Temporal and Spectral Analysis of Spectrum Hole Distributions in an LTE Cell

Rui Zou, Wenye Wang and Huaiyu Dai
Department of Electrical and Computer Engineering
NC State University
Raleigh, NC 27606, USA
Emails: {rzou, wwang, hdai}@ncsu.edu

***Abstract*—Dynamic Spectrum Access (DSA) is proposed to improve spectrum efficiency by enabling opportunistic access of underutilized spectrum resources. The key to successful DSA operations is the correct understanding of spectrum hole distributions. Though huge amounts of studies have been conducted on spectrum tenancy due to the significance of spectrum hole distributions, there are still two overlooked aspects. One is the measurement resolution, and the other is the spectrum distribution in the spectral perspective. Since the spectrum hole analysis relies on the measurement data, we decode the LTE downlink control information to obtain the spectrum tenancy at the same time-frequency granularity with LTE scheduling. We analyze the spectrum hole distributions in fine resolutions along both the temporal and the spectral dimensions, and investigate the performance of two widely used spectrum tenancy models, the Markov and the on/off models, in terms of their capabilities on capturing the distributions of spectrum holes. Our observations include but are not limited to the following. The spectrum holes follow the power law distributions when examined in the LTE scheduling unit from both the time and the frequency perspectives. Both Markov and on/off models should be fitted to the spectrum tenancy along the frequency perspective to achieve their best performance.**

***Index Terms*—dynamic spectrum access, spectrum hole distribution, measurement granularity, power law.**

I. Introduction

The low usage of valuable spectrum resources has been identified in many studies [1]. To improve spectrum efficiency by designing Dynamic Spectrum Access (DSA) systems, the first step is to discover where the available spectrum slices are, i.e., the spectrum holes left unoccupied by Primary Users (PUs). Numerous measurement and modeling studies have been conducted to investigate spectrum tenancy in various spectrum bands and locations [2], [3]. One typical approach to gain insight into spectrum hole distributions is to carry out large scale spectrum measurement campaigns, such as the measurement in Chicago and London [1], [4], which adopt the energy detection method to scan a wide range of spectrum bands in large chunks with commercial spectrum analyzers. Based on the measurement data, various spectrum tenancy models have been proposed, including Markov models [5], linear regression models [6], queueing theory based models [7], and game theory models [8], [9].

Despite the abundant existence of measurement and modeling studies on spectrum hole distributions, there are still two overlooked aspects. First, existing studies on spectrum hole distributions are based on coarse measurement of spectrum activities in wide spectrum bands, so the analysis and results derived from such data sets cannot capture the volatile changes of the spectrum activities in most wireless communication systems. For example, the revisit time is 75 seconds in one measurement campaign [10]. However, the median duration of mobile phone calls lasts only 51 seconds [11], meaning that such measurement time granularity has good chance to deem large percent of mobile calls using no spectrum resources at all. Second, existing analytical models on spectrum hole distributions mostly focus on the temporal perspective [12], not taking into account the fact that spectrum resources are often designed as two dimensional grids which can be analyzed along both the frequency and the time horizons.

To improve on the aforementioned aspects that have long been overlooked, in this paper we answer the research question of *how the spectrum holes are distributed in both temporal and spectral domains when examined at fine resolutions*. To address this question, we target the spectrum usage of the LTE system whose spectrum resources are structured into time-frequency grids in two dimensions. Different from existing measurement campaigns that measure spectrum tenancy in coarse time-frequency granularities using the energy detection method, we measure the spectrum tenancy of an LTE cell in the resolutions at which the spectrum resources are scheduled by the base stations. Equipped with the fine-grained spectrum tenancy data, we analyze the distributions of the spectrum holes from both the temporal and the spectral perspectives. To investigate whether existing spectrum usage models are able to capture the spectrum hole distributions in both the time and the frequency domains, we study the performance of two widely adopted spectrum tenancy models by comparing the spectrum hole distributions in the measurement data and the synthetic data sets generated by the models.

Improvements on the measurement granularity of spectrum tenancy are achieved by employing the decoding based method [13]. Specifically, we set up an Software Defined Radio (SDR) testbed to decode the downlink control information aired by the LTE base station. Since the spectrum resource assignments are included in the downlink control messages, we obtain the LTE spectrum usage at the same time-frequency resolutions with the dynamic scheduling of LTE systems. The time resolution is 1 millisecond (ms), and the frequency resolution is 180 kHz. Moreover, we gather the spectrum tenancy in different traffic conditions, so spectrum usage levels in the measurement data ranges from 9% to 90%. Thus, the results and findings based on such data sets are applicable to most spectrum usage conditions. Based on the fine-grained spectrum tenancy, we analyze the spectrum hole distributions in both the temporal and spectral domains. We find that the temporal spectrum hole distributions in different frequency channels in the LTE cell are identical to one another, and the spectrum usage levels substantially affect the spectrum hole distributions. Both the temporal and the spectral spectrum hole distributions follow the power law distribution, regardless of the frequency channels and the traffic conditions.

Since most existing models for spectrum hole distributions are based on coarse spectrum tenancy measurement in the time perspective, we study the performance of two widely used models, the Markov model and the on/off model in terms of their capabilities on capturing the spectrum hole distributions. We apply the two models to our measurement data along both the temporal and the spectral dimensions, and generate synthetic spectrum usage based on the models. Comparisons between the spectrum hole distributions of the measurement data and those synthetic data sets are quantified using Kolmogorov Smirnov (K-S) tests. Based on the comparative studies, we observe that both Markov and on/off models perform better when they are fitted to spectrum tenancy along the frequency perspective. The on/off model achieves the best performance when applied to fit the spectral spectrum holes with power law distributions.

The contributions of this paper are recapitulated as follows.

- We analyze the spectrum hole distributions in both the temporal and the spectral dimensions based on fine-grained measurement of LTE spectrum tenancy. We discover that the spectrum holes follow the power law distributions when examined in the LTE scheduling unit from both the time and the frequency perspectives.
- We compare the performance of the Markov and the on/off models in terms of characterizing spectrum hole distributions. Both models should be fitted to the spectrum tenancy along the frequency perspective to achieve their best performance. The on/off model performs the best when applied to fit the spectral spectrum holes with the power law distribution.

II. Analysis on Spectrum Hole Distributions

In this section, we first explain how we obtain the LTE spectrum tenancy at the same time frequency granularity as the spectrum resources are scheduled at the base station. Then, the distributions of the spectrum holes are analyzed in both the time and the the frequency dimensions under different spectrum usage levels.

A. Data collection

To obtain LTE spectrum usage at fine granularities, we set up a measurement system and collect our spectrum tenancy data based on the decoding method proposed in [14]. Specifically, the decoding method utilizes the fact that the physical layer downlink control information in LTE systems is not protected by any encryption schemes. LTE base stations broadcast to all the users the decisions on how the spectrum resources are assigned every millisecond via a physical layer control channel in clear texts. Thus, the control information is decodable to reveal the spectrum usage decisions at the same time and frequency granularities with LTE scheduling. The decoded data fields are contained in the Downlink Control Information (DCI) in LTE terminology.

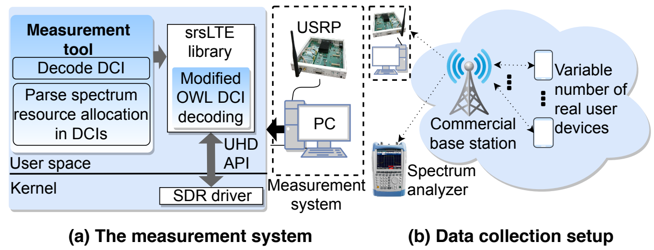


Fig. 1. The measurement system and the measurement setting.

We implement the spectrum tenancy decoder in the user space of a Linux computer. The two main function blocks are decoding the DCIs, and parsing the spectrum tenancy in DCI messages. Many existing LTE routines have been implemented by the srsLTE [15], including DCI decoding functions provided by OWL [14]. The radio front end is realized by a SDR system. Through the SDR driver API, the decoder calls USRP Hardware Driver (UHD) version 3.9.7 to communicate with the SDR device [16]. The SDR system includes a USRP X310 mother board and two SBX-120 wide-band daughter-boards. The SDR boards contain function blocks to convert analog signals to complex samples. The host computer has a quad-core CPU and 16 GB memory, running Ubuntu 16.04. The software and the hardware configurations of the measurement tool is also illustrated in Fig. 1(a).

The measurement setting is depicted in Fig. 1(b). We use a spectrum analyzer to search and verify the existence of a nearby LTE cell with the best signal to noise ratio. The downlink system bandwidth of the cell is 10 MHz that accommodates 50 LTE Resource Blocks (RB) which will be referred to as channels. These channels are frequency-wise orthogonal to one another, and each of them occupies 180 kHz. We collect LTE spectrum tenancy data of the cell for 24 hours, obtaining a total of 4.32×10^9 binary spectrum usage data. Each spectrum usage is either marked as unused by zero, or occupied by one.

B. Two dimensional analysis of spectrum holes

In this subsection, we investigate the distributions of spectrum holes in both the time and the frequency dimensions. When we analyze the spectrum holes along the time dimension, we study the time lengths of spectrum holes and the time lengths in between two consecutive holes in different channels independently. Similarly, the distributions of the sizes of the spectrum holes along the frequency dimension are also studied, together with the distributions of the distances between the holes measured in the number of channels.

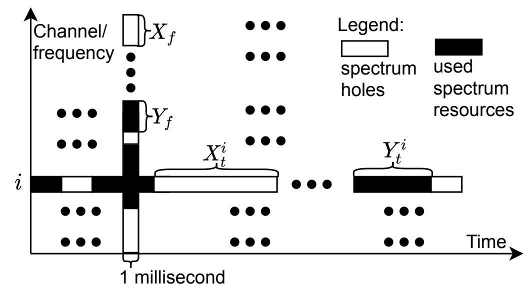


Fig. 2. Illustrations for the denotations.

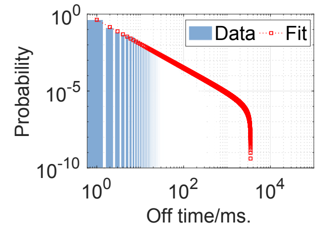
The definitions and the denotations of the random variables whose distributions will be investigated are illustrated in Fig. 2. The 50 LTE RBs are independently scheduled channels that experience tenancy and vacancy of integer numbers of milliseconds. Thus, the sizes of spectrum holes on channel i in the time domain are considered as a discrete random variable denoted as X_t^i, and the interval size between spectrum holes on that channel is a random variable denoted as Y_t^i. Moreover, the spectrum holes are also studied from the frequency perspective, where the size of the spectrum holes is measured in the number of adjacent occupied channels in the same millisecond, a discrete random variable denoted as X_f. The size of spectrum resources between frequency domain spectrum holes is another discrete random variable Y_f.

1) Distributions of spectrum holes: We first look into the shape and the fitting function of spectrum hole distributions. We choose a time period during which the overall spectrum usage level is around 50% to avoid the impacts of extremely high or low spectrum activities. The distributions of spectrum holes in our measure data are presented by the blue bars in Fig. 3. The time lengths of spectrum holes and the intervals in between them are referred to as off time and on time in the figures, and they are all shown in the unit of one millisecond. Similarly, the size of spectrum holes in the frequency domain and the size of occupied channels between the holes are denoted as off sizes and on sizes in the figures, and the sizes are in the unit of one channel or 180 kHz.

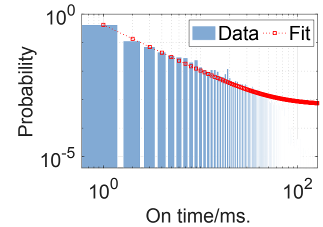
Since the off times take values from a large range and the probabilities of the off times vary widely, the horizontal and vertical axes of Fig. 3a and 3b are in the log scale. We observe from those figures that the trend of LTE off time distributions can be best captured by a power law function with a constant, which takes the form

$$\mathbb{P}(X = x) = ax^b + c, \tag{1}$$

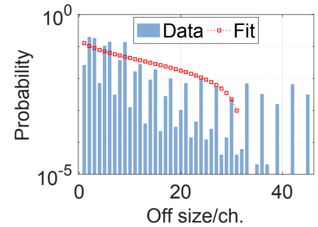
where a, b and c are constants. For the off times X_t, a, b and c take the values of 0.426, -1.565 and -0.000122, respectively. The distribution of the time intervals between off times is characterized by the function $\mathbb{P}(Y_t = x) = 0.4199x^{-1.632} - 0.000641$. The parameters in the fitted distribution functions are estimated using Maximum Likelihood Estimation (MLE).



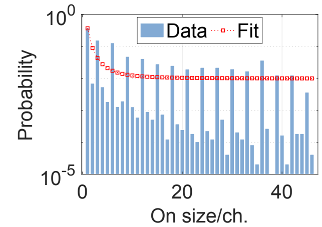
(a) Off time distributions.



(b) On time distributions.



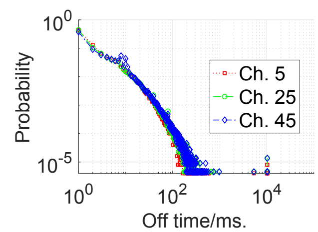
(c) Off size distributions.



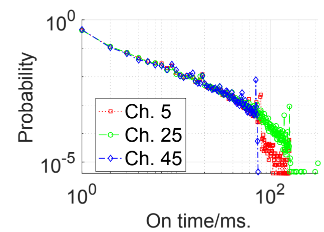
(d) On size distributions.

Fig. 3. Distributions of spectrum holes and the fitted power law functions in two dimensions.

When we examine the spectrum hole distributions in the spectral perspective, we observe that the off sizes and the on sizes are under 50, the total number of LTE channels in the system. As shown in Fig. 3c and 3d, the trends of the probability distributions of the on and the off sizes in the spectral domain also follow the power law function, despite some fluctuations. The fitted functions are plotted as the red lines in Fig. 3. The takeaway is that the spectrum holes and the intervals between them follow power law distributions in both the temporal and the spectral domains.



(a) Off time distributions.

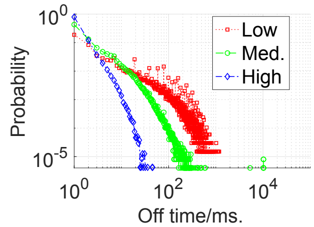


(b) On time distributions.

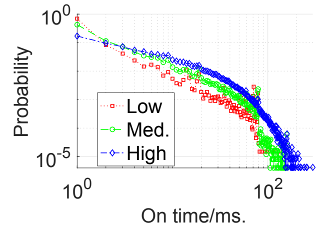
Fig. 4. Distributions of spectrum holes in different LTE channels.

2) Spectrum hole distributions in different channels: To investigate the temporal distributions of spectrum holes in different frequency bands, we analyze the off times in three LTE RBs, RB 5, 25 and 45. The distributions of the on times and off times are illustrated in Fig. 4. Though these three channels are well separated in frequency in a system with 50 RBs in total, their spectrum hole distributions are almost identical to one another as shown by the overlapping plots of the distributions of the spectrum holes in the three channels in Fig. 4. Thus, the observation is that the temporal spectrum hole distributions in different RBs are the same, and they all follow the power law distribution.

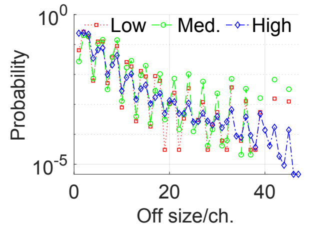
3) Impact of spectrum usage levels: We study the spectrum hole distributions in different spectrum usage levels. The



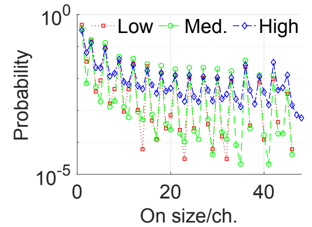
(a) Off time distributions.



(b) On time distributions.



(c) Off size distributions.



(d) On size distributions.

Fig. 5. Distributions of spectrum holes in different spectrum usage levels.

spectrum usage level is defined as the ratio between the number of occupied RBs and the total amount of RBs in 10^6 ms. Specifically, we collect the sizes of spectrum holes in both temporal and spectral domains under spectrum usage levels of 6%, 50% and 89%. The distributions of the spectrum holes conditioned on the three different spectrum usage levels are illustrated in Fig. 5 where plots for in the three cases are shown in red, green and blue, respectively.

Different from the distributions for various channels shown in Fig. 4 where the lines are mostly overlapped, the distributions of spectrum holes are obviously affected by the overall spectrum usage levels as the plots of the distributions are clearly separated. For the temporal spectrum hole distributions, the off times show higher probabilities to take small values below 5 when spectrum usage is high, and the slope of the blue line is also steeper to allow faster decrease of probabilities as the off time grows. The on time distributions demonstrate the opposite effects of spectrum usage on off times, because the plots of the on time distributions show a less steeper decline of probabilities as the overall spectrum usage level increases. In the spectral perspective, higher spectrum usage stabilizes the spectrum hole distributions, because the probabilities fluctuate in smaller ranges as the spectrum usage intensifies. Besides, the off sizes become much less likely to take large values near 50 as the spectrum usage increases. Thus, the spectrum usage levels exert substantial impacts on spectrum hole distributions. Spectrum holes in both temporal and spectral domains are enlarged by the decrease of spectrum usage, but the shape of the distribution is unchanged as it is still characterized by the power law function.

To sum up, the spectrum hole distributions follow the power law function along both the frequency and the time perspectives. The spectrum hole distributions in different frequency bands are highly similar, and they are sensitive to the variations of the overall spectrum usage levels. Though the average sizes of spectrum holes change with the spectrum usage levels, the shape of the spectrum hole distributions can always be captured by the power law function.

III. Comparative study of different models

Since we have analyzed the spectrum hole distributions in both temporal and spectral domains, in this section we examine how well existing spectrum tenancy models capture the distributions of spectrum holes. Due to the pivotal role of spectrum tenancy models in DSA systems, many spectrum tenancy models have been proposed [2], and one of their key goals is to generate synthetic spectrum usage data with the same spectrum hole distributions as the actual measurement. Two well-known and widely used spectrum tenancy models are compared, the Markov model and the on/off model. We apply these two models to the measurement data along both the time and the frequency dimensions, resulting in four models, the Markov model in the frequency domain, the Markov model in the time domain, the on/off model in the frequency domain, and the on/off model in the time. These four models are abbreviated as MF, MT, OF and OT, respectively.

A. Estimating model parameters

Though Markov models are theoretically possible to contain large amount of elements in its state space, accommodating all 50 channels in practice results in an state space that requires too many transition probabilities to learn. Thus, we apply the Markov model in temporal and spectral dimensions separately. The MF model is obtain by characterizing LTE spectrum tenancy along the frequency dimension with a discrete time Markov chain. The state of the Markov chain is considered as the binary spectrum tenancy in three previous channels, and we assume that the transition probabilities among different states are only dependent on the current state. To collect the empirical state transition probabilities from the measurement data, we transpose the binary spectrum tenancy matrix into a vector by serializing along the frequency dimension. Then, the transition probabilities are extracted from the spectrum tenancy vector. The only difference between the MT model and the MF model is that MT collects the empirical state transition probabilities along the time dimension. For both MF and MT models, we collect the empirical transition probabilities from the spectrum tenancy of 50 channels in 10^6 time slots during which the overall spectrum usage around is around 50%. The transition probability of $\mathbb{P}(s_{n+1} = a|s_n = b)$ is approximated by $|\{s_i|s_i = b, s_{i+1} = a\}|/|\{s_i|s_i = b\}|$, the ratio between the number of the appearances of state b followed by state a and the number of all appearances of state b. The $|\cdot|$ denotes the cardinality of a set.

To obtain the parameters for the OF model, we first serialize the binary spectrum tenancy measurement along the frequency direction. Then, we extract the time lengths of busy and idle time intervals. We assume that the on times and off times are independent and identically distributed (i.i.d.). Since we have learned from the previous section that the spectrum hole distributions follow the power law function in the spectral perspective, we obtain the parameters in equation (1) by MLE fitting in the same way we fit the parameters in Fig. 3. As for the parameters in the OT model, we apply the same method to temporal spectrum usage in an LTE channel. We extract the model parameters of OF and OT from the same spectrum usage matrix from which MF and MT parameters are learned for fair performance comparisons.

B. Performance evaluations

To evaluate the performance of the four spectrum tenancy models in terms of their capabilities of capturing the distributions of the spectrum holes in both the spectral and temporal domains, we first generate synthetic spectrum tenancy with the four models. The size of the synthetics data is 50 channels by 10^6 ms, the same as the measurement data based on which the parameters in the four models are learned. Then, we analyze the spectrum whole distributions in the four synthesized data sets, and compare the spectrum hole distributions of the synthetic data sets with those in the measurement data.

The spectrum hole distributions of synthetic data are compared against their counterpart obtained from the measurement data, and we measure the differences between the distributions via the K-S test, used in [17] for the same purpose as well. K-S test is a tool for comparing the closeness of two distributions. We employ K-S test to quantify the differences between the distribution of spectrum holes in the measurement and the distribution obtained from the synthetic data sets. The empirical distribution $F_e(x)$ of a random variable Z that has n observed samples Z_i is

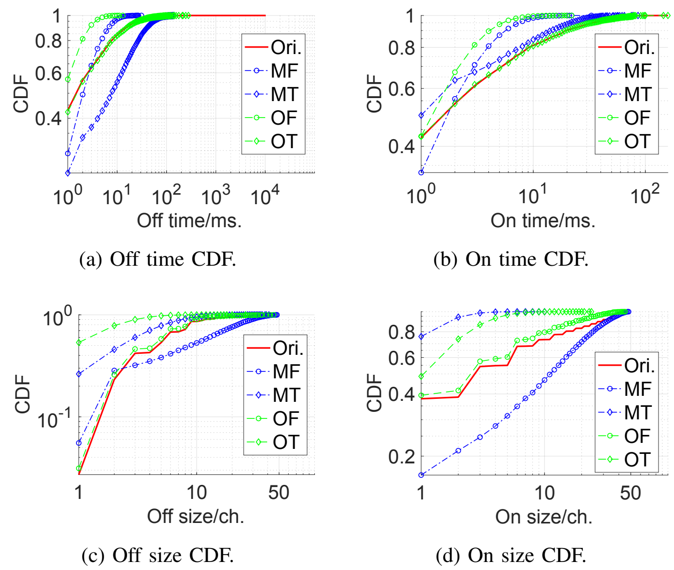
$$F_e(x) = \mathbb{P}(Z < x) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{\{Z_i<x\}}. \tag{2}$$

The upper bound of the difference between the empirical distribution and another empirical distribution function $F_0(x)$ is D,

$$D = sup|F_e(x) - F_0(x)|. \tag{3}$$

If the two empirical distributions F_e and F_0 are identical, the distribution of the random variable D in this case, denoted as D^*, is independent of the distribution function of the random variable Z. Let G be the cumulative distribution function (CDF) of D^*. The p value is defined as $p = 1 - G(D)$, so the larger the p value, the more likely D obeys the distribution of D^*, meaning that $F_e(x)$ and $F_0(x)$ are more likely to be the same. A threshold value $p = 0.05$ is chosen, so the null hypothesis that the two data sets follow the same distribution F_0 is accepted when $p \geq 0.05$.

The spectrum hole distributions of the measurement data and the four synthetic data sets are depicted in Fig. 6. The temporal distributions are presented in Fig. 6a and 6b. Similar to previous spectrum distribution plots in the temporal perspective, both the horizontal and the vertical axes are shown in the log scale. The spectrum holes and the intervals in between them are in the unit of one millisecond. The CDFs of the measurement and synthetic data of MF, MT, OF and OT models are shown in red lines, blue circles, blue diamonds,



(a) Off time CDF. (b) On time CDF.

(c) Off size CDF. (d) On size CDF.

Fig. 6. CDF comparisons of spectrum hole distributions in time and frequency dimensions in different data sets.

green circles and green diamonds, respectively. Based on the comparisons of the CDF plots, the OT model achieves the most similar spectrum hole distributions with the measurement data. OT model outperforms Markov models because the latter entail geometric distribution of spectrum holes which actually follows power law distributions. Another observation is that MF achieves better performance than MT in temporal spectrum distributions, so Markov models should be applied along the frequency dimension if the state space is limited.

Fig. 6c and 6d demonstrate the distribution comparisons in the spectral domain, and corresponding relationships between the lines and the data sets are the same with those in Fig. 6a and 6b. The spectral CDFs of spectrum holes are plotted with linear scale horizontal axis and log scale vertical axis. We observe that the OF model achieves the best performance since the distributions of spectral domain spectrum holes are most closely captured by the OF model.

TABLE I

D values of K-S tests between original and synthetics data.

Model	Off time	On time	Off size	On size
MF	0.1374	0.1903	0.3544	0.3315
MT	0.3032	0.0963	0.2786	0.5554
OF	0.2916	0.2488	0.0505	0.0798
OT	0.0067	0.0058	0.5520	0.4152

To quantify the distances between the spectrum hole distributions of the measurement data and the distributions obtained from those synthetic data sets, we tabulate the D values between those distributions in Table I. As shown by the highlighted row, the OF model achieves the overall best result because of the smallest D values in spectrum hole distribution in frequency domain. Though the D values of the OF model in temporal spectrum hole distributions are not the smallest, those D values are not far from the best.

In summary, the takeaway of the performance comparison is as follows. To best capture the spectrum hole distributions in both the temporal and the spectral domains with a simple model, we should adopt the on/off model with the random variables following the power law function, and apply this model along the frequency dimension of the measurement data. If a Markov model is adopted, it also needs to be applied to the spectrum tenancy data along the frequency domain to best characterize the spectrum hole distributions.

IV. Related work

Spectrum hole distributions are obtained by analyzing the spectrum tenancy data, so the spectrum usage data should have fine resolutions in order to achieve accurate spectrum hole distributions. However, early efforts in spectrum usage measurement based on the principle of energy detection often result in poor time-frequency granularity. For example, the Chicago measurement campaign reports spectrum holes in the time-frequency grids of 500 minutes by 10 MHz [1]. Though more recent energy detection based measurements achieve improved resolution, they still cannot reach the same granularity with LTE scheduling, i.e., 1 ms by 180 kHz. According to Table VI in [18], recent energy detection based measurements achieve time granularity from several to tens of seconds. Thus, we adopt the decoding based measurement method that is able to obtain the spectrum tenancy at the same resolution as they are scheduled by the base station.

Though spectrum holes appear in the two dimensional time-frequency grids, spectrum tenancy are often analyzed and modeled from the time perspective alone. The busy and the idle periods of wireless local area network are characterized by the Gaussian mixture model in [19]. In [20], the authors propose a new time domain model for duty cycles. Though some studies do have examined the spectrum tenancy in both time and frequency domains, such as [21], they often rely on low resolution measurement and only study the correlations of spectrum tenancy in different frequency bands.

V. Conclusion

We investigate spectrum hole distributions in fine time-frequency resolutions from both the temporal and the spectral perspectives. We analyze the fine-grained measurement data and observe that the spectrum holes follow the power law distributions in both the time and the frequency perspectives, when the spectrum usage is examined in the LTE scheduling unit of 1 ms by 180 kHz. When applied to model the spectrum tenancy, the Markov and the on/off models should be fitted to the spectrum tenancy along the frequency perspective to achieve their best performance on capturing the distributions of spectrum holes. When the on/off time intervals follow the power law distribution, the on/off model outperforms the Markov model in characterizing spectrum hole distributions.

References

[1] M. A. McHenry, P. A. Tenhula, D. McCloskey, D. A. Roberson, and C. S. Hood, "Chicago spectrum occupancy measurements & analysis and a long-term studies proposal," in *Proceedings of the first international workshop on Technology and policy for accessing spectrum*. ACM, 2006, p. 1.

[2] Y. Chen and H.-S. Oh, "A survey of measurement-based spectrum occupancy modeling for cognitive radios," *IEEE Communications Surveys & Tutorials*, vol. 18, no. 1, pp. 848–859, 2016.

[3] R. Zou and W. Wang, "Change detection based segmentation and modeling of lte spectrum tenancy," in *2019 IEEE Global Communications Conference (GLOBECOM)*. IEEE, 2019, pp. 1–6.

[4] A. Palaios, J. Riihijärvi, O. Holland, and P. Mähönen, "A week in london: Spectrum usage in metropolitan london," in *2013 IEEE 24th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*. IEEE, 2013, pp. 2522–2527.

[5] M. Lopez-Benitez and F. Casadevall, "Time-dimension models of spectrum usage for the analysis, design, and simulation of cognitive radio networks," *IEEE transactions on vehicular technology*, vol. 62, no. 5, pp. 2091–2104, 2013.

[6] H. Mosavat-Jahromi, Y. Li, L. Cai, and J. Pan, "Prediction and modeling of spectrum occupancy for dynamic spectrum access systems," *IEEE Transactions on Cognitive Communications and Networking*, pp. 1–1, 2020.

[7] S. Saad, I. Tarrad, and A. Ammar, "Multi-channel preemptive priority model for spectrum mobility in cognitive radio networks," *International Journal of Electrical and Computer Engineering (IJECE)*, vol. 8, no. 6, pp. 5169–5177, 2018.

[8] S. Chowdhury, "Resource allocation in cognitive radio networks using stackelberg game: A survey," *Wireless Personal Communications*, pp. 1–18, 2021.

[9] L. An, A. Chakrabortty, and A. Duel-Hallen, "A stackelberg security investment game for voltage stability of power systems," in *2020 59th IEEE Conference on Decision and Control (CDC)*. IEEE, 2020, pp. 3359–3364.

[10] D. Chen, S. Yin, Q. Zhang, M. Liu, and S. Li, "Mining spectrum usage data: a large-scale spectrum measurement study," in *Proceedings of the 15th annual international conference on Mobile computing and networking*. ACM, 2009, pp. 13–24.

[11] P. O. V. De Melo, L. Akoglu, C. Faloutsos, and A. A. Loureiro, "Surprising patterns for the call duration distribution of mobile phone users," in *Joint European Conference on Machine Learning and Knowledge Discovery in Databases*. Springer, 2010, pp. 354–369.

[12] K. Umebayashi, M. Kobayashi, and M. López-Benítez, "Efficient time domain deterministic-stochastic model of spectrum usage," *IEEE Transactions on Wireless Communications*, vol. 17, no. 3, pp. 1518–1527, 2018.

[13] R. Zou and W. Wang, "U-ciman: Uncover spectrum and user information in lte mobile access networks," in *IEEE INFOCOM 2020-IEEE Conference on Computer Communications*. IEEE, 2020, pp. 1459–1468.

[14] N. Bui and J. Widmer, "OWL: a Reliable Online Watcher for LTE Control Channel Measurements," in *ACM All Things Cellular (MobiCom Workshop)*, Nov. 2016.

[15] I. Gomez-Miguelez, A. Garcia-Saavedra, P. D. Sutton, P. Serrano, C. Cano, and D. J. Leith, "srslte: an open-source platform for lte evolution and experimentation," in *Proceedings of the Tenth ACM International Workshop on Wireless Network Testbeds, Experimental Evaluation, and Characterization*. ACM, 2016, pp. 25–32.

[16] E. Research, "Uhd," November 2017. [Online]. Available: https://kb.ettus.com/UHD

[17] S. Geirhofer, L. Tong, and B. Sadler, "A measurement-based model for dynamic spectrum access in wlan channels," 2006.

[18] M. Höyhtyä, A. Mämmelä, M. Eskola, M. Matinmikko, J. Kalliovaara, J. Ojaniemi, J. Suutala, R. Ekman, R. Bacchus, and D. Roberson, "Spectrum occupancy measurements: A survey and use of interference maps," *IEEE Communications Surveys & Tutorials*, vol. 18, no. 4, pp. 2386–2414, 2016.

[19] A. Gupta, S. Agarwal, and S. De, "A new spectrum occupancy model for 802.11 wlan traffic," *IEEE Communications Letters*, vol. 20, no. 12, pp. 2550–2553, 2016.

[20] K. Umebayashi, M. Kobayashi, and M. López-Benítez, "Efficient time domain deterministic-stochastic model of spectrum usage," *IEEE Transactions on Wireless Communications*, vol. 17, no. 3, pp. 1518–1527, 2017.

[21] M. Wellens, J. Riihijärvi, and P. Mähönen, "Empirical time and frequency domain models of spectrum use," *Physical Communication*, vol. 2, no. 1-2, pp. 10–32, 2009.