, the fact that $|\beta_k|^2 \propto a_{\rm i}^{-2}$ leads to enhanced particle creation when $a_{\rm i} \ll 1$ and the fractional change in scale factor is large.

The total number density and energy density of the created particles are given by equations (2.24) and (2.27), respectively, with $a_{\rm f} = 1$. For the purposes of an estimate, let us use equation (2.38). Although this form does not hold over the entire range of integration, it should be a fair approximation for $k \approx \rho$, which is expected to give the dominant contribution to the integrals. In addition, we use $E_{\rm out} \approx k$. This leads to

$$n \approx \frac{(1-a_{\rm i}^2)^2 m^4}{16\pi a_{\rm i}^2 \rho}, \tag{2.42}$$

and

$$u \approx \frac{(1-a_{\rm i}^2)^2 m^4}{32\pi^2 a_{\rm i}^2}. \tag{2.43}$$

If we take the ratio of these two expressions, we may estimate the average energy of the created particles to be

$$E_{\rm ave} = \frac{u}{n} \approx 2\pi\rho, \tag{2.44}$$

so this average particle energy is proportional to the inverse expansion time, ρ.

2.2.2. The Parker and Toms model: effects of non-conformal coupling. There is a similar model, given by Parker and Toms [2, 11], with statically bounded expansion which provides an exact solution for the creation of massless, minimally coupled scalar particles, and hence of gravitons as will be discussed in section 2.5. If one defines a new time coordinate τ in the metric equation (2.10) by ${\rm d}t = a^3\,{\rm d}\tau$, then the metric becomes

$${\rm d}s^2 = -a^6(\tau){\rm d}\tau^2 + a^2(\tau){\rm d}\mathbf{x}^2, \tag{2.45}$$

and the solutions of the massless, minimally coupled Klein–Gordon equation $\Box\varphi = 0$ can be written as

$$f_{\mathbf{k}}(\mathbf{x},\tau) = \frac{{\rm e}^{{\rm i}\mathbf{k}\cdot\mathbf{x}}}{\sqrt{(2\pi)^3}}\chi_k(\tau). \tag{2.46}$$

Note that there is no explicit scale factor in this expression, in contrast to equation (2.11). Now $\chi_k(\tau)$ satisfies the equation

$$\frac{{\rm d}^2\chi_k}{{\rm d}\tau^2} + k^2 a^4(\tau)\chi_k = 0. \tag{2.47}$$

The scale factor used by Parker and Toms can be expressed in a form analogous to equation (2.29)

$$a^4(\tau) = \frac{1}{2}[1 + a_{\rm i}^4 + (1-a_{\rm i}^4)\tanh(\rho\tau)], \tag{2.48}$$

and also expands from an initial value of $a_{\rm i} < 1$ to a final value of $a_{\rm f} = 1$, as illustrated in figure 1, but now as a function of τ. Equation (2.47) becomes

$$\frac{{\rm d}^2\chi_k}{{\rm d}\tau^2} + \frac{1}{2}k^2\left[1 + a_{\rm i}^4 + (1-a_{\rm i}^4)\tanh(\rho\tau)\right]\chi_k(\tau) = 0, \tag{2.49}$$

which is the same as equation (2.30) if we make the substitutions

$$k \to 0, \quad m \to k \quad a_{\rm i}^2 \to a_{\rm i}^4, \quad \eta \to \tau \tag{2.50}$$

in the latter. As a consequence, the solutions for the in and out modes, and also the Bogolubov coefficients, may be found from those in section 2.2.1 by this substitution, except here there will be no overall factors of $a_{\rm i}$. Now the Bogolubov coefficients are given by equations (2.33) and (2.34) without the leading $1/a_{\rm i}$ factor, and where now

$$\omega_{\rm in} = ka_{\rm i}^2, \qquad \omega_{\rm out} = k, \tag{2.51}$$

so here $\omega_\pm = k(1 \pm a_{\rm i}^2)/2$. Now we have the result

$$|\beta_k|^2 = \frac{\sinh^2[\pi k(1-a_{\rm i}^2)/(2\rho)]}{\sinh(\pi k/\rho)\,\sinh(\pi k a_{\rm i}^2/\rho)}. \tag{2.52}$$

In the high frequency limit, $k > ka_{\rm i}^2 \gtrsim \rho$, we have

$$|\beta_k|^2 \approx {\rm e}^{-2\pi k a_{\rm i}^2/\rho}. \tag{2.53}$$

Equations (2.38)–(2.40), and (2.53) all illustrate that the creation rate of particles with energies larger than the inverse expansion time, ρ, is exponentially suppressed. Parker [11] has noted that these exponential factor are similar to those which appear in thermal spectrum at finite temperature. Note, however, that here the particles are created in a pure quantum state and hence have zero entropy until decoherence occurs. This issue will be treated in section 3. Several other authors have discussed cases where an approximately thermal spectrum of particles is created [12–15].

As in the previous subsection, we may use equation (2.53) to obtain estimates for the total number and energy density of the created particles. The results are

$$n \approx \frac{\rho^3}{8\pi^5 a_{\rm i}^6} \tag{2.54}$$

and

$$u \approx \frac{3\rho^4}{16\pi^6 a_{\rm i}^8}. \tag{2.55}$$

The associated estimate for the mean energy of the created particles is

$$E_{\rm ave} = \frac{u}{n} \approx \frac{3\rho}{2\pi a_{\rm i}^2}. \tag{2.56}$$

The inverse powers of $a_{\rm i}$ in the above expressions indicate that a large change in scale factor, where $a_{\rm i} \ll 1$, is especially effective in creating massless minimally coupled scalars and gravitons.

2.3. A perturbative approach

It is difficult to solve equation (2.12) for the mode functions in all but a few examples, such as those just discussed. However, there is a perturbative method, developed by Zeldovich and Starobinsky [16] and by Birrell and Davies [17], which is often useful. The first step is to rewrite equation (2.12) as

$$\frac{{\rm d}^2\chi_k}{{\rm d}\eta^2} + [\omega_{\rm in}^2 + U(\eta)]\chi_k = 0, \tag{2.57}$$

where

$$U(\eta) = V(\eta) - a_{\rm i}^2 m^2 = m^2[a^2(\eta) - a_{\rm i}^2] + a^2(\eta)\left(\xi - \frac{1}{6}\right)R(\eta). \tag{2.58}$$

Note that $U(\eta) \to 0$ as $\eta \to -\infty$. Next we rewrite equation (2.57) as an integral equation:

$$\chi_k(\eta) = \chi_k^{\rm (in)}(\eta) - \omega_{\rm in}^{-1}\int_{-\infty}^{\eta} U(\eta')\ \sin\left[\omega_{\rm in}(\eta - \eta')\right]\ \chi_k(\eta'){\rm d}\eta', \tag{2.59}$$

where here $\chi_k^{\rm (in)}(\eta)$ is understood to be the asymptotic form given in equation (2.16). This integral equation is equivalent to equation (2.57) plus the boundary condition that $\chi_k(\eta) \sim \chi_k^{\rm (in)}(\eta)$ for $\eta \to -\infty$. We now wish to assume that U is sufficiently small that we may iterate this integral equation to lowest order by replacing $\chi_k(\eta')$ by $\chi_k^{\rm (in)}(\eta')$ in the integrand. If we compare the resulting formula in the limit $\eta \to \infty$ with the right-hand side of equation (2.20), the Bogolubov coefficients may be read off:

$$\alpha_k \approx \frac{a_{\rm f}}{a_{\rm i}}\left[1 - \frac{i}{2\omega_{\rm in}}\int_{-\infty}^{\infty} U(\eta)\,{\rm d}\eta\right], \tag{2.60}$$

and

$$\beta_k \approx \frac{ia_{\rm f}}{2a_{\rm i}\omega_{\rm in}}\int_{-\infty}^{\infty} {\rm e}^{-2i\omega_{\rm in}\eta}\ U(\eta)\,{\rm d}\eta. \tag{2.61}$$

We can see that the above procedure is a first order expansion in m^2 and in $(\xi - \frac{1}{6})R$, or more precisely, the dimensionless ratios of these two quantities to the inverse squared expansion time scale. In the limit that $m^2 = (\xi - \frac{1}{6})R = 0$, there is no particle creation, as expected. Otherwise, equation (2.61) tells us that the particle creation will be peaked at the values of $\omega_{\rm in}$ associated with the peak of the Fourier transform of $U(\eta)$, which is of the order of the characteristic inverse time scale, in conformal time, of the expansion. Strictly, the perturbative approach assumes that U is always small, which requires that the scale factor not change by large factors if $m \neq 0$, in which case $\omega_{\rm out} \approx \omega_{\rm in}$.

We can compare the perturbative result with the exact solution in the Bernard and Duncan model. In this case, the squared scale factor is given by equation (2.29) and $\xi = 1/6$. The integral in equation (2.61) may be performed explicitly. The result contains a term proportional to $\delta(\omega_{\rm in})$, which would seem to describe created particles with zero energy, and which does not appear in the exact solution. If we assume that this term should not be present, and drop it, we then find

$$|\beta_k|^2 \approx \frac{\pi^2(1 - a_{\rm i}^2)^2 m^4}{4\omega_{\rm in}^2 a_{\rm i}^2 \rho^2}\ {\rm e}^{-2\pi\omega_{\rm in}/\rho}, \tag{2.62}$$

where we have set $a_{\rm f} = 1$ and assumed that $\omega_{\rm in} \gtrsim \rho$. This result agrees with equation (2.38) in the small mass limit, where $\omega_{\rm in} \approx k + O(m^2)$ and $\omega_{\rm out} \approx \omega_{\rm in}$.

Let us now consider the case where $m = 0$. In this case, the mean number density becomes

$$n = \frac{\left(\xi - \frac{1}{6}\right)^2}{16\pi a_{\rm f}^3}\int_{-\infty}^{\infty} a^4(\eta)R^2(\eta)\,{\rm d}\eta, \tag{2.63}$$

and the energy density becomes

$$u = -\frac{\left(\xi - \frac{1}{6}\right)^2}{32\pi^2 a_{\rm f}^4}\int_{-\infty}^{\infty}{\rm d}\eta_1\int_{-\infty}^{\infty}{\rm d}\eta_2 \times\left\{\ln(|\eta_1 - \eta_2|\mu)\frac{\rm d}{{\rm d}\eta_1}\left[a^2(\eta_1)R(\eta_1)\right] \times\frac{\rm d}{{\rm d}\eta_2}\left[a^2(\eta_2)R(\eta_2)\right]\right\}. \tag{2.64}$$

Here μ is an arbitrary quantity with the dimensions of mass; u is independent of μ provided that $a^2(\eta)R(\eta) \to 0$ as $\eta \to \pm\infty$. The approximation which is being used in the $m = 0$ case amounts to perturbation around the conformally invariant theory in powers of $(\xi - \frac{1}{6})$. Note that here n and u are independent of $a_{\rm i}$, and we have restored a general value of $a_{\rm f}$.

2.4. Estimates of magnitude of the particle creation

The rate of particle creation is not precisely defined, but in the massless case one can view the integrand of equation (2.63) as an approximate number density creation rate per unit conformal time η. The energy density creation rate is more complicated, because equation (2.64) involves a double time integration, and because the integrand depends upon the arbitrary constant μ. However, if one estimates a characteristic energy of the created particles from the Fourier transform in equation (2.61), then this may be combined with the approximate number density creation rate to estimate the energy density creation rate. However, in the case that $m \neq 0$, even the particle number is expressed as a double time integral [17], so there no obvious candidate of a number creation rate.

More generally, we may combine the perturbative method with the exact solutions given in sections 2.2.1 and 2.2.2 to make some estimates of the number and spectrum of the created particles. First, equations (2.25) and (2.61) tell us that the number density of particles created into a given interval

of wavenumber or energy is proportional to the square of a Fourier component of the function $U(\eta)$ associated with that energy interval. Let η_C be the characteristic timescale associated with the scale factor $a(\eta)$. For example, in the case of equation (2.29), we could take $\eta_C = 1/\rho$.

We have found in the exact solutions in sections 2.2.1 and 2.2.2 that the creation of particles with energies large compared to $1/\eta_C$ is exponentially suppressed, as seen in equations (2.38)–(2.40), and (2.53). The feature also appears in the perturbative results in section 2.3 due to exponential decay of Fourier transforms at high frequencies for a wide class of functions. More generally, we can expect the energy spectrum of the created particles to be peaked near energies $E \propto 1/\eta_C$, as illustrated in equations (2.44) and (2.56).

The particle creation rate depends upon the cosmological expansion rate, which is in turn determined by the matter equation of state. The effects of a stiff equation of state have been discussed by Lankinen and Vilia [18].

Note that this review has focused on the quantum creation of bosons, especially scalar particles. The creation of fermions in an expanding Universe was treated by Parker [7], where anti-commutation relations replace the commutation relation used here in section 2.1. In general, one can expect the creation rate of fermions to be somewhat less than for bosons of similar mass and coupling to curvature. This is due to the effects of the exclusion principle for fermions.

2.5. Quantum creation of gravitons

Although there is not yet a universally accepted quantum theory of gravity, there are limits in which gravity may be quantized unambiguously. One such limit is the quantization of tensor perturbations of an expanding Universe, such as that of equation (2.10). This is treated, for example, in [19]. If one imposes the transverse tracefree gauge, then all of the gauge freedom is removed, leaving only the two physical degrees of freedom associated with the two independent choices of polarization. The metric perturbations in this gauge satisfy the scalar wave equation, equation (1.2), with $m = \xi = 0$, that is the equation for a massless, minimally coupled scalar field [20]. Because of the two polarization degrees of freedom, we can view gravitons in a spatially flat expanding Universe as being equivalent to a pair of massless minimally coupled fields. A somewhat different approach using Hamiltonian methods was given by Berger [21].

A key point is that the wave equation for gravitons, unlike that for photons, is not conformally invariant. The means that in general gravitons will be created by the cosmological expansion, as was emphasized by Grishchuk [22]. For example, the creation of gravitons by the Parker–Toms model of section 2.2.2 may be obtained by multiplying the number density or energy density of created massless scalar particles by two. Similarly, the perturbative method of section 2.3 may be used to obtain the number density or energy density of the created gravitons. In this case, set $\xi = 0$ in equations (2.63) and (2.64), and then multiply the result by two to account for the two graviton polarization states. An explicit example will be treated in section 4.2.

2.6. The issues of initial conditions and local definitions of particle number

Here we have used initial and final flat regions to give asymptotic definitions of particle number. In section 4.1.2, we will discuss a more natural definition of initial particle number in the context of inflationary cosmology. In general, particle number is not well defined during the expansion. Expressions such as equation (2.63) give a total number of created particles as a time integral, and it is tempting to interpret the integrand as a number creation rate. However, this should be done with caution. In general, one should not expect the instantaneous number of particles during the expansion to be well defined. Nonzero particle creation occurs when the particle number definitions in the initial and final regions differ. Certainly particle creation is also linked to the time dependence of the background and the lack of a conserved energy on such a background.

There are limits in which an approximate local definition may be given. When the wavelength of the particle is small compared to the local radius of spacetime curvature, then the particle is locally in an approximately flat region of spacetime, and particle number is meaningful. This may be made more precise in the context of an adiabatic or WKB approximation. Equation (2.12) has a solution of the form

$$\chi_k(\eta) = \frac{1}{\sqrt{2W_k(\eta)}}\ \exp\left(-i\int^{\eta} W_k(\eta')\,\mathrm{d}\eta'\right), \qquad (2.65)$$

where $W_k(\eta)$ is a solution of

$$[W_k(\eta)]^2 = k^2 + V(\eta) - \frac{1}{2}\left(\frac{W_k''}{W_k} - \frac{3(W_k')^2}{2W_k^2}\right), \qquad (2.66)$$

where $W_k' = \mathrm{d}W_k/\mathrm{d}\eta$. When the expansion rate is sufficiently slow that the derivative terms in the above equation may be ignored, we have the familiar WKB approximation

$$\chi_k(\eta) \approx \frac{1}{\sqrt{2\omega_k(\eta)}}\ \exp\left(-i\int^{\eta} \omega_k(\eta')\,\mathrm{d}\eta'\right), \qquad (2.67)$$

where

$$\omega_k(\eta) = \sqrt{k^2 + V(\eta)}. \qquad (2.68)$$

This approximation is valid when both $(a')^2$ and $|a''(\eta)|$ are small compared to $[\omega_k(\eta)]^2$. Note that this can be viewed as either a slow expansion, or a high frequency approximation.

Note that equation (2.67) is a positive frequency solution, with a slowly varying frequency. It may be taken to define the adiabatic vacuum state, and hence adiabatic particle number. So long as the WKB approximation holds, this form for $\chi_k(\eta)$ remains a positive frequency mode, and hence no particle creation occurs and particle number remains constant. Thus, the WKB solutions may be used for an approximate local definition of particle number. This approach has been used by several authors [23–28].

Another attempt to give a local definition of particle number is the Hamiltonian diagonalization approach of Grib and Mamaev [29]. Here one defines a Hamiltonian, diagonalizes it by a Bogolubov transformation, and defines particles in the

new Fock space after diagonalization. The physical motivation for this definition is not clear, and it has been critically discussed by Fulling [30], who argues that the choice of Hamiltonian is ambiguous. However, Fulling also notes that in some cases Hamiltonian diagonalization is essentially equivalent to the WKB approach.

Several other criteria for defining particular classes of quantum states have been discussed in [31–34].

3. Quantum state of the created particles, decoherence, and entropy

3.1. The quantum state

Let us return to the Bogolubov transformation, equation (2.7) for particle creation in a spatially flat Universe. In this case, we have

$$a_{\mathbf{k}} = \alpha_k^* b_{\mathbf{k}} - \beta_k^* b_{-\mathbf{k}}^\dagger, \tag{3.1}$$

where $a_{\mathbf{k}}$ is the annihilation operator for a particle in mode $\mathbf{k}$ in the in-region, and $b_{\mathbf{k}}$ is the corresponding operator in the out region. Recall that the quantum state of the system, when no particles are initially present, is the in-vacuum $|0\rangle_{\rm in}$, for which $a_{\mathbf{k}}|0\rangle_{\rm in} = 0$. The effect of the expansion of the Universe is to create pairs of particles in the out-region, where one member of each pair is in mode $\mathbf{k}$, and the other in mode $-\mathbf{k}$. For a moment we ignore all other modes, and write the quantum state in the out-region as

$$|0\rangle_{\rm in} = \sum_{n=0}^{\infty} c_n |n,n\rangle, \tag{3.2}$$

where $|m,n\rangle$ a number eigenstate in the out-region with m particles in mode $\mathbf{k}$, as well as n particles in mode $-\mathbf{k}$. Thus

$$b_{\mathbf{k}}^\dagger b_{\mathbf{k}} |m,n\rangle = m|m,n\rangle, \qquad b_{-\mathbf{k}}^\dagger b_{-\mathbf{k}} |m,n\rangle = n|m,n\rangle. \tag{3.3}$$

Now we have

$$\begin{aligned} a_{\mathbf{k}}|0\rangle_{\rm in} &= \sum_{n=0}^{\infty} c_n \left[\alpha_k^* \sqrt{n}|n-1,n\rangle - \beta_k^* \sqrt{n+1}|n,n+1\rangle\right] \\ &= \sum_{n=0}^{\infty} (\alpha_k^* \sqrt{n+1} c_{n+1} - \beta_k^* \sqrt{n+1} c_n)|n,n+1\rangle = 0, \end{aligned} \tag{3.4}$$

which leads to the recurrence relation

$$c_n = \frac{\beta_k^*}{\alpha_k^*} c_{n-1} = \left(\frac{\beta_k^*}{\alpha_k^*}\right)^n c_0. \tag{3.5}$$

The normalization of the state $|0\rangle_{\rm in}$ yields

$$\sum_{n=0}^{\infty} |c_n|^2 = |c_0|^2 \sum_{n=0}^{\infty} \left|\frac{\beta_k}{\alpha_k}\right|^{2n} = \frac{|c_0|^2}{1-|\beta_k/\alpha_k|^2} = |c_0|^2 |\alpha_k|^2 = 1, \tag{3.6}$$

where the second from last step follows from equation (2.22).

This argument determines the state $|0\rangle_{\rm in}$ in the out-Fock space up to an overall phase, so we have

$$|0\rangle_{\rm in} = c_0 \sum_{n=0}^{\infty} \left(\frac{\beta_k^*}{\alpha_k^*}\right)^n |n,n\rangle, \tag{3.7}$$

where $|c_0| = 1/|\alpha_k|$. From this expression, we see that the state of the system in the out region is a superposition of all possible number of pairs of particles, where within each pair, the two particles have opposite momenta. Furthermore, the probability of finding n pairs in this state is

$$P_n = |c_n|^2 = |\alpha_k|^{-2} \left|\frac{\beta_k}{\alpha_k}\right|^{2n}. \tag{3.8}$$

So far, we have considered only one value of $\mathbf{k}$, but particles are expected to be created into all modes for which $\beta_k \neq 0$. We may generalize the above discussion to the case of multiple values of $\mathbf{k}$. The result may be written as

$$|0\rangle_{\rm in} = c_0 \sum_{\{n_{\mathbf{k}}\}} \prod_{\mathbf{k}} \left(\frac{\beta_k^*}{\alpha_k^*}\right)^{n_{\mathbf{k}}} |\{n_{\mathbf{k}}\}\rangle. \tag{3.9}$$

Here $\{n_{\mathbf{k}}\}$ denote a set of occupation numbers; for each mode $\mathbf{k}$ there is non-negative integer $n_{\mathbf{k}}$ which gives the number of particles in mode $\mathbf{k}$ and the number of particle in mode $-\mathbf{k}$. In our enumeration, we may require $k_z \geqslant 0$ to avoid over counting. The overall constant c_0 satisfies

$$|c_0| = \left(\prod_{\mathbf{k}} |\alpha_k|\right)^{-1}. \tag{3.10}$$

The quantum state of the created particles is a multi-mode squeezed vacuum state.

3.2. Correlations, entropy and decoherence

The quantum state, equation (3.9), is a highly entangled state with numerous quantum correlations. The two-particle sector of this state describes pairs of particles with equal and opposite momenta. In effect, each created particle is entangled with a partner moving in the opposite direction. In addition, there are sectors of the state with all possible numbers of pairs, which are in turn correlated with one another. So long as the state of the system is a pure quantum state, then its entropy is zero. However, one can define a formal entanglement entropy, or von Neumann entropy, for such a state. This is discussed by Lin *et al* [35], who show that the von Neumann entropy for the state equation (3.9) may be written as

$$S_{\rm vN} = \sum_{\mathbf{k}} [(n_{\mathbf{k}}+1)\ \ln(n_{\mathbf{k}}+1) - n_{\mathbf{k}}\ \ln\ n_{\mathbf{k}}], \tag{3.11}$$

where $n_{\mathbf{k}} = |\beta_k|^2$ is the mean number of particle in mode $\mathbf{k}$. As before, we are assume quantization in a box of coordinate volume $\mathcal{V}$, and in the large volume limit, the sum on discrete modes is replaced by an integral, as in equation (2.23). This shows that $S_{\rm vN}$ is an extensive quantity proportional to $\mathcal{V}$, as expected.

The physical meaning of entanglement entropy is that it may be converted into thermodynamic entropy by decoherence, which is the loss of phase information in the quantum state through interaction with an environment. The details of this process need not concern us, so long as the mean number of particles per mode is approximately constant. Consider the case of a massless scalar field, and assume that after decoherence, $n_{\mathbf{k}}$ is replaced by a Planck factor:

$$n_{\mathbf{k}} \to \frac{1}{e^{\omega/(k_B T)} - 1}, \tag{3.12}$$

where $\omega = |\mathbf{k}|$ and k_B is Boltzmann's constant. Then S_{vN} becomes the usual thermodynamic entropy S_T at temperature T,

$$S_{vN} = S_T = \frac{2\pi^2}{45}\mathcal{V}(k_B T)^3. \tag{3.13}$$

This is the expected entropy of massless scalars at temperature T and is one-half of the familiar expression for the entropy of electromagnetic blackbody radiation.

4. Applications to cosmology

4.1. Inflation

4.1.1. Essentials of inflation. Inflationary models postulate a period of accelerating expansion in the early Universe [36, 37]. These models successfully solve some previous cosmological puzzles, the horizon and flatness problems, and predict a nearly scale invariant spectrum of initial perturbations for the post-inflationary Universe as well as a Universe which is very close to the spatially flat model of equation (2.10). For a recent review, see [38]. During inflation, the spacetime metric is approximately that of de Sitter space, given by

$$a(t) = e^{Ht} = a(\eta) = -\frac{1}{H\eta}, \quad \eta < 0, \tag{4.1}$$

where H is the Hubble parameter, a constant with dimensions of inverse time. de Sitter space is a solution of Einstein's equations with a positive cosmological constant, or, equivalently, a matter stress tensor which is proportion to the metric tensor and an energy density of

$$\rho_I = \frac{3H^2}{8\pi}. \tag{4.2}$$

There are many versions of inflationary cosmology, but one of the simplest assumes a classical scalar field (the 'inflaton') with an effective potential which has a flat region, where the first derivative of the potential with respect to the field is small. This field obeys an equation analogous to that of a classical particle in this potential. When the field is in the flat region, its energy density is approximately constant and equal to ρ_I. This is the inflationary period, which is assumed to end at a finite value of $t = t_R$ or of $\eta = \eta_R$. As inflation ends, the energy in the scalar field is converted into ordinary matter particles. If these particle are relativistic, then the subsequent epoch of the Universe is a radiation dominated expansion, where

$$a(t) \propto (t - t_R)^{1/2}, \quad \text{or} \quad a(\eta) \propto \eta - \eta_R. \tag{4.3}$$

Apart from a successful resolution of the horizon and flatness problem, one of the great successes of inflationary cosmology is the prediction of a nearly scale invariant spectrum on initial density perturbations, which has been confirmed by several observations, especially the temperature fluctuations of the cosmic microwave background (CMB). These density perturbations arise from quantum fluctuations of the inflaton field during inflation [39–43]. For a review, see [44]. This effect may be visualized as follows: the inflaton is a nearly classical field, but has small vacuum fluctuations, which cause some spatial regions to evolve slightly faster than nearby regions. The regions which evolve faster transition from the de Sitter phase to a radiation dominated phase somewhat sooner and begin redshifting sooner. At later times, these regions have a lower temperature and smaller energy density compared to regions which ended inflation later. In addition to these density, or scalar, perturbations, inflation also predicts tensor perturbations in the form of primordial gravity waves which will be discussed in section 4.1.6.

The creation of matter at the end of inflation ('reheating') is usually assumed to occur due to self-coupling of the inflaton field and its coupling to other matter fields. This reheating period necessarily requires a finite interval of time, during which the spacetime metric makes a smooth transition from that of de Sitter space to that of a radiation dominated Universe. For recent reviews of reheating, see [45, 46]. In addition to matter creation by field couplings, there should be at least some particle creation by the mechanism described above in section 2, which is the principle topic of this review. This is sometimes called gravitational particle creation to distinguish it from creation by field couplings.

4.1.2. Initial conditions: the Bunch–Davies vacuum. One of the requirements for well-defined quantum particle creation are clear definitions for the in and out vacua. In our discussion in section 2, both the initial and final regions were taken to be flat spacetime regions. In the context of cosmology, the final approximately flat region is not a problem, but the initial one is problematic. Fortunately, inflation seems to offer a natural resolution of this difficulty. An essential feature of a long period of inflationary expansion is that it effectively erases the memory of whatever preceeded it through redshifting of any preexisting matter. Regardless of the quantum state of the matter fields at the beginning of inflation, the effective state rapidly approaches the vacuum state as inflation proceeds. For massless conformal fields there is no ambiguity in the definition of the vacuum in a conformally flat spacetime, such as that of equation (2.10), and the natural definition is called the conformal vacuum. For massive or other nonconformal fields, there is generally a choice of vacuum in de Sitter space which is de Sitter invariant, and respects the spacetime symmetries of de Sitter space. This is often called the Bunch–Davies vacuum [47], and is the state which any initial state will asymptotically approach after an extended period of inflation. There is a subtlety in the case of the massless, minimally coupled scalar field in that the two point function suffers from an infrared divergence, so in a strict sense, the Bunch–Davies vacuum does not exist for this field. However, there are a family of quantum

states which are infrared finite and any initial state of the massless, minimally coupled scalar field will approach one of these states [48]. These states are also close to the Bunch–Davies state in the sense that expectation values of quantities such as the stress tensor, which are free of infrared divergences, due to the presence of space or time derivatives, are the same in all of these states as in the Bunch–Davies state. Thus it is reasonable to take the Bunch–Davies vacuum as the in-vacuum for discussions of quantum particle creation during the transition from inflationary expansion to the radiation dominated Universe. Note, however, that particles in very high frequency modes in the initial state could have an effect if these modes are not sufficiently redshifted by the inflationary expansion. This possibility was discussed by Agullo and Parker [49], who consider the effect of stimulated emission produced by such particles.

4.1.3. Estimates of gravitational particle creation during reheating. Here we wish to estimate the magnitude of the number and energy densities of particles which are created by the transition from de Sitter expansion to a radiation dominated epoch. We follow the treatment in [50], using the perturbation method of section 2.3. Consider the case of massless, non-conformally coupled scalar particles, and set $a = a_{\rm f} = 1$ at the end of reheating, $\eta = \eta_R$. The scalar curvature vanishes in a spatially flat radiation dominated Universe, so take $R = 0$ for $\eta > \eta_R$. The particle creation effectively ceases at $\eta = \eta_R$, and we can estimate the number density of created particles in the subsequent phase from equation (2.63) to be

$$n \approx \frac{\left(\xi - \frac{1}{6}\right)^2}{16\pi a^3}\int_{-\infty}^{\eta_R} a^4(\eta) R^2 \, {\rm d}\eta = \frac{\left(\xi - \frac{1}{6}\right)^2 H^3}{12\pi a^3}, \qquad (4.4)$$

where we have used the fact that $R = 12H^2$ during inflation, and the factor of a^{-3} describes the dilution of the particle number density by the cosmological expansion after their creation. If we assume that the transition occurs on a comoving time scale of $\Delta t \lesssim 1/H$, then it is shown in [50] that equation (2.64) implies that the energy density of the created particle is

$$\rho \approx \frac{\left(\xi - \frac{1}{6}\right)^2 H^4}{8\pi^2 a^4} \ln[1/(H\Delta t)]. \qquad (4.5)$$

This implies that the mean energy of the created particles is of the order of $H \ln[1/(H\Delta t)]$. Note that, unlike the number density, the energy density and the mean energy per particle diverge as $\Delta t \to 0$. This reflect a general feature that very rapid changes in spacetime geometry can lead to violent particle creation.

The inflaton energy density, ρ_I, which drives the expansion, is related to H by the Einstein equation:

$$H^2 = \frac{8\pi\rho_I}{3\sqrt{\rho_{\rm Pl}}}, \qquad (4.6)$$

where $\rho_{\rm Pl} \approx \left(10^{19}\ {\rm GeV}\right)^4$ is the Planck density. We can express our estimate for the energy density of the created particles just after the end of inflation as

$$\rho \approx (1 - 6\xi)^2 \frac{\rho_I^2}{\rho_{\rm Pl}}. \qquad (4.7)$$

If, for example, we were to take $\rho_I \approx \left(10^{15}\ {\rm GeV}\right)^4$, then we obtain the estimate

$$\rho \approx (1 - 6\xi)^2 \left(10^{11}\ {\rm GeV}\right)^4. \qquad (4.8)$$

This energy density is much less than ρ_I, and would hence be negligible if there is efficient reheating. However, if other reheating mechanisms are not efficient, then particle creation by the gravitational field could play a significant role in cosmological evolution.

However, a very weak coupling between the inflaton and other fields has some phenomenological advantages, as it facilitates a very long period of slow roll needed to have enough inflationary expansion to solve the horizon and flatness problems. Peebles and Vilenkin [51] have discussed this issue and constructed a plausible inflationary model in which gravitational particle creation plays a key role. Some more recent work on the role of gravitational particle creation in various reheating models has been given, for example, in [52–57]. Dissipation and entropy production have been discussed in [58–60].

4.1.4. De Sitter particle creation?. There have been many papers written claiming that de Sitter space itself is unstable to runaway quantum particle creation. See, for example, Mottola [61]. These claims are controversial, and have been disputed by Marolf and Morrison [62, 63], who give arguments in favor of the quantum stability of de Sitter space. The model described in the previous subsection, which leads to equation (4.5), seems to support the quantum stability, at least so far as quantum particle creation is concerned. Here the energy density of created particles depends upon the transition time from de Sitter to the radiation dominated Universe, but does not depend upon the duration of the de Sitter phase. If runaway particle creation were occurring, one would expect this energy density to grow with increasing duration of inflation. This appears to be a situation where the questions of initial conditions and definition of particle number, discussed in section 2.6, play an important role.

4.1.5. Dark matter and baryogeneis from gravitational particle creation. One of the mysteries of modern cosmology is the nature of the dark matter, which composes about 25% of the mass density of the Universe. This is in contrast to only about 5% due to stars and other luminous matter. The dark matter reveals its presence only through its gravitational effects. Furthermore, the fraction of dark matter appears to grow as one probes larger length scales. For example, the dynamics of galaxy clusters reveal a larger dark matter to luminous matter ratio than do the rotation curves of individual spiral galaxies.

One plausible explanation of the dark matter is that it consists of weakly interacting, massive particles. The difficulty in directly detecting dark matter particles requires that they interact only very weakly with ordinary matter particles. However,

this makes formation of dark matter in the early Universe more difficult. One possibility is that the dark matter couples only through the gravitational interaction, which is vastly weaker than the other known forces of nature. In this case, gravitational particle creation becomes a promising model for the origin of the dark matter, and has been explored by many authors [64–73]. Several models have been explored, including ones where the present dark matter particles were directly created by the expansion of the Universe at the end of inflation, as described above. Another possibility is that very massive particles were initially created, and subsequently decayed into lighter particles [64, 65, 69]. Massive particles created at the end of inflation have also been proposed as models for gamma rays [74] and ultra high energy cosmic rays [75–77].

Gravitational particle creation could also play a role in baryogenesis, the creation of a net baryon number in the Universe. This has been discussed in [78, 79].

4.1.6. Graviton creation in inflation and tensor perturbations. The transition from de Sitter expansion to a subsequent radiation dominated phase will not only create matter particles, but also gravitons, which can later appear as primordial gravity waves which will contribute to the net energy density of the Universe, but also lead to anisotropic, tensor perturbations of the CMB. This possibility was first suggested by Starobinsky [80], who used a somewhat heuristic procedure of matching a graviton mode function in de Sitter space to one in the radiation dominated phase. In the transverse tracefree gauge, this amounts to matching two different solutions of equation (2.12) with different choices of $a(\eta)$ but $m = 0$ and $\xi = 0$. The resulting mode in the radiation dominated phase is interpreted as describing a classical, stochastic gravity wave. A similar procedure was used in [81–84], where estimates of the resulting anisotropy of the CMB were given. Later Abbott and Harari [85] and Allen [86] gave more rigorous treatments using the Bogolubov coefficient approach.

After their creation, the gravitons are in a squeezed vacuum state of the form of that found in section 3.1. At this point, they are far from describing a classical gravity wave, as the expectation values of the metric and curvature perturbation vanish in such a state. In order to become a classical wave, the gravitons need to decohere into a state closer to a coherent state. Once this has occurred, the resulting classical waves can produce anisotropies in the temperature and polarization of the CMB which are potentially observable, but have not yet been detected.

In many models of inflation, the inflaton field is expected to oscillate around a minimum value as reheating proceeds. These field oscillations can drive metric oscillations, which can in turn lead to gravitational creation of gravitons and of other particles. This is distinct from particle creation by direct coupling of matter fields to the istanton, gravitational particle creation by metric oscillations at the end of inflation was discussed by Vilenkin [87] in the context of the Starobinsky model [36], and by Suen and Anderson [88] in other versions of inflation. Graviton creation was discussed in [89] as a mechanism for inflaton decay. Several other authors have discussed various aspects of gravitons created in inflationary models [90–97].

4.1.7. Effects of anisotropy and inhomogeneity. It has been noted by several authors that anisotropic expansion can lead to enhanced particle creation. Before the inflationary model was proposed as a solution to the remarkable observed isotropy of the Universe, particle creation was investigated as a damping mechanism for anisotropic expansion [16, 98–101]. The effects of small anisotropy may be studied in the perturbative method discussed in section 2.3. Anisotropy results in a nonzero Weyl tensor, $C_{\alpha\beta\mu\nu}$. For the case of massless scalar particles, equation (2.63) for the mean number density of created particles, now becomes [16, 17]

$$n = \frac{1}{16\pi a_{\rm f}^3}\int_{-\infty}^{\infty} a^4(\eta) \times \left[\frac{1}{60}C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu} + \left(\xi - \frac{1}{6}\right)^2 R^2(\eta)\right] {\rm d}\eta. \quad (4.9)$$

Thus the anisotropy produces a contribution to the average particle creation rate which is proportional to the square of the Weyl tensor. Note that particles are created even in the conformal limit, $\xi = 1/6$. More generally, massless conformal particles, including photons and fermions [102], can be created in an anisotropic Universe. Anisotropy can also enhance entanglement entropy, as was recently discussed by Pierini *et al* [103].

Although inflation removes the immediate motivation for anisotropy damping mechanisms, it leaves other questions unresolved. One of these is the issue of the set of initial conditions which can give rise to inflation. This has been discussed by several authors, including Calzetta [104], who argues that particle creation can enlarge this set of initial conditions.

The treatments of anisotropy summarized above still assume a homogeneous Universe. However, they may be generalized to include the effects of small inhomogeneities. This was done by Frieman [105] and by Cespedes and Verdaguer [106].

4.2. Late time particle creation

After inflation has ended, and the Universe has entered a radiation or a matter dominated epoch, gravitational particle creation would normally be expected to become very small, as the spacetime curvature has become relatively small, and the expansion time correspondingly large. However, there are some possible exceptions, including emergent cosmology models. These models postulate a Universe which undergoes a very long, and possibly infinite, period of oscillations before emerging into an inflationary Universe. Emergent models have been discussed by Bag *et al* [107], who show that quantum creation of gravitons during the oscillatory phase places strong constraints on such models.

Another possible source of late time metric oscillation arise from effective actions for gravity which contain terms quadratic in the curvature. Such terms can arise either in modified classical theories of gravity or from one loop quantum

corrections, and can produce rapid metric oscillations around an expanding background spacetime [108]. These oscillations can produce gravitons, as was discussed in [109, 110]. The scale factor, including the oscillations, can be taken to be

$$a(t) = a_{\rm B}(t)[1 + A_0\ \cos(\omega_0 t)], \tag{4.10}$$

where A_0 and ω_0 are the amplitude and angular frequency of the oscillations, and $a_{\rm B}(t)$ is the background scale factor. The graviton creation rate may be calculated using the perturbative approach described in section 2.3. On time scales short compared to the background expansion time, both the graviton number density and energy density seen by a local comoving observer grow linearly in time:

$$n_{\rm g} = \frac{A_0^2\omega_0^4 t}{16\pi}, \tag{4.11}$$

and

$$\rho_{\rm g} = \frac{A_0^2\omega_0^5 t}{32\pi}. \tag{4.12}$$

Comparison of these two expressions reveals that the mean energy per graviton is $\omega_0/2$. This result may be understood from local energy conservation. In general, energy is not conserved in a time-dependent background spacetime, but here we may take the rate of variation of $a_{\rm B}(t)$ to be small compare to the oscillatory factor. In this case, we effectively have oscillations around an average background of flat spacetime, and the time averaged energy is conserved. The metric oscillation may be viewed as a classical gravity wave at angular frequency ω_0, or equivalently, as a coherent state of gravitons with energy ω_0. This energy is converted into a pair of quantum created gravitons, each with energy $\omega_0/2$. This effect is similar to that of parametric nonlinear down conversion in nonlinear optics. (See, for example, [111].)

The energy density of the created gravitons contributes to net energy density which determines the overall expansion rate of the Universe, and may be constrained by cosmological observations. This in turn places constraints on the parameters A_0 and ω_0 and on the underlying modification of gravity theory.

Another context in which quantum particle creation could become important in the late Universe would if the effective equation of state of the matter in the Universe were of the form $p = w\rho$, where $w < -1$, sometime called phantom matter. The equation of state causes such rapid expansion that curvature singularity arises, the 'Big Rip'. Nunes and Pavon [112] have suggested that quantum particle creation could give rise to such an equation of state. However, Dimopoulos [113] argues that the backreaction from particle creation could have the effect of increasing the value of w, and preventing the Big Rip.

5. Backreaction

In this section, we briefly address the backreaction of particle creation on the background spacetime geometry. Here we assume a semiclassical version of general relativity in which the expectation value of the stress tensor operator of the quantum field acts as part of the source of the gravitational field. Thus Einstein equation in this theory takes the form

$$G_{\mu\nu} = 8\pi G(T^{\rm cl}_{\mu\nu} + \langle\hat{T}_{\mu\nu}\rangle), \tag{5.1}$$

where $G_{\mu\nu}$ is the Einstein tensor of the classical background spacetime, G is Newton's constant, $T^{\rm cl}_{\mu\nu}$ is the stress tensor of any classical matter present, $\hat{T}_{\mu\nu}$ is the quantum stress tensor operator, and $\langle\hat{T}_{\mu\nu}\rangle$ is its suitably defined expectation value in some quantum state. Here the phrase 'suitably defined' hides a great deal of technical complexity, as the formal expectation value suffers from ultraviolet divergences which need to be removed by a combination or regularization and renormalization. Here we give only a quick summary, and refer the reader to, for example [1, 2], and the papers cited therein for more details.

Regularization is a formal prescription for removing the divergences, but leaving a result which depends upon one or more undetermined parameters. The conceptually cleanest regularization methods are covariant ones which respect the general covariance of the theory, such as dimensional regularization. In this case, the regularized expectation value is finite, but contains four terms which would diverge if the regulator were removed. These terms are proportion to the spacetime metric tensor $g_{\mu\nu}$, the Einstein tensor, and two tensors which are quadratic in the curvature, $H^{(1)}_{\mu\nu}$ and $H^{(2)}_{\mu\nu}$. The latter two tensors may be taken to be the functional derivatives of R^2 and of $R_{\mu\nu}R^{\mu\nu}$, respectively, with respect to the metric tensor. The regulator dependence may be absorbed by renormalization, which is a redefinition of the cosmological constant, Newton's constant, and two constants which are introduced as coefficients of R^2 and of $R_{\mu\nu}R^{\mu\nu}$ in the gravitational action. In writing equation (5.1), we have assumed that the renormalized values of all of these constants have been set to zero, apart from G, whose renormalized value is that measured by the Cavendish experiment. However, the renormalized expectation value can still contain finite terms proportional to each of the four tensors, $g_{\mu\nu}$, $G_{\mu\nu}$, $H^{(1)}_{\mu\nu}$, and $H^{(2)}_{\mu\nu}$. The latter two terms can cause particular problems because they involve fourth derivatives of the metric tensor and lead to a fourth-order Einstein equation, which can suffer from instabilities or oscillations as discussed in section 4.2. One resolution to this problem has been discussed by Simon [114], who argues that higher order theories can be viewed as resulting from approximations to a nonlocal theory, and may be treated in perturbation theory with the imposition of constraints which eliminate the unstable solutions.

More generally, it is not possible to uniquely distinguish contributions to $\langle\hat{T}_{\mu\nu}\rangle$ coming from created particles from those due to vacuum energy effects. This is related to the intrinsic ambiguity in defining particle number in a dynamic spacetime. However, we may make some order of magnitude estimates of the combined effects of particles and vacuum energy. Let ℓ be the characteristic length scale associated with the spacetime curvature. In the case of an expanding spatially flat Universe, such as that described by equation (2.29), ℓ is the characteristic expansion time scale, $1/\rho$. The Einstein and

Ricci tensors are of order $1/\ell^2$,

$$G_{\mu\nu} \approx R_{\mu\nu} \approx O\left(\frac{1}{\ell^2}\right), \tag{5.2}$$

while $\langle \hat{T}_{\mu\nu} \rangle$ is typically of order $1/\ell^4$, so

$$G\langle \hat{T}_{\mu\nu} \rangle \approx O\left(\frac{\ell_{\rm P}^2}{\ell^4}\right), \tag{5.3}$$

where $\ell_{\rm P}$ is the Planck length. Thus, we expect the backreaction from quantum effects on the background geometry to be small if $\ell \gg \ell_{\rm P}$. However, as noted in section 4.1.3, even though the energy density of particles created at the end of inflation may be small compared to the vacuum energy during inflation (see equation (4.7)), the effects of the created particles can still be significant.

Although $\langle \hat{T}_{\mu\nu} \rangle$ can contain contributions both from created particles and from vacuum energy which are hard to distinguish, there are cases where this distinction may be made. One case is an out region after particle creation has ceased, as in equation (2.29) when $\eta \gg \rho$. Here spacetime is essentially flat so there is no vacuum energy effect, but there can be a nonzero energy density of created particles. Another case is that of a conformally invariant field. Here there is no particle creation, but $\langle \hat{T}_{\mu\nu} \rangle \neq 0$ during the expansion due to vacuum energy.

In general, covariant regularization and renormalization of the stress tensor in an expanding Universe is very difficult to perform explicitly. An alternative approach is the adiabatic regularization method of Parker and Fulling [115]. This is based upon a generalization of the WKB method given by Chakraborty [116]. The key idea is to start with the solution of equation (2.12) given by equations (2.65) and (2.66). To lowest order, one takes $W_k(\eta) \approx \omega(\eta)$, given by equation (2.68), but then successively iterates equation (2.66) to obtain a sequence of better approximations to $W_k(\eta)$. The leading terms in this sequence will generate terms in the stress tensor proportional to the divergent contributions. These terms are subtracted from the mode functions computed by some other means, such as numerically. This leads to finite integrals over all modes for the stress tensor components. This method is especially well suited to numerical evaluation of the energy density, and has been used by several authors [117–119].

The treatment of the backreaction on the background spacetime geometry requires a careful consideration of the choice of gauge. This is also an issue when discussing the backreaction of classical perturbations, and has been discussed, for example, by Geshnizjani and Brandenberger [120], who give a description in terms of a modified local expansion rate.

6. Analog models

In this section, we consider several effects which are analogous in some way to particle creation by gravitational fields.

6.1. Schwinger effect

An explicit example of quantum particle creation by a classical external field is the Schwinger effect [121], the creation of electron–positron pairs by a constant electric field. The rate of pair creation per unit time per unit volume may be written as

$$R = \frac{\alpha^2}{\pi^2} E^2 \sum_{n=1}^{\infty} n^{-2} \exp\left(-\frac{\pi n m^2}{eE}\right). \tag{6.1}$$

Here m and e denote the mass and magnitude of the electric charge of the electron, respectively, $\alpha = e^2/(4\pi)$ is the fine structure constant, and E is the magnitude of the electric field in Lorentz–Heaviside units. First we note that the pair creation is exponentially suppressed if $m^2 \gg eE$. This is analogous to the suppression of the creation of massive particles by the cosmological expansion, as illustrated in equation (2.39), for example. In the electromagnetic case, this leads to a critical electric field, $E_{\rm cr} = m^2/e \approx 1.3 \times 10^{18}$ V m^{-1}, above which the spontaneous electron positron pair creation rate becomes large. This type of particle creation may be derived by a Boglubov coefficient approach, of the type outlined in section 2.1 [122]. The Schwinger effect has not yet been observed in the laboratory. However, there may be greater chance of observing pair creation in a time dependent laser field than in a static electric field [123].

One can understand the above expression for $E_{\rm cr}$ from a simple heuristic argument. The work on by an electric field E moving a charge e through a distance d is, from Newtonian mechanics, $W = eEd$. If we set this work to be of the order of the electron rest mass energy and $d = \lambda_{\rm C} = 1/m$ to be the electron Compton wavelength, then we obtain $E \approx E_{\rm cr}$.

There is a key difference between the Schwinger formula, equation (6.1), and the results for gravitational particle creation discussed in section 2.2. The former depends upon a coupling constant, e, while the latter do not, presumably reflecting the universality of the gravitational interaction. Anderson *et al* [124, 125] have argued that gravitational particle creation in global de Sitter spacetime, as opposed to an inflationary Universe, is analogous to the Schwinger effect. However, as discussed in section 4.1.4, particle creation in de Sitter spacetime does not seem to be well-defined.

6.2. Particle creation by moving mirrors

A simple example of quantum particle creation was given by Fulling and Davies [126, 127]. This consists of a moving mirror in two-dimensional spacetime coupled to a massless scalar field, φ. The field is assumed to satisfy a boundary condition on the worldline of the mirror, such as $\varphi = 0$. For a given mirror trajectory, it is possible to construct exact solutions of the wave equation which satisfy this boundary condition. Let $v = t + x$ and $u = t - x$ be null coordinates which are constant upon null rays moving to the left and to the right, respectively. A null ray of fixed v which reflects off of the mirror becomes a ray of fixed u. The relation between the values of v and of u is a function determined by the mirror's trajectory. (See figure 2.) Let

$$v = G(u), \tag{6.2}$$

or, equivalently,

$$u = g(v) = G^{-1}(v). \tag{6.3}$$

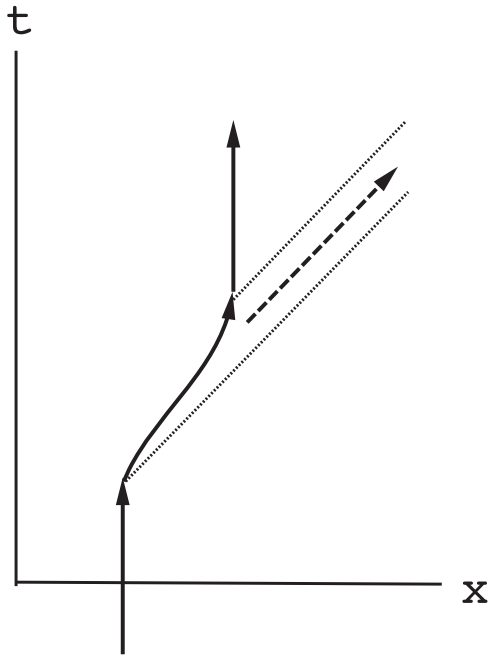

Figure 2. A moving mirror in two-dimensional spacetime accelerates for a finite period of time. The quantum radiation emitted to the right of the mirror propagates in the spacetime region between the dotted lines. There is also radiation emitted to the left which is not shown.

The mode functions which satisfy the massless wave equation and which vanish on the worldline of the mirror are

$$f_k(x) = \frac{1}{\sqrt{4\pi\omega}} \left(e^{-i\omega v} - e^{-i\omega G(u)} \right). \tag{6.4}$$

The incoming positive frequency wave, $e^{-i\omega v}$, is reflected from the mirror and becomes an outgoing wave, $e^{-i\omega G(u)}$, which is a superposition of positive frequency ($e^{-i\omega u}$) and negative frequency ($e^{i\omega u}$) parts.

Fulling and Davies [126] show that the flux of energy radiated to the right is

$$F(u) = \langle T^{xt} \rangle = \frac{1}{48\pi} \left[3 \left(\frac{G''}{G'} \right)^2 - 2 \left(\frac{G'''}{G'} \right) \right]. \tag{6.5}$$

This flux may also be expressed in terms of the instantaneous mirror velocity $v(t)$ as

$$F = -\frac{(1-v^2)^{1/2}}{12\pi(1-v)^2} \frac{d}{dt} \left[\frac{\dot{v}}{(1-v^2)^{3/2}} \right]. \tag{6.6}$$

In the nonrelativistic limit,

$$F \approx -\frac{\ddot{v}}{12\pi}. \tag{6.7}$$

Note that F may be either positive or negative. In the latter case, one has an example of the negative energy in quantum field theory.

Unfortunately, the simple solution for the moving mirror radiation of a massless field in two-dimensional spacetime depends upon the special conformal properties of this case and does not generalize to massive fields or to four-dimensional spacetime. In the four-dimensional case, there are exact solutions available for special trajectories [128, 129], and approximate solutions for general trajectories [130], but no general, exact solutions. However, the technique of mapping between ingoing and outgoing rays is crucial in the derivation of particle creation by black holes [131].

Particle creation by moving mirrors is sometimes called the dynamical Casimir effect. An analog version of this effect has been observed in a metamaterial [132]. The possible effects of actual moving mirrors in an optical cavity have been discussed in [133].

6.3. Creation of squeezed light in nonlinear optics

As noted above, the quantum state of the particles created by a time-dependent gravitational field is a multi-mode squeezed vacuum state. In recent decades, it has become possible to create squeezed states of photons in the laboratory using nonlinear optical materials [134, 135]. In this section, we will examine how this is done, and show that the basic physical process is essentially the same as particle creation by an expanding Universe or by a moving mirror.

First we review selected aspects of nonlinear optics. Recall that in SI units, the electric displacement vector **D** in a material is related to the electric field **E** and polarization (electric dipole moment per unit volume) **P** by

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}, \tag{6.8}$$

where ϵ_0 is a constant. This relation holds in both isotropic and anisotropic materials. In a nonlinear material, the Cartesian components of **P** become nonlinear function of those of **E**, and may be expanded in a power series of the form

$$P_i = \epsilon_0 \left(\chi^{(1)}_{ij} E_j + \chi^{(2)}_{ijk} E_j E_k + \chi^{(3)}_{ijkl} E_j E_k E_l + \cdots \right), \tag{6.9}$$

where sums on repeated indices are understood. The coefficients in this expansion are tensors, and are called susceptibilities.

For simplicity, consider the case where the three vectors, **D**, **E**, and **P** may be taken to all be in the same direction, which we take to be the z-direction, so

$$D_i = \delta_{iz} D, \qquad E_i = \delta_{iz} E, \qquad P_i = \delta_{iz} P. \tag{6.10}$$

Note that this does not require isotropy, but does require that certain components of the susceptibility tensors vanish, e.g. $\chi^{(1)}_{xz} = 0$, etc. Now equation (6.9) may be written as

$$P = \epsilon_0 \left(\chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \cdots \right), \tag{6.11}$$

where

$$\chi^{(1)} = \chi^{(1)}_{zz}, \qquad \chi^{(2)} = \chi^{(2)}_{zzz}, \qquad \chi^{(3)} = \chi^{(3)}_{zzzz}. \tag{6.12}$$

Here $\chi^{(1)}$ is the familiar linear susceptibility, $\chi^{(2)}$ is the second order susceptibility, which can only be nonzero in a material whose unit cell lacks spatial inversion symmetry, and $\chi^{(3)}$ is the third order susceptibility.

Now let the electric field be a sum,

$$E = E_0 + E_1 \tag{6.13}$$

of a background field E_0 and a perturbation E_1, where $|E_1| \ll |E_0|$, and assume similar expressions for D and P. If we substitute these forms into equations (6.8) and (6.9), and expand to first order in the perturbation, we find

$$D_1 = \epsilon_{\rm eff} E_1, \tag{6.14}$$

where

$$\epsilon_{\rm eff} = \epsilon_0[(1 + \chi^{(1)}) + 2\chi^{(2)}E_0 + 3\chi^{(3)}E_0^2 + \cdots]E_1 \tag{6.15}$$

is an effective dielectric function which will appear in the wave equation for the perturbation E_1. If the classical field E_0 is a function of time, then $\epsilon_{\rm eff}$ will be also. If we quantize the perturbation, in general there will be mixing of the positive and negative frequency modes of the quantum field and hence quantum creation of photons.

Consider the geometry illustrated in figure 3. Here the classical field E_0 is linearly polarized in the z-direction, propagating in the x-direction, and has angular frequency ω_0. It passes through a slab of nonlinear material and induces a space and time-dependent effective dielectric function, $\epsilon_{\rm eff}(x,t)$. A vacuum mode $f_\omega(y,t)$ of the quantized electromagnetic field, with angular frequency ω, enters the slab propagating in the y-direction. Before entry, it is a pure positive frequency mode. Inside the slab, it obeys a wave equation containing $\epsilon_{\rm eff}(x,t)$, which leads to $f_\omega(y,t)$ becoming a superposition of positive and negative frequency components when it exits the slab. This is analogous to the effect of the expansion of the Universe, discussed in section 2, and leads to quantum creation of pairs of photons in a squeezed vacuum state. In both cases, particle number is not well-defined in the time-dependent region, and the energy needed to create particles comes from the time-dependent background. The mean number of photons created into mode $f_\omega(y,t)$ will be $|\beta|^2$, where β is the amplitude of the negative frequency component of $f_\omega(y,t)$ in the final region. The angular frequency of the created photons depends both upon the frequency ω_0 of the classical field, and upon the order of the relevant nonlinear susceptibility. If second order nonlinearity, described by $\chi^{(2)}$, dominates, then $\omega = \omega_0/2$. This was the case in the experiment reported in [135], so a pair of photons, each with energy $\omega_0/2$, are created by a classical oscillating background with angular frequency ω_0. This is the exact analog of the creation of pairs of gravitons by an oscillating background described in section 4.2. More general discussions of the effects of a time-dependent effective dielectric function have been given in [136, 137].

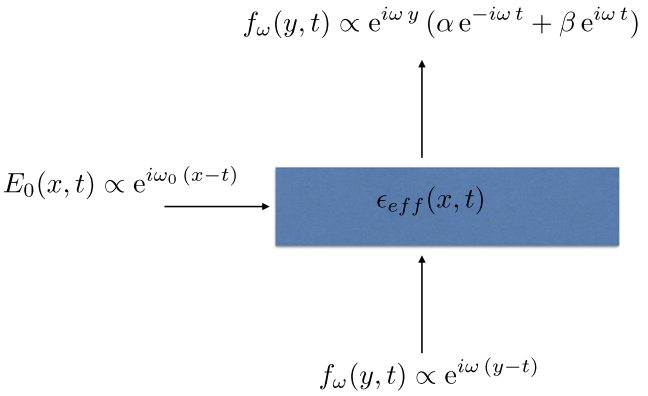

Figure 3. The quantum creation of photons into a squeezed vacuum state in a nonlinear material is illustrated. The shaded region is a slab of nonlinear material in which a classical field with angular frequency ω_0 propagates. This creates a time dependent effective dielectric function, $\epsilon_{\rm eff}$. A vacuum mode f_ω of the quantized electric field also propagates through the slab in a different direction. This mode enters the material as a pure positive frequency mode, but exits as a superposition of positive and negative frequency parts, leading to quantum photon creation. Both the classical field and the photons are assumed to be linearly polarized in the direction out of the page.

6.4. Other analog models

The various models discussed in the previous subsections can all be described as examples of the phenomenon of quantum vacuum friction, of which there are other examples, such as atoms moving near a surface [138]. The collapse or expansion of a Bose–Einstein condensate bears some similarities to the cosmological expansion, and the excitations created in the condensate are analogous to cosmological particle creation [139, 140]. Several other analogies between superfluids and cosmology were discussed by Volovik [141]. The creation of squeezed states of ions in a trap as an analog model was discussed by Fey *et al* [142]. In another model, Walud *et al* [143] have discussed the creation of low mass bosons by an oscillating charged particle as a possible means for axion detection, as well as an analog of cosmological particle creation.

7. Summary

This review has covered the quantum creation of particles by the expansion of the Universe using the formalism of quantum field theory in curved spacetime. A quantum field theory requires both a set of local field operators, as well as a characterization of the set of quantum states. The latter is especially subtle in an expanding Universe, where no unique local definition of particles, and hence of the vacuum state exists. We have dealt with this ambiguity by use of quasilocal definitions in regions where the expansion rate is sufficiently small, and by asymptotic definitions when available. However, it is this ambiguity which leads to the phenomenon of quantum particle creation. The definition of a particle in an initial region can differ from that in a final region, so a quantum state without particles initially may later contain particles. This was discussed in section 2.1 using the formalism of Bogolubov transformations. The particle creation is also due to the non-conservation of field energy in a time-dependent spacetime, but while the creation is occurring only approximate definitions of local particle number are meaningful. The inherent ambiguity of the concept of particle number in a time dependent situation has observable consequences, as illustrated by the creation of squeezed states described in section 6.3. This review has concentrated on

the creation of bosons, and especially scalar particles for simplicity. Some exact results for particular model universes, as well as approximate results of more general spacetimes were reviewed, which allowed us to make some estimates of the particle creation rate.

The quantum state of the created particles was treated in section 3, where we found that these particles exhibit a high degree of correlation and entanglement. We saw that the quantum state is a multi-mode squeezed vacuum state, which is a superposition of various numbers of pairs of particles. Each pair contains particles with opposite momenta of equal magnitude. Interactions can lead to decoherence and entropy production.

The application of quantum particle creation to various cosmological models was discussed. In particular, quantum particle creation could play a role in the creation of matter in the Universe after the end of an inflationary era, as discussed in section 4.1. There has been considerable work recently on models in which the dark matter arose from quantum particle creation. Gravitons could be among the created particles, and a bath of relic gravitons might leave a detectable imprint on the cosmological microwave background in the form of tensor perturbations. Cosmological anisotropy, inhomogeneity, and rapid metric oscillations are other effects which could enhance particle creation.

The backreaction of the created particle on the background spacetime was discussed in section 5. In a loose sense, the energy to create the particle comes from the classical time-dependent gravitational field of the expanding Universe. However, the energy of gravitational field is difficult to define clearly. A better description of the backreaction comes from the semiclassical theory of gravity, where the renormalized expectation value of the quantum matter stress tensor acts as the source of a classical gravitational field.

Several analog models for cosmological particle creation were treated in section 6. These include photon creation by moving mirrors and other time-dependent media. Some of these models are accessible to experimental confirmation, which effectively provides support for the basic formalism reviewed in this article. One example is the creation of squeezed states of light in nonlinear optical materials discussed in section 6.3 which is now routinely performed in the laboratory, and gives rise to squeezed vacuum states of the form expected in cosmological particle creation. This, and several other topics treated in this review, are currently active areas of research.

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Data availability statement

No new data were created or analysed in this study.

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