Unified Waveform Design in Joint Communications and Sensing with Clutters: Shannon or Cramer-Rao?

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Abstract—**In joint communications and sensing (JCS), the performance metrics belong to the areas of information theory and estimation theory, respectively. The lack of unified design criterion prohibits gaining intuitions and unifying the mathematical structure in the analysis and design of JCS. This paper endeavors to unify both performance metrics of communications and sensing in either information-theoretic or estimation-theoretic framework. In the information-theoretic framework, the mutual information between the target and received signal, conditioned on the transmitted signal, is maximized while that of clutter and signal is minimized, which is formulated as an optimization problem. For the estimation-theoretic framework, the differential relationship between mutual information and minimum mean square error (MMSE) is leveraged to unify both communications and radar sensing in terms of MMSE. Both criteria are evaluated and compared using numerical simulations.**

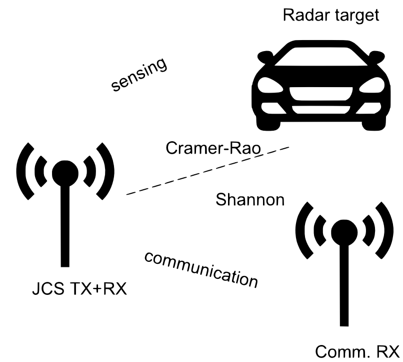

Fig. 1: Illustration of JCS mechanism.

I. Introduction

In recent years, the technique of joint communications and sensing (JCS) has attracted substantial attentions in academia and industry, due to its potential applications in various cyber physical systems, such as autonomous driving and unmanned aerial vehicular networks in which both communications and sensing are essential [1]–[5]. As illustrated in Fig. 1, a JCS transceiver sends electromagnetic (EM) wave modulated by data, and delivers the information to the destined communication receiver in the forward propagation. When the EM wave is reflected by certain target(s), the JCS transceiver infers the target information from the backward propagated EM wave. Both functions are accomplished in the same round of transmission, using the same waveform and sharing the same bandwidth and power. Therefore, a joint design for both communications and sensing in JCS will substantially improve the efficiency of spectrum and power.

However, communications and sensing have different goals, namely 'pushing' data to the destination and 'pulling' information from the environment, thus making the joint optimization incoherent. Moreover, the two technologies, communications and sensing, are based on different theories: information theory is the corner stone of communications, while radar sensing is based on estimation theory. Despite many intrinsic relations between information theory and estimation theory [6], they use different performance metrics (e.g., mutual information and mean square error (MSE)) and different arguments (e.g., random coding and Cramer-Rao bound). Therefore, it is desirable to integrate the two distinct tasks in the same framework, either information theory (Shannon) or estimation theory (Cramer-Rao). Note that in the rate-distortion theory [7], information in terms of Shannon entropy and estimation error in terms of distortion are coherently related. A framework similar to the rate-distortion theory has been proposed in [8] for the theoretic analysis of JCS. However, the results are highly abstract, due to the simplified model, and cannot be applied to practical JCS system directly.

In this paper, we discuss integrated frameworks incorporating communications and sensing:

- Information theoretic framework: It is in [9] that information theory is first applied in the design of radar sensing waveforms, by maximizing the mutual information of the received signal and the radar target information. Using this framework, JCS can be considered as a pure communication system (JCS transceiver to communication receiver, and radar target to JCS transceiver), thus facilitating unified analysis and design.
- Estimation theoretic framework: A bridge between communications and estimation has been disclosed in [10]: in additive noise channels the derivative of mutual information $I(X;Y)$ between variables X and Y with respect to the signal-to-noise ratio (SNR) equals the minimum mean-square error (MMSE) of estimating X given Y. Therefore the MMSE of estimation indicates the marginal utility of SNR for increasing the mutual information. By using MMSE metrics in JCS, the system performance is measured by the estimation error of radar sensing and the SNR efficiency of communications.

In this paper, we will consider both unified frameworks and make comparisons. The trade-off between communications and sensing, as well as the mitigation of clutters, will be formulated as optimization problems, using unified metrics in

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information theory or estimation theory. The performance will be demonstrated and compared using numerical simulations.

The remainder of this paper is organized as follows. In Section II the related works are introduced. The signal model for JCS is given in Section III. Then, the unified information-theoretic and estimation-theoretic frameworks are discussed in Sections IV and V, respectively. Finally, numerical results and conclusions are provided in Sections VI and VII, respectively.

II. Related Works

For the technique of JCS, comprehensive surveys can be found in [1]–[5]. Theoretical studies have been carried out for JCS in [8], which integrates both information and estimation theoretic metrics similarly to the rate-distortion theory [7]. The relationship between information-theoretic measures and estimation-theoretic metrics has been explored by statisticians for a long time (see Chapter 12 of [7]), e.g., the Stein's Lemma that relates the asymptotic performance of hypothesis testing to the Kullback-Leibler's distance that is of more information theoretic flavor. In particular, the relationship between mutual information and MMSE is established in the Guo-Shamai-Verdu identity [11]. A comprehensive introduction to information-theoretic signal processing has been given in [6]; e.g., using information-theoretic metric to explain data sufficiency, or leveraging the maximum entropy principle for estimating parameters. It provides motivation but not direct solution to the design of JCS. The application of Kullback-Leibler's distance in statistical inference is also discussed in [12]. An alternative bridge between information-theoretic measure and statistical inference (including estimation) is the information geometry [13]; however, there has not been a clue for how to apply information geometry in JCS. Moreover, most studies on JCS have not taken clutters that are of critical importance in radar sensing into account. The corresponding formulation of waveform diversity subject to clutter has been studied in [14].

III. System Model

In this section, we introduce the JCS system model, which includes the models of the signal, and the channels of radar sensing and communications.

A. Signal Model

We denote by $x(t)$ the transmitted signal, and by $y_c(t)$ and $y_s(t)$ the received signals at the communication and sensing receivers, respectively. For simplicity of analysis, we consider scalar signals, namely single-antenna systems. The extension to the multi-antenna case will be non-trivial and is left to our future study.

We further assume that the transmitted signal is a wide-sense stationary process (WSS) within each information symbol (radar pulse) period. Therefore, the transmitted signal has the following spectral representation (pp.109, [15]):

$$x(t) = \int_{\mathbb{R}} e^{jwt} Z_x(dw), \qquad t \in [0, T_p], \tag{1}$$

where T_p is the pulse duration, Z_x is the stochastic measure of real number. We further assume that Z_x is absolutely continuous with respect to the Borel measure μ_B of complex numbers. The corresponding Radon-Nikodym derivative $\frac{dZ_x}{d\mu_B}$ at w is the random frequency spectrum of $x(t)$, which is denoted by $X(jw)$. Moreover, $X(jw)$ is mutually independent at different frequencies. The WSS modeling of waveform covers two practical waveforms:

- Orthogonal frequency division multiplexing (OFDM): When $X(jw)$ is supported at discrete frequency points and assumes discrete complex numbers, such as quadrature amplitude modulation (QAM), at each frequency point (subcarrier), it is essentially an OFDM waveform.
- Noise waveform: When $X(jw)$ is supported in all the frequency band and each $X(jw)$ is Gaussian distributed with possibly different variances, the waveform is colored noise in the time domain, It can be used to model the code division division multiplexing (CDMA) waveform and noise radar [16].

The total power of the waveform is denoted by P_t.

B. Radar Sensing Channel

We consider a single radar target, whose state is denoted by T, which may have different physical meanings, depending on the specific applications of radar sensing[1]:

- Detection: The existence of target is indicated by a binary T: $T = 0$ for nonexistence and $T = 1$ for existing target. The received signal is given by
$$y_s(t) = \alpha T x(t - \tau) + n_s(t), \tag{2}$$
where τ is the round trip time if the target exists, α is the signal attenuation, and n_s is the noise at the JCS receiver.
- Ranging: The target has been detected and is considered as a point in the space. The received is given by
$$y_s(t) = \alpha x(t - \tau) + n_s(t), \tag{3}$$
and T is the roundtrip time τ.
- Inference: The target is no longer a simple point and needs to be characterized by an impulse response h_t. Therefore, the received signal is given by
$$y_s(t) = h_t * x(t) + n_s(t). \tag{4}$$
The target state T is then the impulse response h_t. Note that the range information determined by the roundtrip time τ is incorporated in the impulse response h_t (e.g., for single-point target $h_t(t) = \alpha\delta(t - \tau)$).

We also consider clutters that form interference to the signal reflected from the target. We denote by h_{cl} the impulse response of the clutters. Therefore, the received signal at the JCS transceiver is given by

$$y_s(t) = (h_t + h_{cl}) * x(t) + n_s(t). \tag{5}$$

[1]In this paper we consider static radar targets and thus exclude the possibility of Doppler shift.

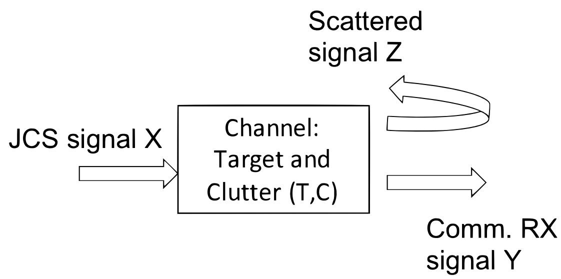

Fig. 2: Information-theoretic formulation for waveform optimization.

C. Communication Channels

We assume additive Gaussian noise fading channel, such that the received signal is given by

$$y_c(t) = x * h_c(t) + n_c(t), \tag{6}$$

where h_c is the impulse response of the communication channel, which includes the case of multi-path propagation, and n_c is the communication noise.

D. Sampling

For facilitating the subsequent analysis, we assume that the continuous-time signals are sampled with sufficient sampling rate. The frequency spectrum and time domain signal are approximately related by Discrete Fourier Transform (DFT).

IV. SHANNON

In this section, in the spirit of Shannon, we use the unified framework of information theory for both communications and sensing in JCS. It is a natural selection for communications. When applied to radar sensing, we consider the information on the radar target fetched by the reflected EM wave, instead of the estimation error. Note that the information-theoretic design criterion for radar sensing waveform design was originally proposed in [9].

A. Optimization Formulation

As illustrated in Fig. 2, we consider the environment including the radar target and clutter as the communication and sensing channels, whose outputs are Y and Z, namely the signals received by the communication receiver and JCS transceiver, respectively. The outputs are ruled by the conditional probability distribution $P(Y|X,T,C)$ and $P(Z|X,T,C)$, where C is the status of clutters. We further assume that the prior distributions of T and C are known in advance, which could be provided by the target tracking procedure such as Kalman filtering, as well as historic statistics.

Then, we formulate the information-theoretic waveform synthesis as the following optimization problem:

$$\begin{aligned} & \max_{P(X)} I(X;Y) \\ s.t. \quad & I(Z;T|X) \geq \gamma_1, I(Z;C|X) \leq \gamma_2, \\ & Var(X) \leq P_t, \end{aligned} \tag{7}$$

where the objective function is to maximize the mutual information between the communication channel input and output, namely the constrained channel capacity, while the first two constraints are to guarantee the information of target in the reflected signal Z while preventing the interference from the clutters, and the last constraint is on the transmit power. Note that the conditions on X in the mutual informations in the constraints are due to the full knowledge of X at the JCS transceiver.

To facilitate the solution to (7), we consider the frequency domain analysis, by sampling the random spectrum $X(jw)$ and obtaining M samples $\mathbf{x} = (X_1, ..., X_M)$. Then, the corresponding frequency samples of the outputs are denoted by M-vectors $\mathbf{y}$ and $\mathbf{z}$, respectively. This frequency domain analysis is especially valid for systems with OFDM signaling. The optimization depends on the realizations of conditional probabilities $P(Y|X,T,C)$ and $P(Z|X,T,C)$. For the frequency domain signals, we consider

$$\begin{cases} \mathbf{y} = \mathbf{H}_c\mathbf{x} + \mathbf{n}_c \\ \mathbf{z} = \mathbf{H}_t\mathbf{x} + \mathbf{H}_{cl}\mathbf{x} + \mathbf{n}_s \end{cases}, \tag{8}$$

where $\mathbf{H}_c$, $\mathbf{H}_t$ and $\mathbf{H}_{cl}$ are $M \times M$ diagonal matrices whose diagonal elements are the frequency responses of the communication channel, target reflection and clutter, respectively, and $\mathbf{n}_c$ and $\mathbf{n}_s$ are the frequency-domain noises received at the communication receiver and JCS transceiver, whose variances are denoted by $\{\sigma_{c,n}^2\}_{n=1,...,M}$ and $\{\sigma_{s,n}^2\}_{n=1,...,M}$, respectively. The relations in (8) are determined by the time-domain convolutions in (5) and (6). For exploring solutions in practical systems, we have the following assumptions:

- We assume that $\mathbf{H}_c$ is known, which is valid due to channel estimation in practical communication systems. The diagonal elements are $\{\sigma_{c,1}^2, \sigma_{c,2}^2, ..., \sigma_{c,M}^2\}$.
- We assume that $\mathbf{H}_t$ is random with variance $\{\sigma_{t,1}^2, \sigma_{t,2}^2, ..., \sigma_{t,M}^2\}$. We model the elements in $\mathbf{H}_{cl}$ as circular symmetric complex Gaussian random variables with different variances $\{\sigma_{cl,1}^2, ..., \sigma_{cl,M}^2\}$. We assume that the variances are known to the JCS transceiver. This is valid since the variances are determined by the corresponding scenario (sea waves or clouds) while the realizations are random due to the random positions and scattering surfaces of the clutters.

B. Optimal Solution without Clutter

When the first two constraints are omitted, it is well known that Gaussian distribution for X maximizes the objective function $I(X;Y)$ given the linear channel in (8) [7]. For the constraint of $I(Z;T|X)$ without clutter, the following proposition shows the optimal distribution of X for statistical radar inference:

Proposition 1. *When the elements in $\mathbf{H}_t$ are independent and Gaussian distributed and $\mathbf{H}_{cl} = 0$, the optimal X to maximize the mutual information $I(Z;T|X)$ satisfies the following classical water-filling power allocation:*

$$|X_n^*|^2 = \left(\lambda - \frac{\sigma_{s,n}^2}{\sigma_{t,n}^2}\right)^+, \; n = 1, ..., M, \tag{9}$$

where the constant λ guarantees $\sum_{n=1}^{M}|X_n^*|^2 = P_t$.

Proof. For the mutual information between Z and T, we have

$$\begin{aligned} I(Z;T|X) &= H(Z|X) - H(Z|T,X) \\ &= H(Z|X) - H(N_s), \end{aligned} \quad (10)$$

where the second term is the entropy of the noise and thus being independent of the input X and target information T. Therefore, to maximize $I(Z;T|X)$, we simply maximize the conditional entropy $H(Z|X)$. Due to the linear channel model in (8), we have that the maximal entropy is achieved by Gaussian distributions with variances $\{|X_n(\mathbf{H}_t)_{nn}|^2\}_{n=1,...,M}$. Then, we consider $X_n(\mathbf{H}_t)_{nn}$ as the received signal with Gaussian input of power $|X_n|^2$ and channel gain $\sigma_{t,n}^2$. It concludes the proof by applying the classical water-filling for the multi-channel power allocation. □

From the above conclusion, we observe that the optimal power spectrum of X for sensing is deterministic and the maximal mutual information $I(Z;T|X)$ is given by

$$I_{\max}(Z;T|X) = \sum_{n=1}^{M}\log\left(1 + \frac{|X_n^*|^2\sigma_{t,n}^2}{\sigma_{s,n}^2}\right). \quad (11)$$

However, when the mutual information for communications (the object function) is taken into account, there exist conflicts between communications and sensing. When the subcarriers with large variances or gains are diffierent for communication and sensing channels, a compromise is needed for the two functions.

According to Prop. 1, the optimal distribution of X_n is Gaussian. Considering the variable $x_n = |X_n|^2$, $n = 1, ..., M$ and taking derivative for the generalized objective function:

$$I(X;Y) + \lambda_1 I(Z;T|X) + \lambda_2 Var(X), \quad (12)$$

where λ_1 and λ_2 are the Lagrange multipliers, we obtain

$$\frac{\sigma_{c,n}^2}{x_n\sigma_{c,n}^2 + \sigma_n^2} + \frac{\lambda_1\sigma_{t,n}^2}{x_n\sigma_{t,n}^2 + \sigma_s^2} = \lambda_2, \ n = 1, ..., M, \quad (13)$$

when $x_n > 0$, and when the solution to (13) is negative, we set $x_n = 0$. By scanning different values of λ_1 and λ_2, we obtain feasible solutions that maximize the objective function, if there is any. We notice that when the radar sensing constraint does not exist, the solution is simply the water-filling one. However, the addition of the radar sensing function complicates the solution.

C. Optimal Solution with Clutter

When clutters are taken into account, the Gaussian distribution of the frequency-domain signal is not necessarily optimal, since it also maximizes the information of the clutter $I(Z;C|X)$. The optimal distribution is difficult to obtain, which requires numerical computations. To simplify the analysis, we assume that the distribution of each dimension of x is Gaussian. Then it is easy to verify

$$I(Z;T|X) = \sum_{n=1}^{M}\log\left(1 + \frac{\sigma_{t,n}^2 x_n}{\sigma_{cl,n}^2 x_n + \sigma_{s,n}^2}\right), \quad (14)$$

and

$$I(Z;C|X) = \sum_{n=1}^{M}\log\left(1 + \frac{\sigma_{cl,n}^2 x_n}{\sigma_{t,n}^2 x_n + \sigma_{s,n}^2}\right), \quad (15)$$

since the clutters can be considered as Gaussian noise, while the powers are determined by the transmitted signals. In (14), the mutual information $I(Z;T|X)$ is dependent on the signal-to-interference-and-noise ratios (SINRs) over different subcarriers, while in (15), the mutual information $I(Z;C|X)$ is determined by the corresponding interference-to-signal-and-noise ratios (ISNRs).

We consider the sum of $I(Z;T|X)$ and $I(Z;C|X)$:

$$\begin{aligned} & I(Z;T|X) + I(Z;C|X) \\ = & \sum_{n=1}^{M}\log\left(\frac{(\sigma_{t,n}^2 + \sigma_{cl,n}^2)x_n + \sigma_n^2}{\sigma_{cl,n}^2 x_n + \sigma_{s,n}^2}\frac{(\sigma_{t,n}^2 + \sigma_{cl,n}^2)x_n + \sigma_{s,n}^2}{\sigma_{ct,n}^2 x_n + \sigma_{s,n}^2}\right) \\ \leq & \sum_{n=1}^{M}\log\left(\frac{(\sigma_{t,n}^2 + \sigma_{cl,n}^2)^2}{\sigma_{t,n}^2\sigma_{cl,n}^2}\right), \end{aligned} \quad (16)$$

which becomes tight as $x_n \to \infty$ (namely high SNR). Therefore, the sum of $I(Z;T|X)$ and $I(Z;C|X)$ is upper bounded by (16). Maximizing $I(Z;T|X)$ can limit the upper bound of $I(Z;C|X)$. Therefore, for practical applications, we can reduce the constraints to the first one (on $I(Z;T|X)$).

Taking derivative with respect to the generalized objective function in (12), we obtain

$$\begin{aligned} \lambda_2 &= \frac{|H_{c,n}|^2}{x_n|H_{c,n}|^2 + \sigma_n^2} + \frac{\lambda_1(\sigma_{t,n}^2 + \sigma_{cl,t}^2)}{x_n(\sigma_{t,n}^2 + \sigma_{cl,t}^2) + \sigma_{s,n}^2} \\ &- \frac{\lambda_1\sigma_{cl,t}^2}{x_n\sigma_{cl,t}^2 + \sigma_{s,n}^2}, \qquad n = 1, ..., M. \end{aligned} \quad (17)$$

The corresponding solution is obtained by searching different values of λ_1 and λ_2.

V. Cramer-Rao

In this section, we unify the performance metrics of communications and sensing in JCS as MMSE, which is of critical importance in estimation and whose bound is provided by the the Cramer-Rao bound.

A. Bridging Mutual Information and MMSE

MMSE is natural for radar sensing, but indirect for communications. To unify the performance metrics, we need the following theorem for bridging the estimation performance metric and communications:

Theorem 1 ([11]). *For random variable X of arbitrary distribution and variance P, and the outcome of additive Gaussian noise channel $Y = X + N$, where N is Gaussian distributed with variance σ_n^2, the mutual information between X and Y, namely $I(X;Y)$, is related to the MMSE of estimating X from Y via*

$$\frac{d}{d\gamma}I_\gamma(X;Y) = MMSE(\gamma), \quad (18)$$

where $\gamma = \frac{P}{\sigma_n^2}$ is the SNR of X, and I is considered as a function of γ.

From (18), we observe that the MMSE of estimating X from Y (as a metric of sensing) represents the marginal utility of mutual information (as a metric for communications) in terms of SNR. Therefore, a low value of MMSE indicates that the performance improvement has saturated and thus causes inefficient utilization of transmit power.

When MMSE is used as the unified metric for communications and sensing, we observe that the conflict of interest between communications and sensing falls in the selection of distribution of $\{|X_n|^2\}_{n=1,...,M}$. According to [11], the Gaussian distribution maximizes the MMSE of estimation, given a fixed SNR. For communications, it is desirable to use Gaussian distribution since it improves the increasing rate of mutual information; meanwhile, the Gaussian distribution also maximizes the sensing MMSE. Therefore, a trade-off in terms of the signal distribution is needed.

B. Optimization Formulation

Based on the above relationship between mutual information and MMSE, we can formulate the estimation-theoretic waveform synthesis in the following optimization problem:

$$\begin{aligned} & \min_{P(X)} MMSE(X|Y) \\ s.t. \quad & MMSE(T|Z,X) \leq \gamma_3 \text{ and } Var(X) \leq P_t. \end{aligned} \quad (19)$$

The rationale of the optimization formulation includes

- The performance of sensing is more stringent. Therefore, we set the MMSE of target information T (conditioned on the transmitted signal X and received signal Z), as the constraint of the optimization. Note that this is valid only when T is continuous (e.g. the matrix $\mathbf{H}_s$).
- The communication performance is usually more flexible. Therefore, we set the MMSE of transmit signal $\mathbf{x}$, which indicates the increasing slope, as the objective function, for the best-effort performance. The reason for minimizing the objective function is to minimize the marginal utility of SNR.
- The impact of clutter is incorporated in the constraint on $MMSE(T|Z,X)$.

For simplicity of analysis, we assume that the sensing noise has the same power σ_s^2 over different subcarriers. We cite the following lemma for the analytic expression of MMSE:

Lemma 1 (Eq. (6.27) [17]). *For received signal* $\mathbf{y} = \mathbf{S}\mathbf{A}\mathbf{x} + \mathbf{n}$, *where* $\mathbf{A}$ *is diagonal,* $\mathbf{S}$ *is a generic matrix, and* $\mathbf{n}$ *consists of independent elements with identical power* σ_s^2*, the MMSE of* $\mathbf{x}$ *is given by*

$$MMSE(\mathbf{x}) = trace\left[\mathbf{I} + \sigma_s^{-2}\mathbf{A}\mathbf{R}\mathbf{A}\right]^{-1}, \quad (20)$$

where $\mathbf{R} = \mathbf{S}^H\mathbf{S}$.

Based on Lemma 1, we obtain

$$MMSE(X|Y) = trace\left[\mathbf{I} + \sigma_s^{-2}\mathbf{R}_c\right]^{-1}, \quad (21)$$

where $\mathbf{R}_c = \mathbf{H}_c\mathbf{H}_c^H$, and

$$MMSE(T|X,Z) = trace\left[\mathbf{I} + \mathbf{W}^{-1}\mathbf{R}_x\mathbf{W}^{-1}\right]^{-1}, \quad (22)$$

where $\mathbf{W} = \mathbf{H}_{cl} + \sigma_s^2\mathbf{I}$ and $\mathbf{R}_x = diag(\{|X_n|^2\}_{n=1,...,M})$.

Thanks to the diagonal structures of $\mathbf{H}_c$ and $\mathbf{H}_{cl}$, (21) is further simplified to

$$MMSE(X|Y) = \sum_{n=1}^{M} \frac{\sigma_s^2}{\sigma_s^2 + \sigma_{c,n}^2 x_n}. \quad (23)$$

and (22) is simplified to

$$MMSE(T|Z,X) = \sum_{n=1}^{M} \frac{\sigma_s^2 + \sigma_{cl,n}^2 x_n}{\sigma_s^2 + (\sigma_{t,n}^2 + \sigma_{cl,n}^2)x_n}. \quad (24)$$

Then, the optimal solution is determined by the following equation:

$$\begin{aligned} \lambda_4 &= -\frac{\sigma_s^2\sigma_{c,n}^2}{(\sigma_s^2 + \sigma_{c,n}^2 x_n)^2} \\ &- \frac{\lambda_3\sigma_{t,n}^2\sigma_s^2}{(\sigma_s^2 + (\sigma_{t,n}^2 + \sigma_{cl,n}^2)x_n)^2}, \ n = 1,...,M, \end{aligned} \quad (25)$$

where λ_3 and λ_4 are Lagrange multipliers.

VI. Numerical Results

In this section, we use numerical simulations to demonstrate and compare the proposed criteria of JCS.

A. Simulation Setup

We consider 256 samples in the frequency domain. Then, for the OFDM signaling, this means 256 subcarriers. We assume 64-QAM for OFDM waveform and Gaussian distribution of signal in the WSS noise waveform. The communication channel gains are generated from an exponential distribution, while the clutter frequency response is set to be a rectangular function between subcarrier 30 and 100. We set the transmit power P_t to be 1, while the thermal noise power is set to 0.01 (thus an SNR of 20dB).

When solving Equations (17) and (25), we range various values of the Lagrange multipliers. For each combination of the multipliers, we obtain the solution x_n. Only solutions that satisfies $\sum_{n=1}^{M} x_n = P_t$ are kept. The solutions whose performance dominated by other solutions are discarded. The remaining points of performances (channel capacity and MSE) form the boundary of feasible performance region. Note that the performance metric MSE is that of the received sensing signal, instead of the target parameters. The map from the target parameter to the received sensing signal is nonlinear, thus making the design difficult, which will be left to our next-stage study.

B. WSS Noise Waveform

The performance boundaries obtained from the information-theoretic and estimation-theoretic criteria are plotted in Fig. 3 for the WSS noise waveform, in which the frequency spectrum samples are Gaussian distributed. The points above the boundaries are feasible. We observe that the boundaries resulting from the two criteria are substantially different. It is difficult to conclude which criterion is better.

When there is no clutter, namely $\mathbf{H}_{cl} = 0$, the corresponding boundaries are plotted in Fig. 4. We observe that the pattern

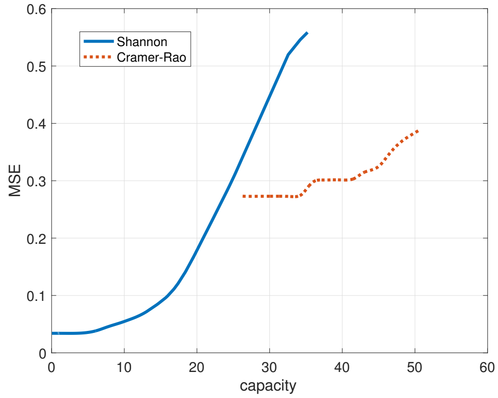

Fig. 3: Comparison between information-theoretic and estimation-theoretic criteria in terms of optimal trade-off boundary for WSS noise waveform.

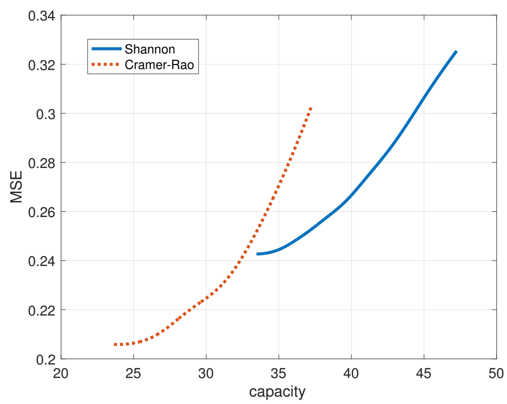

Fig. 4: Comparison between information-theoretic and estimation-theoretic criteria in terms of optimal trade-off boundary for WSS noise waveform, when there is no clutter.

is completely changed. Unfortunately, we are still unable to analyze the solutions analytically and cannot explain the underlying reason for patterns.

C. OFDM Waveform

The corresponding performance boundaries for OFDM waveforms are plotted in Fig. 5. We observe that the performance is worsened, due to the modulation of QAM, instead of Gaussian distribution of signaling. However, the pattern is still simular to that of the WSS noise waveforms.

VII. Conclusions

In this paper, we have proposed two frameworks for unifying the performances of communication and sensing in JCS in either information-theoretic (Shannon) or estimation-theoretic (Cramer-Rao) manners. The information-theoretic one defines sensing objective function based on mutual information, while the estimation-theoretic framework is based on the Guo-Shamai-Verdu identity that equalizes the MMSE and the increasing slope (with respect to the SNR) of mutual information. Constrained optimization problems have been formulated. Numerical computations have been carried out to compare both schemes in the context of joint communications and radar ranging. It is still unclear which criterion results in a better performance of JCS.

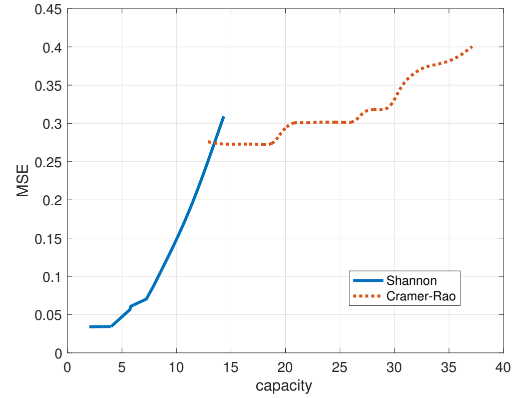

Fig. 5: Comparison between information-theoretic and estimation-theoretic criteria in terms of optimal trade-off boundary for OFDM waveform.

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