Dual-Function Multiplexing for Waveform Design in OFDM-Based Joint Communications and Sensing: An Edgeworth Box Framework

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***Abstract*—In joint communications and sensing (JCS), which is a potential technology for the 6G wireless communication networks, the multiplexing of communication and sensing functions is of critical importance. In the signaling framework of orthogonal frequency division multiplexing (OFDM), if all subcarriers are used for communications (which can also be used for sensing as a byproduct), the randomness of data will add significant uncertainty to the sensing results; meanwhile, if deterministic signals are used for all subcarriers, in order to optimize the sensing performance, the function of communications is invalidated due to the loss of randomness. Therefore, it is proposed to multiplex the communication and sensing functions in different OFDM subcarriers. The mutual benefits of communication and sensing subcarriers are analyzed, in which communication subcarriers provide extra bandwidth and power for sensing, while sensing subcarriers with deterministic sensing signals are used as pilots for communication channel estimation. The allocation of power and subcarriers for communications and sensing is solved using the Edgeworth Box in economics. Numerical simulations are used to demonstrate the proposed multiplexing scheme in JCS.**

I. INTRODUCTION

As a potential technology in 6G cellular systems, joint communications and sensing (JCS) have received substantial interest in recent years [1]–[5]. In JCS, the same round of electromagnetic (EM) wave delivers data messages to the corresponding communication receiver in the forward propagation, while bringing environmental information to the JCS transceiver in the backward propagation upon reflection/scattering. For marrying the two functions of communications and sensing that are developed and designed independently in the history, it is of critical importance to understand the mutual benefits and conflict of interest between communications and sensing. For the bitter side of the marriage, the conflict of interest mainly includes the randomness in communications and determinism in sensing.

However, a moment of reflection finds the mutual benefits between communications and sensing in the forward and backward propagations of the EM wave:

- Communications: The main function of communication is to deliver data in the forward propagation. However, when communication signal is reflected by targets, the backward propagation benefits the radar sensing, despite the inherent randomness (which is essentially pseudo-random for the JCS transceiver since it knows the modulated data). Moreover, the modulation of communication data provides bandwidth (thus the possibility of pulse compression for sensing) and power (for combating noise).
- Sensing: The main function of sensing is in the backward channel. However, when deterministic sensing signal is received by the communication receiver, it can be used as pilots for channel estimation, thanks to the deterministic signal also known to the communication receiver.

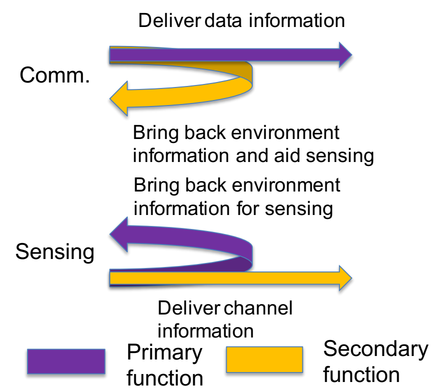

Fig. 1. Illustration of mutual benefits in JCS

The above mutual benefits of communications and sensing are illustrated in Fig. 1, where communication benefits sensing in terms of extra bandwidth and power, in the backward channel while sensing benefits communication in terms of pilots for channel estimation, in the forward channel.

To fully exploit the mutual benefits of communication and sensing signals, we consider the signal structure in orthogonal frequency division multiplexing (OFDM) for JCS. Based on the OFDM signaling structure, we will study the waveform synthesis by allocating power and phase to different subcarriers, in order to maximize the overall system performance in a Pareto manner[1]. To this end, we leverage the Edgeworth Box in economics [6], where the communications and sensing are considered as two agents, each having a utility function (channel capacity and sensing performance, respectively). Power and bandwidth are considered as two commodities divided between the two agents, which are also exchangeable. The Edgeworth

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[1] Pareto efficiency, or Pareto optimality, is a state at which resources cannot be reallocated to make one individual better off without making at least one individual worse off.

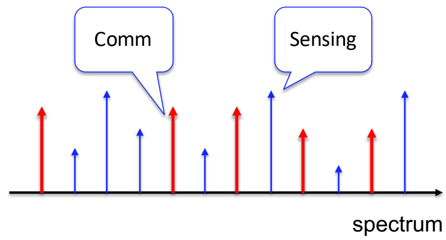

Fig. 2. The interleaved-subcarrier structure of communications and sensing

box model helps to obtain the exchange price between the two commodities, as well as the Pareto allocation, which provides an elegant framework for the resource allocation and waveform synthesis in JCS.

The remainders of the paper are organized as follows. Researches related to this paper are elaborated in Section II. The system model is introduced in Section III. Then, the analysis on the mutual benefits between communications and sensing is carried out in Section IV. Given the performance gains, the Edgeworth Box based Pareto resource allocation is discussed in Section V. Numerical results and conclusions are given in Sections VI and VII, respectively.

II. Related Works

The readers are referred to [1]–[5] for comprehensive surveys on JCS. The design of OFDM waveform for radar sensing is to optimize the amplitude of each subcarrier [7]. Meanwhile, substantially more studies have been paid to OFDM communications, due to its fundamental role in 4G and 5G cellular communication systems [8]. There have been various studies on JCS using OFDM signaling. In [9], the OFDM waveform is optimized for the coexistence of communication and radar systems, in order to minimize the power consumption. In [1], it is proposed to divide each subcarrier by the corresponding information symbol, in order to remove the data dependency of radar sensing[2]. However, these studies use pure data communication signals for sensing, without exploring the possibility of multiplexing dedicated sensing waveforms.

III. System Model

In this section, we introduce the system model of JCS with OFDM signaling.

A. Signal Model

We consider the structure of interleaved subcarriers in OFDM signaling for JCS with a single antenna, as illustrated in Fig. 2. We assume M subcarriers with initial carrier frequency f_c and frequency spacing δ_f. The subsets of subcarriers dedicated to sensing and communications are denoted by $\mathcal{S}_s$ and $\mathcal{S}_c$, respectively. The definitions of the two subcarrier subsets are known to both communication receiver and JCS transceiver, based on a hand-shaking protocol. The transmitted signal is given by

$$s(t)=\sum_{m=0}^{M-1} X_m e^{j(2\pi(f_c+m\delta_f)t+\phi_m)},\ t\in[0,T_p], \qquad (1)$$

where T_p is the pulse duration, X_m is a quadratic amplitude modulation (QAM) symbol if it is allocated for communications, or a deterministic value also known to the communication receiver, if it is allocated for sensing, and ϕ_m is the corresponding phase. For simplicity, we assume that the modulation and coding in the given communication subcarrier set $\mathcal{S}_c$ are operated independently of the sensing procedure, using the standard procedure in traditional communication systems, thanks to the separation between communication and sensing signals in different carriers. The total transmit power is denoted by P_t. The powers allocated to sensing and communications are denoted by P_s and P_c, respectively. Obviously we have $P_t = P_s + P_c$. We denote by P_m the power over subcarrier m, which equals $E[|X_m|^2]$. The constraint on the total power is given by $\sum_{m=0}^{M-1} P_m = P_t$.

B. Channel Model

For simplicity, we consider a single significant target for the radar sensing and a set of reflectors that substantially impact the communication channel gains but incur minor reflections to the JCS transceiver (which can be considered as clutters), except for the significant target. The scenario will be extended to multiple targets when multiple antennas are used, in our future study. For the sensing function, we model the roundtrip channel as the attenuation and delay of signal; i.e., the received signal is given by

$$r_s(t)=\alpha s(t-\tau), \qquad (2)$$

where τ is the round trip time and α is the amplitude attenuation due to path loss. This is valid for static reflectors. The Doppler shift due to moving targets will be studied in our future research.

For communications, we model the channel using the frequency response $H(jw)$. Therefore, the received baseband signal on the n-th subcarrier is given by

$$Y_m = X_m H_m + N_m, \qquad (3)$$

where the complex scalar channel gain $H_m = H(j2\pi(f_c + m\delta_f))$ and N_m is the complex Gaussian thermal noise. For simplicity, we consider frequency-domain block fading in the frequency spectrum, namely $H_m = H_n$ if $\left\lfloor \frac{m}{N_c} \right\rfloor = \left\lfloor \frac{n}{N_c} \right\rfloor$, where N_c is the number of coherent subcarriers having the same channel gains and is assumed to divide M, thus defining $M_c = \frac{M}{N_c}$. We assume $E[N_m^2] = \sigma_n^2$ for all m and $\{N_m\}_{m=1,\dots,M}$ are mutually independent.

C. Performance Metrics

We consider the following performance metrics for communications and sensing, respectively:

- Communications: The communication performance is featured by the channel capacity C, which characterizes the

[2]However, this might not be a good strategy, since the division results in a set of sinusoidal subcarriers, thus losing the bandwidth brought by the random data.

maximum rate of reliable data transmissions. It is well known that the sum capacity of multiple subcarriers is given by

$$C = \sum_{m \in \mathcal{S}_c} \log\left(1 + \frac{|H_m|^2 P_m}{\sigma_n^2}\right). \quad (4)$$

- Sensing: The performance of sensing is characterized by the ambiguity function (AF) [10], which indicates the resolution of radar sensing. For a radar signal x, the AF is defined as

$$\chi(\tau, f) = \int_{-\infty}^{\infty} x(t)x^*(t-\tau)e^{-j2\pi f(t-\tau)}dt. \quad (5)$$

For simplicity, we consider only the time-domain autocorrelation r in the AF χ since we focus on only the function of ranging, namely

$$r(\tau) \quad = \quad \int_{\tau}^{T_p} x(t)x^*(t-\tau)dt. \quad (6)$$

We consider the samples of the autocorrelation $r[k] = r(kT_c)$, namely the k-th sample of the autocorrelation function and $T_c = \frac{T_p}{M}$ is the chip period[3]. For quantifying the performance, we use the integrated sidelobe level (ISL) [10] to characterize the radar sensing performance:

$$\xi = \sum_{k=-(M-1), k\neq 0}^{M-1} |r[k]|^2 = 2\sum_{k=1}^{M-1} |r[k]|^2, \quad (7)$$

where $M-1$ is the number of sidelobes taken into account. A smaller ISL indicates less confusion among radar targets, and prevents strong sidelobes from dominating weak targets or resulting in a wrong range.

IV. Analysis of Mutual Benefits

In this section, we analyze the mutual benefits between the signals in the subcarrier sets of communications and sensing.

A. Communications for Sensing

We first evaluate the benefit of communication for the task of sensing by proving the following conclusion.

1) ISL Assessment:

Proposition 1. *For the proposed JCS scheme, in which the dedicated sensing and communication signals are allocated to different subcarriers, the ISL is given by*

$$\xi = \xi_s + \xi_c + \xi_{s,c} - 2P_s P_c, \quad (8)$$

where ξ_s (ξ_c) is the ISL when all communication (sensing) subcarriers are set to zero, and the cross term $\xi_{s,c}$ given by

$$\xi_{s,c} \quad = \quad \sum_{p=0}^{M-1} \Phi_p^s \Phi_p^c, \quad (9)$$

where Φ_p^s (Φ_p^c) equals Φ_p when all communication (sensing) subcarriers are set to zero.

[3]The M chips are the IDFT of the M symbols on the subcarriers.

2) Benefit of Communication for Sensing: The benefit of communication subcarriers for the purpose of sensing consists of the increase of bandwidth for lowering the ISL and the enhancement of power for combating noise. Note that, if the power enhancement is not taken into account, setting zero power for all subcarriers will result in a perfect ISL=0. To incorporate both performance gains, we consider the following performance gain for sensing, brought by the incorporation of communication subcarriers:

$$\mathcal{G}_{c\to s} = \frac{r_{s+c}^2[0]}{\xi_{s+c} + \sigma_n^4} - \frac{r_s^2[0]}{\xi_s + \sigma_n^4}, \quad (10)$$

where the subscript $s + c$ means that both the sensing and communication subcarriers are used for sensing, while the subscript s means that only the sensing subcarriers are used for sensing. Note that the metric $\frac{r^2[0]}{\xi+\sigma_n^4}$ is similar to signal-to-interference-and-noise ratio (SINR), where the mainlobe $r[0]$ is considered as the signal power and ξ is the power of self-interference.

B. Sensing for Communications

Now we evaluate the benefit of the signal over the sensing subcarriers for communications in terms of channel estimation[4]. Consider the channel estimation for the n-th coherent subset of subcarriers, in which the channel gains $\{H_m\}_{m=(n-1)N_c+1,\ldots,nN_c}$ are identical. Suppose that there are K_n sensing subcarriers within one coherent subset as the channel estimation pilots, allocated to the coherent subset for channel estimation, each having identical power Q_n. Then, the mean square error (MSE) of channel estimation $\hat{H}$ is given by

$$E\left[\delta H_m^2\right] = \frac{\sigma_n^2}{K_n Q_n}. \quad (11)$$

Therefore, when detecting the symbol over subcarrier m, we use the matched filter output:

$$Y_m \hat{H}_m^* = X_m H_m^2 + X_m H_m \delta H + N_m \hat{H}_m^*. \quad (12)$$

Therefore, the signal-to-noise ratio (SNR) in $Y_m\hat{H}^*$ is given by

$$\begin{aligned} \gamma_m &= \frac{P_m|H_m|^4}{P_m H_m^2 E[\delta|H_m|^2] + \sigma_n^2(|H_m|^2 + E[\delta H_m^2])} \\ &= \frac{KP_m Q_n |H_m|^4}{P_m H_m^2 \sigma_n^2 + \sigma_n^2 K_n |H_m|^2 Q_n + \sigma_n^4} \\ &\approx \frac{KP_m Q_n |H_m|^4}{P_m |H_m|^2 \sigma_n^2 + \sigma_n^2 K_n |H_m|^2 Q_n} = \frac{P_m |H_m|^2}{\frac{P_m \sigma_n^2}{K_n Q_n} + \sigma_n^2}, \end{aligned} \quad (13)$$

where the approximation is valid in the the high SNR regime. We notice that, in the denominator of (13), the term $\frac{P_m \sigma_n^2}{KQ_n}$ represents the impact of channel estimation error on the SNR of communication subcarriers, which can be mitigated using greater K_n (more sensing subcarriers as pilots) and Q_n (more power allocated to sensing subcarriers).

[4]Sensing could be beneficial to communications in other aspects, such as receiver positioning for beam alignment, which is beyond the scope of this paper.

For facilitating further analysis, we approximate (13) by

$$\begin{aligned}\gamma_m &= \frac{P_m|H_m|^2}{\sigma_n^2}\frac{1}{1+\frac{P_m}{K_nQ_n}} \\ &\approx \frac{P_m|H_m|^2}{\sigma_n^2}\left(1-\frac{P_m}{K_nQ_n}\right), \quad (14)\end{aligned}$$

which means that the SNR is reduced by a proportion of $\frac{P_m}{K_nQ_n}$. Therefore, given the high-SNR assumption, the channel capacity of subcarrier m is reduced by

$$\delta C_m \approx \log\left(1-\frac{P_m}{K_nQ_n}\right) \approx -\frac{P_m}{K_nQ_n}. \quad (15)$$

Assuming that $K_n > 0$ and the same power P_n is allocated to all the communication subcarriers in coherent block n, we obtain the total channel capacity gain due to the sensing subcarriers as pilots:

$$\begin{aligned}\delta C \approx& \sum_{n=1}^{M_c}(N_c-K_n)\left(\left(\log\left(\frac{P|H_n|^2}{\sigma_n^2}\right)\right)-C_{nh}\right) \\ &-\sum_{n=1}^{M_c}K_n\log\left(\frac{P|H_n|^2}{\sigma_n^2}\right)-\sum_{n=1}^{M_c}\frac{(N_c-K_n)P_n}{K_nQ_n}, \quad (16)\end{aligned}$$

where C_{nh} is the noncoherent channel capacity without channel state information (CSI).

C. Waveform Design

The signals over the sensing subcarriers cannot be made adaptive to the instantaneous signals over the communication subcarriers, since it needs to be deterministic as pilot signals for communications[5]. It can be optimized such that the overall time-domain signal has a good mean ISL. We consider traditional time-domain radar sensing waveforms such as Golomb codes or Zadoff-Chu codes [10], denoted by $x^*[n]$, $n = 0, ..., N-1$. When the communication signals are given by $\{X_k\}_{k\in\mathcal{S}_c}$, we design the sensing signals $\{X_k\}_{k\in\mathcal{S}_s}$, in order to minimize the difference between the synthesized waveform and the target waveform:

$$\mathbf{x}_s^* = \arg\min_{\mathbf{x}_s} E\|\mathbf{F}_s\mathbf{x}_s+\mathbf{F}_c\mathbf{x}_c-\mathbf{x}^*\|_2^2, \quad (17)$$

where $\mathbf{x}_s$ is the vector obtained by stacking the elements in $\{X_k\}_{k\in\mathcal{S}_s}$, $\mathbf{F}_s$ and $\mathbf{F}_c$ are the columns of discrete Fourier transform (DFT) matrix corresponding to the indices of sensing and communication subcarriers, respectively. The objective function (17) is given by

$$\begin{aligned}MSE &= \mathbf{x}_s^T\mathbf{F}_s^T\mathbf{F}_s\mathbf{x}_s+E\left[\mathbf{x}_c^T\mathbf{F}_c^T\mathbf{F}_c\mathbf{x}_c\right]+(\mathbf{x}^*)^T\mathbf{x}^* \\ &- \mathbf{x}_s^T\mathbf{F}_s^T\mathbf{x}^*-(\mathbf{x}^*)^T\mathbf{F}_s\mathbf{x}^*. \quad (18)\end{aligned}$$

Taking derivative with respect to $\mathbf{x}_s$, we obtain

$$\mathbf{F}_s^T\mathbf{F}_s\mathbf{x}_s^* = \mathbf{F}_s^T\mathbf{x}^*, \quad (19)$$

where the optimal solution is independent of the communication signals $\mathbf{x}_c$. Notice that the above analysis did not incorporate the power constraint of $\mathbf{x}_s$, namely $\|\mathbf{x}_s\|_2^2 \le P_s$. However, this can be easily solved by renormalizing the power of $\mathbf{x}_s^*$ to P_s, since $\mathbf{x}^*$ minimizes the ISL regardless of the power.

[5]If the dedicate sensing signal can be made adaptive to the communication signal, the sensing performance can be improved; however, it cannot be used as pilots due to its randomness.

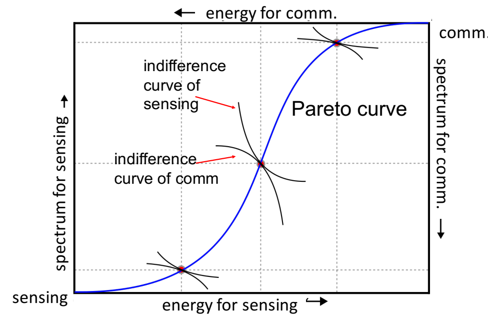

Fig. 3. Illustration of Edgeworth Box

V. Edgeworth Box JCS

In this section, we discuss the Pareto front for the proposed JCS scheme using the tool of Edgeworth Box in economics.

A. Edgeworth Box

In the Edgeworth Box [6], as illustrated in Fig. 3, each agent, communication or sensing, has its own indifference curves in the spectrum-power plane, namely its performance metric (data rate or MSE) does not change on the same curve. The tangent point of two indifference curves belonging to different agents is a Pareto point, at which communications and sensing may reach a bargain. The set of the tangent points forms a Pareto curve, which characterizes the tradeoff in JCS.

B. Algorithm for Pareto Front

The algorithm for computing the Pareto front using the Edgeworth Box is summarized in Procedure 1.

Algorithm 1 Edgeworth Box based Pareto Front Computation

Divide the power-bandwidth plane into grid.
for Each grid point (power-bandwidth combination) **do**
  Compute the corresponding sensing SINR and capacity.
end for
Determine the sampled values of SINR and capacity.
for Each value of sensing SINR θ and each capacity C **do**
  Find the grid points with sensing SINR close to θ.
  Find the grid points with capacity close to C.
  Calculate the slopes of tangent at each of the point.
end for
for Each indifference curve of SINR **do**
  Find the point whose sensing SINR and capacity slopes are close.
  Record this point as a Pareto point.
end for
Output the Pareto points.

VI. Numerical Results

In this section, we use numerical results to illustrate the theoretical framework and analytic conclusions.

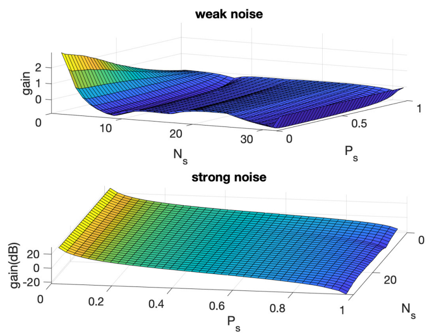

Fig. 4. Performance gain in sensing with weak and strong noises

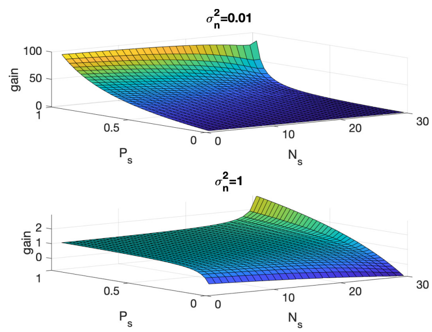

Fig. 5. Performance gain in communications with weak and strong noises

A. Performance Gains Due to Mutual Benefits

We first numerically calculated the performance gains of sensing and communications due to the aid of dedicated communication and sensing signals. In Fig. 4, we plotted the performance gain metric $\mathcal{G}_{c\rightarrow s}$ in (10), which is normalized by the performance without the aid of communication signals (thus setting the signals over the communication subcarriers to be zero). We consider 256 subcarriers and assume that the coherent block width N_c is 32. We set $P_t = 1$ and consider P_s on 40 uniform grid points between 0 and 1. The data modulation is assumed to be 256-QAM. The number of sensing subcarriers within each coherent block ranges from 1 to 31. The results of $\sigma_n^2 = 0.01$ (weak) and $\sigma_n^2 = 1$ (strong) are computed. We observe that, when the noise power is weak, the gain could be negative. For these cases, the total power ($r[0]$) is increased, while the ISL is also increased due to the power of communication signals. When the noise is weak, the denominator of each ratio in (10) is dominated by ISL. Therefore, the introduced communication signal does not necessarily help to reduce the ISL.

The performance gains of channel capacity are plotted in Fig. 5, where both $\sigma_n^2 = 0.01$ (weak) and $\sigma_n^2 = 1$ (strong) are considered. We observe that the performance gain is also positive, and is more significant when the noise is weak. This is reasonable since the channel estimation error will play a more important role. We also observe that the performance gain drops when P_s becomes smaller, or when the number of sensing subcarriers decreases.

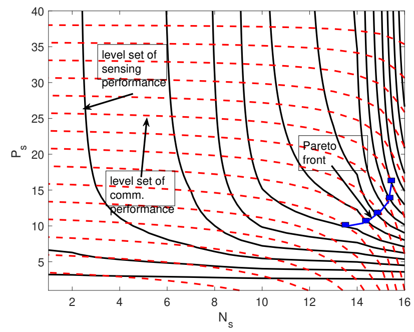

Fig. 6. Pareto analysis based on Edgeworth box

B. Edgeworth Box based Pareto Analysis

The Edgeworth Box analysis for Pareto front is plotted in Fig. 6. We use the same simulation setup as in Figures 4 and 5. The noise power is set to an intermediate value of 0.1. We obtained the performances of sensing (the SINR-like metric in (10)) and communication (capacity) for each point on the 40×32 grid. The level sets of the sensing and communication performances are plotted. The tangent points of the level sets are identified and then connected to form the Pareto front, as illustrated in the figure.

VII. Conclusions

In this paper, we have studied the multiplexing of dedicated sensing and communication signals in different subcarriers in OFDM-based JCS. In JCS the dedicated sensing and communication signals could be beneficial to each other. For the allocation of power and bandwidth between communications and sensing, the Edgeworth Box in economics has been applied to calculate the Pareto front of the system performance.

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