Systematic Doping of SC-LDPC Codes

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2022 IEEE International Symposium on Information Theory (ISIT) | 978-1-6654-2159-1/22/$31.00 ©2022 IEEE | DOI: 10.1109/ISIT50566.2022.9834590

***Abstract*—In this paper, we examine variable node (VN) doping to mitigate the error propagation problem in sliding window decoding (SWD) of spatially coupled LDPC (SC-LDPC) codes from the point of view of the encoding process. More specifically, in order to simplify the process of generating an encoded sequence with some number of doped code bits, we propose to employ systematic encoding and to limit doping to systematic bits only. Numerical results show that doping of systematic bits only achieves comparable performance to employing general (nonsystematic) encoding and full doping of all the code bits at each doping position, while benefiting from a much simpler encoding process. We then show that the inherent rate loss due to doping can be reduced by doping only a fraction of the variable nodes at each doping position with only a minor impact on performance.**

***Index Terms*—sliding window decoding, decoder error propagation, variable node doping, fractional doping, systematic doping**

I. Introduction

Capacity approaching spatially coupled low-density parity-check (SC-LDPC) codes, also known as LDPC convolutional codes [1], combine the best features of both regular and irregular LDPC block codes (LDPC-BCs) [2], [3]. The excellent performance of SC-LDPC codes relies mainly on two important features. One is the fact that they exhibit *threshold saturation*, i.e., the suboptimal belief propagation (BP) iterative decoding threshold of SC-LDPC code ensembles over memoryless binary-input symmetric-output channels coincides with the maximum a posteriori probability (MAP) threshold of their underlying LDPC-BC ensembles, thereby allowing an SC-LDPC code to achieve the optimum performance of its underlying LDPC-BC with suboptimal decoding complexity. The other is that *sliding window decoding* (SWD) can be employed to reduce decoding latency, memory, and complexity [4]. In order to achieve the best possible performance over a range of signal-to-noise ratios (SNRs), Huang et al. have shown empirically in [5] that the *decoder window size* W should be at least six times the decoding constraint length. However, in practice, lower latency operation is often desirable, thereby necessitating a smaller window size. In this case, infrequent but severe decoder error propagation can sometimes occur when using SWD. More specifically, during the sliding window decoding process, when a decoding error occurs, the decoding of subsequent symbols can also be affected, and a continuous string of decoding errors can result. This *decoder error propagation* phenomenon can result in unacceptable performance loss, particularly for a continuous (streaming) transmission scenario or a large frame length.

The effect of error propagation on SWD of SC-LDPC codes was first mentioned in [6], whereas the first detailed study of error propagation in SWD was done for the related class of braided convolutional codes in [7], [8]. In this work, three effective approaches (window extension, resynchronization, and retransmission) were proposed to prevent the decoder from experiencing error propagation. For SWD of SC-LDPC codes, Klaiber et al. [9] proposed adapting the number of decoder iterations and/or shifting the window position in order to limit the effects of error propagation, both of which involve altering the decoding procedure. Using a different approach that involves altering the encoding procedure, Zhu et al. proposed *check node (CN) doped SC-LDPC codes* in [10], which employ reduced-degree CNs spaced throughout the coupling chain to help the decoder recover from error propagation. Similar to [9], however, this also requires altering the decoding procedure whenever a doping position is reached. More recently, Zhu et al. proposed *variable node (VN) doped SC-LDPC codes* by fixing the code bits corresponding to certain VNs spaced throughout the coupling chain to a predetermined value [11]. Sololovskii et al. subsequently studied the finite length scaling behavior of VN doped SC-LDPC codes on the binary erasure channel (BEC) in [12] and showed that doping effectively works by initiating a *decoding wave* at a doping position similar to what is observed at the beginning of an undoped coupling chain. Unlike CN doping and the techniques of [9], VN doping allows recovery from error propagation without altering the decoding procedure, although fixing the value of certain VNs presents encoding challenges. Also, the required pre-determined distribution of doped positions in VN doping lacks flexibility and may not completely eliminate error propagation. Hence an adaptive VN doping strategy for SC-LDPC codes that relies on the availability of a noiseless binary feedback channel was proposed in [13]. Finally, a general model for computing the error rate of SWD of SC-LDPC codes and predicting the performance improvement achievable with doping was recently presented in [14].

In this paper, we build on our previous work on VN doping by introducing *systematic doping* to simplify the process of encoding as well as the procedure for recovering the decoded information sequence. Systematic doping employs systematic encoding and only dopes a fraction of the VNs, i.e., the systematic bits at each doping position. This allows the doping to be done prior to encoding, thus simplifying the encoding process, and also results in a straightforward procedure for recovering the decoded information sequence. This is particularly advantageous in the case of adaptive doping, where these operations must be performed "on the fly".

In Section II, we briefly review the error propagation problem and VN doping of SC-LDPC codes. Then, motivated by a desire to reduce the rate loss due to doping, *fractional VN doping*[1] is examined in Section III. In Section IV, the systematic VN doping strategy, which makes use of fractional doping, is presented as a means of simplifying the encoding process and the procedure for recovering the decoded information sequence. Numerical results for the binary-input additive white Gaussian noise (AWGN) channel, showing that (i) systematic encoding combined with fractional systematic doping can perform as well as general (nonsystematic) encoding combined with full doping of all the code bits at each doping position, and (ii) fractional doping can reduce rate loss with only a minor degradation in performance, are given in Section V. Finally, some concluding remarks are presented in Section VI.

II. Review of VN Doped SC-LDPC Codes

In this paper, we consider SC-LDPC codes constructed by coupling together a sequence of L disjoint (J,K)-regular LDPC-BC *protographs* into a single coupled chain, where infinite L results in an unterminated coupled chain and finite L results in a terminated coupled chain. A *graph lifting factor* M using randomly chosen permutation matrices is then applied to the coupled chain to produce an ensemble of (J,K)-regular SC-LDPC codes. Due to the fact that the structured irregularity at the boundaries of a spatially coupled chain is responsible for the threshold saturation effect and thus the capacity-approaching performance of SC-LDPC codes, the VN doped SC-LDPC codes proposed in [11] introduced occasional VNs with fixed values in the coupled chain to emulate the structured irregularity at the boundaries and thus to end any possible error propagation without having to alter the decoding process. Without loss of generality, we now use (3,6)-regular SC-LDPC codes as an example to briefly review the construction of VN doped SC-LDPC codes.

Based on the protograph representation shown in Fig. 1(a) of a sequence of (3,6)-regular LDPC-BCs lifted from the simple 1×2 *base matrix* $B_{(3,6)} = [3\ 3]$, a (3,6)-regular spatially coupled chain with *coupling memory* $m_s = 2$ is formed by redirecting some of the edges connected to VNs in each protograph to CNs in the $m_s = 2$ neighboring protographs (see [16] for detail) of this edge-spreading technique, an example of which is shown in Fig. 1(b) . To introduce doping, the reduced degrees are achieved by fixing (setting to 0) the values of occasional VNs in the coupled chain (thereby causing a small rate loss penalty) as shown Fig. 1(b), where each time unit represents a block of $2M$ coded symbols and the *decoding constraint length* is given by $v_s = 2M(m_s + 1)$. The VNs at time $t = \tau_1$ (the green empty circles) are doped by setting the $2M$ coded bits corresponding to these VNs to be "0". As a result, the CNs at times $t = \tau_1, \tau_1 + 1$, and $\tau_1 + 2$ (colored red) act like degree 4, rather than degree 6, CNs, thus emulating the structured irregularity at the boundaries without actually altering the graph structure or the decoding process. Similarly, if the VNs at times $t = \tau_2$ are doped, the CNs at times τ_2, τ_2+1, and τ_2+2 (colored red) act like degree 4 CNs. Finally, Fig. 1(c) illustrates this SWD decoding process, where a window of size W time units slides from left to right, each time stopping to decode the block of $2M$ target symbols at the left end of the window by performing iterative BP decoding across the window. During the decoding process, the LLRs of the doped bits are set to their maximum (known) values.

[1]Doping only a fraction of the nodes at a given position was also employed in [15] as a means of emulating termination in the decoding of tail-biting SC-LDPC codes.

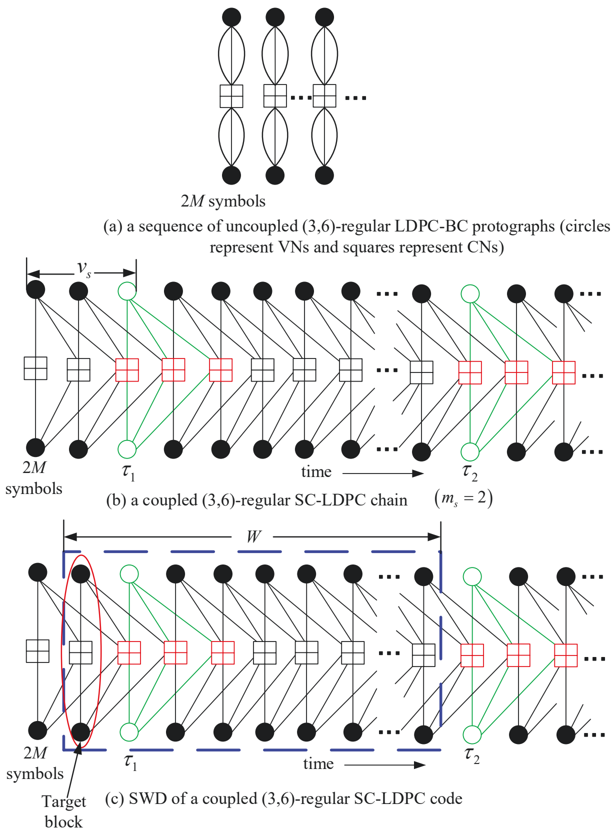

Fig. 1. VN doping for a (3,6)-regular SC-LDPC code with occasional fixed variable nodes spaced throughout the coupled chain.

In general, if n_c and n_v denote the total number of CNs and the total number of *unknown* VNs, respectively, the *design rate* of VN doped SC-LDPC codes with frame length L and d doped VNs is given by

$$R_{\text{doped}} = 1 - \frac{n_c}{n_v} = 1 - \left(\frac{L + m_s}{L - d/2M} \right)(1 - R), \qquad (1)$$

where $R = 1 - J/K$ is the design rate of the uncoupled LDPC-BC protograph [16]. Compared to the design rate $R_L = 1 - \left(\frac{L+m_s}{L}\right)(1 - R)$ of undoped SC-LDPC codes [16], we see from (1) that the design rate of VN doped SC-LDPC codes is smaller, i.e., VN doping results in some rate loss. In order to reduce the amount of rate loss, in the next section we examine fractional doping, in which, instead of doping all $2M$ bits at a doping position, only a fraction of the bits are doped.

III. Fractional Doping

The fractional doping process is illustrated in Fig. 2, where the slashed circles represent the fractionally doped nodes and the solid circles represent undoped nodes. At each time unit, the two protograph nodes represent a total of $2M$ bits. Let $0 \leq \delta \leq 1$ represent the fraction of doped bits at each doping position. Then, at each doping position, for example, at time τ_1, $2\delta M$ bits will be doped, as shown in Fig. 2, where we note that $\delta = 0$ corresponds to no doping and $\delta = 1$ corresponds to full doping. We classify the $2M$ bits at each time unit into two sets: a doped set $\mathcal{D}$ and an undoped set $\bar{\mathcal{D}}$.

Now consider SWD of SC-LDPC codes (see [16] for details). In the case of fractional doping, let L_i^t, $1 \leq i \leq 2M$, denote the channel *log-likelihood ratio* (LLR) used for decoding of the ith bit at time unit t. Then we have

$$L_i^t = \begin{cases} \Gamma, & i \in \mathcal{D} \\ L_i^{t,\text{ch}}, & i \in \bar{\mathcal{D}} \end{cases}, \tag{2}$$

where $L_i^{t,\text{ch}}$ denotes the received channel LLR of the ith bit at time unit t and $\Gamma = +10$ is chosen to denote the known LLR value, corresponding to a doped symbol 0. Note that (2) has the effect of assigning certainty to the doped bits during the decoding process.

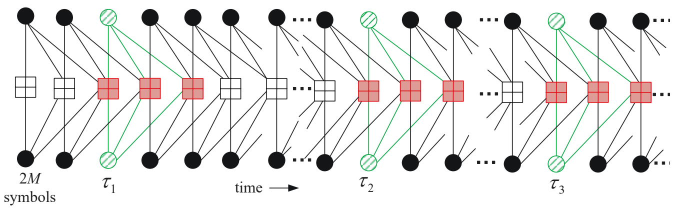

Fig. 2. Fractional VN doping for a (3,6)-regular SC-LDPC code with occasional fixed variable nodes spaced throughout the coupled chain.

In the case of fractional doping, we can choose which bits at a protograph node to dope and which to leave undoped. In this paper, we employed two fractional doping patterns: *adjacent* doping and *periodic* doping. These two options are illustrated in Fig. 3 for doping fraction $\delta = 0.5$, where the white circles represent doped bits and the black circles represent undoped bits. As we can see from Fig. 3, in adjacent doping $2\delta M$ consecutive bits are doped, whereas in periodic doping, the $2\delta M$ doped bits are spaced uniformly across the $2M$ bits at a time unit. At a doping position, in the case of $\delta = 0.5$, we see that adjacent doping is equivalent to doping all the VNs at one protograph node and no VNs at the others, whereas periodic doping spreads the doped VNs evenly over both protograph nodes.

IV. Systematic Doping

As noted above, VN doping involves fixing certain bits in the encoded sequence to have known values. This implies that an encoder for VN doped SC-LDPC codes must be designed to

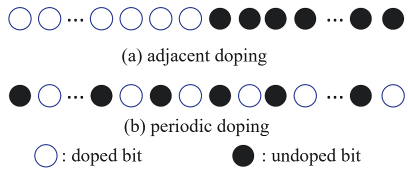

Fig. 3. Doping patterns for fractional doping with $\delta = 0.5$.

ensure that the value of the encoded bits in the doped positions remains constant for all possible information sequences. This requirement has the effect that certain information sequences are invalid inputs to the encoder, thus resulting in rate loss.

In order to see this, consider an example of a general (nonsystematic) encoder in which the length K information sequence $\mathbf{u} = (u_0, u_1, \ldots, u_{K-1})$ produces the length N encoded sequence $\mathbf{v} = (v_0, v_1, \ldots, v_{N-1})$ and a particular encoded bit, say v_j, must be a "0". Since every encoded bit is the sum of some subset of information bits, v_j can be expressed as $v_j = 0 = u_{j1} + u_{j2} + \cdots + u_{jk}$, where the indices $j_1, j_2, \ldots, j_k \in \{0, 1, \ldots, K-1\}$ represent the subset of information bits that contributes to v_j. It follows that changing the value of any one of the bits in this subset, while leaving the others unchanged will change the value of v_j from "0" to "1", which implies that all information sequences that contain this particular subset of information bits are invalid. Moreover, all information sequences containing any combination of these bits that gives odd parity are also invalid. As a consequence, we see that, in general, not all 2^K possible information sequences are valid when code doping is used, which leads to rate loss.

Based on the above discussion, we observe that designing a non-systematic encoder with doped code bits (or a systematic encoder with doped parity bits) in general leads to a highly complex encoding process. Moreover, in the decoding of LDPC codes, the decoding process results in an estimated code sequence, which must then be inverted according to the same highly complex encoding rule in order to recover the estimated information sequence. Furthermore, in the case of adaptive VN doping [13], these complex encoding and encoder inverse operations must be done "on the fly", whenever the feedback channel requests the insertion of doped bits into the encoded sequence. This difficulty motivates us to restrict our attention to systematic encoding rules and to limit doping to systematic bits only, which we refer to as *systematic doping*. This follows from the fact that doping can now be done directly on the information sequence, prior to encoding, thus greatly simplifying the encoding process, and that the encoder inverse operation is trivial in this case, since all the information bits appear unchanged as code bits in the encoded sequence, and thus the estimated information sequence can be determined directly from the estimated code sequence[2].

[2]We note that such a strategy can be implemented with only minor modifications to the usual process by occasionally fixing input symbols to a standard systematic encoder and removing those symbols after recovering the decoded information sequence

Since only a fraction of the bits (depending on the design rate R) at each position are systematic, a systematic doping strategy necessitates spreading fractional doping over a span of several positions in order to dope the same number of bits as full doping of all the protograph nodes at any one position. However, as we will see in Sec. V, this can be achieved with essentially no loss in performance, and even fractional systematic doping at one position can sometimes performs as well as full doping.

Again using (3,6)-regular SC-LDPC codes as an example, for which the design rate $R = 1/2$, systematic doping requires spreading a "doping position" over a *doping span* of $s \geq 1/R = 2$ positions, where only systematic VNs are doped at each position. This is illustrated in Fig. 4, where $s = 2$ and the white circles represent the doped systematic protograph nodes.[3] At each doping position, say $t = \tau_1$, where both protograph nodes are shown as fractionally doped in the general (nonsystematic) doping scheme of Fig. 2, in systematic doping only a fraction $\delta = R = 0.5$ of the protograph nodes are doped and a second systematic protograph node is doped at the next position $t = \tau_1 + 1$, making fractional ($\delta = 0.5$) systematic doping over a span of $s = 2$ positions equivalent to full doping at one position (see Fig. 1(b)), which necessarily entails the doping of parity bits. Similarly, at time unit $t = \tau_2$, systematic doping covers a span of $s = 2$ positions.

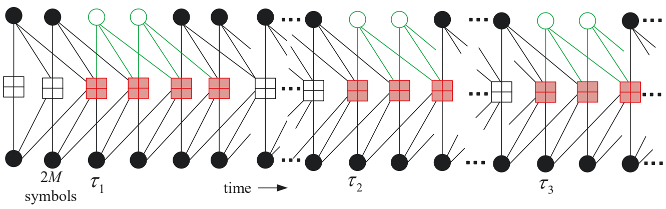

Fig. 4. Systematic VN doping with $\delta = R = 0.5$.

We can apply systematic doping in the same manner to general (J, K)-regular SC-LDPC codes of design rate $R = (K - J)/K$. Here, we consider (3,9)-regular SC-LDPC codes with rate $R = 2/3$ and (4,6)-regular SC-LDPC codes with $R = 1/3$ as examples. In the $R = 2/3$ (3,9)-regular case with coupling memory $m_s = 2$, formed from the 1×3 LDPC-BC base matrix $B_{(3,9)} = [3\ 3\ 3]$, the coupled chain formed by applying the edge-spreading technique to the uncoupled protograph (see [16] for details) is shown in Fig. 5, where we see that there are three protograph nodes at each time unit. Therefore, in order to implement systematic doping, we can consider two options:

- doping span $s = 2$. In this case, systematic doping operates over $s = 2$ time units, as shown in Fig. 6(a), where two systematic protograph nodes are doped at time τ_1 and one systematic protograph node is doped at time $\tau_1 + 1$.

[3]In Fig. 4, we assume that the upper protograph node at each position contains systematic bits only, while the lower protograph node contains parity bits only, which corresponds to the adjacent doping of Fig. 3(a).

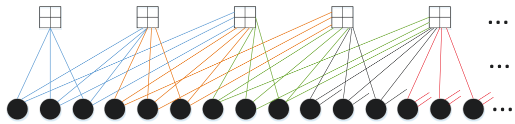

Fig. 5. Coupled chain for (3,9)-regular SC-LDPC codes.

- doping span $s = 3$. In this case, systematic doping operates over $s = 3$ time units, as shown in Fig. 6(b), where one systematic protograph node is doped at times $\tau_2, \tau_2 + 1$, and $\tau_2 + 2$.

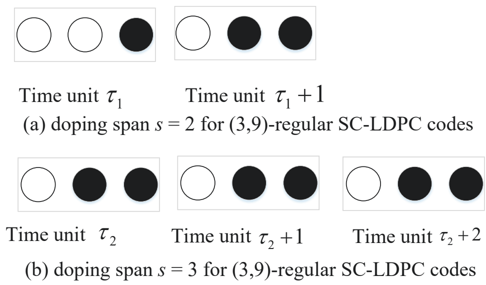

Fig. 6. Systematic doping options for (3,9)-regular SC-LDPC codes.

In the $R = 1/3$ (4,6)-regular case with coupling memory $m_s = 1$, formed from the 2×3 LDPC-BC base matrix

$$B_{(4,6)} = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}, \tag{3}$$

applying edge spreading (see [16]) results in the coupled chain shown in Fig. 7. Since there are three protograph nodes at each time unit, one protograph node can be considered as the systematic node and the other two protograph nodes as the parity check nodes. Thus, for systematic doping, only one protograph node can be doped at each position, which results in the doping pattern shown in Fig. 6(b).

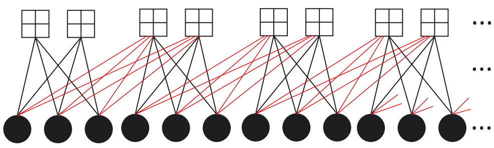

Fig. 7. Coupled chain for (4,6)-regular SC-LDPC codes.

V. Numerical Results

In order to verify the effectiveness of systematic VN doping, we first consider a (3,9)-regular SC-LDPC code with rate $R = 2/3$ and coupling memory $m_s = 2$ lifted from the coupled chain shown in Fig. 5, where we use the systematic doping pattern of Fig. 6(b). The simulation results are presented in Fig. 8, where we see that this systematic doping pattern, which covers three protograph nodes, achieves essentially the same *bit-error-rate* (*BER*) and *block error rate* (*BLER*) performance as full doping of a single position, which involves a much more complex encoding process, and we note that the total number of doped VNs $d = 3\delta s M = 3M$, both for systematic doping ($\delta = 1/3, s = 3$) and full doping ($\delta = 1, s = 1$).

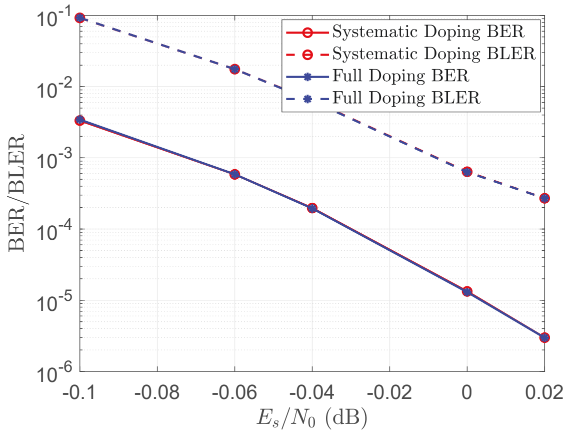

Fig. 8. Performance comparison between full doping and systematic doping for a (3,9)-regular SC-LDPC code with $M = 1000$, $L = 500$, $W = 12$, and $R_{\text{doped}} = 0.665$ on a binary-input AWGN channel with BPSK signaling.

We then consider a (4,6)-regular SC-LDPC code with rate $R = 1/3$ and coupling memory $m_s = 1$ lifted from the coupled chain shown in Fig. 7, where again we use the systematic doping pattern of Fig. 6(b). The simulation results are shown in Fig. 9, where we see only very slight (< 0.025 dB) performance loss for systematic doping at BERs/BLERs $> 10^{-5}/10^{-4}$ compared to the much more complex full doping, and we again note that $d = 3M$ in both cases.

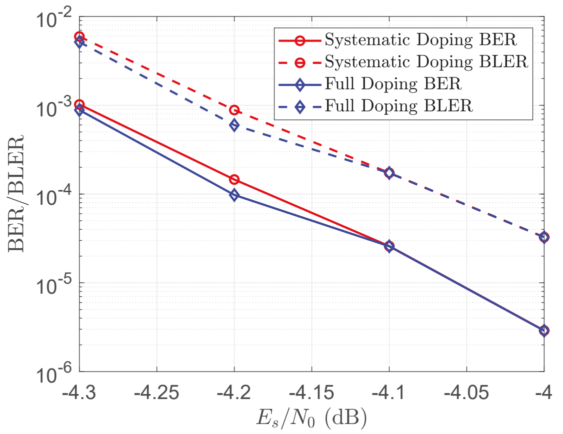

Fig. 9. Performance comparison between full doping and systematic doping for a (4,6)-regular SC-LDPC code with $M = 1000$, $L = 500$, $W = 12$, and $R_{\text{doped}} = 0.331$ on a binary-input AWGN channel with BPSK signaling.

Finally, in order to test the effectiveness of general (systematic or nonsystematic) fractional VN doping at reducing rate loss, we simulated the (3,6)-regular SC-LDPC code of Fig. 1(b) with $M = 2000$, $L = 500$, and $W = 18$ for a binary-input AWGN channel with BPSK signaling at a *signal-to-noise ratio (SNR)* of $E_b/N_0 = 0.9$ dB. A single ($s = 1$) doping position was placed in the middle of the coupled chain and the doping fraction δ was varied between 0 (no doping) and 1 (full doping). From Fig. 10, we see that even a small amount of fractional doping yields significant performance improvement, with $\delta = 0.2$ (20% doping, $R_{\text{doped}} = 0.499$) giving essentially the same result as $\delta = 1.0$ (full doping, $R_{\text{doped}} = 0.497$). This is consistent with the analytical results of [12], where the authors show that a doping point operates like a switch, where a decoding wave is either initiated or it is not, and suggests that the systematic doping results presented above can also be achieved by doping only a single position, thus reducing the rate loss due to doping, as long as R exceeds some critical doping fraction.

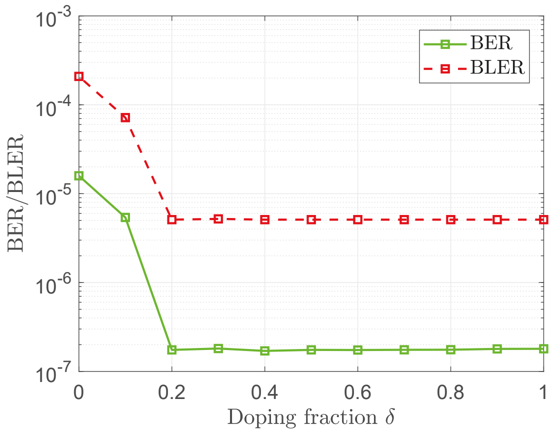

Fig. 10. Performance of a fractionally doped (3,6)-regular SC-LDPC code with $M = 2000$, $L = 500$, $W = 18$ at $E_b/N_0 = 0.9$ dB .

VI. Conclusion

In this paper, we proposed a systematic VN doping strategy which employs systematic encoding and fractional doping of systematic bits only as a way to mitigate the error propagation problem of SWD of SC-LDPC codes while greatly simplifying the complexity of the encoding process (and the corresponding encoder inverse operation required to recover the estimated information sequence). Numerical simulation results show that (i) systematic encoding combined with fractional systematic doping performs essentially as well as general (nonsystematic) encoding combined with full doping of all the code bits at each doping position, while greatly simplifying the encoding process and the procedure for recovering the decoded information sequence, and (ii) fractional doping can be used to reduce the rate loss due to doping with only a minor impact on performance.

Acknowledgment

This material is based upon work supported by the National Science Foundation under Grant Nos. CCF-2145917, OIA-1757207, HRD-1914635, and NSFC under Grant No. 61701368.

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