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High-Order Leader-Follower Tracking Control under Limited Information Availability †

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Abstract

Limited information availability represents a fundamental challenge for control of multi-agent systems, since an agent often lacks sensing capabilities to measure certain states of its own and can exchange data only with its neighbors. The challenge becomes even greater when agents are governed by high-order dynamics. The present work is motivated to conduct control design for linear and nonlinear high-order leader-follower multi-agent systems in a context where only the first state of an agent is measured. To address this open challenge, we develop novel distributed observers to enable followers to reconstruct unmeasured or unknown quantities about them-selves and the leader and on such a basis, build observer-based tracking control approaches. We analyze the convergence properties of the proposed approaches and validate their performance through simulation.

KEYWORDS:

Leader-follower tracking; multi-agent systems; distributed observer; distributed control; high-order dynamics

1 | INTRODUCTION

Cooperative autonomy based on multi-agent systems (MASs) is finding ever-increasing application in different fields. This has driven a surge of research on distributed cooperative control for different tasks, including consensus, leader-follower tracking, synchronization, rendezvous, flocking, optimization, learning, formation control, and coverage control^{1–10}. Most of the current literature considers agents governed by first- or second-order models. Although such low-order models are useful as well as amenable to control design, they are often found inadequate to characterize agents with more complex higher-order dynamics. It is also neither trivial nor easy to extend low-order cooperative control designs to high-order systems. Recent years have thus seen a growing amount of work on high-order MAS control synthesis¹¹.

The study¹² takes a lead in investigating high-order MAS consensus, presenting a distributed consensus control algorithm. This subject has since attracted considerable research efforts, with many studies proposed to deal with various challenges, e.g., directed communication topologies^{13,14}, switching topologies^{13,15}, output feedback design^{16–21}, bipartite consensus^{22,23}, external disturbances^{24,25}, switching or heterogeneous dynamics²⁶, quantization²⁷, and constrained energy budget⁸, to name a few. Besides consensus, high-order leader-follower tracking has emerged as another problem of great interest. A basic form of the problem is introduced in¹², which assumes that the leader agent continuously broadcasts its state information to all the followers. A consensus-based control algorithm is then developed therein to make the followers track the leader. The study²⁸ considers a more general setting where only a subset of the followers can receive information from the leader. It proposes a leader-follower tracking control method and proves that followers with small degrees must be informed by the leader to ensure

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tracking convergence. Further, nonlinear dynamics constitutes a stronger challenge for high-order leader-follower tracking. The investigations^{29,30} propose to adaptively estimate the nonlinearity inherent in an agent's dynamics using neural networks and then offset it in the control run. A few well-known nonlinear control techniques, including backstepping control and iterative nonlinear control, have also found use in addressing tracking problems of nonlinear multi-agent systems^{31–33}. In³⁴, a finite-time tracking control approach is developed for a high-order nonlinear MAS with actuator saturation, and the work in³⁵ studies the problem of finite-time higher-order tracking with mismatched disturbances.

Despite the importance, the above studies generally assume that a follower can obtain a large amount of information to make control decisions. For example, it is required in ^{12,28,29,34} that a follower must know all of its own states, all of the states of its neighbors, and if connected with the leader, all of the leader's states. This requirement can be hardly met in a real world where relevant sensors can be unavailable²⁸. One can also find similar requirements in studies about high-order consensus control. This hence motivates us to explore a more realistic setting—when only the first state of every agent (leader and followers) is measured. It is unsurprising that the low information availability will increase the difficulty for tracking control. In addition, the present literature often requires that the leader's dynamics is input-free^{29–33,36}, which comes as another limitation in practice. To overcome the challenges, we propose to exploit the notion of observer-based control design and present two major contributions. First, we design an observer-based tracking control approach for linear high-order MASs. We propose a set of distributed observers to compensate for the limited information, allowing a follower to comprehensively estimate the leader's maneuver input and states along with its own states. These observers are then combined with a nominal controller to form an observerembedded tracking controller. We further characterize the convergence of tracking when the proposed controller is applied. As the second contribution, the study extends to the more challenging case when the agents' dynamics is not only high-order but also nonlinear. Extrapolating the design for the linear case, we develop an observer-based tracking control approach and analyze its convergence. Compared to the literature, our work is distinct in two aspects. First, the control design requires very limited information—only every agent's first state. Second, the leader's dynamics is input-driven, and the input is only known by the followers that communicate with the leader.

Our work is further related with two lines of research. 1) Leader-follower tracking via output feedback, which generally considers a state-space model and uses local state or parameter observers to estimate certain unknown quantities^{37–42}. Differing from them, our study designs distributed observers to help the followers cooperate to infer the leader's input and states, removing the restriction that the leader's maneuver input must be either zero or known by all the followers. 2) Observer-based first- and second-order tracking control. The literature includes various kinds of observers designed to allow a follower to estimate its own velocity^{43,44}, the disturbances acting on it³⁵, its velocity relative to the leader⁴⁵, the leader's input^{46–48}, or the leader's velocity^{49,50}. These results nonetheless cannot be readily generalized to the high-order MASs. Finally, the conference version of this study in ⁵¹ considers only agents with linear high-order dynamics. We make substantial expansion in this paper to investigate MASs with generic-form nonlinear high-order dynamics.

The rest of the paper is organized as follows. Section 2 introduces the notation about graph theory. Section 3 formulates the problem of leader-follower tracking control for a linear high-order MAS, develops an observer-based tracking control approach, and analyzes its convergence. Section 4 proceeds to study the tracking problem for a nonlinear high-order MAS. Section 5 then offers a numerical simulation example to validate the proposed design. Finally, Section 6 gathers our conclusions.

2 + NOTATION

We use a graph to delineate the information exchange topology among the leader and followers. First, consider a network composed of *N* independent followers, the topology of which is modeled as an undirected graph. The follower graph is expressed as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set that contains unordered pairs of nodes. A path is a sequence of connected edges in a graph. The follower graph is connected if there is a path between every pair of nodes. The neighbor set of node *i* is denoted as \mathcal{N}_i . The adjacency matrix of \mathcal{G} is $A = [a_{ij}] \in \mathbb{R}^{N \times N}$, which has non-negative elements. The element $a_{ij} > 0$ if and only if $(i, j) \in \mathcal{E}$, and moreover, $a_{ii} = 0$ for all $i \in \mathcal{V}$. For the Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N \times N}$, $l_{ij} = -a_{ij}$ if $i \neq j$ and $l_{ii} = \sum_{k \in \mathcal{N}_i} a_{ik}$. The leader is numbered as node 0 and can send information to its neighboring followers. Then, we define a graph $\overline{\mathcal{G}} = (\overline{\mathcal{V}, \overline{\mathcal{E}})$ for the entire network, where $\overline{\mathcal{V}} = \{0\} \cup \mathcal{V}$, and $\overline{\mathcal{E}} \subseteq \overline{\mathcal{V}} \times \overline{\mathcal{V}}$ is the edge set for all nodes. The leader is globally reachable in $\overline{\mathcal{G}}$ if there is a path in graph $\overline{\mathcal{G}}$ from every node 0 to node *i*. We denote the leader adjacency matrix associated with $\overline{\mathcal{G}}$ by $B = \text{diag}(b_1, \dots, b_N)$, where $b_i > 0$ if the leader is a neighbor of agent *i* and $b_i = 0$ otherwise.

3 | LEADER-FOLLOWER TRACKING WITH LINEAR HIGH-ORDER DYNAMICS

In this section, we show the leader-follower tracking problem for linear high-order MASs and develop an observer-based tracking control method as well as convergence analysis.

3.1 | Problem Formulation

Consider an MAS composed of N + 1 agents, among which agent 0 is the leader and agents 1 to N are followers. Each agent has *l*th-order dynamics ($l \ge 3$) expressed as

$$\dot{x}_{i,m} = x_{i,m+1}, \quad m = 1, 2, \dots, l-1,$$
(1a)

$$\dot{x}_{i,l} = u_i,\tag{1b}$$

for i = 0, 1, ..., N, where $x_{i,m} \in \mathbb{R}$ is the *m*th state of agent *i*, and u_i the maneuver input. The objective is to design a distributed control law u_i such that follower *i* for i = 1, 2, ..., N can convergently track the leader with $\lim_{t\to\infty} |x_{i,m}(t) - x_{0,m}(t)| = 0$ for m = 1, 2, ..., l.

Here, we assume that only $x_{i,1}$ for i = 0, 1, ..., N is available. That is, only the first state of an agent is measured, regardless of whether it is the leader or a follower. This assumption considerably relaxes the usual requirement in the literature that substantial states of an agent must be measured. However, it also implies that the accessible information about the agents is rather limited, which makes it more challenging to design an effective distributed tracking controller.

3.2 | The Proposed Algorithm

We develop an observer-based control algorithm to enable convergent tracking in the above setting. To begin with, we consider the following controller for follower *i*:

$$u_{i} = -k_{1} \left[\sum_{j \in \mathcal{N}_{i}} a_{ij}(x_{i,1} - x_{j,1}) + b_{i}(x_{i,1} - x_{0,1}) \right] - \sum_{m=2}^{l} k_{m}(\hat{x}_{i,m} - \hat{x}_{0,i,m}) + \hat{u}_{0,i},$$
(2)

for i = 1, ..., N, where k_m for m = 1, 2, ..., l are gain parameters, $\hat{x}_{0,i,m}$ and $\hat{u}_{0,i}$ are follower *i*'s estimates of the leader's state $x_{0,m}$ and input u_0 , respectively, and $\hat{x}_{i,m}$ is follower's estimate of its own state $x_{i,m}$. The motivation underlying (2) is to drive follower *i* toward its neighbors and the leader simultaneously. When all the followers do this, they can track the leader in a collective manner. Next, we design the observers so as to obtain the estimates as needed in (2).

A distributed input observer is first introduced to enable the followers to estimate u_0 , which is given by

$$\dot{a}_{0,i} = -\sum_{j \in \mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) - b_i(\hat{u}_{0,i} - u_0) - d_i \cdot \text{sgn}\left[\sum_{j \in \mathcal{N}_i} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_i(\hat{u}_{0,i} - u_0)\right],\tag{3a}$$

$$\dot{d}_{i} = \tau_{i} \left| \sum_{j \in \mathcal{N}_{i}} a_{ij} (\hat{u}_{0,i} - \hat{u}_{0,j}) + b_{i} (\hat{u}_{0,i} - u_{0}) \right|,$$
(3b)

for i = 1, ..., N, where d_i is an adaptive gain, and τ_i a positive scalar. For (3a), the first two terms on its right-hand side are used to drive the input estimation $\hat{u}_{0,i}$ to approach u_0 while maintaining consistency with the neighboring followers; further, its third term serves as further correction with an adaptive gain given in (3b) to enhance the convergence.

We further propose another distributed observer for the followers to estimate $x_{0,m}$ for m = 2, 3, ..., l:

$$\dot{z}_{0,i,2} = -b_i c_{0,2} z_{0,i,2} - b_i^2 c_{0,2}^2 x_{0,1} - c_{0,2} \sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_{0,i,2} - \hat{x}_{0,j,2}) + \hat{x}_{0,i,3},$$
(4a)

$$\hat{x}_{0,i,2} = z_{0,i,2} + b_i c_{0,2} x_{0,1}, \tag{4b}$$

$$\dot{z}_{0,i,m} = -c_{0,m} z_{0,i,m} - c_{0,m}^2 \hat{x}_{0,i,m-1} + \hat{x}_{0,i,m+1},\tag{4c}$$

$$\hat{x}_{0,i,m} = z_{0,i,m} + c_{0,m} \hat{x}_{0,i,m-1}, \quad m = 3, 4, \dots, l-1,$$
(4d)

$$\dot{z}_{0,i,l} = -c_{0,l} z_{0,i,l} - c_{0,l}^2 \hat{x}_{0,i,l-1} + \hat{u}_{0,i},\tag{4e}$$

$$\hat{x}_{0,i,l} = z_{0,i,l} + c_{0,l} \hat{x}_{0,i,l-1}, \tag{4f}$$

for i = 1, ..., N, where $z_{0,i,m}$ and $c_{0,m}$ for m = 2, 3, ..., l are the observer's internal states and gain parameters, respectively. The development of (4) is inspired by ⁵², in which a centralized disturbance observer is designed for a single plant. Significantly transforming the original design, we develop the above observer, which has a distributed structure that is uniquely suitable for the considered MAS setting. Here, the observer in (4a)-(4b) attempts to estimate the leader's second state $x_{0,2}$. In (4a), $-\sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_{0,i,2} - \hat{x}_{0,j,2})$ tries to keep the estimation consistent between follower *i* and its neighbors; then, (4b) performs the estimation to obtain $\hat{x}_{0,i,2}$ using the internal variable $z_{0,i,2}$ and the leader's first state $x_{0,1}$. Following similar lines in (4a)-(4b), the observers in (4c)-(4f) are designed to estimate the leader's rest higher-order state.

Finally, we design the following observer such that follower *i* can estimate its own states $x_{i,m}$ for m = 2, 3, ..., l:

$$\dot{z}_{i,2} = -r_2 z_{i,2} - r_2^2 x_{i,1} + \hat{x}_{i,3},\tag{5a}$$

$$\hat{x}_{i,2} = z_{i,2} + r_2 x_{i,1},\tag{5b}$$

$$\dot{z}_{i,m} = -r_m z_{i,m} - r_m^2 \hat{x}_{i,m-1} + \hat{x}_{i,m+1},$$
(5c)

$$\hat{x}_{i,m} = z_{i,m} + r_m \hat{x}_{i,m-1}, \ m = 3, 4, \dots, l-1,$$
(5d)

$$\dot{z}_{i,l} = -r_l z_{i,l} - r_l^2 \hat{x}_{i,l-1} + u_i, \tag{5e}$$

$$\hat{x}_{i,l} = z_{i,l} + r_l \hat{x}_{i,l-1},\tag{5f}$$

for i = 1, ..., N, where $z_{i,m}$ and $r_{i,m}$ for m = 2, 3, ..., l are the internal states and gain parameters, respectively.

Putting together (2)-(5), we obtain a distributed observer-based control algorithm to achieve high-order leader-follower tracking. Its convergence is analyzed in Section 3.3.

Remark 1. We highlight a comparison between the proposed approach and the study of output-feedback leader-following tracking control in $^{37-42}$. These references use different observers to help a follower estimate its own states or certain parameters. These observers are local observers as they are designed to estimate local unknown quantities. Compared with them, the proposed approach focuses more on distributed observer design—the observers in (3)-(4) have a distributed structure to enable the followers to collectively infer the leader's input and states by information exchange. This new design allows the followers to keep tracking the input-driven leader, setting it apart from the references that restrictively require the leader to be input-free or the followers to have at least certain knowledge of the leader's input.

3.3 | Convergence Analysis

This section characterizes the convergence property for the algorithm proposed above. Before proceeding further, we make the following assumption:

Assumption 1. The leader's input $u_0 \in C^1$ with $|\dot{u}_0| \le w < \infty$, where w is unknown.

This assumption is reasonable and justifiable due to the fact that control actuations are usually smooth and subject to rampup/down limits.

As the convergence depends on the estimation and tracking errors, we lay out the definitions of these errors first. For the observer in (3), we define the estimation error as $e_{u,i} = \hat{u}_{0,i} - u_0$. Its dynamics is

$$\dot{e}_{u,i} = -b_i e_{u,i} - \sum_{j \in \mathcal{N}_i} a_{ij} (e_{u,i} - e_{u,j}) - d_i \cdot \text{sgn} \left[\sum_{j \in \mathcal{N}_i} a_{ij} (e_{u,i} - e_{u,j}) + b_i e_{u,i} \right] - \dot{u}_0.$$
(6)

Further, define $e_u = \begin{bmatrix} e_{u,1} & e_{u,2} & \cdots & e_{u,N} \end{bmatrix}^{\top}$. It then follows from (6) that

$$\dot{e}_u = -He_u - D \cdot \operatorname{sgn}(He_u) - \dot{u}_0 \mathbf{1},\tag{7}$$

where H = B + L and $D = \text{diag}\{d_1, \dots, d_N\}$. It is seen that the signum function term in (7) is discontinuous, measurable and locally bounded. Therefore, (7) admits a Filippov solution according to ⁵³, which is governed by the differential inclusion $\mathcal{K}[\cdot]$:

$$\dot{e}_u \in \mathcal{K} \left[-He_u - D \cdot \operatorname{sgn}(He_u) - \dot{u}_0 \mathbf{1} \right].$$
(8)

Now, we consider the observer in (4). Defining $e_{0x,i,m} = \hat{x}_{0,i,m} - x_{0,m}$, we have

$$\dot{e}_{0x,i,2} = -c_{0,2}b_i e_{0x,i,2} + e_{0x,i,3}c_{0,2} \sum_{j \in \mathcal{N}_i} a_{ij}(e_{0x,i,2} - e_{0x,j,2}),$$
(9a)

$$\dot{e}_{0x,i,m} = -c_{0,m}c_{0,2}b_ie_{0x,i,2} + e_{0x,i,m+1} - c_{0,m}c_{0,2}\sum_{j\in\mathcal{N}_i}a_{ij}(e_{0x,i,2} - e_{0x,j,2}), \quad m = 3, 4, \dots, l-1,$$
(9b)

$$\dot{e}_{0x,i,l} = -c_{0,l}c_{0,2}b_ie_{0x,i,2} + e_{u,i} - c_{0,l}c_{0,2}\sum_{j\in\mathcal{N}_i}a_{ij}(e_{0x,i,2} - e_{0x,j,2}).$$
(9c)

Define $e_{0x,m} = \begin{bmatrix} e_{0x,1,m} & e_{0x,2,m} & \cdots & e_{0x,N,m} \end{bmatrix}^{\mathsf{T}}$ and $e_{0x} = \begin{bmatrix} e_{0x,2}^{\mathsf{T}} & e_{0x,3}^{\mathsf{T}} & \cdots & e_{0x,l}^{\mathsf{T}} \end{bmatrix}^{\mathsf{T}}$. Then, (9) can be written into a compact form as below:

$$\dot{e}_{0x} = F_1 e_{0x} + \ell_1, \tag{10}$$

where

$$F_{1} = \begin{bmatrix} -c_{0,2}H & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -c_{0,l-1}c_{0,2}H & 0 & \cdots & 0 & I \\ -c_{0,l}c_{0,2}H & 0 & \cdots & \cdots & 0 \end{bmatrix}, \ \mathcal{\ell}_{1} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ e_{u} \end{bmatrix}$$

Proceeding further, we define $e_{x,i,m} = \hat{x}_{i,m} - x_{i,m}$ for the observer in (5) and have

$$\begin{aligned} \dot{e}_{x,i,2} &= -r_2 e_{x,i,2} + e_{x,i,3}, \\ \dot{e}_{x,i,m} &= -r_m r_2 e_{x,i,2} + e_{x,i,m+1}, \quad m = 3, 4, \dots, l-1, \\ \dot{e}_{x,i,l} &= -r_l r_2 e_{x,i,2}. \end{aligned}$$

Define $e_{x,m} = [e_{x,1,m} \ e_{x,2,m} \ \cdots \ e_{x,N,m}]^{\top}$ for m = 2, 3, ..., l and $e_x = [e_{x,2}^{\top} \ e_{x,3}^{\top} \ \cdots \ e_{x,l}^{\top}]^{\top}$. Then, $\dot{e}_x = F_2 e_x$,

where

$$F_{2} = \begin{bmatrix} -r_{2}I & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -r_{l-1}r_{2}I & 0 & \cdots & 0 & I \\ -r_{l}r_{2}I & 0 & \cdots & \cdots & 0 \end{bmatrix}$$

Finally, to investigate the state tracking error, we define it as $e_{i,m} = x_{0,m} - x_{0,m}$. The dynamics of $e_{i,m}$ is

$$\dot{e}_{i,m} = e_{i,m+1}, \quad m = 1, 2, \dots, l-1$$

$$\dot{e}_{i,l} = -k_1 \left[\sum_{j \in \mathcal{N}_i} a_{ij}(e_{i,1} - e_{j,1}) + b_i e_{i,1} \right] - \sum_{m=2}^l k_m e_{i,m} - \sum_{m=2}^l k_m (e_{x,i,m} - e_{0x,i,m}) + e_{u,i}.$$

Define $e_m = [e_{1,m} \ e_{2,m} \ \cdots \ e_{N,m}]^{\top}$ for m = 1, 2, ..., l, and $e = [e_1^{\top} \ e_2^{\top} \ \cdots \ e_l^{\top}]^{\top}$. Here, *e* is the global tracking error with its dynamics governed by

$$\dot{e} = F_3 e + \ell_3,\tag{12}$$

where

$$F_{3} = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I \\ -k_{1}H & -k_{2}I & \cdots & -k_{l}I \end{bmatrix}, \ell_{3} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -\sum_{m=2}^{l} k_{m}(e_{x,m} - e_{0x,m}) + e_{u} \end{bmatrix}.$$

As a main result, the following theorem outlines the convergence property of the proposed algorithm.

(11)

Theorem 1. Suppose that the controller in (2)-(5) is applied to (1). If Assumption 1 holds, then

$$\lim_{t \to \infty} e_u(t) = 0. \tag{13}$$

Further, $\lim_{t \to \infty} e_{0x}(t) = 0$ if there exist $c_{0,2}, c_{0,3}, \dots, c_{0,l} > 0$ such that the polynomials

$$h_i(s) = s^{l-1} + c_{0,2}s^{l-2}\lambda_i(H) + c_{0,2}\lambda_i(H)\sum_{z=0}^{l-3} c_{0,l-z}s^z$$
(14)

for i = 1, 2, ..., N are Hurwitz stable, and $\lim_{t\to\infty} e_x(t) = 0$ if there exist $r_2, r_3, ..., r_l > 0$ such that the polynomial

$$s^{l-1} + r_2 s^{l-2} + r_2 \sum_{z=0}^{l-3} r_{l-z} s^z$$
(15)

is Hurwitz stable. Finally, the global tracking error $\lim_{t\to\infty} e(t) = 0$ if there exist k_m for m = 1, 2, ..., l such that the polynomials

$$s^{l} + k_{1}\lambda_{i}(H) + \sum_{z=2}^{l} s^{z-1}k_{z}$$
(16)

for i = 1, 2, ..., N are Hurwitz stable.

Proof: To prove (13), let us consider a Lyapunov functional candidate, $V_1 = \bar{V}_1(e_u) + \tilde{V}_1(d_i)$, where

$$\bar{V}_1(e_u) = \frac{1}{2} e_u^\top H e_u, \ \tilde{V}_1(d_i) = \sum_{i=1}^N \frac{(d_i - \beta)^2}{2\tau_i},$$

with $\beta \ge w$. The set-valued Lie derivative of $\bar{V}_1(e_u)$ denoted by $\mathcal{L}\bar{V}_1$ along (8) is given by

$$\begin{split} \mathcal{L}\dot{V}_{1} &= \mathcal{K}\left[-e_{u}^{\top}H^{2}e_{u} - e_{u}^{\top}HD \cdot \operatorname{sgn}(He_{u}) - e_{u}^{\top}H\dot{u}_{0}\mathbf{1}\right] \\ &= \mathcal{K}\left[-\sum_{i=1}^{N}d_{i}\left(\sum_{j\in\mathcal{N}_{i}}a_{ij}(\hat{u}_{0,i}-\hat{u}_{0,j}) + b_{i}(\hat{u}_{0,i}-u_{0})\right)^{\top} \\ &\cdot \operatorname{sgn}\left(\sum_{j\in\mathcal{N}_{i}}a_{ij}(\hat{u}_{0,i}-\hat{u}_{0,j}) + b_{i}(\hat{u}_{0,i}-u_{0})\right) - e_{u}^{\top}H^{2}e_{u} - e_{u}^{\top}H\dot{u}_{0}\mathbf{1}\right] \\ &\leq -\sum_{i=1}^{N}d_{i}\left|\sum_{j\in\mathcal{N}_{i}}a_{ij}(\hat{u}_{0,i}-\hat{u}_{0,j}) + b_{i}(\hat{u}_{0,i}-u_{0})\right| - e_{u}^{\top}H^{2}e_{u} + w||He_{u}||_{1}, \end{split}$$

where the fact that $\mathcal{K}[f] = \{f\}$ if f is continuous is used. It then is obtained that $\dot{V}_1 \in \mathcal{L}\dot{V}_1^{53}$. Hence,

$$\begin{split} \dot{V}_{1} &= \dot{V}_{1} + \dot{\tilde{V}}_{1} = \dot{V}_{1} + \sum_{i=1}^{N} \frac{(d_{i} - \beta)\dot{d}_{i}}{\tau_{i}} \\ &\leq -\sum_{i=1}^{N} d_{i} \left| \sum_{j \in \mathcal{N}_{i}} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_{i}(\hat{u}_{0,i} - u_{0}) \right| \\ &+ \sum_{i=1}^{N} (d_{i} - \beta) \left| \sum_{j \in \mathcal{N}_{i}} a_{ij}(\hat{u}_{0,i} - \hat{u}_{0,j}) + b_{i}(\hat{u}_{0,i} - u_{0}) \right| - e_{u}^{\top} H^{2} e_{u} + w \| H e_{u} \|_{1} \\ &= -e_{u}^{\top} H^{2} e_{u} - (\beta - w) \| H e_{u} \|_{1}. \end{split}$$

Note that H is positive definite⁵⁴. This, together with $\beta \ge w$, indicates $\dot{V}_1 \le 0$. Thus, $V_1(e_u)$ is non-increasing, implying that e_u and d_i are bounded. It follows from (3b) that d_i is monotonically increasing. This means that d_i should converge to some finite value. In the meantime, $V_1(e_u)$ reaches a finite limit as it is decreasing and lower-bounded by zero. If denoting $s(t) = \int_0^t e_u^{\mathsf{T}}(\tau) H^2 e_u(\tau) d\tau$, we see that $s(t) \le V_1(0) - V_1(t)$ by integrating $\dot{V}_1(e_u) \le -e_u^{\mathsf{T}} H^2 e_u$. Hence, $\lim_{t\to\infty} s(t)$ exists and is finite. Due to the boundedness of e_u and \dot{e}_u , \ddot{s} is also bounded. This implies that \dot{s} is uniformly continuous. Then, $\lim_{t\to\infty} \dot{s}(t) = 0$ by Barbalat's Lemma⁵⁵. Therefore, we can obtain $\lim_{t\to\infty} e_u(t) = 0$.

To prove the asymptotic stability of e_{0x} , we use the Schur complement, we can find out that the characteristic polynomial of F_1 is $\prod_{i=1}^{N} h_i(s)$, where $h_i(s)$ is shown in (14). Note that $\lim_{t\to\infty} \ell_1(t) = 0$ due to (13). Hence, we have $\lim_{t\to\infty} e_{0x}(t) = 0$ based on the input-to-state stability (ISS) theory⁵⁵. Following similar lines, we can obtain that $\lim_{t\to\infty} e_x(t) = 0$ given (15), and further prove that $\lim_{t\to\infty} e(t) = 0$ if (16) holds.

Remark 2. The proposed controller only requires the neighboring followers to interchange $x_{i,1}$, $\hat{u}_{0,i}$ and $\hat{x}_{0,i,2}$, because the observers can locally estimate other quantities necessary for control. This greatly reduces the amount of data to be exchanged between agents and makes the design more advantageous in terms of communication costs.

4 | HIGH-ORDER TRACKING FOR NONLINEAR DYNAMICS

We have investigated leader-follower tracking for a linear high-order MAS in the previous section. Given the importance of nonlinear leader-follower MASs, this section moves forward to study the case when an MAS has nonlinear high-order dynamics. We will develop an observer-based tracking control algorithm and analyze its convergence properties.

Suppose that agent *i*'s dynamics is governed by

$$\dot{x}_{i,m} = x_{i,m+1} + f_m(\overline{x_{i,m}}), \quad m = 1, 2, \dots, l-1,$$
(17a)

$$\dot{x}_{i,l} = u_i + f_l(\overline{x_{i,l}}),\tag{17b}$$

for i = 0, 1, ..., N, where $f_m(\overline{x_{i,m}}) : \mathbb{R}^m \to \mathbb{R}$ for m = 1, 2, ..., l are nonlinear functions with $\overline{x_{i,m}} = (x_{i,1}, x_{i,2}, ..., x_{i,m})$. Following Section 1, we assume that only $x_{i,1}$ is measured and continue to hold Assumption 1. The control design objective here is still to enable convergent tracking, i.e., $\lim_{t\to\infty} |x_{i,m}(t) - x_{0,m}(t)| = 0$ for m = 1, 2, ..., l and i = 1, 2, ..., N.

To achieve the above objective, we propose the following distributed controller:

$$u_i = -k_1(x_{i,1} - \hat{x}_{0,i,1}) - \sum_{m=2}^{l} k_m(\hat{x}_{i,m} - \hat{x}_{0,i,m}) + \hat{u}_{0,i}.$$
(18)

This controller must be supplemented by corresponding observers. It is noted first that the observer in (3) can also be applied to obtain $\hat{u}_{0,i}$ here, so we continue to use it for the distributed estimation of u_0 . We then construct the following state observer to allow follower *i* to estimate $x_{0,m}$ for m = 1, 2, ..., l:

$$\dot{\hat{x}}_{0,i,1} = -c_{0,1} \left[\sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_{0,i,1} - \hat{x}_{0,j,1}) + b_i (\hat{x}_{0,i,1} - x_{0,1}) \right] + \hat{x}_{0,i,2} + f_1(\overline{\hat{x}_{0,i,1}}),$$
(19a)

$$\dot{z}_{0,i,2} = -b_i c_{0,2} z_{0,i,2} - b_i^2 c_{0,2}^2 x_{0,1} + \hat{x}_{0,i,3} + f_2(\overline{\hat{x}_{0,i,2}}) - c_{0,2} \sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_{0,i,2} - \hat{x}_{0,j,2}) - b_i c_{0,2} f_1(x_{0,1}),$$
(19b)

$$\hat{x}_{0,i,2} = z_{0,i,2} + b_i c_{0,2} x_{0,1}, \tag{19c}$$

$$\dot{z}_{0,i,m} = -c_{0,m} z_{0,i,m} - c_{0,m}^2 \hat{x}_{0,i,m-1} + \hat{x}_{0,i,m+1} + f_m(\hat{x}_{0,i,m}) - c_{0,m} f_{m-1}(\hat{x}_{0,i,m-1}),$$
(19d)

$$\hat{x}_{0,i,m} = z_{0,i,m} + c_{0,m} \hat{x}_{0,i,m-1}, \quad m = 3, 4, \dots, l-1,$$
(19e)

$$\dot{z}_{0,i,l} = -c_{0,l} z_{0,i,l} - c_{0,l}^2 \hat{x}_{0,i,l-1} + \hat{u}_{0,i} + f_l(\overline{\hat{x}_{0,i,l}}) - c_{0,l} f_{l-1}(\overline{\hat{x}_{0,i,l-1}}),$$
(19f)

$$\hat{x}_{0,i,l} = z_{0,i,l} + c_{0,l} \hat{x}_{0,i,l-1}, \tag{19g}$$

where $\hat{x}_{0,i,m} = (\hat{x}_{0,i,1}, \hat{x}_{0,i,2}, \dots, \hat{x}_{0,i,m})$. To make a follower able to estimate its own states, we develop an observer as follows:

$$\dot{z}_{i,2} = -r_2 z_{i,2} - r_2^2 x_{i,1} + \hat{x}_{i,3} + f_2(x_{i,1}, \hat{x}_{i,2}) - r_2 f_1(x_{i,1}),$$
(20a)

$$\hat{x}_{i,2} = z_{i,2} + r_2 x_{i,1},\tag{20b}$$

$$\dot{z}_{i,m} = -r_m z_{i,m} - r_m^2 \hat{x}_{i,m-1} + \hat{x}_{i,m+1} + f_m(x_{i,1}, \hat{x}_{i,m}) - r_m f_{m-1}(x_{i,1}, \hat{x}_{i,m-1}),$$
(20c)

$$\hat{x}_{i,m} = z_{i,m} + r_m \hat{x}_{i,m-1}, \quad m = 3, 4, \dots, l-1,$$
(20d)

$$\dot{z}_{i,l} = -r_l z_{i,l} - r_l^2 \hat{x}_{i,l-1} + u_i + f_l(x_{i,1}, \hat{x}_{i,l}) - r_l f_{l-1}(x_{i,1}, \hat{x}_{i,l-1}),$$
(20e)

$$\hat{x}_{i,l} = z_{i,l} + r_l \hat{x}_{i,l-1}, \tag{20f}$$

where $\overline{\hat{x}_{i,m}} = (\hat{x}_{i,2}, \hat{x}_{i,3}, ..., \hat{x}_{i,m}).$

Integrating the above observers in (3), (19)-(20) into the controller in (18) will yield a complete observer-based tracking controller. Before going further to analyze its effectiveness, we make the following assumption:

Assumption 2. There exist $\rho_m \ge 0$ such that

$$|f_m(\xi) - f_m(\epsilon)| \le \rho_m ||\xi - \epsilon||, \ m = 1, 2, \dots, l$$

where $\xi, \epsilon \in \mathbb{R}^m$.

Assumption 2 implies that the nonlinear functions must be of Lipschitz class. It is commonly used in the literature on nonlinear MAS control and can be satisfied by many practical systems.

The following theorem shows the main result about convergence of the proposed controller.

Theorem 2. Assume that Assumptions 1 and 2 hold and that the controller proposed above is applied to (17). The state tracking error converges to zero, i.e., $\lim_{t\to\infty} |x_{i,m}(t) - x_{0,m}(t)| = 0$ for m = 1, 2, ..., l and i = 1, 2, ..., N, if there exist $c_{0,m}$, r_n and k_m for m = 1, 2, ..., l and n = 2, ..., N such that the polynomials (14), (15) and

$$\left(s^{l} + \sum_{z=1}^{l} s^{z-1} k_{z}\right)^{T}$$
(21)

are Hurwitz stable, and if there exist matrices $Q_i > 0$ and $\eta_i > 0$ for i = 1, 2, 3 such that

$$F_4^{\ I}Q_1 + Q_1F_4 = -\eta_1I, \tag{22a}$$

$$F_2^{\top}Q_2 + Q_2F_2 = -\eta_2 I, \tag{22b}$$

$$F_6^{\mathsf{T}}Q_3 + Q_3F_6 = -\eta_3I, \tag{22c}$$

$$\sum_{i=1}^{l} \|P_{0x,i}\| < \min\left\{\frac{\eta_1}{2\|Q_1\|}, \frac{\eta_3}{2\|Q_3\|}\right\},$$
(22d)

$$\sum_{i=2}^{l} \|P_{x,i}\| < \frac{\eta_2}{2\|Q_2\|},\tag{22e}$$

where $P_{0x,i} = \text{diag}\{\rho_1 I, \rho_2 I, \dots, \rho_i I, 0I, 0I, \dots, 0I\}, P_{x,i} = \text{diag}\{\rho_2 I, \rho_3 I, \dots, \rho_i I, 0I, 0I, \dots, 0I\}$ for $i = 1, 2, \dots, l$, and

$$F_{4} = \begin{bmatrix} -c_{0,1}H & I & 0 & \cdots & \cdots & 0 \\ 0 & -c_{0,2}H & I & 0 & \cdots & 0 \\ \vdots & \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ \vdots & -c_{0,l-1}c_{0,2}H & 0 & \cdots & 0 \end{bmatrix}, F_{6} = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & I \\ -k_{1}I & -k_{2}I & \cdots & \cdots & -k_{l}I \end{bmatrix}.$$

Proof: Let us define $e_{0x} = \begin{bmatrix} e_{0x,1}^{\top} & e_{0x,2}^{\top} & \cdots & e_{0x,l}^{\top} \end{bmatrix}^{\top}$, where $e_{0x,m}$ follows the same definition as in Section 3, and define $f_{0x,m} = \begin{bmatrix} \cdots & f_m(\overline{\hat{x}_{0,i,m}}) - f_m(\overline{x_{0,m}}) & \cdots \end{bmatrix}^{\top}$ for $i = 1, 2, \cdots, N$. By (17) and (19), we have

$$\dot{e}_{0x} = F_4 e_{0x} + \ell_4, \tag{23}$$

where $\ell_4 = \left[f_{0x,1}^{\top} \cdots f_{0x,l-1}^{\top} f_{0x,l}^{\top} + e_u^{\top} \right]^{\top}$. We choose a Lyapunov candidate function

$$V_2(e_{0x}) = \frac{1}{2} e_{0x}^\top Q_1 e_{0x},$$

for which there exist $\alpha_1, \alpha_2 > 0$ such that

$$|\alpha_1||e_{0x}||^2 \le V_2(e_{0x}) \le \alpha_2||e_{0x}||^2$$

By (22a), we have

$$\begin{split} \dot{V}_{2} &= \frac{1}{2} e_{0x}^{\mathsf{T}} (Q_{1} F_{4} + F_{4}^{\mathsf{T}} Q_{1}) e_{0x} + e_{0x}^{\mathsf{T}} Q_{1} \ell_{4} \\ &\leq -\frac{1}{2} \eta_{1} \|e_{0x}\|^{2} + \|e_{0x}\| \|Q_{1}\| \|\ell_{4}\| \\ &\leq -\frac{1}{2} \eta_{1} \|e_{0x}\|^{2} + \|e_{0x}\| \|Q_{1}\| (\|P_{0x,1}e_{0x}\| + \|P_{0x,2}e_{0x}\| \\ &+ \dots + \|P_{0x,l}e_{0x}\| + \|e_{u}\|) \\ &= -\left(\frac{1}{2} \eta_{1} - \|Q_{1}\| \sum_{i=1}^{l} \|P_{0x,i}\|\right) \|e_{0x}\|^{2} + \|e_{0x}\| \|Q_{1}\| \|e_{u}\|, \end{split}$$

Define

$$\sigma_1 = \frac{1}{2}\eta_1 - \|Q_1\| \sum_{i=1}^l \|P_{0x,i}\|, \ \mathcal{X}(\|e_u\|) = \frac{\|Q_1\|\|e_u\|}{\sigma_1\theta_1}$$

for any $0 < \theta_1 < 1$. By (22d), one can see that $\sigma_1 > 0$. It can be verified that

$$||e_{0x}|| \ge \mathcal{X}(||e_u||) \Rightarrow \dot{V}_2 \le -\sigma_1(1-\theta_1)||e_{0x}||^2.$$

Hence, V_2 is an ISS-Lyapunov function, implying that the system (23) is ISS⁵⁵. Then, we have $\lim_{t\to\infty} e_{0x} = 0$ since $\lim_{t\to\infty} e_u = 0$ as indicated in (13).

Define $f_{x,m} = \left[\cdots f_m(x_{i,1}, \overline{x_{i,m}}) - f_m(x_{i,1}, \overline{x_{i,m}}) \cdots \right]^{\mathsf{T}}$ for $i = 1, 2, \dots, N$. and continue to adopt e_x as defined in (11). According to (20), its dynamics can be expressed as

$$\dot{e}_x = F_2 e_x + \ell_5, \tag{24}$$

where F_2 was defined in (11), and $\ell_5 = \left[f_{x,2}^{\mathsf{T}} \cdots f_{x,l}^{\mathsf{T}} \right]^{\mathsf{T}}$. Following similar lines to the above, we can prove that $\lim_{t \to \infty} e_x = 0$ if (22b) and (22e) hold.

We proceed to consider the global tracking error when the controller in (18) is applied. We define $f_m = \left[\cdots f_m(\overline{x_{i,m}}) - f_m(\overline{x_{0,m}}) \cdots\right]^{\mathsf{T}}$ for $i = 1, 2, \cdots, N$. The dynamics of the tracking error $e_{i,m} = x_{i,m} - x_{0,m}$ is

$$\dot{e}_{i,m} = e_{i,m+1} + f_m(\overline{x_{i,m}}) - f_m(\overline{x_{0,m}}),$$
(25a)

$$\dot{e}_{i,l} = -\sum_{m=1}^{l} k_m e_{i,m} - \sum_{m=2}^{l} k_m e_{x,m} + \sum_{m=1}^{l} k_m e_{0x,m} + e_{u,i} + f_l(\overline{x_{i,l}}) - f_l(\overline{x_{0,l}}),$$
(25b)

for m = 1, 2, ..., l - 1 and i = 1, 2, ..., N. The notation of e in (12) is still adopted here. Now, combining (19), (20) and (25), the closed-loop system is indicated into a compact structure as below:

$$\dot{e} = F_6 e + \ell_6 + \ell_7, \tag{26}$$

where $\ell_6 = \begin{bmatrix} f_1^\top & \cdots & f_l^\top \end{bmatrix}^\top$ and

$$\ell_{7} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -\sum_{m=2}^{l} k_{m} e_{x,m} + \sum_{m=1}^{l} k_{m} e_{0x,m} + e_{u} \end{bmatrix}.$$

For (26), we can use the ISS theory to prove that it is asymptotically stable if (22c)-(22d) hold. Therefore, $\lim_{t\to\infty} e(t) = 0$ is established as $\lim_{t\to\infty} \ell_7(t) = 0$ due to (7), (23) and (24). We conclude that $\lim_{t\to\infty} |x_{i,m}(t) - x_{0,m}(t)| = 0$. Finally, we highlight that the Hurwitz stable polynomials (14), (15) and (21) would ensure the existence of solutions for the Lyapunov equations in (22a)-(22c). This concludes the proof.

Remark 3. Theorem 2 can be briefly explained as follows. The conditions (22a) and (22d) ensure that the distributed observers in (19) are asymptotically stable; the conditions (22b) and (22e) ensure that the local state observers in (20) are asymptotically stable; further, the conditions (22c) and (22d) ensure that the global closed-loop tracking errors converge to zero. When the polynomials (14), (15) and (21) are Hurwitz stable, F_4 , F_2 , and F_6 will be stable, and then the Lyapunov equations in (22a)-(22c) will admit solutions.



FIGURE 1 Topology of the MAS in the simulation.

Remark 4. The above design can be extended to the case when the nonlinear functions $f_m(\cdot)$ is unknown but admits approximation by a known function with bounded error. Specifically, suppose that there exist $g_m(\cdot)$ and $\varphi_m \ge 0$ such that

$$\left|g_m(\phi) - f_m(\phi)\right| \le \varphi_m, \ m = 1, 2, \dots, l,$$

for any $\phi \in \mathbb{R}^m$. We then can replace $f_m(\cdot)$ in (19)-(20) by $g_m(\cdot)$ and obtain a tracking controller based on the approximate nonlinearity. It can be proven that this controller will lead to bounded-error tracking under certain mild conditions. The analysis is omitted here for the sake of space.

5 | NUMERICAL STUDY

This section presents a numerical simulation example to show the effectiveness of the proposed design. For the sake of space, we only illustrate the case of nonlinear leader-follower tracking. Consider a third-order MAS including one leader and five followers. The agents interchange information based on a communication topology shown in Fig. 1. Here, node 0 is the leader, and nodes 1 to 5 are followers. The leader transmits data to only follower 1, and the followers maintain bidirectional communication with their neighbors. The agents' dynamics is as described in (17), for which $f_m(\overline{x_{i,m}}) = \cos(\overline{x_{i,m}})^{\mathsf{T}} \mathbf{1} = \sum_{k=1}^{m} \cos(x_{i,k})$. The leader's maneuver input is set to be $u_0 = \sin(0.2\pi t)$. When implementing the proposed observer-based controller, we select $c_{0,1} = c_{0,2} = c_{0,3} = 5$, $r_2 = r_3 = 4$ and $k_1 = k_2 = k_3 = 3$. Such a gain set is verifiable to make the convergence conditions satisfied. The simulation results are summarized in Fig. 2. Figs. 2(a)-2(c) illustrate followers' and the leader's state trajectories, showing that the followers can manage to catch up with and then keep tracking the leader, despite they differ in initial states. Fig. 2(d) shows the estimation of the leader's input by the followers. For each follower, the estimation can quickly converge to the actual values. Meanwhile, the followers can also effectively estimate the leader's states using the designed observer, with the estimation errors approaching zero as shown in Figs. 2(e)-2(g). Figs. 2(h) and 2(i) further present the followers' estimation of their own unmeasured states. These results validate that the proposed design can ensure convergent tracking, despite nonlinearity and limited information availability.

6 | CONCLUSION

We studied leader-follower tracking control for high-order MASs in this paper. While this problem has recently attracted growing attention, the previous studies generally require all the states of an agent to be measured, even though the measurement information can be practically limited by the availability of sensors. Here, we focused on the challenging but more realistic setting where only the first state of an agent is measured. We designed novel distributed observers, by which a follower can reconstruct unknown or unmeasured quantities about itself and the leader, and then performed distributed observer-based controller synthesis. We conducted the design for both linear and nonlinear MASs and characterized the convergence properties. A simulation result demonstrated the effectiveness of our design. Our future work will be directed towards extending the results to directed graphs and completely unknown nonlinearity.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.



FIGURE 2 Third-order nonlinear MAS profiles: (a) leader's and followers' state trajectory profiles of $x_{i,1}$ for i = 0, 1, ..., N; (b) leader's and followers' state trajectory profiles of $x_{i,2}$ for i = 0, 1, ..., N; (c) leader's and followers' state trajectory profiles of $x_{i,3}$ for i = 0, 1, ..., N; (d) leader's input profile and the estimation by each follower; (e) leader's state trajectory profile of $x_{0,1}$ and the estimation by each follower; (f) leader's state trajectory profile of $x_{0,2}$ and the estimation by each follower; (g) leader's state trajectories of $x_{i,2}$ for i = 1, 2, ..., N; (i) followers' estimation of their own state trajectories of $x_{i,3}$ for i = 1, 2, ..., N.

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