Identifying student resources for reasoning microscopically about heat and temperature

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We identify three conceptual resources that introductory physics students in our sample commonly use when reasoning microscopically about thermal physics topics: 1) differences will eventually even out, 2) macroscopic changes connect to microscopic collisions, and 3) when something is hotter (colder), its molecules are moving faster (slower). We report the prevalence of these resources, as well as the prevalence of microscopic thinking, in 624 written responses to three different heat and temperature questions administered to introductory physics students at four different colleges and universities. This work complements past research identifying common student difficulties in using microscopic models for heat and temperature, and it adds to the small but growing body of literature that focuses on student resources for heat and temperature, identifying ideas in student thinking that are sensible and continuous with formal physics. By reporting common student resources, we aim to assist instructors in noticing, appreciating, and building on student ideas during introductory thermal physics instruction.

I. INTRODUCTION

One way in which Physics Education Research (PER) has served instructors is through systematic investigations of student ideas about physics topics. By identifying common student ideas about heat and temperature, for example, researchers provide instructors with Knowledge of Student Ideas (KSI), which, along with physics content knowledge and knowledge of instructional strategies, supports an instructor in facilitating student learning [1-3].

Extensive work has been done investigating student ideas about heat and temperature in introductory physics [4-20]. In investigations that included microscopic models for understanding heat and temperature concepts, researchers found that students mistakenly associate volume with number of particles, assume collisions between particles lead to heat production or changes in internal energy, and fail to correctly interpret or apply particle flux in thinking about pressure [11-18]. While previous research primarily focused on identifying student difficulties or misconceptions, a few studies have taken a resources lens, seeking to identify "seeds of science" in student ideas, which rather than needing to be replaced, may be nurtured by instruction and grown into canonically correct physics concepts [19-21]. These studies have identified students' conceptual resources about heat and temperature, including that heat transfer is directional, that temperature is associated with energy, and that systems naturally move 'toward' equilibrium [22-25]. Our study combines the focus on microscopic reasoning of some previous difficulties-oriented heat and temperature research and the *resources* approach of more recent studies by identifying the promising ideas present in students' microscopic thinking about heat and temperature.

We explore the question: What conceptual resources are common in student responses to heat and temperature questions when microscopic thinking is used? To answer this, we analyzed 624 written student responses to three heat and temperature questions administered at four different colleges and universities. We identified three resources for reasoning microscopically about heat and temperature: 1) differences will eventually even out. 2) macroscopic changes connect to microscopic collisions, and 3) when something is hotter (colder), its molecules are moving faster (slower). By defining and offering examples of these resources, including their connections to canonically correct physics concepts, we aim to add to instructor KSI and therefore aid instructors in anticipating and building on student ideas during heat and temperature instruction.

II. THEORETICAL FRAMEWORK: RESOURCES

In resources theory, a resource is a piece of knowledge that is activated in context-sensitive ways, sometimes in concert with other resources, to form an idea, explanation, argument, or theory [19,21,26-32]. Researchers have theorized extensively about the development, structure, and

role of resources, and have used resources theory to highlight the dynamic, emergent, complex-systems-like nature of student thinking [27].

Our work draws extensively from resource theory's orientation toward student thinking as fundamentally sensible and continuous with formal physics [19,21,27-29,31,32], seeking to make visible the continuities between students' thinking and formal physics, even and especially when that thinking does not use the language of formal physics or is incorrect. Our work also builds from resource theory's definition of learning, which involves changing the structure or activation of resources, by reorganizing, refining, or increasing the degree of formality of resources [19,21,26,28,31,32]. Our primary aim in identifying resources is to provide instructors with knowledge that they can use to build from student ideas in instruction.

III. METHODS

The resources we report in this paper were identified in written student responses to three questions about thermal phenomena: the hot to cold question, the piston question, and the cold tire question (Figure 1). All three were constructed for the purposes of this project, with different conceptual targets in mind. The hot to cold question was meant to target mechanisms for energy transfer in thermal phenomena; the piston question, microscopic reasoning about the first law of thermodynamics; and the cold tire question, microscopic explanations for changes in (macroscopic) thermal quantities. We chose these questions from among our data set because student responses frequently relied on microscopic reasoning, which was the focus of our analysis.

In the hot to cold question, hot objects have higher temperatures, which corresponds to particles with higher

Hot to cold question: You may have heard that

"heat" or "thermal energy" transfers from hot to cold objects, and not the other way around. Why is this the case? How do you make sense of this phenomenon?

Piston question:

An ideal gas is in a container that is closed on the top by a movable piston. The gas is now heated, so that the temperature goes up. As this happens, the piston moves up. Use ideas about microscopic gas particles and interactions to explain why this happens.

Cold tire question:

It became suddenly very cold last night. You start your car in the morning to go to work and get a warning light indicating that your tire pressure is low. You check the tires carefully, and there is no sign of any puncture. You remember from class that the ideal gas law (PV=nRT) says that temperature and pressure are related, so you hypothesize that because there was a sudden decrease in temperature, there must also have been a decrease in pressure.

U1 version ending.

What is another way (using You also remember a class discussion something other than the ideal gas law) to make sense of what happened with your

U3 version ending:

about a gas being made of many small particles. How can you use this particle picture of a gas to make sense of what happened to your tire?

FIG. 1. Questions used in our study

kinetic energy. When two objects come into contact, one hot and one cold, the interactions between the higher- and lowerenergy particles transfer some of the energy to the colder object. In the *piston* question, as the temperature goes up, the speed of the particles of the gas increases, and they collide with the piston with greater "oomph" as they move upward. This means that the force exerted by the gas on the piston increases; the gravitational force on the piston by the Earth remains the same, so there is a net upward force on the piston, and it moves up. For the cold tire question, "tire pressure" is the pressure of the gas on the tire. Microscopically, pressure is the force of the collisions of the gas particles with the wall of the tire, per unit area. When the temperature decreases, the speed of the particles decreases, and the force that particles exert as they collide with the wall also decreases.

We analyzed a total of 624 written responses from introductory physics courses at four US colleges and universities. U1 and U2 are large public institutions, U1 in the Pacific Northwest US and U2 in the Midwest. U3 is a mid-sized public community college in the Western US. U4 is a small private institution in the Pacific Northwest. Students answered the questions on homework and quizzes, before and after instruction on heat and temperature. The course response rates (i.e. number of responses divided by the number of students enrolled in the course) were 73%-86% for questions asked at U1; 33% for U2; 78-89% for U3; and 90% for U4. Ranges indicate that students from more than one course or quarter answered questions. The low response rate for U2 reflects that students turned in responses as part of a group assignment at that institution, with one response given per group, rather than one response given per individual.

The racial and/or ethnic demographics for the colleges/ universities in our study versus all college/university students are shown in Figure 2. Figure 2 suggests that the institutions in our study are not racially and/or ethnically representative of the population of college-bound freshmen in the US. By this measure, the universities in our study serve more Asian and Asian American students, more multiracial students, fewer Hispanic or Latinx students, fewer Black or African American students, and fewer white students than are in the general population of college students. The median parental income of the students at colleges/universities in our study is also higher than the national average. This sampling limits the generalizability of our results; though the resources we identified are common among the students in our sample, we cannot speak to their commonality in the population of introductory physics students writ large.

We used responses to the questions in Figure 1 to create an emergent coding scheme [35] that captured some common resources that the students in our sample used to reason microscopically about thermal phenomena. Author AA, in consultation with author TH, conducted preliminary analyses of student responses to each question, looking for ideas that they considered to be continuous with relevant

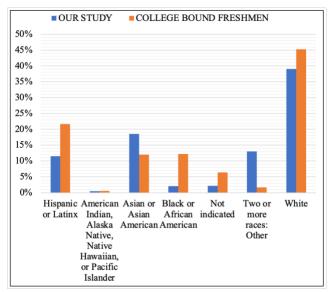


FIG. 2. Racial and/or ethnic demographics of institutions in our sample (blue) versus all college-bound freshmen (orange). Blue bars were constructed using demographic data provided by offices of institutional research or institutional websites, weighted by sample size. Orange bars were constructed using data from Kanim and Cid [33]. As explained by activist Kat Lazo [34], neither Hispanic nor Latinx are racial groups, and these two identities are not the same. "Hispanic" is a descriptor for people of Spanish-speaking origins, and "Latinx" is a descriptor for people with origins in Latin America. The former focuses on language, the latter on geographic location.

formal physics concepts, even if not stated in formal terms. They then identified patterns across questions, producing a coding scheme of three resources. This pattern-seeking approach foregrounds a model of generalizability that emphasizes recurrence across multiple sources of heterogeneity [36].

Authors AA and LB then independently coded student responses to the three questions. As a first step, they independently coded whether each response did or did not contain microscopic thinking. The agreement was near perfect (Cohen's kappa = 0.99). Responses that did contain microscopic thinking were then coded for the use of each of the three resources. Resources can be and often are activated in concert; thus, a single response could receive no code, one code, or many codes. As a measure of inter-rater agreement, we took the normalized difference between the total number of possible codes and the total number of disagreements between the two coders. We used percentage agreement rather than a standard statistical measure of agreement (e.g., Cohen's kappa) because our codes are not independent or mutually exclusive [37,38]. The percentage agreement on resource codes for the full data set was 96%. Disagreements were not resolved through discussion; instead, the percentages reported in Tables I and II represent the fraction of responses for which both authors were in agreement.

TABLE I. Prevalence of microscopic reasoning by question and institution

	Hot to Cold			Piston		Cold Tire	
	U1	U2	U3	U1	U4	U1	U3
Total N	258	18	39	89	37	159	24
Micro N	88	10	26	74	26	81	23
Micro %	34%	56%	67%	83%	70%	51%	96%

IV. RESULTS AND DISCUSSION

A. Prevalence of microscopic reasoning

We considered a student response to contain microscopic reasoning if it used atoms, molecules, or other particulate descriptions of an object or substance, either in drawings or in words. Responses that mention microstates or basic states but do not otherwise contain particulate descriptions (e.g. "A system tends to move toward having the most basic states, or the greatest entropy.") were *not* categorized as microscopic thinking, as it was often unclear if students understood these terms as referring to particle configurations or microscopic descriptions. The prevalence of microscopic reasoning varied across questions and institutions, as seen in Table 1. We believe several factors are at play here. First, the three questions used in our research prompt microscopic thinking to differing degrees. Some of our questions ask explicitly for students to use microscopic ideas and others do not, and we see the prevalence of microscopic thinking varying in a way that appears consistent with these constraints. Second, we expect variability due to the nature of resources: resources are activated in context sensitive ways [19,20,31]. For example, the likelihood of particular resources to be activated may depend on the scenario in the question prompt, the physics topics a student has most recently engaged with during instruction, the thinking elicited by other questions given at the same time as our research questions, etc. Finally, different student populations who have experienced different instruction may also vary in the prevalence or accessibility of a particular resource or way of thinking. By sampling from different institutions, we expect to witness variation.

B. Microscopic heat and temperature resources

Within student responses that used microscopic reasoning, we identified three common resources. The prevalence of each resource, by question and institution, is given in Table II. As with the variation in prevalence of microscopic reasoning, we expect variation in the prevalence of these three resources due to the contextual nature of resources and the heterogeneity of our sampling. To be considered common, a resource needed to be present in student responses to multiple questions and in at least 10% of responses that included microscopic thinking for at least one question. We will next describe each of these resources, with examples of how they are used in both correct and incorrect responses. We will also discuss their connections to formal physics ideas and relationships to prior research.

TABLE II. Percentage of microscopic responses containing each resource (1, 2, 3) by question and institution

	Hot to Cold			Piston		Cold Tire	
	U1	U2	U3	U1	U4	U1	U3
1	36%	10%	23%	7%	12%	4%	0%
2	42%		65%				74%
3	65%	90%	92%	77%	96%	68%	91%

1. Differences will eventually even out

Student responses commonly included the idea that objects or systems tend toward uniform distributions of quantities such as pressure, temperature, energy, force, density, molecular speed, and molecular flux. This resource, differences will eventually even out, does not always involve microscopic modeling in and of itself; it is included in our findings here because it was often used in conjunction with microscopic reasoning. It can describe a quantity inside a system coming to balance with a quantity outside the system (e.g. "The average thermal energy of the tire changes to match the average thermal energy of the surrounding environment."). It can also describe a quantity that varies within a system becoming more uniform (e.g. "The equilibrium state will be reached when each atom has the same thermal energy and it will even out.").

The resource differences will eventually even out is used in support of ideas that are both consistent and inconsistent with formal physics. Using the examples above: we do think of systems as coming into thermal equilibrium with their environments, as is implied by the first example, but we do not think of each atom as having equal energy when equilibrium is reached, as described in the second example. What is shared by both examples is the abstract resource that systems tend toward uniformity. This resource is continuous with formal physics ideas of systems tending toward equilibrium configurations. At the macroscopic level, that might be internal and external pressure balancing as a piston moves; at the microscopic level, that might be the particle speed distributions of ideal gases converging to the Maxwell-Boltzmann distribution.

The differences will eventually even out resource also has connections to student resources identified in other studies. Loverude identified the resource systems naturally move 'toward' equilibrium rather than away in the thinking of upper division thermal physics students, which may be a more specific or formal version of differences will eventually even out [24]. Abraham et al. identified the resource heat transfer is directional in the thinking of introductory physics students, which includes reasoning that the direction in question is towards an equilibrium [22]. At a higher level of abstraction, diSessa discusses the intuitive idea of equilibration, used by high school students in reasoning about heat and temperature topics, which includes the tendency to return to balanced configurations [25]. We believe differences will eventually even out may be tapping in to this more basic idea.

2. Macroscopic changes connect to microscopic collisions

Many student responses drew connections between changes at the macroscopic level—such as changes in temperature, pressure, volume, and heat flow—and collisions at the microscopic level. In some cases those collisions were happening between particles within a substance, and in other cases those collisions were happening as particles struck the container walls or the particles of neighboring substances. The connections were often, but not always, explicitly causal. In all these cases we considered students to be using the second resource we identified: macroscopic changes connect to microscopic collisions. For example, in response to the piston question, one student wrote, "When the gas is heated, this gives the particles more energy and they move with more speed and collide with each other with more force. Not only do they collide with each other with more force, but also the walls of the container and the piston. This is what causes the piston to rise." Here, the student describes particles colliding with the piston with more force as causing the macroscopic movement of the piston. In response to the *cold tire* question, one student writes, "As temperature lowers, gaseous molecules move slower. Therefore, they hit the tire less frequently causing pressure to lower." Here, the student describes a decrease in macroscopic pressure as being caused by less frequent microscopic collisions between the gas molecules and the tire.

Neither example above gives a *complete* picture of how we model microscopic collisions as causing changes in volume or pressure in physics; we would want to consider changes to both frequency of collisions as well as impulse per collision when molecules are moving faster or slower. Nonetheless, the resource *macroscopic changes connect to microscopic collisions* is continuous with the kinetic theory of gases, as students are looking to microscopic collisions to explain changes in macroscopic properties.

This resource relates to some of the common student difficulties identified in previous research. Several studies report that students mistakenly connect collisions between particles to an increase in internal energy [14,16-18]. While Robertson & Shaffer note that some students attribute pressure of an ideal gas to particle-wall collisions [12], which we see as an appropriate use of the resource macroscopic changes connect to microscopic collisions, Kautz et al. describe many ways that students fail to completely and correctly use this microscopic model of pressure [17]. We believe the resource we have identified and these related difficulties identified by others complement each other in the space of Knowledge of Student Ideas (KSI). It is potentially helpful for an instructor to see both the resourcefulness in students' making connections between microscopic collisions and macroscopic changes in order to build on those ideas and also the potential pitfalls that students can be guided around or through in the application of those models [39].

3. When something is hotter (colder), its molecules are moving faster (slower)

The final, and most common, resource we identified was that when something is hotter, its molecules are moving faster, or its opposite, when something is colder, its molecules are moving slower. "Something is hotter (colder)" included both descriptions of substances being hotter/colder and having higher/lower temperatures or thermal energies, and "its molecules are moving faster (slower)" included descriptions of particles moving faster/slower and having higher/lower speeds, velocities or kinetic energies. For example, one student writes, "...an increase in temperature is caused by the gas particles speeding up." Another student writes, "...the atoms of a hot object have a higher kinetic energy than that of a cold object." This resource is continuous with the formal physics conception of temperature as a measure of the average kinetic energy of the particles in a substance.

We believe this resource is closely related to the resource hotter objects have more energy, identified by Abraham et al. [22]. Young & Meredith also identified that students connect temperature with kinetic energy [23], and Leinonen et al. identify that thermal energy is associated with the motion of particles [14]. Our study supports their findings and adds to the literature on these student ideas by reporting the prevalence of this resource in larger samples and at multiple institutions.

V. CONCLUSIONS

In this study, we identified three resources introductory physics students use when they reason microscopically about heat and temperature: 1) differences will eventually even out, 2) macroscopic changes connect to microscopic collisions, and 3) when something is hotter (colder), its molecules are moving faster (slower). These resources were common in student written responses to three different heat and temperature questions given at four different colleges and universities. These findings support, complement, and expand previous work on student conceptions about heat and temperature topics. Instructors may benefit from this research by increasing their Knowledge of Student Ideas (KSI), therefore better positioning themselves to anticipate and notice these resources in the thinking of their students, and then build on these resources in their instruction.

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