Upper limits on branching ratios of the lepton-flavor-violating decays $\tau \to \ell\gamma\gamma$ and $\tau \to \ell X$

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From analysis of data produced by the *BABAR* experiment, the first upper bounds (90% C.L.) were obtained on the branching ratios $\mathrm{Br}(\tau \to e\gamma\gamma) < 2.5 \times 10^{-4}$ and $\mathrm{Br}(\tau \to \mu\gamma\gamma) < 5.8 \times 10^{-4}$. In addition, improved upper bounds (95% C.L.) were found on branching ratios $\mathrm{Br}(\tau \to eX) < 1.4 \times 10^{-3}$ and $\mathrm{Br}(\tau \to \mu X) < 2.0 \times 10^{-3}$, where X is an undetected weakly interacting boson with mass $m_X < 1.6\ \mathrm{GeV}/c^2$.

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I. INTRODUCTION

The violation of lepton family number has been firmly established by the observation of neutrino oscillations, which also implies charged lepton family (flavor) number violation (CLFV). Although no CLFV has been observed yet, it is of fundamental interest, and searches for CLFV processes continue to be pursued. In the Standard Model (SM) extended to include massive neutrinos (generically denoted the νSM), the branching ratios for CLFV decays such as $\mu \to e\gamma$, $\mu \to e\gamma\gamma$, $\mu \to ee\bar{e}$, $\tau \to \ell\gamma$, and $\tau \to \ell\ell'\bar{\ell}'$, where $\ell = e, \mu$ and $\ell' = e, \mu$, are many orders of magnitude below the level where they could be observed in existing or planned experiments. This means that searches for these decays and similar CLFV processes are of great interest as probes of physics beyond the νSM (BSM).

Other CLFV processes which are not present in the SM may involve new weakly interacting bosons (X). For example, searches for $\mu \to eX$ were reported in [1–5]. The emission of an X boson has also been searched for in π^+ decays [6–8] and K^+ decays [9–12]. The ARGUS experiment [13] at DESY reported limits for $\mathrm{Br}(\tau \to \ell X)/\mathrm{Br}(\tau \to \ell\nu_\tau\bar{\nu}_\ell)$.

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Current upper bounds[1] on some CFLV τ decay modes are listed in Table I. In this paper, we used existing data to set the first upper limits on the branching ratios of the CLFV decays $\tau \to e\gamma\gamma$ and $\tau \to \mu\gamma\gamma$. We also examined the branching ratios for the decays $\tau \to \ell X$.

II. THEORETICAL BACKGROUND

A. CLFV in the νSM

To accommodate the observed neutrino oscillations and associated violation of lepton family number in the neutrino sector, the (renormalizable) SM Lagrangian can be modified by adding a number n_s of electroweak-singlet neutrino fields $\nu_{i,R}$, $i = 1, \ldots, n_s$, conventionally written as right-handed chiral fermions. With these, Yukawa terms are formed with the left-handed lepton doublets which, via the vacuum expectation values of the Higgs field, yield Dirac-type mass terms for neutrinos. The electroweak-singlet neutrinos also generically lead to Majorana mass terms of the form $\sum_{i,j} M^{(R)}_{ij} \nu^T_{i,R} C \nu_{j,R} + \mathrm{H.c.}$ The diagonalization of this combination of Dirac and Majorana mass terms yields the neutrino mass eigenstates. The resultant unitary transformation relating the left-handed chiral components of the mass eigenstates of the neutrinos, $\nu_{i,L}$, to the weak eigenstates, $\nu_{a,L}$, is given by

$$\nu_{a,L} = \sum_i U^{(\nu)}_{ai} \nu_{i,L}. \tag{2.1}$$

[1]Unless otherwise indicated, all experimental upper limits are given at the 90% confidence level (C.L.).

TABLE I. Upper limits on the branching ratios for some CLFV decays of the τ lepton [14].

Br(decay) upper limit
$\mathrm{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8}$
$\mathrm{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$
$\mathrm{Br}(\tau \to ee\bar{e}) < 2.7 \times 10^{-8}$
$\mathrm{Br}(\tau \to e\mu\bar{\mu}) < 2.7 \times 10^{-8}$
$\mathrm{Br}(\tau \to \mu e\bar{e}) < 1.8 \times 10^{-8}$
$\mathrm{Br}(\tau \to \mu\mu\bar{\mu}) < 2.1 \times 10^{-8}$
$\mathrm{Br}(\tau \to e\pi^0) < 0.80 \times 10^{-7}$
$\mathrm{Br}(\tau \to \mu\pi^0) < 1.1 \times 10^{-7}$

The property that $U^{(\nu)}$ is different from the identity gives rise to neutrino oscillations and the associated violation of lepton family number in the neutrino sector. (There is, in general, also violation of total lepton number in the νSM, due to the presence of Majorana mass terms.)

The diagonalization of the charged lepton mass matrix involves another unitary matrix $U^{(\ell)}$, and the product of (the adjoint of) $U^{(\ell)}$ and $U^{(\nu)}$ determines the form of the weak charged current:

$$J_\lambda = \bar{\ell}_L \gamma_\lambda \nu_{\ell,L} = \sum_{a,i} \bar{\ell}_{a,L} \gamma_\lambda U_{ai} \nu_{i,L}, \tag{2.2}$$

where U is the lepton mixing matrix,

$$U = U^{(\ell)\dagger} U^{(\nu)}. \tag{2.3}$$

As an example of CLFV in the νSM, the branching ratio for $\ell_a \to \ell_b + \gamma$ is [15,16]

$$\mathrm{Br}(\ell_a \to \ell_b \gamma) = \frac{3\alpha_{em}}{32\pi} \left| \sum_i U_{ai} U^*_{bi} \frac{m^2_{\nu_i}}{m^2_W} \right|^2, \tag{2.4}$$

where $\alpha_{em} = e^2/(4\pi)$ is the fine structure constant, and a, b are family or generation indices, with $\ell_1 \equiv e$, $\ell_2 \equiv \mu$, and $\ell_3 \equiv \tau$. Using current data on neutrino masses and lepton mixing, the resultant SM predictions for the branching ratios for the decays $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ have values $\lesssim 10^{-53}$, far below a level that could be observed in any existing or planned experiment. In passing, we recall the current upper limits on CLFV muon decays, $\mathrm{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ from the MEG experiment at PSI [17], $\mathrm{Br}(\mu \to e\gamma\gamma) < 0.72 \times 10^{-10}$ from the Crystal Box experiment at LAMPF [18], and $\mathrm{Br}(\mu \to ee\bar{e}) < 1.0 \times 10^{-12}$ from the SINDRUM experiment at SIN/PSI [19]. Since the decay $\ell_a \to \ell_b\gamma\gamma$ involves emission of a second photon, as compared with $\ell_a \to \ell_b\gamma$, it follows that for the νSM, up to logarithmic terms,

$$\mathrm{Br}(\ell_a \to \ell_b\gamma\gamma) \sim \alpha_{em} \mathrm{Br}(\ell_a \to \ell_b\gamma). \tag{2.5}$$

[Similarly, $\mathrm{Br}(\ell_a \to \ell_b \ell_c \bar{\ell}_c) \sim \alpha_{em} \mathrm{Br}(\ell_a \to \ell_b\gamma)$ in the νSM.]

B. Possible physics beyond the Standard Model contributing to $\tau \to \ell\gamma$ and $\tau \to \ell\gamma\gamma$

Although CLFV processes are predicted to be unobservably small in the νSM, there are many models of physics beyond the νSM that generically predict CLFV at observable rates. While none of these models has been confirmed by experiment, they remain of interest since they address incomplete aspects of the SM. One such aspect concerns the Higgs mass. There is a fine-tuning problem associated with this quantity since one-loop corrections to the Higgs mass squared are quadratically sensitive to the highest mass scale in an ultraviolet completion of the SM, such as a grand unified theory. Two early ideas for BSM physics that addressed this problem were supersymmetry (SUSY) and dynamical electroweak symmetry breaking (EWSB), and both of these generically predicted CLFV (as well as a number of flavor-changing neutral-current processes) at observable levels. For example, supersymmetric extensions of the SM predicted the decay $\mu \to e\gamma$ to occur at observable levels [20–23], and this is also true of $\tau \to \ell\gamma$. Early SUSY models with light neutralinos $\tilde{\chi}$ allowed the decay $\mu \to e\tilde{\chi}\tilde{\chi}$, which would be distinct from SM μ decay [24]. Substantial contributions to $\mu \to e\gamma\gamma$ and $\tau \to \ell\gamma\gamma$ would also be expected in such SUSY theories. Although searches for supersymmetric particles at the Fermilab Tevatron and at the CERN Large Hadron Collider (LHC) have yielded null results so far, there still remains the possibility of supersymmetry characterized by a SUSY-breaking scale that is larger than the electroweak scale.

Dynamical EWSB models also predict CLFV processes at possibly observable levels [25–27]. A relevant property of reasonably ultraviolet-complete dynamical EWSB models is the generic presence of sequential stages of breaking of an asymptotically free chiral gauge symmetry in the ultraviolet. The feature that the third generation is associated with the lowest of these scales could give rise to enhanced CLFV processes involving the τ lepton [28]. Modern versions of dynamical EWSB models typically involve quasiconformal behavior, which can result naturally from an approximate infrared fixed point of the renormalization group equations describing the strongly coupled vectorial gauge interaction [29–31]. In general, in these models, the observed Higgs is a composite state. These dynamical EWSB models are tightly constrained by precision electroweak data, the observed agreement of the Higgs boson with SM predictions, and, more generally, the nonobservation of any BSM Higgs properties at the LHC.

A large variety of other BSM theories predict CLFV effects at potentially observable levels. These could have the potential to alter the νSM relation (2.5). For example, in theories with doubly charged leptons, the ratio of branching ratios $\mathrm{Br}(\tau \to \ell\gamma\gamma)/\mathrm{Br}(\tau \to \ell\gamma)$ can be substantially enhanced relative to the $O(\alpha_{em})$ relation in Eq. (2.5), just as was true of the ratios $\mathrm{Br}(\mu \to ee\bar{e})/\mathrm{Br}(\mu \to e\gamma)$ and

$\mathrm{Br}(\mu \to e\gamma\gamma)/\mathrm{Br}(\mu \to e\gamma)$ [32]; some recent studies of theories with doubly charged leptons [33] provide experimental constraints. These theories could also lead to an enhancement of $\tau \to \ell\pi^0$, which, via the $\pi^0 \to \gamma\gamma$ decay, could contribute to a $\ell\gamma\gamma$ final state and hence to an overall $\tau \to \ell\gamma\gamma$ decay.

Of particular interest for $\tau \to \ell X$ decays are models with a light pseudo-Nambu-Goldstone boson (NGB) or a massless NGB that can couple to fermions in a flavor-violating manner [34,35]. These arise in models that hypothesize a "horizontal" symmetry mixing SM fermions transforming in the same manner under the SM gauge group, $G_{\mathrm{SM}} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, namely the sets $(e, \mu, \tau)_L$, $(e, \mu, \tau)_R$, and so forth for the neutrinos and quarks. With the hypothesized generational (i.e., family) symmetry taken to be global, a consequence would be that the spontaneous breaking of the flavor symmetry would lead to massless, spinless NGB(s) (often called familons). In the presence of some explicit breaking of the generational symmetry, the spontaneous breaking yields light NGB(s), with mass(es) determined by the relative sizes of explicit and spontaneous symmetry breaking. These NGBs are often called "axionlike particles" (ALPS). CLFV effects may also be associated with spontaneous breaking of total lepton number and resultant majorons [36–39]. Some more recent studies and reviews include [40–49].

Models featuring extra Z' vector bosons with flavor-nondiagonal couplings can yield CLFV effects at observable levels (e.g., [50,51]). These could contribute to CLFV decays such as $\tau \to \ell\gamma\gamma$ and $\tau \to \ell\gamma$. CLFV processes have also been studied in models with extra (spatial) dimensions and fermion fields having localized wave functions in these extra dimensions [52,53]. An appeal of these models is that they can produce a strong hierarchy in SM fermion masses via moderate separation of fermion wave function centers in the extra dimensions [54,55]. Connections between reported anomalies in B meson decays, $e - \mu$ universality violation, and models with CFLV have been discussed in a number of studies and are reviewed, e.g., in [56]. Although dark matter is, in principle, independent of CFLV, there may be connections between these in certain models [57].

III. $\tau \to \ell\gamma\gamma$

In the following we discuss the angular distribution expected for the decay products $\tau \to \ell\gamma\gamma$. Then, we use existing data on searches for $\tau \to e\gamma$ and $\tau \to \mu\gamma$ from the *BABAR* experiment at SLAC [58] to derive the first upper limits on the branching ratios for these decays, $\tau \to e\gamma\gamma$ and $\tau \to \mu\gamma\gamma$. (See also the recent result from Belle [59], which improves slightly on the upper limit on $B(\tau \to \mu\gamma)$ in [58].) In abstract notation, these decays are of the form $\ell_a \to \ell_b\gamma\gamma$ with the generational indices $a = 3$ and $b = 1, 2$, respectively.

A. Angular distribution for $\tau \to \ell\gamma\gamma$

A calculation of the decay rate for $\mu \to e\gamma\gamma$ was originally carried out in 1962 by Dreitlein and Primakoff [60] and was applied to the data on searches for $\mu \to e\gamma$ to set the upper bound $\mathrm{Br}(\mu \to e\gamma\gamma) < 5 \times 10^{-6}$. This procedure including a general expression for the angular distribution of the photons (discussed below) was applied by Bowman *et al.* [61] to data from two contemporaneous experiments searching for $\mu \to e\gamma$ [62,63] to derive the upper limit $\mathrm{Br}(\mu \to e\gamma\gamma) < 5 \times 10^{-8}$ [64–68].

For our analysis, we do not assume a particular BSM theory, but instead use an effective field theory method. In general, for a decay of the form $\ell_a \to \ell_b\gamma\gamma$, the operators that contribute to leading order to the effective Lagrangian $\mathcal{L}_{\mathrm{eff},\ell_a\ell_b\gamma\gamma}$ are lepton bilinears contracted with $F_{\alpha\beta}F^{\alpha\beta}$ or $F_{\alpha\beta}\tilde{F}^{\alpha\beta}$ [with coefficients given in Eq. (3.2) below], where $F^{\alpha\beta}$ is the electromagnetic field strength tensor and $\tilde{F}^{\alpha\beta} = (1/2)\epsilon^{\alpha\beta\lambda\rho}F_{\lambda\rho}$ is its dual. At more suppressed levels, there are additional operators involving derivatives. In d spacetime dimensions, the mass dimension of an operator $\mathcal{O}$ comprised of a lepton bilinear, a product of FF or $F\tilde{F}$, and n_∂ derivatives, is $\dim(\mathcal{O}) = 2d - 1 + n_\partial$. It follows that the coefficient $c_\mathcal{O}$ has mass dimension $\dim(c_\mathcal{O}) = -(d - 1 + n_\partial)$, i.e., for the physical case $d = 4$, $\dim(c_\mathcal{O}) = -(3 + n_\partial)$. One can thus write

$$c_\mathcal{O} = \frac{\bar{c}_\mathcal{O}}{(\Lambda_\mathcal{O})^{3+n_\partial}}, \tag{3.1}$$

where $\bar{c}_\mathcal{O}$ is dimensionless and $\Lambda_\mathcal{O}$ denotes a scale of BSM physics responsible for the appearance of the operator $\mathcal{O}$. Since in both of the decays $\tau \to \ell\gamma\gamma$ with $\ell = e$ or $\ell = \mu$, $m_\ell \ll m_\tau$, the only mass that enters into the phase space kinematics of the decay is m_τ. Because the derivatives yield factors of momenta in the amplitude, and the sizes of these momenta are set (in the τ rest frame) by m_τ, it follows that the contribution of an operator with n_∂ derivatives is suppressed by the factor $(m_\tau/\Lambda_\mathcal{O})^{n_\partial}$. The agreement of the νSM with current data implies that the scales $\Lambda_\mathcal{O}$ are much larger than $m_{W,Z}$, and hence $(m_\tau/\Lambda_\mathcal{O})^{n_\partial} \lll 1$. Therefore, operators with derivatives are expected to make a negligible contribution to the amplitude for $\tau \to \ell\gamma\gamma$. The effective Lagrangian for $\tau \to \ell\gamma\gamma$ can then be written, retaining non-negligible terms, as

$$\begin{aligned}\mathcal{L}_{\mathrm{eff},\tau\ell\gamma\gamma} = {} & c_{\tau\ell\gamma\gamma,LR;FF}[\bar{\ell}_L\tau_R]F_{\alpha\beta}F^{\alpha\beta} + c_{\tau\ell\gamma\gamma,RL;FF}[\bar{\ell}_R\tau_L]F_{\alpha\beta}F^{\alpha\beta} + c_{\tau\ell\gamma\gamma,LR;F\tilde{F}}[\bar{\ell}_L\tau_R]F_{\alpha\beta}\tilde{F}^{\alpha\beta} \\ & + c_{\tau\ell\gamma\gamma,RL;F\tilde{F}}[\bar{\ell}_R\tau_L]F_{\alpha\beta}\tilde{F}^{\alpha\beta} + \mathrm{H.c.},\end{aligned} \tag{3.2}$$

where the subscript LR on $c_{\tau\ell\gamma\gamma,LR;FF}$ refers to the chirality structure in the associated lepton bilinear, $[\bar{\ell}_L\tau_R]$, and similarly for the other coefficients. Without loss of generality, we will introduce a single effective mass scale $\Lambda_{\tau\ell\gamma\gamma}$ to characterize the CFLV physics responsible for the decay $\tau \to \ell\gamma\gamma$; any differences in the actual mass scales characterizing different operators $\mathcal{O}$ are absorbed into the values of the dimensionless coefficients $\bar{c}_{\mathcal{O}}$. Then Eq. (3.1) reads

$$c_{\mathcal{O}} = \frac{\bar{c}_{\mathcal{O}}}{\Lambda^3_{\tau\ell\gamma\gamma}} \tag{3.3}$$

for each of the coefficients $c_{\tau\ell\gamma\gamma,LR;FF}$, $c_{\tau\ell\gamma\gamma,RL;FF}$, $c_{\tau\ell\gamma\gamma,LR;F\tilde{F}}$, and $c_{\tau\ell\gamma\gamma,RL;F\tilde{F}}$ in $\mathcal{L}_{\mathrm{eff},\tau\ell\gamma\gamma}$. Let us denote the four-momenta of the τ, the final-state charged lepton ℓ, and the two photons as p_τ, p_ℓ, k_1, and k_2, respectively, and Lorentz-scalar products of two four-vectors as $p_\tau \cdot p_\ell$, etc. Let us further denote the matrix element for this decay as $\mathcal{M}_{\tau\to\ell\gamma\gamma}$. As usual, the amplitude is Bose-symmetrized with respect to the interchange of the identical bosons (photons) in the final state. With the above input $\mathcal{L}_{\mathrm{eff},\tau\ell\gamma\gamma}$, the square of the amplitude has a kinematic factor $(p_\tau \cdot p_\ell)(k_1 \cdot k_2)^2 = m_\tau E_\ell[E_{\gamma_1}E_{\gamma_2}(1-\cos\theta_{\gamma_1\gamma_2})]^2$, and the differential decay rate is

$$\frac{d\Gamma_{\tau\to\ell\gamma\gamma}}{dE_{\gamma_1}dE_{\gamma_2}d\cos\theta_{\gamma\gamma}} \propto \left(\frac{\sum_{\mathcal{O}}|\bar{c}_{\mathcal{O}}|^2}{\Lambda^6_{\tau\ell\gamma\gamma}}\right)E_\ell(E_{\gamma_1}E_{\gamma_2})^2(1-\cos\theta_{\gamma\gamma})^2, \tag{3.4}$$

where

$$\sum_{\mathcal{O}}|\bar{c}_{\mathcal{O}}|^2 = |\bar{c}_{\tau\ell\gamma\gamma,LR;FF}|^2 + |\bar{c}_{\tau\ell\gamma\gamma,RL;FF}|^2 + |\bar{c}_{\tau\ell\gamma\gamma,LR;F\tilde{F}}|^2 + |\bar{c}_{\tau\ell\gamma\gamma,RL;F\tilde{F}}|^2, \tag{3.5}$$

E_ℓ, E_{γ_1}, and E_{γ_2} are the energies of the daughter lepton ℓ and the two photons, respectively, and $\theta_{\gamma\gamma}$ is the angle between the 3-momenta of the photons [i.e., $\cos\theta_{\gamma\gamma} = (\vec{k}_1 \cdot \vec{k}_2)/(E_{\gamma_1}E_{\gamma_2})$] in the τ rest frame.

We note that a two-photon final state could also arise as a radiative correction to the decay $\tau \to \ell\gamma$, via emission of the second photon from the initial τ or from the final-state ℓ, where $\ell = e$ or μ. An event of this type would have an angular distribution different from that of an event in which the two photons originated directly as a consequence of the BSM physics, and the associated $\mathcal{L}_{\mathrm{eff},\tau\ell\gamma\gamma}$ in Eq. (3.2). Events in which a second photon is emitted as a radiative correction to a $\tau \to \ell\gamma$ decay were considered by the *BABAR* experiment [58], were modeled by the event simulation programs used in that experiment, and were taken into account in their upper limits on $\mathrm{Br}(\tau \to \ell\gamma)$.

B. Study of $\tau \to \ell\gamma\gamma$ based on *BABAR* limits on $\tau \to \ell\gamma$

The *BABAR* experiment searches for $\tau \to \ell\gamma$ decays [58] were performed at the SLAC PEP-II e^+e^- storage rings, primarily using center-of-mass (c.m.) energy $\sqrt{s} \simeq 10.6$ GeV at the $\Upsilon(4S)$ resonance. The *BABAR* detector is described in Ref. [69]. Charged particles were reconstructed as tracks with a silicon vertex tracker and a drift chamber inside a 1.5 T solenoidal magnet. A CsI(Tl) electromagnetic calorimeter identified electrons and photons, and a ring imaging Cherenkov detector identified charged pions and kaons. The flux return of the solenoid was instrumented with resistive plate chambers, and limited streamer tubes were used to identify muons.

Events ascribed to the reaction $e^+e^- \to \tau^+\tau^-$ were selected, and events of the form $\tau^\pm \to \ell^\pm\gamma$, were identified by a ℓ,γ pair with an invariant mass and total energy in the c.m. frame close to $m_\tau = 1.777$ GeV$/c^2$ and $\sqrt{s}/2$, respectively. Another $\tau^\pm$ decay in the opposite detector hemisphere was used as a tag. Important backgrounds arose from the reaction $e^+e^- \to \tau^+\tau^-\gamma$ yielding a hard photon when one τ underwent a SM decay to an ℓ and a neutrino antineutrino pair. Other backgrounds for the $\tau \to \ell\gamma$ search arose from the reaction $e^+e^- \to \ell^+\ell^-\gamma$ and from hadronic τ decays with particle misidentification.

The signal-side hemisphere was required to contain one photon with c.m. energy > 1 GeV, with no other photon with energy > 100 MeV in the laboratory frame. The signal had to contain one track identified as an electron or muon within the calorimeter acceptance with c.m. momentum less than $0.77\sqrt{s}/2$. Muons were also required to have momentum greater than 0.7 GeV$/c$ in the laboratory frame. In addition, the cosine of the opening angle between the signal track and signal photon was required to be less than 0.786 characterizing the back-to-back distribution of $\tau \to \ell\gamma$ events in the τ rest frame. Neural net cuts were also applied to the *BABAR* data.

Signal decays were identified by two kinematic variables: the energy difference $\Delta E = E^{\mathrm{c.m.}}_{\ell\gamma} - \sqrt{s}/2$, where $E^{\mathrm{c.m.}}_{\ell\gamma}$ is the c.m. energy of the $\ell\gamma$ pair, and the beam energy constrained τ mass (mEC), obtained from a kinematic fit after requiring the c.m. τ energy to be $\sqrt{s}/2$; the origin of the γ candidate was assigned to the point of closest approach of the signal lepton track to the e^+e^- collision axis [58].

Limits on the decays $\tau \to e\gamma\gamma$ and $\tau \to \mu\gamma\gamma$ were obtained using the results of the *BABAR* experiment searching for $\tau \to e\gamma$ and $\tau \to \mu\gamma$ decays. Using Eq. (3.4), we simulated 1×10^7 events for each $\tau \to \ell\gamma\gamma$ process applying momentum and energy resolutions (smearing) for the charged track and photons as reported by Ref. [69] and applying the cuts indicated above (except for the neural net cuts) to select events. Then, without the resolution effects applied, we constructed the mEC and ΔE variables for the $\tau \to \ell\gamma\gamma$ events which passed the cuts and were within the *BABAR* detector acceptance. The mEC and ΔE variables were then smeared according to their reported resolutions [58].

Figure 1 shows a plot of mEC vs ΔE for simulated $\tau \to e\gamma\gamma$ events after the cuts and resolution smearing. Compared with $\tau \to e\gamma$ in Ref. [58], the plot of mEC vs ΔE for $\tau \to e\gamma\gamma$ is widely distributed due to the requirement for the second gamma to have $E < 100$ MeV if in the signal side hemisphere or to be outside the detector acceptance. The red ellipse in Fig. 1 represents the signal region for $\tau \to e\gamma$ used by the *BABAR* analysis including the observed shift in position due to radiative effects [58]. This elliptical region contains the simulated $\tau \to e\gamma\gamma$ events which would have been classified as consistent with the $\tau \to e\gamma$ signal representing an efficiency of $\epsilon_{e\gamma\gamma} = 1.2 \times 10^{-4}$ compared to $\epsilon_{e\gamma} = 0.50$ for our simulation efficiency for $\tau \to e\gamma$. The estimated uncertainty in the ratio $\epsilon_{e\gamma}/\epsilon_{e\gamma\gamma}$ (used below) is approximately 10%.

To obtain the limits on $\tau \to \ell\gamma$, *BABAR* used the numbers of observed events and the numbers of the expected background events in the signal ellipse leading to $\mathrm{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8}$ and $\mathrm{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$. For $\tau \to e\gamma$ ($\tau \to \mu\gamma$), 0 (2) events were observed and the expected background was 1.6 ± 0.4 (3.6 ± 0.7). In order to avoid complications of estimating the expected backgrounds for $\tau \to \ell\gamma$ in the presence of $\tau \to \ell\gamma\gamma$ decays [70], we used a conservative approach and based the following limits on only the number of events observed by *BABAR* in the signal ellipses: $\mathrm{Br}'(\tau \to e\gamma) < 6.1 \times 10^{-8}$ and $\mathrm{Br}'(\tau \to \mu\gamma) < 9.1 \times 10^{-8}$.

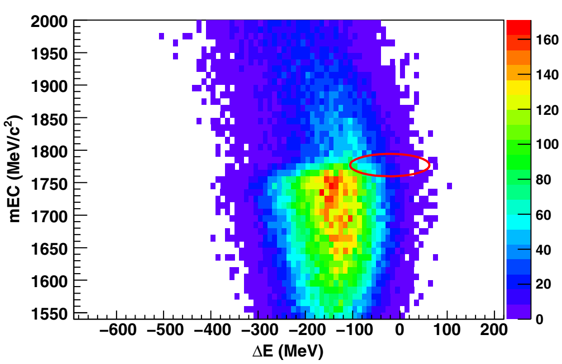

FIG. 1. mEC vs ΔE for simulated $\tau \to e\gamma\gamma$ events. The red ellipse indicates the signal region where events would have passed cuts for $\tau \to e\gamma$ [58].

Then, we found the limit

$$\mathrm{Br}(\tau \to e\gamma\gamma) < \frac{\mathrm{Br}'(\tau \to e\gamma) \times \epsilon_{e\gamma}}{\epsilon_{e\gamma\gamma}} = 2.5 \times 10^{-4}. \quad (3.6)$$

For the $\tau \to \mu\gamma\gamma$ case, we had $\epsilon_{\mu\gamma\gamma} = 7.2 \times 10^{-5}$ and $\epsilon_{\mu\gamma} = 0.46$ resulting in

$$\mathrm{Br}(\tau \to \mu\gamma\gamma) < \frac{\mathrm{Br}'(\tau \to \mu\gamma) \times \epsilon_{\mu\gamma}}{\epsilon_{\mu\gamma\gamma}} = 5.8 \times 10^{-4}. \quad (3.7)$$

The estimated uncertainty in the ratio $\epsilon_{\mu\gamma}/\epsilon_{\mu\gamma\gamma}$ is approximately 10%. Concerning sensitivity to new physics, our upper bounds (3.6) and (3.7) probe BSM scales $\Lambda_{\tau\ell\gamma\gamma} \sim O(10^2)$ GeV if the $|\bar{c}_{\mathcal{O}}| \sim O(1)$.

We note that the upper bounds $\mathrm{Br}(\tau \to e\pi^0) < 0.80 \times 10^{-7}$ and $\mathrm{Br}(\tau \to \mu\pi^0) < 1.2 \times 10^{-7}$ from Belle [71] and $\mathrm{Br}(\tau \to \mu\pi^0) < 1.1 \times 10^{-7}$ from *BABAR* [72] may also be used to obtain limits on $\tau \to \ell\gamma\gamma$. However, our evaluation of these processes led to limits on $\tau \to \ell\gamma\gamma$ that were 2 orders of magnitude less sensitive than those presented in Eqs. (3.6) and (3.7).

IV. $\tau \to \ell X$

In this section we obtain new constraints on the decays $\tau \to \ell X$ where X is a weakly interacting neutral boson that escapes without being detected. The latter condition is satisfied if the lifetime τ_X is sufficiently long or if X decays invisibly. Theoretical motivations for searching for such emission were discussed in Sec. II.

The signature for the decay $\tau \to \ell X$ is a monochromatic peak in the energy of the daughter lepton ℓ in the τ rest frame at the value

$$E_\ell = \frac{m_\tau^2 + m_\ell^2 - m_X^2}{2m_\tau} \quad (4.1)$$

where m_X is the mass of the X particle. This type of search involves an analysis of the energy or momentum spectrum of the daughter lepton in the τ decay. A different approach to setting an upper limit on $\mathrm{Br}(\tau \to eX)$ and $\mathrm{Br}(\tau \to \mu X)$ is based on the fact that if such events occurred and were included together with events from the corresponding SM leptonic decays of the τ, they would alter the observed rates of the respective decays.

Measurements of the individual branching ratios for $\tau \to \nu_\tau e\bar{\nu}_e$ and $\tau \to \nu_\tau \mu\bar{\nu}_\mu$ have been carried out, with the results [14]

$$\mathrm{Br}(\tau \to \nu_\tau e\bar{\nu}_e) = 0.1782 \pm 0.0004 \quad (4.2)$$

and

$$\mathrm{Br}(\tau \to \nu_\tau \mu \bar{\nu}_\mu) = 0.1739 \pm 0.0004. \tag{4.3}$$

The measured branching ratios (4.2) and (4.3) and the τ lifetime $\tau_\tau = (2.903 \pm 0.005) \times 10^{-13}$ s [14] can be used to obtain the decay rates to compare with SM calculations. Using the formulation in [73], the calculated values for the branching ratios [denoted by superscript (c)] are $\mathrm{Br}^{(c)}(\tau \to \nu_\tau e \bar{\nu}_e) = 0.17781 \pm 0.00031$ and $\mathrm{Br}^{(c)}(\tau \to \nu_\tau \mu \bar{\nu}_\mu) = 0.17293 \pm 0.00030$. Then, the ratios of experimental to calculated decay rates are [74,75]

$$S_{\tau\to e} = \Gamma_{\tau\to e}/\Gamma^{(c)}_{\tau\to e,\mathrm{SM}} = 1.0022 \pm 0.0028 \tag{4.4}$$

and

$$S_{\tau\to \mu} = \Gamma_{\tau\to \mu}/\Gamma^{(c)}_{\tau\to \mu,\mathrm{SM}} = 1.0056 \pm 0.0029 \tag{4.5}$$

with the following 95% C.L. [76] limits

$$S_{\tau\to e} < 1.008 \tag{4.6}$$

and

$$S_{\tau\to \mu} < 1.011. \tag{4.7}$$

Equations. (4.6) and (4.7) correspond to the 95% C.L. limits on the branching ratios of $\tau \to \ell X$ relative to $\tau \to \ell \nu \bar{\nu}$

$$\frac{\mathrm{Br}(\tau \to eX)}{\mathrm{Br}(\tau \to \nu_\tau e \bar{\nu}_e)} < 0.008 \tag{4.8}$$

and

$$\frac{\mathrm{Br}(\tau \to \mu X)}{\mathrm{Br}(\tau \to \nu_\tau \mu \bar{\nu}_\mu)} < 0.011. \tag{4.9}$$

These limits are plotted in Fig. 2 along with the previous results from the ARGUS experiment [13]. Using the measured $\tau \to \nu_\tau \ell \bar{\nu}_\ell$ branching ratios in Eqs. (4.2) and (4.3), we found $\mathrm{Br}(\tau \to eX) < 1.4 \times 10^{-3}$ and $\mathrm{Br}(\tau \to \mu X) < 2.0 \times 10^{-3}$.

Our new upper bounds (4.8) and (4.9) yield improved lower bounds on the weighted decay constants $F_{\tau\ell}$, $\ell = e, \mu$, appearing in the effective Lagrangian for $\tau \to \ell X$. For

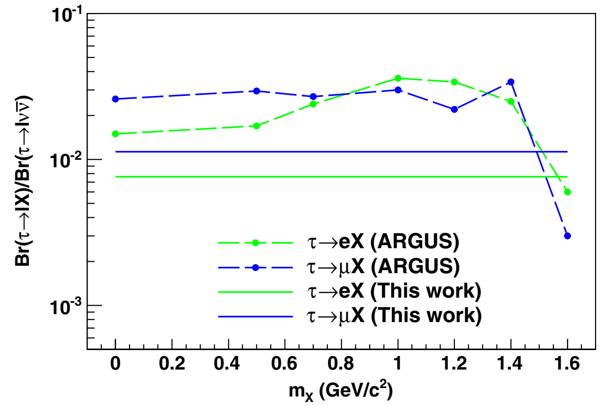

FIG. 2. Limits (95% C.L.) on $\mathrm{Br}(\tau \to \ell X)/\mathrm{Br}(\tau \to \ell \nu_\tau \bar{\nu}_\ell)$ from ARGUS [13] (filled points) and this work (solid lines).

example, in the notation of Table 1 of Ref. [49], at an illustrative mass $m_X = 0.6$ GeV, our bounds increase the lower limit on $F_{\tau e}$ from 4.3×10^6 GeV to $\sim 7 \times 10^6$ GeV and increase the lower limit on $F_{\tau\mu}$ from 3.3×10^6 GeV to $\sim 6 \times 10^6$ GeV. The limits found for $\tau \to \ell X$ decays also apply to three-body decays of the form $\tau \to \ell XX$, for which no previous bounds have been reported.

V. CONCLUSIONS

Using an analysis of data from searches for $\tau \to e\gamma$ and $\tau \to \mu\gamma$ performed by the *BABAR* experiment, we have obtained the first upper limits on the branching ratios $\mathrm{Br}(\tau \to e\gamma\gamma)$ and $\mathrm{Br}(\tau \to \mu\gamma\gamma)$. We have also presented improved upper limits on $\mathrm{Br}(\tau \to \ell X)$ where ℓ denotes e or μ and X is a weakly interacting boson with mass $m_X < 1.6\ \mathrm{GeV}/c^2$ that escapes detection. We expect that these decay modes can be searched for with considerably higher sensitivity at Belle II [77].

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