Robust Graph Localization for Underwater Acoustic Networks

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***Abstract*—We consider the problem of robust localization of a set of underwater network nodes, based on pairwise distance measurements. Localization plays a key role in underwater network optimization, as accurate node positioning enables location-aware scheduling, data routing, and geo-referencing of the collected underwater sensor data. State-of-the-art graph localization approaches include variations of the classical multidimensional scaling (MDS) algorithm, modified to handle unlabelled, missing, and noisy distance measurements. In this paper, we present MAD-MDS, a robust method for graph localization from incomplete and outlier corrupted pair-wise distance measurements. The proposed method first conducts outlier excision by means of Median Absolute Deviation (MAD). Then, MAD-MDS performs rank-based completion of the distance matrix, to estimate missing measurements. As a last step, MAD-MDS applies MDS to the reconstructed distance matrix, to estimate the coordinates of the underwater network nodes. Numerical studies on both sparsely and fully connected network graphs as well as on data from past sea experiments corroborate that MAD-MDS attains high coordinate-estimation performance for sparsely connected network graphs and high corruption variance.**

***Index Terms*—Underwater acoustics; graph localization; robust MDS; missing data; corrupted distance measurements**

I. Introduction

ADVANCES in marine technologies led to an increasing number of remote sensing applications for underwater exploration. Seabed mapping, water quality monitoring, remote command-and-control of subsea vehicles, wireless diver-to-diver communication and coordinated operation between swarms of autonomous unmanned systems are just a few applications that can be supported by the deployment of underwater wireless networks. Fundamental to the success of underwater communications and networking is robust localization and tracking of underwater network nodes.

Underwater acoustic localization is traditionally performed locally at the tracked node. Common approaches rely on channel measurements such as received signal strength indicator (RSSI) [1], time-of-arrival (ToA) [2] and time-difference of arrival (TDoA) [3], or bearing measurements [4] –referred as base-line approaches. Efforts have been carried out to compensate for multipath distortions [5], sound speed variations [6], and anchors' location mismatches [7].

Underwater networks are usually sparse and encompass a few hop connections. Consequently, traditional long baseline (LBL) techniques (cf. [8]) are not suitable due to localization ambiguities that arise when a single node is not able to connect to at least three neighbor nodes. Short baseline (SBL) and ultra-short baseline (USBL) systems (cf. [9]) rely on the deployment of multiple transponders (in the form of a tetrahedral or pyramidal array) and utilize ToA and angle-of-arrival (AoA) measurements to estimate the position of an underwater source from a single point, however outlier measurements due to long delays and non line-of-sight multi-path propagation may significantly affect their accuracy [10], [11]. An alternative solution for *under-ranked* localization is to leverage the spatial diversity of mobile nodes in the network [12] or collect acoustic channel fingerprints [13], [14] to localize a node. Such methods rely on bathymetry and matched field processing to estimate the position of a source yet, their performance is sensitive to medium fluctuations [15] that may create mismatches in sound speed or seabed maps.

In this work, we aim to resolve positioning ambiguities that arise due to heavily corrupted ranging measurements and sparse connectivity in an underwater acoustic network by means of *robust graph localization*. Graph localization considers a network of nodes as a connected graph and leverages measurements or bounds on pairwise ranges to estimate their relative positions. To achieve global positioning, known coordinates from at least two reference/anchor nodes in the network can be used to shift and rotate the estimated graph and simultaneously estimate the coordinates of all network nodes [16]. The most commonly used graph localization technique is multidimensional scaling (MDS)– an eigenvalue-decomposition based algorithm– that exploits the properties of Euclidean distance matrices [17] to reconstruct node coordinates from pairwise distance measurements. Variants of MDS include embedding bounds on the link delays, and techniques to reduce its sensitivity to link distance measurement errors [17], [18], [19]. An alternative approach to MDS is to formalize graph localization as a convex optimization problem with proximity-imposed connectivity constraints [20], [21], [22]. In [23] we use information about the relative locations of the network nodes as bounds to link distance measurements to increase the resilience of the positioning algorithm to sparsely connected network graphs. Still, for large networks (of tens of nodes) stationed over large areas (of several km), high-complexity optimization based solutions do not allow for on-line positioning. For this, we revert to dimensionality reduction methods, while taking into consideration heavily corrupted

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distance measurements and sparse connectivity constraints imposed on the network graph.

We propose to enhance the robustness of MDS to highly corrupted and missing distance measurements by first implementing an algorithm to detect and remove faulty ranging measurements by means of median absolute deviation (MAD). Then, we exploit the rank properties of Euclidean distance matrices (EDMs) to complete the missing distance measurements. Subsequently, after denoising and completing missing distance measurements in the EDM, we reconstruct the coordinates of the network nodes by eigenvalue decomposition. We evaluate the coordinate-estimation performance of MAD-MDS in terms of Root-Mean-Square-Error (RMSE) with numerical simulations on both sparse and fully-connected ad-hoc mesh network topologies as well as on data collected from past sea experiments. Simulation results show considerable performance gains over classical MDS and robust principal component analysis for MDS, especially when ranging errors and proximity-imposed connectivity constraints result in highly corrupted and missing distance measurements, respectively.

II. Problem Statement

We consider a set of N nodes, arranged in a mesh multi-hop network. Each node is equipped with an underwater acoustic modem. Network nodes exchange communication signals, thus each node can estimate its distance from every other node that is in close proximity. Then, distance measurements are gathered at a centralized processing node, that is responsible to conduct graph localization. Our goal is to estimate the D-dimensional *relative coordinates* of each node in the network graph. Given the geo-location of two (or more) nodes, the estimated relative coordinates can also be translated to UTM coordinates.

Underwater network nodes may experience diverse forms of disturbance (due to the large geographical separation of the network nodes), while the formed network graph may also contain missing connections (i.e., missing graph edges) due to proximity-imposed connectivity constraints. Consequently, traditional centralized trilateration solutions do not directly apply to the problem. We assume that nodes establish network connectivity, e.g., through a topology discovery process [24], such that multi-hop connectivity exists between any pair of nodes. During this network initialization, each pair of nodes evaluates the propagation delay of their joint transmission (through e.g., [25]). This delay translates into an evaluation of the distances between the graph's vertices. However, in practice, for a few pairs of nodes, distance measurements may experience large errors due to multi-path propagation and impulsive underwater noise [26], [27], [28] (sparse outliers) or may be unavailable due to communication range limitations (missing measurements).

The problem of retrieving network node coordinates from pair-wise distance measurements has been long studied in the literature and can be solved by means of Multi-Dimensional Scaling (MDS) [29]. The problem has also been studied for incomplete and noisy measurements [17]. Additionally, robust reformulations of the problem have considered the case of heavy/outlying corruption [30], [31], [32], [33], [34]. At this point we notice, that while most works consider either missing entries, or heavily corrupted data, it is the joint occurrence of both events that makes the problem most challenging. The reason is that apart from their direct impact to MDS, corrupted entries can also significantly mislead the EDM completion process. More recently, [35] addressed this problem, by jointly conducting outlier removal and completion before MDS by means of Robust Principal-Component Analysis (RPCA) [36].

Algorithm 1: Proposed MAD-MDS

Given: $\check{\mathbf{D}}$, $\mathbf{W}$

$\widetilde{\mathbf{D}} \leftarrow \texttt{MAD}(\check{\mathbf{D}}, \mathbf{W}, \tau)$ ▷ Remove corruptions

$\widehat{\mathbf{D}} \leftarrow \texttt{RBC}(\widetilde{\mathbf{D}}, \mathbf{W}, D)$ ▷ Complete missing entries

$\widehat{\mathbf{X}} \leftarrow \texttt{MDS}(\widehat{\mathbf{D}}, D)$ ▷ Estimated coordinates

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III. System Model

We denote by $\mathbf{x}_i \in \mathbb{R}^{D\times 1}$ the D-dimensional coordinate vector of node $i \in [N] := \{1, 2, \ldots, N\}$ and define the coordinate matrix $\mathbf{X} := [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N] \in \mathbb{R}^{D\times N}$. Accordingly, we denote by $\mathbf{D} \in \mathbb{R}_+^{N\times N}$ the true squared-distance matrix of the graph, such that

$$[\mathbf{D}]_{i,j} := \|\mathbf{x}_i - \mathbf{x}_j\|^2 \tag{1}$$

for every $i, j \in [N]$.

For every $i \in [N]$, node i measures its distance from all other nodes and sends them to the processing node, which collects all distance measurements in $\check{\mathbf{D}} \in \mathbb{R}^{N\times N}$. We assume that $\mathbf{X}$ remains invariant during the formation of $\check{\mathbf{D}}$.

We denote by $\mathbf{W} \in \{0, 1\}^{N\times N}$ the graph connectivity map. That is, $[\mathbf{W}]_{i,j}$ is 1 if nodes i and j are connected (i.e., they lie within the communication range of each other) and 0 otherwise. Then, for every (i, j), it holds that

$$[\check{\mathbf{D}}]_{i,j} = [\mathbf{W}]_{i,j}(|[\mathbf{D}]_{i,j} + [\mathbf{E}]_{i,j}|) \tag{2}$$

where $\mathbf{E} \in \mathbb{R}^{N\times N}$ is the matrix of distance-estimation errors. For every (i, j), $[\mathbf{E}]_{i,j}$ will be drawn from a distribution determined by the interference profile of the link between node i and node j. In this work, we are particularly interested in the case that some of the entries of $\mathbf{E}$ are of very high variance (i.e., incomplete $\check{\mathbf{D}}$ also contains sparse outliers). In the following section, we propose a robust method to retrieve $\mathbf{X}$ from $\check{\mathbf{D}}$, in practice.

IV. Proposed Graph Localization Method

In this work, we present MAD-MDS, a robust method for graph-localization from possibly incomplete and severely corrupted distance measurements. MAD-MDS works as follows.

First, we detect the corrupted distance measurements in $\check{\mathbf{D}}$ by evaluating their Median Absolute Deviation (MAD) [37], [38]. Then, we remove these measurements from $\check{\mathbf{D}}$ (i.e., treat them as missing), and obtain the "outlier-free" but incomplete distance matrix $\widetilde{\mathbf{D}}$. Next, we apply to $\widetilde{\mathbf{D}}$ Rank-Based Completion (RBC) [17] to recover the missing distances (i.e., those not measured due to connectivity limitations and those discarded as corrupted). The resulting matrix $\widehat{\mathbf{D}}$ is, in principle, completed and corruption-free.

The proposed algorithm is summarized in Alg. 1. In the following, we present individually the three steps of the proposed graph localization method (i.e., corruption removal with MAD, completion with RBC, and coordinate estimation with MDS) in mathematical detail.

Algorithm 2: MAD corruption removal [37]

Given: $\check{\mathbf{D}}$, $\mathbf{W}$, τ
Initialize $\widetilde{\mathbf{D}} \leftarrow \check{\mathbf{D}}$
Form $\mathbf{d}$ $\triangleright$ sorted measurements in $\widetilde{\mathbf{D}}$ (not diagonal)
$\mu \leftarrow \text{median}(\mathbf{d})$
$m \leftarrow \text{median}(\texttt{abs}(\mathbf{d} - \mathbf{1}_M \mu))$
For every $(i, j) \in [N]^2$ with $j > i$
  If $[\mathbf{W}]_{i,j} = 1$ && $|[\widetilde{\mathbf{D}}]_{i,j} - \mu| > \tau \cdot m$
    Remove from $\widetilde{\mathbf{D}}$ entries (i, j) and (j, i)
    $[\mathbf{W}]_{i,j} \leftarrow 0$, $[\mathbf{W}]_{j,i} \leftarrow 0$
Return $\widetilde{\mathbf{D}}$ and $\mathbf{W}$

A. Step 1: Outlier Excision via MAD

Let $0 < M \leq (N^2 - N)/2$ be the number of collected squared-distance measurements in $\check{\mathbf{D}}$, sorted in vector $\mathbf{d}$. We start with computing the median of $\mathbf{d}$, median$(\mathbf{d})$. Then, for each entry of $\mathbf{d}$, we compute its absolute distance from the median, forming vector

$$\mathbf{v} = \texttt{abs}(\mathbf{d} - \mathbf{1}_M \text{median}(\mathbf{d})) \tag{3}$$

such that $[\mathbf{v}]_i = |[\mathbf{d}]_i - \text{median}(\mathbf{d})|$, for every $i \in [M]$. Next, we compute the Median Absolute Deviation (MAD)

$$m = \text{median}(\mathbf{v}) = \text{median}(\texttt{abs}(\mathbf{d} - \mathbf{1}_M \text{median}(\mathbf{d}))). \tag{4}$$

Then, for the i-th distance measurement d_i, $i \in [M]$, we decide that it is outlying and remove it from $\check{\mathbf{D}}$, if

$$|d_i - \text{median}(\mathbf{d})| > \tau \cdot m \tag{5}$$

where τ is a parameter of the MAD method, that is tuned based on the expected sparsity of the heavily corrupted measurements (typically, τ takes values between 8 and 10). The MAD step is also summarized in Alg. 2. The use of MAD for outlier detection is presented in detail in [37], [38].

B. Step 2: EDM Completion and Denoising with RBC

This is an iterative method for matrix completion, based on eigenvalue truncation [17] and the rank property of EDMs (i.e., the rank of an EDM for points in $\mathbb{R}^D$ is at most $D + 2$). First, $\widetilde{\mathbf{D}}$ is initialized to $\mathbf{0}_{N \times N}$. Then, $\widetilde{\mathbf{D}}$ is set to be equal to $\check{\mathbf{D}}$, wherever $\check{\mathbf{D}}$ is non-empty. The next steps are iteratively repeated until convergence in $\widetilde{\mathbf{D}}$, or termination: (i) Do eigenvalue decomposition (EVD) of $\widetilde{\mathbf{D}} \overset{\text{EVD}}{=} \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T$; (ii) Update $\widetilde{\mathbf{D}} = [\mathbf{U}]_{:,1:D+2}[\mathbf{\Lambda}]_{1:D+2,1:D+2}[\mathbf{U}]_{:,1:D+2}^T$; (iii) Set to 0 the diagonal and/or negative entries of $\widetilde{\mathbf{D}}$. The RBC step is summarized in Alg. 3.

C. Step 3: Classical MDS

As the last step, we apply MDS to the completed estimated distance matrix $\widehat{\mathbf{D}}$ (symmetric, with non-negative entries, and zeros in the main diagonal). The method is presented in detail in [29] and summarized here for completeness.

We define $\mathbf{J} = \mathbf{I}_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T$ and, motivated by the fact that $\mathbf{X}^T\mathbf{X} = -\frac{1}{2}\mathbf{J}\mathbf{D}\mathbf{J}$, we form an estimate of the *Gram matrix*

$$\mathbf{G} = -\frac{1}{2}\mathbf{J}\widehat{\mathbf{D}}\mathbf{J} \tag{6}$$

Algorithm 3: RBC [17]

Given: $\widetilde{\mathbf{D}}$, $\mathbf{W}$, D
Initialize $\widehat{\mathbf{D}} \leftarrow \mathbf{0}_{N \times N}$
For every $(i, j) \in [N]^2$
  If $[\mathbf{W}]_{i,j} = 1$, $[\widehat{\mathbf{D}}]_{i,j} \leftarrow [\widetilde{\mathbf{D}}]_{i,j}$
Repeat (until convergence or maximum iterations)
  $\widehat{\mathbf{D}} \leftarrow \text{EVThreshold}(\widehat{\mathbf{D}}, D + 2)$
For every $(i, j) \in [N]^2$
  If $[\mathbf{W}]_{i,j} = 1$, $[\widehat{\mathbf{D}}]_{i,j} \leftarrow [\widetilde{\mathbf{D}}]_{i,j}$
  If $i = j$ or $[\widehat{\mathbf{D}}]_{i,j} < 0$, $[\widehat{\mathbf{D}}]_{i,j} \leftarrow 0$
Return $\widehat{\mathbf{D}}$

function *EVThreshold* $(\mathbf{A}, r)$
  $\{\mathbf{U}, \mathbf{\Lambda}\} \leftarrow \text{EVD}(\mathbf{A})$
  $[\mathbf{\Lambda}]_{i,i} \leftarrow 0$, for every $i > r$
Return $\mathbf{A} \leftarrow \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T$

Algorithm 4: Classical MDS [29]

Given: $\widehat{\mathbf{D}}$, D
$\mathbf{J} \leftarrow \mathbf{I}_M - \frac{1}{N}\mathbf{1}\mathbf{1}^T$
$\mathbf{G} \leftarrow -\frac{1}{2}\mathbf{J}\widehat{\mathbf{D}}\mathbf{J}$
$\{\mathbf{U}, \mathbf{\Lambda}\} \leftarrow \text{EVD}(\mathbf{G})$
Return $\widetilde{\mathbf{X}} \leftarrow [\mathbf{\Lambda}]_{1:D,1:D}[\mathbf{U}]_{:,1:D}^T$

and obtain the estimated relative network node coordinates

$$\widehat{\mathbf{X}} = [\mathbf{\Lambda}]_{1:D,1:D}[\mathbf{U}]_{:,1:D}^T \tag{7}$$

where $\mathbf{G} \overset{\text{EVD}}{=} \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T$ (for $\mathbf{U}^T\mathbf{U} = \mathbf{U}\mathbf{U}^T = \mathbf{I}_N$ and $[\mathbf{\Lambda}]_1 \geq \ldots \geq [\mathbf{\Lambda}]_N \geq 0$). Alg. 4 presents a pseudocode for MDS.

The obtained relative coordinates in $\widehat{\mathbf{X}}$ correspond to the original ones, up to some transformation (rotation, flipping, translation), since the absolute position and orientation of the points are lost when we operate on distances. For many applications, such as scheduling network node transmissions, the relative coordinates within the graph suffice. For other tasks, absolute coordinates are needed. This orientation/translation ambiguity is resolved by aligning the obtained coordinates in $\widehat{\mathbf{X}}$ with a set of anchors (i.e., nodes whose coordinates are fixed and known).

V. Performance Analysis

We evaluate the localization performance of the proposed MAD-MDS graph localization approach in terms of RMSE with numerical studies on (i) random ad-hoc mesh network graphs and (ii) an underwater network topology from past sea experiments. Additionally, we compare its performance to state-of-the art approaches that apply MDS on missing [17] and missing/corrupted distance data [35].

$\widehat{\mathbf{X}}$ in (7) is an estimate of $\mathbf{X}$, up to rotation/translation. To compare the two coordinate matrices and evaluate RMSE we need to "align" them. To accomplish that, we first zero-center $\widehat{\mathbf{X}}_s = \widehat{\mathbf{X}} - \widehat{\mathbf{X}}\frac{1}{N}\mathbf{1}\mathbf{1}^T$ and $\mathbf{X} - \mathbf{X}\frac{1}{N}\mathbf{1}\mathbf{1}^T$. Then, we find rotation matrix $\mathbf{Q} = \mathbf{U}\mathbf{V}^T$ where $\mathbf{U}$ and $\mathbf{V}$ are the left- and right-hand singular vectors of $\widehat{\mathbf{X}}_s\mathbf{X}_s^T$. Finally, we form the aligned coordinate estimates $\widehat{\mathbf{X}}_{sr} = \mathbf{Q}^T\widehat{\mathbf{X}}_s + \mathbf{X}\frac{1}{N}\mathbf{1}\mathbf{1}^T$. For all

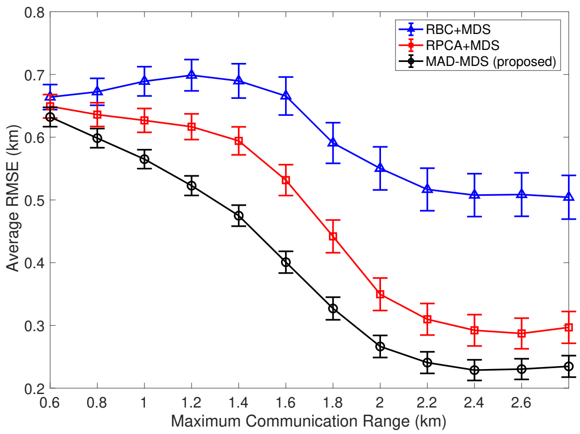

Fig. 1. Average RMSE performance (measured in km) for a network of $N = 5$ underwater acoustic nodes and varying maximum communication range from 600 m to 2.8 km.

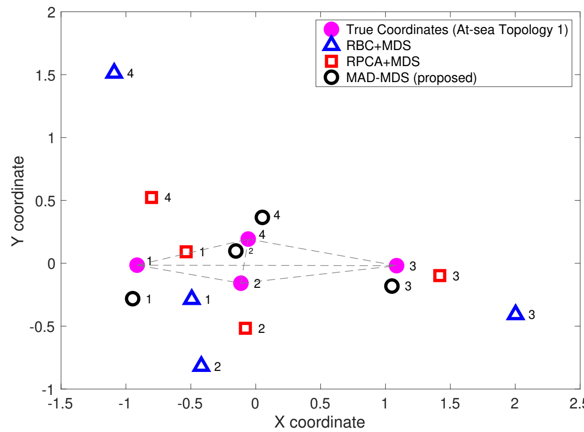

Fig. 3. Snapshot of true $\mathbf{X}$ vs reconstructed point coordinates $\widehat{\mathbf{X}}_{sr}$ for at sea topology 1 [39] and distance measurement error standard deviation $\sigma = 10$.

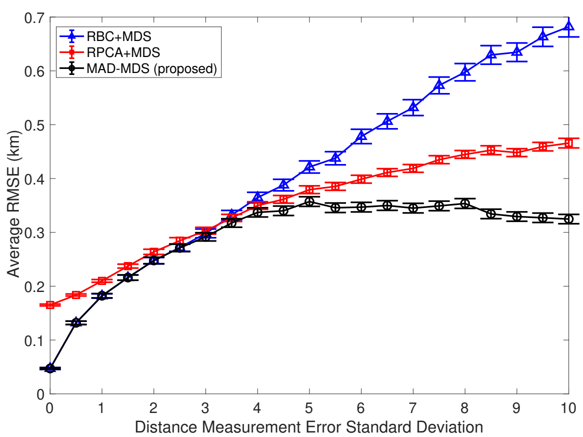

Fig. 2. Average RMSE performance (measured in km) for at-sea topology 1 [39] and different distance measurement error deviation.

methods, RMSE is calculated as $\text{RMSE} = \sqrt{\frac{1}{N}\|\widehat{\mathbf{X}}_{sr} - \mathbf{X}\|_F^2}$, averaged over 1000 realizations.

A. Simulations with Synthetic Data

Our simulation setup considers 1000 Monte-Carlo realizations for 12 different network graphs, each considering uniform deployment of $N = 5$ nodes in an area of $2000\,\text{m} \times 2000\,\text{m}$. The nodes are randomly arranged in different mesh network configurations. Link connectivity is controlled by setting a threshold on the proximity range between pairs of nodes. In Fig. 1, we plot the average RMSE performance (in km) versus the communication range of the nodes, for the proposed MAD-MDS, RBC with MDS [17], and RPCA with MDS [35]. As the communication range increases, the distance measurement matrix $\check{\mathbf{D}}$ becomes denser and, accordingly, the localization performance tends to improve for all methods –most emphatically for range greater than 1.5km. An estimated range between each pair of nodes in the network is given to all algorithms with an error randomly set by a generalized Gaussian distribution. Based on our prior work [40], the selected modeling approach captures the wide range of error distributions that may appear during the construction of the distance matrix. In particular, we consider that each entry in the distance matrix is corrupted from low-variance noise (modeled as zero-mean Gaussian with $\sigma = 0.05$) with probability 0.9 and high-variance Gaussian noise with $\sigma = 10$ with probability 0.1. We observe that the proposed algorithm exhibits superior RMSE performance for both sparse and fully connected network graphs that contain sparse outlier measurements.

B. Simulations with Data from Past Sea Experiments

In this section, we demonstrate the applicability of our approach to realistic conditions. In particular, we utilize the true coordinates and time-varying binary topology of an underwater mesh network that was deployed in a port area in Israel during a past sea experiment [39]. The experiment involves four nodes changing their position within the Haifa port to create different topology setups, while communicating with a self-made underwater acoustic modem. Experimental data logs can be downloaded from: https://sites.google.com/marsci.haifa.ac.il/asuna. Fig. 2 depicts the average RMSE of MAD-MDS for topology 1 in [39], which considers a fully-connected network of four nodes. We observe that MAD-MDS performs similar to MDS after RBC completion, for small error variance, however for as distance error measurement variance increase the proposed algorithm keeps the error below 300 m. Fig. 3 shows the 2-D coordinates of all network nodes for this topology. We observe that MAD-MDS can still recover the positions of all networks nodes (with an error of approximately 300 m) even in the presence of high-variance error in the collected distance measurements.

VI. Conclusions

We presented a method for robust graph localization in underwater acoustic networks from incomplete and outlier corrupted pair-wise distance measurements. The proposed method conducts outlier excision, matrix completion, and multi-dimensional scaling and it is shown to outperform state-of-the-art counterparts, especially for the cases where network connectivity is sparse (i.e. missing graph edges) and the distance measurements experience high corruption variance.

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