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Approximate multiplication in young children prior to multiplication instruction

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ABSTRACT

Prior work indicates that children have an untrained ability to approximately calculate using their approximate number system (ANS). For example, children can mentally double or halve a large array of discrete objects. Here, we asked whether children can perform a true multiplication operation, flexibly attending to both the multiplier and multiplicand, prior to formal multiplication instruction. We presented 5- to 8-year-olds with nonsymbolic multiplicands (dot arrays) or symbolic multiplicands (Arabic numerals) ranging from 2 to 12 and with nonsymbolic multipliers ranging from 2 to 8. Children compared each imagined product with a visible comparison quantity. Children performed with above-chance accuracy on both nonsymbolic and symbolic approximate multiplication, and their performance was dependent on the ratio between the imagined product and the comparison target. Children who could not solve any single-digit symbolic multiplication equations (e.g., 2×3) on a basic math test were nevertheless successful on both our approximate multiplication tasks, indicating that children have an intuitive sense of multiplication that emerges independent of formal instruction about symbolic multiplication. Nonsymbolic multiplication performance mediated the relation between children's Weber fraction and symbolic math abilities, suggesting a pathway by which the ANS contributes to children's emerging symbolic math competence. These findings may inform future

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educational interventions that allow children to use their basic arithmetic intuition as a scaffold to facilitate symbolic math learning.

Introduction

Early arithmetic is of great importance in determining readiness for more advanced mathematics and serves as a strong predictor of later academic achievement (Duncan et al., 2007). As one of the four basic mathematical operations, multiplication is an essential arithmetic skill learned in primary education. It constitutes a foundation for many other mathematical operations such as division and exponentiation. In school math curriculum, multiplication instruction often focuses primarily on rote learning of multiplication tables. It takes years for most children to memorize the set of exact, single-digit multiplication facts (De Visscher and Noël, 2014). Unlike subtraction and addition which rely more on calculation (Fayol & Thevenot, 2012), symbolic multiplication is thought to greatly involve automatic retrieval of multiplication facts. Brain imaging studies indicate that addition and subtraction calculation rely heavily on the intraparietal sulcus, whereas multiplication further recruits verbal brain regions such as the left angular gyrus and left middle temporal to superior temporal gyri (Lee, 2000; Polspoel, Peters, Vandermosten, & De Smedt, 2017). These findings suggest that multiplication facts are retrieved via verbal processing during multiplication calculation even into adulthood.

However, before children memorize multiplication tables, they may rely on simpler schemas for multiplication. One early model for multiplication is repeated addition. For example, 3 times 4 can be interpreted as 4 + 4 + 4 (Fischbein, Deri, Nello, & Marino, 1985). Another early model of children's understanding of multiplication originates in one-to-many correspondence, which is defined by a constant relation between two quantities, formally put as $x = f(y)$. For example, when a child is presented with the word problem, "Tom bought three sweets. One sweet cost three pence. How much money did he spend?", the child could map three pence to one sweet and solve this problem as "three threes" (Park & Nunes, 2001). These early models of multiplication are highly dependent on the problem structure and size of the number combinations used (Mulligan, 1992).

In addition to these early models of multiplication, children may develop their understanding of the multiplication concept by grounding it within their approximate number system (ANS). The ANS allows adults, infants, and animals to represent large sets of items approximately (Feigenson, Dehaene, & Spelke, 2004). A hallmark of the ANS is that the discriminability of two numerosities is dependent on the ratio between them rather than their absolute difference, following Weber's Law (Barth, Kanwisher, & Spelke, 2003; Izard & Dehaene, 2008). A handful of studies demonstrate that young children, even before the onset of formal schooling, are capable of performing arithmetic calculations on large nonsymbolic numerosities (Barth, Beckmann, & Spelke, 2008; Barth et al., 2006; McCrink, Shafto, & Barth, 2017; McCrink & Wynn, 2004). The accuracy with which children perform these nonsymbolic, approximate calculations is dependent on the ratio between the estimated outcome and the comparison quantity, indicating that children used imprecise representations generated by the ANS. For example, Barth et al. (2006) showed that 5-year-olds were capable of adding nonsymbolic visual sets and that their performance did not depend on non-numerical continuous quantities or the adoption of any non-arithmetic strategies. Moreover, preschoolers' approximate number knowledge appears to bolster their understanding of basic logical relationships of arithmetic, which allows preschoolers to solve multistep approximate addition and subtraction problems presented in either symbolic or nonsymbolic format (Gilmore & Spelke, 2008).

Beyond addition and subtraction, young children can also perform a scaling operation over nonsymbolic representations of large arrays of discrete objects (Barth, Baron, Spelke, & Carey, 2009; McCrink et al., 2017; McCrink & Spelke, 2010, 2015). In these studies, children without knowledge of symbolic multiplication and division are nevertheless able to learn that a magic wand doubles, halves, quadruples, or quarters an array of dots (Barth et al., 2009; McCrink & Spelke, 2010, 2015). McCrink and Spelke (2010) presented children with a video of an array of objects, which were then hidden behind an occluder. The video displayed a magic wand waving at the occluder that doubled or quadrupled the array. Children were instructed to compare the occluded array with a visible comparison array and judge which array was greater. Children performed with above-chance accuracy on this task, and their accuracy was dependent on the ratio between the imagined doubled or quadrupled array and the visible comparison target array, suggesting that children used their ANS to solve this task. However, scaling by a factor is only one specific case of the multiplicative operation. The scaling transformation solely requires the extraction of the constant proportional relationship between the multiplicative factor and the product, whereas true multiplication or division requires that the multiplier or divisor and the multiplicand or dividend be allowed to vary.

A recent study by Szkudlarek, Zhang, DeWind, and Brannon (2021) examined the ability of 6- to 9-year-old children and college undergraduates to perform symbolic and nonsymbolic approximate division. In their novel approximate division task, both the divisor and dividend were allowed to vary. Specifically, participants were presented with either dots (nonsymbolic format) or Arabic numerals (symbolic format) representing pollen pieces that were evenly distributed onto flower petals. During a demonstration phase, children were shown animations where the pollen pieces were always equally distributed among the petals. Their job was to compare the quotient (quantity on one flower petal) with a comparison target to choose the large quantity. Whereas the quotient was visible in the demonstration trials, on subsequent experimental trials the division operation was obscured by fog before the pollen pieces were distributed onto the petals so that participants needed to imagine how many dots landed on each petal and then compare their imagined quotient with a new visible quantity. Both the starting number of pollen pieces (dividend) and the number of flower petals (divisor) varied across trials, differentiating this experimental design from previous ones where children only needed to divide by a constant scaling factor. Results indicated that elementary school children could intuitively divide in both nonsymbolic and symbolic formats. Importantly, successful approximate division calculation was not dependent on formal division knowledge and was associated with sharper ANS acuity.

The current study was designed to investigate whether children are also capable of performing a true multiplication operation prior to formal schooling in the multiplication operation. To ask this question, we adapted the task used by Szkudlarek et al. (2021) for multiplication rather than division. Children aged 5–8 years were presented with nonsymbolic multiplicands (dot arrays) ranging from 2 to 12 and nonsymbolic multipliers ranging from 2 to 8. To solve the task, children needed to compare their imagined product with a visible comparison quantity. To assess whether children's successful approximate multiplication calculation was dependent on knowledge of symbolic multiplication, we quantified children's symbolic multiplication knowledge level with a test of their formal symbolic multiplication knowledge. We predicted that children who did not exhibit any knowledge of symbolic multiplication would nevertheless be capable of multiplying approximately.

Our second goal was to address whether children can access their intuitive sense of multiplication when presented with numerals to represent numerical magnitude. If children can integrate their intuitive multiplication knowledge within the symbolic number system, they may be more likely to use their intuitive knowledge of multiplication in the context of a math classroom. To answer this question, we provided children with a symbolic, approximate version of our novel approximate multiplication task. The only distinction between the symbolic and nonsymbolic versions of the task was that in the symbolic version children were presented with numerals (instead of dot arrays) to represent the multiplicand and the comparison target. We predicted that children could extend their intuitive multiplication ability beyond simple concrete representations of quantity to abstract mathematical symbols even without any formal training on symbolic multiplication.

A third goal of the current research was to determine whether children use their ANS to perform approximate multiplication. We assessed whether approximate multiplication performance displayed two different hallmarks of the ANS and whether it correlated with an independent measure of ANS acuity. First, we examined whether accuracy was modulated by the ratio between the imagined product and the comparison quantity. Second, we assessed whether accuracy was modulated by the magnitude of the operands. Operations with numerosities are thought to be aided by a mapping to mental magnitude representations (Gallistel & Gelman, 1992). If an operand is represented approximately, the fuzziness of its representation should increase linearly with numerical magnitude (Pica, Lemer, Izard, & Dehaene, 2004). In this case, approximate multiplication accuracy would decrease with the magnitude of the multipliers. Third, we directly correlated ANS acuity with intuitive multiplication performance. If intuitive multiplication is grounded in the ANS, children with sharper ANS acuity should have higher accuracy on our multiplication tasks.

Our fourth goal was to test whether nonsymbolic approximate multiplication serves as a pathway between ANS acuity and symbolic math abilities. There is an association between sharper ANS acuity and greater symbolic math competence across development (Chen & Li, 2014; Feigenson, Libertus, & Halberda, 2013; Schneider et al., 2017). In the first longitudinal study to show this, individual differences in ANS acuity in 14-year-olds were strongly linked to children's past scores on standardized math tests, extending back to kindergarten even after controlling for IQ (Halberda, Mazzocco, & Feigenson, 2008). Despite previous findings demonstrating a link between the ANS and symbolic math, the mechanism underlying this relation remains unknown. Recent evidence indicates that approximate arithmetic ability in young children might account for unique variance in symbolic math performance that cannot be explained away by ANS acuity (Starr, Roberts, & Brannon, 2016; Szkudlarek & Brannon, in press), suggesting a potential pathway that underlies this link. In Szkudlarek et al. (2021), performance on the nonsymbolic division task mediated the relation between ANS acuity and symbolic math performance in both child and adult participants. Thus, we hypothesized that nonsymbolic multiplication performance would mediate the relation between ANS acuity and symbolic math. Specifically, we hypothesized that sharper ANS acuity would lead to better nonsymbolic multiplication performance. In turn, a better intuitive sense of nonsymbolic multiplication might facilitate the conceptual understanding of the logical principles of the multiplication operation and therefore may precede and support instruction-based symbolic math performance. If this potential mechanism exists, better nonsymbolic multiplication calculation skill originating in sharper ANS acuity should lead to more advanced symbolic math performance.

Our hypotheses, procedures, and main analyses were preregistered on the Open Science Framework (OSF) at https://osf.io/u82fn/ Videos of the approximate multiplication tasks and our data are available in the OSF repository.

Method

Participants

A total of 44 5- to 8-year-old children participated in our experiment (M_{age} = 7.26 years, SD = 0.98; 27 females). We were unable to reach our preregistered sample size of 120 participants due to COVID-19 research closures. This age group was chosen because it consists of children who have learned counting and basic addition but will show variability in knowledge of symbolic multiplication. Written parental consent and children's verbal assent were obtained in accordance with a protocol accepted by the institutional review board in the University of Pennsylvania. An additional 12 children were consented but were excluded from the final sample because they did not complete the two experimental tasks. Children in our sample were from a range of racial and ethnic backgrounds (10.7% Asian, 42.9% Black or African American, 25.0% White, 7.1% Hispanic or Latino, and 14.3% unknown) and a broad socioeconomic spectrum based on annual family income (17.9% $0–25,000, 16.1% $25,000–50,000, 14.3% $50,000–75,000, 1.8% $75,000–100,000, 14.3% $100,000–150,000, 14.3% $150,000+, 3.6% preferred not to answer, and 17.9% are unknown. A subset of the 44 children who completed both experimental tasks completed additional assessments (formal multiplication knowledge test, n = 44; numeral identification task, n = 44; Corsi Block Task, n = 40; dot comparison, n = 36; KeyMath-3 Numeration assessment, n = 37; Woodcock–Johnson Reading cluster, n = 37).

Procedure

Children performed the tasks individually with an experimenter in a quiet room either in their after-school program or in our laboratory. Children completed the nonsymbolic and symbolic multiplication tasks first, with the order of the two tasks being counterbalanced across children (nonsymbolic first, $n = 23$; symbolic first, $n = 21$). The order of the other assessments was dependent on the task duration as well as children's availability. We gave each participant a small toy as a "thank you" gift on completion of all tasks.

Approximate multiplication tasks

Both the nonsymbolic and symbolic approximate multiplication tasks were built as Android apps using APIs in the Android framework and were run on a Samsung Galaxy S2 tablet with a 9.7-inch screen. All children, regardless of whether they were given the nonsymbolic or symbolic version of the approximate multiplication task first, watched a short introduction animation about the background story of the game on the tablet. At the start of the animation, children were introduced to a cartoon bee, "Buzz," who collects food in a garden. Children were told, "Buzz likes a special kind of flower that always has the same number of food pieces on each petal. See, there's the same number on each petal." Three examples of flowers that had the same number of pollen pieces on each petal were displayed. Children then heard, "Buzz collects all of the food pieces from the flower and puts them in his honeypot before eating them." Children were shown Buzz and his honeypot below a flower with two petals, each of which contained three dots. Children were then told, "Buzz takes his honeypot and brings it up to the flower and collects all of the food in his honeypot. All of the food from each petal goes into the honeypot. The same amount of food comes from each petal." Children could see as the two sets of three pollen pieces moved from the petals into the transparent honeypot. Children then watched three more examples of Buzz's food collecting process where each example had a different combination of petal number and pollen pieces on each petal.

Nonsymbolic approximate multiplication

The demonstration phase started right after the introduction animation (Fig. 1A). On demonstration trials, children were able to see the exact number of petals (the multiplier), the exact number

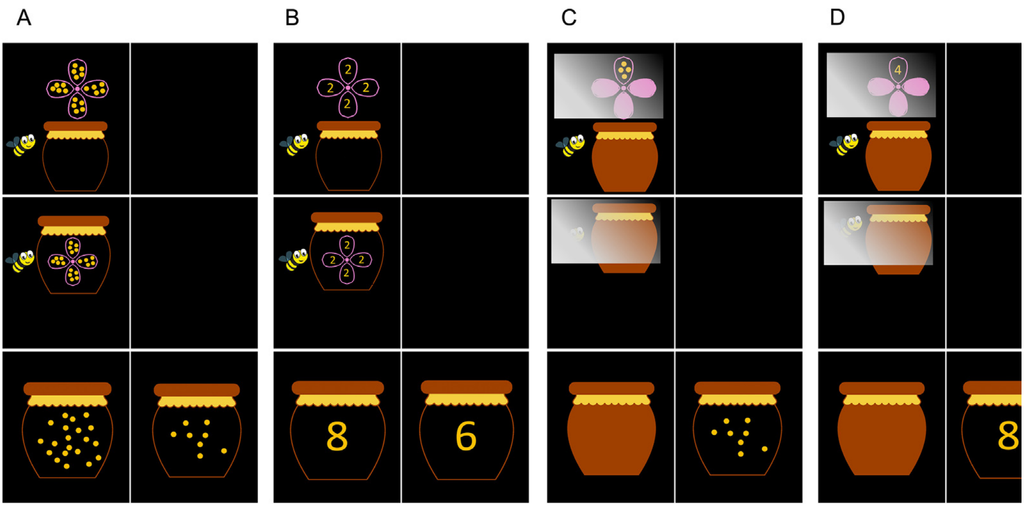

Fig. 1. Diagram of the procedure for the approximate multiplication tasks. (A) Demonstration trials for nonsymbolic version in which children learned that each petal on a given flower has the same number of pollen pieces and that Buzz the bee collects them all with his honeypot. (B) Demonstration trials for symbolic version in which children saw the same numerals on each petal go into Buzz's honeypot and change into the numeral of the product. Children completed 8 demonstration trials for both multiplication tasks by picking the honeypot with greater quantity. (C) Experimental trials for the nonsymbolic version where children could see only one flower petal and needed to mentally multiply to compare their imagined product with the comparison target on the right side. (D) Experimental trials for the symbolic version where the quantity on each petal and the comparison target were Arabic numerals rather than dot arrays.

of dots on each petal (the multiplicand), and the total number of dots that fell into the honeypot (the product). Children completed 8 demonstration trials. The purpose of this phase was to ensure that children could understand the basics of the game. After children watched Buzz collect all the pollen pieces into a transparent honeypot, the flower faded away and the honeypot moved to the left side of the screen. A second honeypot with pollen pieces inside appeared on the right side of the screen as a comparison target. Children were instructed to help Buzz pick the honeypot that contained the larger number of food pieces. If a correct response was made, a happy bee showed up with a positive tone and the written words "Great job!" If an incorrect response was made, a sad bee with a negative tone and the written words "Try again!" appeared as feedback. During the demonstration phase, children saw flowers with 2, 5, or 8 petals. These trials were not included in our data analysis.

After the completion of the demonstration phase, children watched another instruction animation. Children were told, "On foggy days, Buzz can only see one flower petal. But there are still the same number of food pieces on each petal. You have to imagine how many pieces of food are on each petal. Buzz takes his honeypot and collects all the food. All of the food from each petal goes into the honeypot. The same amount of food comes from each petal. If you imagine how many pieces of food were on each petal, you can imagine how many are in the honeypot now." Then children were shown two examples of the foggy day trials with other petal numbers. In experimental trials, children were no longer able to see the total number of dots in the flower; instead, a single flower petal was presented for 2 s. Thus, on experimental trials children needed to mentally multiply the visible dots on one petal by the number of petals to infer how many dots were collected into the opaque honeypot. To respond, children then compared their imagined product with a new visible target quantity and were instructed to choose the array of dots with greater quantity. During testing, children were explicitly instructed not to count or calculate. An experimenter was seated next to children and reminded them not to count if the experimenter noticed any verbal or finger counting by saying: "Remember, this is not a counting game. Try your best to get the right answer as fast as you can!" Children completed 32 trials with feedback during which they saw flowers with 2, 5, and 8 petals and 24 more trials with 3, 4, and 6 petals without feedback to test whether they could generalize to new multipliers. The task recorded children's accuracy and reaction time for each of the experimental trials. This paradigm allowed a test of children's ability to perform true multiplication because both the multiplier (number of petals) and the multiplicand (number of dots on each petal) flexibly varied from trial to trial.

Symbolic approximate multiplication

The symbolic version of the approximate multiplication task contained identical instructions and numerical values as the nonsymbolic version. As shown in Fig. 1, the only difference between the two tasks was that in the symbolic task the multiplicand (number of dots on each petal) and the comparison target (number in the comparison honeypot) were Arabic numerals rather than dot arrays.

Stimulus set for the approximate multiplication tasks

We chose the specific numerical values for the test trials to ensure that participants could not rely on a strategy other than approximate multiplication. Given that the multiplier (number of petals) ranged from 2 to 8 and the multiplicand (number of pollen pieces on each petal) ranged from 2 to 12, we first required that all the target comparison values be within the upper bound of the product value ($8 \times 12 = 96$) so that children could not pick the comparison target merely because it was numerically larger than any product they imagined during the course of the experiment. Second, to prevent children from simply attending to the multiplier or multiplicand, we included trials where the multiplicand and the multiplier were large relative to the range of their potential values, whereas the correct answer was the right comparison target. Specifically, we included 22 of 56 trials where the multiplicand was larger than the median multiplicand value (5), whereas the comparison target was the correct answer, and we included 24 of 56 trials with multipliers larger than the median multiplier value (4.5), whereas the correct answer was the comparison target. In addition, we created the stimulus set to avoid the possibility of children creating a mental model of the average of all the comparison values used in the experiment and making their choice by comparing that median number with each comparison value they saw. To do this, we included 22 of 56 trials where the right-side comparison target was larger than the median number of the comparison value (27.5), whereas the correct answer was the left product. Further analyses of these alternative strategies can be seen in Appendix C.

The numerical values were chosen to create four ratios between the imagined product and the comparison target (Ratio Level 1 ≈ .8, Ratio Level 2 ≈ .6, Ratio Level 3 ≈ .45, and Ratio Level 4 ≈ .35). To allow an assessment of the impact of the magnitude of the multiplier, we further ensured that at each multiplier there was the same number of trials corresponding to each ratio level with equal probability (50%) of correctness for each side. Finally, dot size varied randomly across trials in the nonsymbolic multiplication task (range = 0.1–0.25 cm diameter). Numerical values for experimental trials on both multiplication tasks are shown in Appendix A.

Numeral identification task

Children were presented with the numerals 1 to 30 in random order printed on index cards and were instructed to read the numerals aloud. The accuracy of their responses was recorded by the experimenter.

Formal multiplication knowledge test

We created a formal multiplication knowledge test composed of 16 questions (Appendix B). Of these 16 items, 6 were addition and multiplication word problems, 4 were symbolic addition equations, 4 were symbolic multiplication equations, and 2 examined children's ability to recognize the multiplication symbol (×) and the division symbol (÷). For each question, children were shown a flashcard with relevant pictures or arithmetic equations and the experimenter read the problem aloud to the children. Children's verbal responses were recorded.

Corsi Block Task

The Corsi Block Task was used to assess visuospatial working memory (Corsi, 1972). We aimed to incorporate visuospatial working memory as a control variable in the mediation analysis because arithmetic calculations require spatially represented numerical information to be stored and manipulated, and this largely relies on visuospatial strategies. Furthermore, visuospatial working memory has consistently been found to be correlated with early math performance (St Clair-Thompson & Gathercole, 2006; Toll, Kroesbergen, & Van Luit, 2016). We ran a standardized administration procedure for the Corsi Block-Tapping Task on the Android tablet (Kessels, Van Zandvoort, Postma, Kappelle, & De Haan, 2000). Nine cubes were displayed on the screen. Children saw a hand tap a sequence of blocks and were instructed to touch the blocks in the same sequential order. Correct responses resulted in increases in the length of the sequences. Children's performance was measured by the product of the block span (the length of the last correctly repeated sequence) and the number of correct trials.

Numerosity comparison task

The numerosity comparison task was programmed using the Psychophysics Toolbox extension in MATLAB (Brainard, 1997; Kleiner, Brainard, & Pelli, 2007; Pelli, 1997) and was run on a 15-inch touchscreen laptop computer. Before the start of the task, children heard the following narrative for instructions as they viewed printed slides shown by the experimenter: "This is our friend Dani the Dinosaur. Dani needs your help today. You are going to see circles that have dots in them just like this one. And your job today is to tell Dani which circle has more dots inside. Look at these two. Can you tell Dani which has more dots?" Children were given two practice trials with feedback. In the experimental trials, a colorful readiness cue was presented in the center of the screen. and children were instructed to tap it to start the next trial. Then two circles with dot arrays inside displayed simultaneously on the right and left of the screen for 750 ms. The dot arrays were then occluded, and children's response was measured. The stimulus set was constructed to spread numerical and non-numerical stimulus dimensions evenly along a logarithmic scale, with the distance between values in the space proportional to the ratios of their various features (number, dot size, and spacing), yielding a total of 13 ratio levels. The numerosities used ranged from 8 (2^3) to 32 (2^5), with 11 more powers of 2 spaced evenly between 2^3 and 2^5. All 13 values were rounded to the nearest whole number (e.g., $2^{4.5}$ was rounded to 23). Children completed 100 trials with an adaptive procedure developed by Lindskog, Winman, Juslin, and Poom (2013). The ratio between the two arrays advanced to a harder level if accuracy was above 80% and retreated to an easier level if accuracy was lower than 70% using a 5-trial running average. To increase the reliability of ANS measures, each child's Weber fraction was calculated as a quantitative index of ANS acuity using a novel model that controlled for the influence of non-numerical features of the stimuli (DeWind, Adams, Platt, & Brannon, 2015; DeWind & Brannon, 2016).

KeyMath-3 Diagnostic Assessment

The Numeration subtest of the KeyMath-3 Diagnostic Assessment (Connolly, 2008), which consists of 49 items, was used to assess children's general math abilities such as early number awareness, place value, magnitude of numbers, basic concepts of integers, and fractions. The standardized assessment was administered and scored following basal and ceiling rules given in the manual.

Woodcock–Johnson IV test of cognitive abilities

Children's reading abilities were measured by the Letter–Word Identification and Word Attack subtests of the Woodcock–Johnson IV Tests of Achievement (Schrank, F. A., Mather, N., & McGrew, K. S. (2014)). The Letter-Word Identification subtest required children to identify letters and words. In the Word Attack subtest, children were instructed to read aloud as many nonsense words as possible. The scoring procedure following basal and ceiling rules was administered.

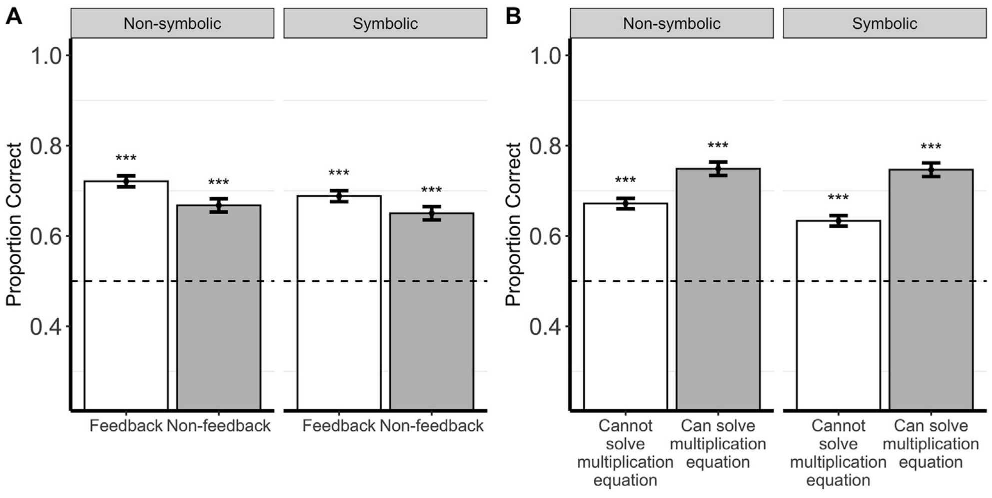

Fig. 2. (A) Accuracy on the nonsymbolic and symbolic multiplication tasks for children. The dashed line denotes chance performance. Error bars represent standard error of the mean. (B) Children's performance on the nonsymbolic and symbolic multiplication tasks broken down by their performance on the formal multiplication knowledge test. The feedback and non-feedback phases are combined for each task. Children who could not solve any multiplication equations performed significantly above chance on both the nonsymbolic and symbolic tasks. Children who could solve at least one multiplication equation performed significantly better than children who could not. ***p < .001.

Results

Nonsymbolic and symbolic approximate multiplication performance

As shown in Fig. 2A, and consistent with our preregistered prediction, children performed the nonsymbolic approximate multiplication task with above-chance accuracy on both the feedback phase (72.1% accuracy, 95% confidence interval (CI) [.69, .75], $t(43) = 13.79$, $p < .001$) and non-feedback phase (66.7% accuracy, 95% CI [.63, .70], $t(43) = 10.14$, $p < .001$). There were 29 children who could not solve any of the four multiplication equations from the formal multiplication assessment. Consistent with our preregistered prediction, and as shown in Fig. 2B, children who could not solve any multiplication equations successfully completed the nonsymbolic multiplication task with above-chance accuracy (feedback: 69.5% accuracy, 95% CI [.66, .73], $t(28) = 10.42$, $p < .001$; non-feedback: 64.0% accuracy, 95% CI [.60, .68], $t(28) = 6.85$, $p < .001$). Children who could solve at least one simple multiplication equation performed with higher accuracy than children who could not solve any multiplication equations (Wilcoxon rank sum test, $M_1 = 74.9\%$, $M_2 = 67.2\%$, $W = 114$, $p < .01$).

On the symbolic multiplication task, consistent with our preregistered prediction, children performed well above chance on both the feedback phase (68.8% accuracy, 95% CI [.64, .74], $t(43) = 7.99$, $p < .001$) and non-feedback phase (65.0% accuracy, 95% CI [.61, .70], $t(43) = 6.76$, $p < .001$) of the task (Fig. 2A). As can be seen in Fig. 2B, children who could not solve multiplication equations nevertheless performed the symbolic multiplication task with above-chance accuracy (feedback: 65.1% accuracy, 95% CI [.60, .70], $t(28) = 6.03$, $p < .001$; non-feedback: 61.0% accuracy, 95% CI [.57, .65], $t(28) = 5.35$, $p < .001$). Similar to results with the nonsymbolic task, children who could solve at least one multiplication equation were more accurate than children who could not solve any multiplication equations (Wilcoxon rank sum test, $M_1 = 74.6\%$, $M_2 = 63.3\%$, $W = 132$, $p = .017$). These data indicate that formal multiplication knowledge is not a prerequisite for engaging in approximate multiplication in either the nonsymbolic or symbolic format but that some symbolic multiplication knowledge may benefit approximate multiplication performance.

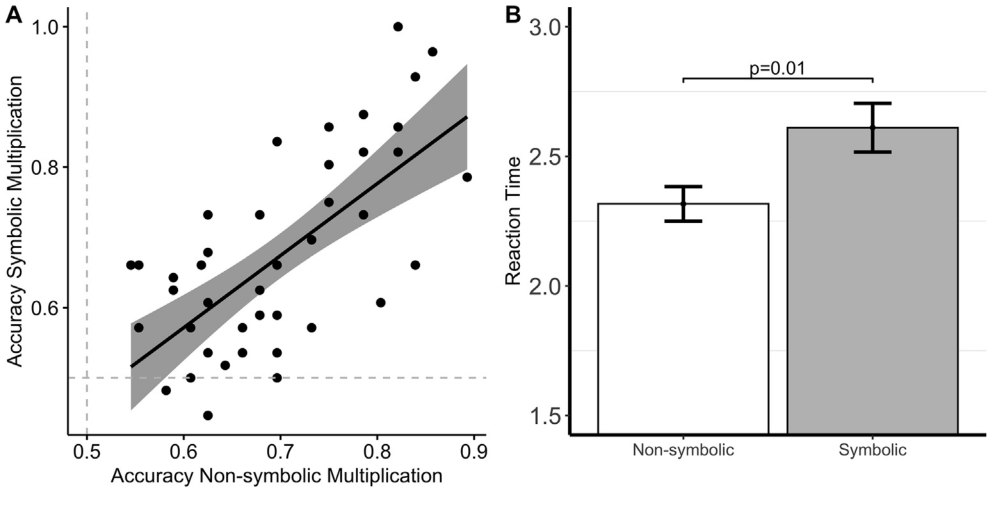

Fig. 3. (A) Performances on the symbolic and nonsymbolic approximate multiplication tasks were highly correlated. Dashed lines denote chance accuracy. (B) Children solved the nonsymbolic task faster than the symbolic task. Error bars indicate standard error of the mean.

Approximate multiplication task format effect

Children's performance on the nonsymbolic and symbolic multiplication tasks were highly correlated [r = .68, $t(42)$ = 5.93, p < .001] (Fig. 3A). Accuracy on the nonsymbolic version was not significantly different from accuracy on the symbolic version [nonsymbolic: 70.0% accuracy; symbolic: 67.2% accuracy; MD = 0.03, $t(43)$ = 1.66, p = .10]. We found no significant task format effect on accuracy even among children unable to identify all the numerals 1 to 30 (n = 14) (Wilcoxon signed rank test, nonsymbolic: 64.1% accuracy; symbolic: 60.0% accuracy; V = 68.5, p = .12). Unlike the accuracy data, there was a significant effect of format on median reaction time, indicating that children were faster to complete the nonsymbolic task than the symbolic task (95% CI [−.32, −.04], $t(43)$ = −2.68, p = .01) (Fig. 3B).

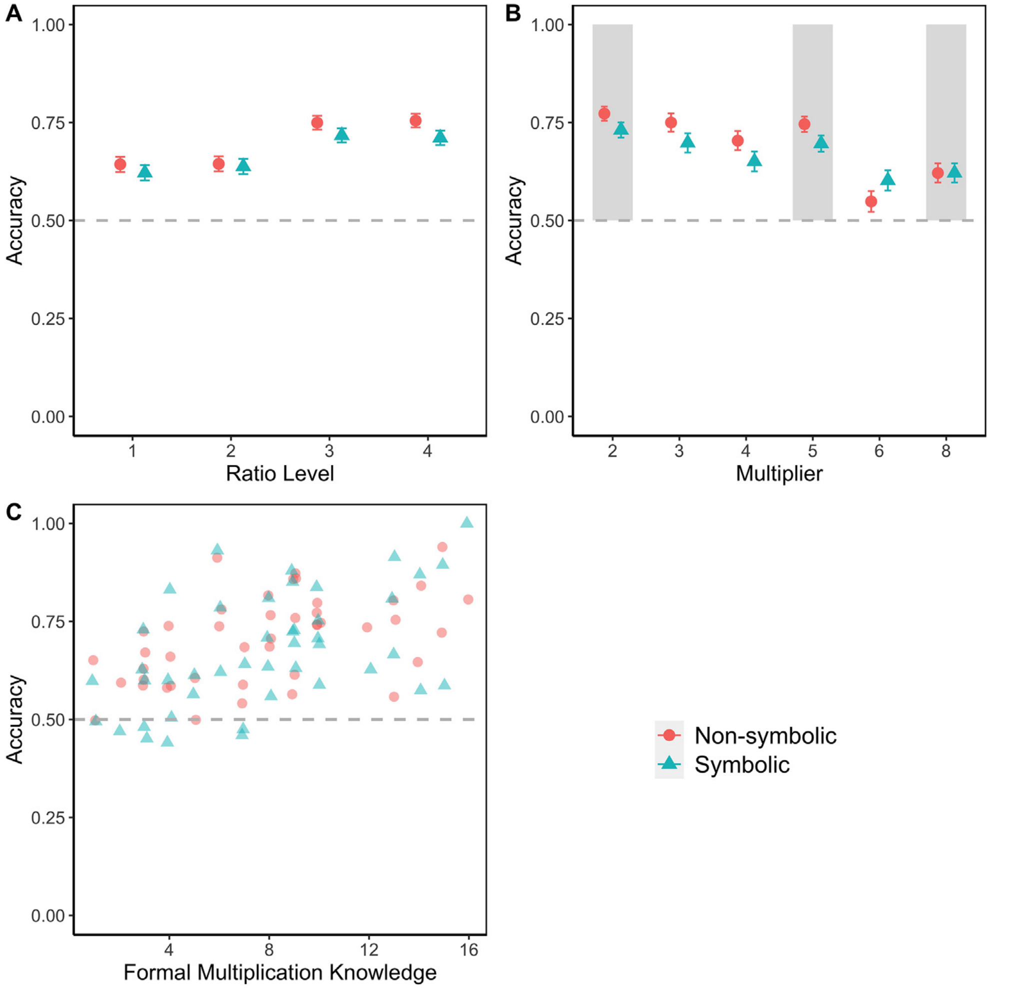

Fig. 4. (A) Mean accuracy across participants on each ratio level between the product and the comparison target. Performance on the nonsymbolic and symbolic multiplication tasks was ratio dependent. The dashed line denotes chance performance. Error bars represent standard error of the mean. Ratio Level 1 ≈ .8; Ratio Level 2 ≈ .6; Ratio Level 3 ≈ .45; Ratio Level 4 ≈ .35. (B) Mean accuracy for both tasks across participants at each multiplier. Children's performance was modulated by the magnitude of multipliers. Gray shaded rectangles denote multipliers involved in the feedback phase. (C) Mean accuracy for each participant as a function of his or her formal multiplication knowledge score.

Predictors of approximate multiplication performance

Our preregistered prediction was that both ratio and multiplier magnitude would modulate performance for both tasks. We fit a generalized linear mixed-effects model (GLMM) following a binomial error distribution predicting children's item level accuracy (correct or incorrect) from the ratio level between the product and the comparison target, children's formal multiplication knowledge (total scores of formal multiplication knowledge test), magnitude of the multipliers, whether the task was in a symbolic or nonsymbolic format, age, and order of the two multiplication tasks while controlling for the random effects of participant. The GLMM was constructed in R using the *lme4* package (Bates, Mächler, Bolker, & Walker, 2014).

As shown in Fig. 4, the ratios between the product and the comparison target, children's formal multiplication knowledge, and magnitude of multipliers all were significant predictors of trial correctness (Akaike's information criterion [AIC] = 5876.5). A 1-unit increase in ratio level between the product and the comparison quantity led to a 13% increase in the odds of a correct response (β_{RL} = .12, SE = .02, z = 7.10, p < .001, 95% CI [.10, .17]). A 1-unit increase in scores of the formal multiplication knowledge test led to a 5% increase in odds of a correct response (β_{MK} = .05, SE = .01, z = 4.08, p < .001, 95% CI [.02, .07]). An increase in the magnitude of multipliers led to a 7% decrease in the odds of a correct response (β_{MM} = −.08, SE = .01, z = −8.16, p < .001, 95% CI [−.09, −.06]). Modeling results revealed no significant fixed effects of age (β_{age} = .00, SE = .05, z = 0.05, p = .96), task format (β_{TF} = −.07, SE = .04, z = −1.93, p = .054), or the order of the two tasks (β_{OD} = −.01, SE = .09, z = −0.10, p = .92).

Given the surprising result that age was not a significant predictor of performance in the GLMM, we looked at the correlation between age and performance in each of the two approximate multiplication tasks. The correlation between age and the approximate multiplication performance was not significant in either format (nonsymbolic: r = .29, 95% CI [−.01, .54], p = 0.06; symbolic: r = .17, 95% CI [−.14, .44], p = .28).

The relation among ANS acuity, approximate multiplication, and symbolic math skills

Our preregistered prediction was that accuracy on the nonsymbolic approximate multiplication task would mediate the relationship between ANS acuity and symbolic math performance. To test for this mediation relation, we first discarded outliers greater or less than 3 times the interquartile range for each measure. This process removed two ANS scores and two KeyMath-3 Numeration scores. We next ran a normality check for each measure and transformed highly skewed distributions to approach normality. Specifically, we conducted natural log transformation for ANS acuity (Shapiro–Wilk, W = .94), scores of Corsi Block (W = .96), and Woodcock–Johnson Reading cluster (W = .94). Table 1 displays the descriptive statistics and the age-standardized correlations between the nonsymbolic and symbolic approximate multiplication performance, ANS acuity, KeyMath-3 Numeration

Table 1
Descriptive statistics and age-standardized Pearson correlation matrix.

		M	*SD*	1	2	3	4	5	6
1	Nonsymbolic multiplication	0.70	0.09						
2	Symbolic multiplication	0.67	0.14	.68***					
3	ANS acuity	0.33	0.13	−.51**	−.47**				
4	KeyMath-3 Numeration	11.30	5.12	.62***	.49**	−.48**			
5	Woodcock–Johnson Reading cluster	46.30	20.47	.47**	.40*	−.42*	.83***		
6	Corsi Block	15.67	13.37	.33	.37*	−.37*	.11	.08	
7	Formal multiplication	7.36	3.82	.51**	.52**	−.49**	.83***	.81***	.06

Note. The bivariate correlations are controlling for age. Approximate number system (ANS) acuity is calculated from the dot comparison task. The nonsymbolic and symbolic multiplication measures are the total accuracy across the feedback and non-feedback trials.
*p < .05.
**p < .01.
***p < .001.

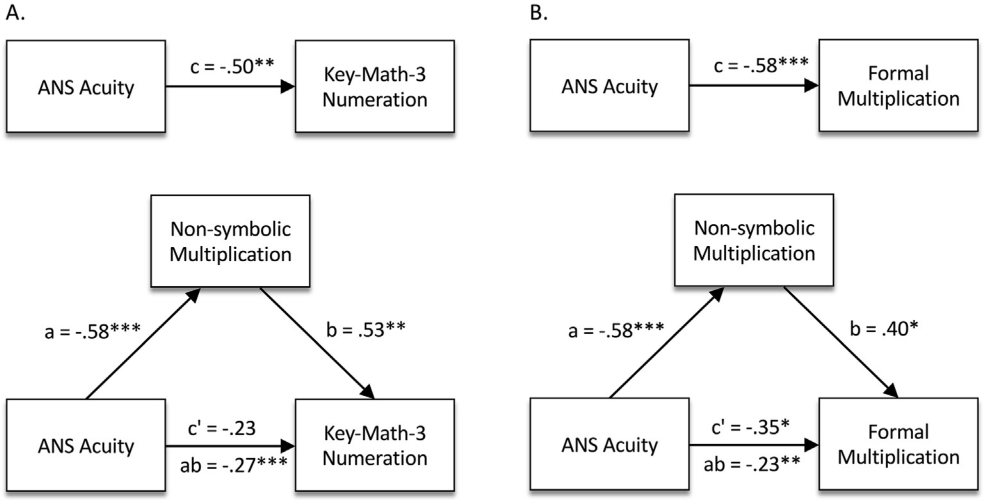

Fig. 5. Mediation models test for whether the associations between approximate number system (ANS) acuity and symbolic math outcome measures were mediated via the nonsymbolic multiplication performance, with *c* representing the total effect before controlling for the mediator, the product of standardized path coefficients *a* and *b* representing the size of the mediation effect (the indirect effect), and *c'* representing the relation of ANS acuity to outcome measures adjusted for the mediator (the remaining direct effect). The mediation framework is constrained by the assumption that $c = ab + c'$. (A) Nonsymbolic multiplication performance mediates the relation between ANS acuity and children's scores on KeyMath-3 Numeration. The remaining direct effect (*c'*) is no longer significant, whereas the indirect effect (*ab*) is significant. (B) The relation between ANS acuity and children's scores on the formal multiplication knowledge test was mediated by nonsymbolic multiplication performance. The unmediated direct effect (*c'*) and the indirect effect (*ab*) both are significant. *$p < .05$, **$p < .01$, ***$p < .001$.

section, score of formal multiplication knowledge test, Corsi Block score, and Woodcock–Johnson Reading cluster. More detailed descriptive information on each measurement can be seen in Appendix D.

To examine whether nonsymbolic approximate multiplication performance mediates the relation between ANS acuity and symbolic math, we ran the preregistered mediation analysis with the KeyMath-3 Numeration section as an outcome measure. Traditionally, standard mediation analyses require strong functional form assumptions to be met (Baron & Kenny, 1986). Given our small sample size, we also used a bootstrap procedure based on a nonparametric resampling test to provide standard errors and confidence intervals (Preacher & Hayes, 2004, 2008). The main feature of this test is that it does not rely on either power or distributional assumptions (Pardo & Román, 2013). As Fig. 5A illustrates, the regression coefficient between ANS acuity and KeyMath-3 Numeration score (standardized $\beta = -.50$, $p < .01$) and the regression coefficient between ANS acuity and nonsymbolic multiplication performance (standardized $\beta = -.58$, $p < .001$) both were significant. Nonsymbolic multiplication accuracy mediated the relation between ANS acuity and KeyMath-3 Numeration score (standardized $\beta = .53$, $p < .01$). ANS acuity was no longer a significant predictor of children's KeyMath-3 Numeration score after controlling for nonsymbolic multiplication (standardized $\beta = -.23$, $p = .15$). The significance of the indirect effect was tested by conducting bootstrapping procedures with 5000 Monte Carlo simulations, and the 95% confidence interval was computed by determining the indirect effect at the 2.5th and 97.5th percentiles (indirect effect = −.27, 95% CI [−.53, −.10], $p < .001$). The direct effect was not significant (direct effect = −.23, 95% CI [−.50, .09], $p = .12$). The proportion of the effect of ANS acuity on the KeyMath-3 Numeration score that goes through the mediator was 0.54 ($p < .01$, 95% CI [.22, 1.31]). The indirect effect remained significant when controlling for the Corsi Block score as a covariate factor (indirect effect = −.25, 95% CI [−.50, −.09], $p < .001$; direct effect = −.26, 95% CI [−.59, .06], $p = .09$; proportion mediated = 0.49, 95% CI [.19, 1.16], $p < .01$). However, the indirect effect was no longer significant when the Woodcock–Johnson Reading assessment was controlled as a covariate factor.

We then conducted the same mediation analysis using our formal multiplication knowledge test as an outcome measure. As shown in Fig. 5B, ANS acuity was a significant predictor of children's scores on the formal multiplication knowledge test (standardized $\beta = -.58$, $p < .001$) and of accuracy on nonsymbolic multiplication (standardized $\beta = -.58$, $p < .001$). The effect of ANS acuity on the score of the formal multiplication knowledge test was mediated by the accuracy on nonsymbolic approximate multiplication (standardized $\beta = .40$, $p = .011$). This significant indirect effect was tested with a bootstrapping method with 5000 resampling simulations (indirect effect = −.23, 95% CI [−.50, −.07], $p = .001$; proportion mediated = 0.40, 95% CI [.14, .90], $p = .002$). The relation between ANS acuity and formal multiplication knowledge remained significant after controlling for the mediator, nonsymbolic multiplication accuracy (direct effect = −.35, 95% CI [−.57, −.05], $p = .030$). As was the case for the KeyMath-3 measure, the indirect effect still held after entering Corsi Block score as a covariate factor into the mediation model (indirect effect = −.23, 95% CI [−.48, −.08], $p < .001$; direct effect = −.38, 95% CI [−.65, −.11], $p = .02$; proportion mediated = .37, 95% CI [.15, .77], $p < .001$), whereas this indirect effect was no longer significant when controlling for the correlation between a child's score of the formal multiplication knowledge test and Woodcock–Johnson Reading cluster.

Discussion

Our study provides the first evidence that young children can intuitively multiply using both large quantities of objects and Arabic numerals prior to formal knowledge of multiplication. Even children who were unable to solve any simple multiplication equations, such as 2×3, were nevertheless capable of performing both approximate multiplication tasks with above-chance accuracy. As predicted, performance on the two multiplication tasks was significantly modulated by the ratio of the product to the comparison target and the numerical magnitude of the multipliers, indicating that children used numerical representations generated by the ANS to solve our tasks.

Previous research demonstrated children's ability to perform scaling operations. In those experiments, children learned that a magic wand resulted in a change in the numerosity of a dot array by a constant multiplicative or fractional factor (Barth et al., 2009; McCrink & Spelke, 2010). The current experiment took this finding a step further to demonstrate that children can engage in true approximate multiplication. Multiplication requires the integration of both a multiplier and a multiplicand in the computation (Barth et al., 2009). The current study allowed both the multiplier and the multiplicand to hold multiple values and thus demonstrated a flexibility with intuitive multiplication that had not yet been observed beyond the special case of the scaling transformation. During non-feedback testing, children switched to multiplying with novel multipliers, demonstrating that they were able to perform a true multiplication operation that is not restricted to specific scaling values.

Our study also provides evidence that children can extend their rapid and intuitive approximate multiplication skills beyond nonsymbolic numerical representations to engage in approximate arithmetic with Arabic numerals. In contrast to recent findings that children performed with higher accuracy on the nonsymbolic version compared with the symbolic version (Szkudlarek & Brannon, in press; Szkudlarek et al., 2021), children's accuracy in the current study did not differ as a function of task format. However, consistent with previous research, children in the current experiment were faster on the nonsymbolic version of the task compared with the symbolic version. This pattern of accuracy and reaction time suggests that children are capable of integrating their intuitive sense of multiplication with their knowledge of numerals but that this integration may require an additional cognitive processing step.

Children often use addition and counting strategies when they are first learning symbolic multiplication (Anghileri, 1989; Ter Heege, 1985). However, we designed our task with the goal of preventing children from relying on counting. First, children were explicitly instructed and supervised not to count. Second, whereas the number of petals ranged from 2 to 8, the number of pollen pieces varied from 4 to 96 with only 2-s exposure duration, leaving insufficient time to count. Furthermore, our reaction time analyses provided strong evidence that children were not using a sequential addition strategy. Specifically, children's reaction time did not increase with the number of petals, indicating that the approximate multiplication process was parallel rather than sequential (Appendix C).

Our findings raise important questions about the underlying cognitive processes of intuitive multiplication computations. Systematic variability in performance likely emerges during both the multiplication calculation process and the comparison of the product with the target comparison quantity (Cordes, Gallistel, Gelman, & Latham, 2007; McCrink, Dehaene, & Dehaene-Lambertz, 2007). During calculation, operands larger than 4 were represented by the ANS, whereas operands smaller than 4 may have been represented more precisely by parallel enumeration (Feigenson et al., 2004). Thus, greater variability in children's representation of larger operands may lead to greater variability in their imagined product, which in turn may lead to worse performance when children compare this product with a comparison target quantity. Consistent with previous findings (McCrink & Spelke, 2010), our analyses indicated that accuracy decreased as the multiplicative operand (number of petals) increased. This operand effect was distinct from the ratio effect. It is an open question how much information is preserved from the initial visible operands to the representation of the product. In other words, how similar is the representation of the products of 2×16 and 8×4? A future study that compares accuracy for trials with different operands but equivalent products may help to identify the source of the operand effect.

Moreover, open questions remain as to the domain-general cognitive skills that affect approximate multiplication ability. Interestingly, there was no significant age-standardized correlation between children's visuospatial working memory and their nonsymbolic approximate multiplication performance. Although clearly there is spatial ability involved in the nonsymbolic calculation, it is unclear how linked children's abstract representation of numerical quantity is to the visual representation of the individual multiplicands. Future work can systematically manipulate the spatial layout of the dots and flower petals or use continuous representations of magnitude, such as circles, to test whether spatial layout of the nonsymbolic stimulus affects the calculation of an approximate product.

We found multiple sources of evidence that children relied on their ANS to solve the approximate multiplication tasks. First, we observed two characteristic signatures of the ANS: ratio dependence and poorer performance as the numerical magnitude of the operands increased. Second, there was a high correlation between accuracy on both tasks and children's ANS acuity measured independently by the numerosity comparison task. Together, these results suggest that both symbolic and nonsymbolic approximate multiplication computation depends on numerical representations generated by children's ANS, which further complements a growing body of studies addressing the role of the ANS in various approximate arithmetic operations (Barth, La Mont, Lipton, & Spelke, 2005; McCrink et al., 2017; McCrink & Spelke, 2015; McCrink & Wynn, 2004; Szkudlarek et al., 2021).

Consistent with recent findings by Szkudlarek et al. (2021), our results indicate that nonsymbolic multiplication performance mediated the relation between ANS acuity and symbolic math performance in young children. We found convergent evidence for this mediation effect with two distinct outcome measures for symbolic math: the KeyMath-3 Numeration section and a questionnaire that assessed formal multiplication knowledge. Collectively, our findings and those of Szkudlarek et al. (2021) and Szkudlarek and Brannon (in press) suggest a pathway by which the ANS contributes to children's emerging symbolic math competence at the beginning of formal arithmetic instruction. The established correlation between symbolic math performance and ANS acuity may reflect variability in children's ability to integrate intuitive arithmetic representations grounded in the ANS with the corresponding symbolic arithmetic concepts.

Furthermore, nonsymbolic multiplication accuracy significantly mediated the relation between ANS acuity and symbolic math even after controlling for visuospatial working memory, suggesting that the mediation effect cannot be attributed to the domain-general skill of visuospatial working memory. However, it is crucial to note that for both math outcome measures, the indirect effect for the mediation model was no longer significant when controlling for the Woodcock–Johnson Reading cluster as a covariate factor. This parallels findings by Szkudlarek et al. (2021) that also found that approximate division no longer mediated the relation between ANS and math when controlling for the Woodcock–Johnson Reading cluster as a covariate factor, although verbal skills did not affect this mediation relation in adults. As is typical in developmental studies (Durand, Hulme, Larkin, & Snowling, 2005; Hart, Petrill, Thompson, & Plomin, 2009), we found a strong correlation between both symbolic math measurements and the Woodcock–Johnson Reading cluster. Therefore, there may have been too little variance left in math ability after controlling for reading skill to detect the mediation effect. It remains possible that this mediation effect would hold when testing our hypothesis with a much larger sample.

The current study has several limitations. First, due to COVID-19 research closures, our sample size was relatively small. This restricted sample size may have obscured a potential increase in accuracy with age on our multiplication tasks. However, despite this small sample size, all our main preregistered predictions held. Second, although we varied dot size randomly across trials, a limitation of our design is that we did not employ rigorous stimulus controls for physical extent in the nonsymbolic multiplication task. A full counterbalance of all non-numerical features was unachievable in the limited number of trials children could complete in the time available. It is well known that continuous quantity variables such as object size, cumulative surface area, and density affect numerical estimation (Gebuis & Reynvoet, 2012; Leibovich, Katzin, Harel, & Henik, 2017). Although it is theoretically possible that children used some combination of continuous and discrete magnitude features on our nonsymbolic multiplication task, children's above-chance accuracy on the symbolic version of the approximate multiplication task demonstrates that children can engage in approximate multiplication using only numerical cues.

In summary, despite the challenges children face in learning multiplication and memorizing multiplication facts (Campbell & Graham, 1985; Koshmider & Ashcraft, 1991; van der Ven, Straatemeier, Jansen, Klinkenberg, & van der Maas, 2015), our study illustrates that they have an intuitive concept of multiplication that is independent of their acquisition of formal multiplication facts and procedures. This intuitive multiplication sense is grounded in the ANS. Indeed, our findings suggest that approximate multiplication might serve as a pathway between ANS acuity and children's symbolic math abilities. An important next step is to test whether educational interventions that harness this intuitive multiplication capacity would improve symbolic arithmetic learning for children before and during early mathematics instruction. Although rote memorization of multiplication times tables is important, approximate multiplication exercises might allow children to use their basic intuition as a stepping-stone to form more advanced conceptual understanding of abstract arithmetic principles even before they are ready for the introduction of precise symbolic multiplication.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

See Table A1.

Table A1
Stimulus set for experimental trials in the approximate multiplication tasks.

Multiplicand	Multiplier	Comparison value	Ratio level	Phase
8	8	49	1	Feedback
4	5	15	1	Feedback
5	5	20	1	Feedback
4	2	6	1	Feedback
4	8	42	1	Feedback
3	5	20	1	Feedback
7	2	18	1	Feedback
5	2	13	1	Feedback
9	8	43	2	Feedback
3	5	9	2	Feedback
12	5	36	2	Feedback
4	2	5	2	Feedback
3	8	40	2	Feedback
5	5	42	2	Feedback
3	2	10	2	Feedback
10	2	33	2	Feedback
11	8	40	3	Feedback
10	5	23	3	Feedback
4	2	4	3	Feedback
3	5	7	3	Feedback
2	8	35	3	Feedback
4	5	44	3	Feedback
12	2	53	3	Feedback
2	2	9	3	Feedback
12	8	34	4	Feedback
3	5	5	4	Feedback
11	8	31	4	Feedback
4	2	3	4	Feedback
2	8	46	4	Feedback
2	5	29	4	Feedback
5	2	29	4	Feedback
10	2	57	4	Feedback
2	3	5	1	Non-feedback
8	6	38	1	Non-feedback
9	4	29	1	Non-feedback
6	3	24	1	Non-feedback
3	6	24	1	Non-feedback
10	4	50	1	Non-feedback
4	3	7	2	Non-feedback
10	6	36	2	Non-feedback
12	4	28	2	Non-feedback
5	3	25	2	Non-feedback
2	6	20	2	Non-feedback
12	4	80	2	Non-feedback
3	3	4	3	Non-feedback
12	6	33	3	Non-feedback
7	4	13	3	Non-feedback
11	3	73	3	Non-feedback
2	6	27	3	Non-feedback
4	4	35	3	Non-feedback
4	3	4	4	Non-feedback
12	6	25	4	Non-feedback
7	4	10	4	Non-feedback
2	3	17	4	Non-feedback
2	6	35	4	Non-feedback
8	4	91	4	Non-feedback

Appendix B

Script: Formal multiplication knowledge test

Stack each card on top of each other directly in front of the child and say the following:

1. Do you know what 2 plus 3 is?
2. If Sam has 4 apples and Kate gives him 2 more, how many apples will he have?
3. What number is twice of 4?
4. What number is twice of 9?
5. You are having pizza with your friends. There are 4 plates. If you put 2 slices on each plate, how many slices are there altogether?
6. Pretend you are a squirrel. There are 3 trees. If you find 5 acorns under each tree, how many acorns did you find altogether?
7. Do you know what this symbol is? $\times$
8. And what about this one? $\div$

Can you solve any of these problems? (continue to stack them on top of each other for the child, but do not read the cards to the child)

9. 6 + 3 = ?
10. 8 + 2 = ?
11. 5 + 5 = ?
12. 12 + 4 = ?
13. 2 $\times$ 3 = ?
14. 2 $\times$ 4 = ?
15. 8 $\times$ 3 = ?
16. 9 $\times$ 5 = ?

Appendix C

The alternative strategy analyses

To rule out the possibility that children were using alternative strategies instead of conducting true approximate multiplication, we first assessed for evidence that children used a "median-split" strategy when merely attending to the range information of the multiplicative operands or the comparison target. For example, if children used this strategy, they would construct an intuitive estimation of the median or average of the comparison target values across trials and make their choice based on the comparison between the target value and the median comparison value. Specifically, they would pick the target value if it was greater than the median comparison value; otherwise, they would pick the imagined product. Based on our stimulus set, children would get 34 of 56 trials correct if they perfectly executed the median-split strategy on the comparison target. Likewise, children were expected to be 60.7% correct (34 of 56 trials) when using this median-split heuristic on the multiplicand and 57.1% correct when applying this strategy to the multiplier. In other words, children should not perform above 60.7% accuracy if they relied exclusively on any of these alternative heuristics. A one-sample t test showed that children's performance was significantly above 60.7% accuracy in both the nonsymbolic and symbolic formats (nonsymbolic: 69.8% accuracy, 95% confidence interval (CI) [.67, .73], $t(43) = 6.53$, $p < .001$; symbolic: 67.2% accuracy, 95% CI [.63, .71], $t(43) = 3.07$, $p < .01$), indicating that children's above-chance performance on approximate multiplication tasks cannot be attributed to reliance on the median-split heuristics.

Another possible alternative strategy is to implement a sequential addition computation rather than performing a true multiplication computation. For example, a trial with 3 petals and 3 dots on each petal can be interpreted as 3 + 3 + 3, whereas a trial with 4 petals and 3 dots on each petal can be solved as 3 + 3 + 3 + 3. As such, the greater the multipliers are, the longer reaction time must be required to compensate for incremental additive steps. Therefore, we modeled the reaction time by conducting a generalized linear mixed-effects model following a gamma distribution. The magnitude of the multiplier (the petal number) was entered as a predictor while controlling for the fixed effects of the ratio level and the task format as well as the random effects of individual participants. Results revealed no significant effects of the multiplier on predicting the reaction time ($\beta = .004$, $SE = .002$, $t = 1.72$, $p = .09$), indicating that the sequential addition process was unlikely.

Appendix D

See Table D1.

Table D1
Descriptive statistics of individual assessments.

	M	*SD*	*Min*	First quartile	*Mdn*	Third quartile	*Max*
ANS acuity	0.33	0.13	0.11	0.25	0.31	0.38	0.78
KeyMath-3 Numeration	11.30	5.12	3.00	7.00	12.00	15.00	22.00
Woodcock–Johnson Reading cluster	46.30	20.47	17.00	28.00	46.00	66.00	85.00
Corsi Block	15.67	13.37	0.00	6.00	12.00	20.00	60.00
Formal multiplication	7.36	3.82	1.00	4.00	8.00	10.00	15.00

Note. Approximate number system (ANS) acuity is calculated from the dot comparison task. Formal multiplication is the scores of the formal multiplication knowledge test.

Appendix E

See Table E1.

Table E1
Zero-order correlation matrix.

		1	2	3	4	5	6	7
1	Nonsymbolic multiplication							
2	Symbolic multiplication	.68***						
3	ANS acuity	−.54***	−.49**					
4	KeyMath-3 Numeration	.60***	.55***	−.50**				
5	Woodcock–Johnson Reading cluster	.52**	.50**	−.50**	.81***			
6	Corsi Block	.42**	.48**	−.41*	.21	.25		
7	Formal multiplication	.51***	.54**	−.58***	.85***	.80***	.27	
8	Age	.29	.17	−.45**	.28	.45**	.27	.40**

Note. The nonsymbolic and symbolic multiplication measures are the total accuracy across the feedback and non-feedback trials. ANS, approximate number system.
*p < .05.
**p < .01.
***p < .001.

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