Transportation Research Part C 132 (2021) 103409

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

Optimal time-differentiated pricing for a competitive mixed traditional and crowdsourced event parking market

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A R T I C L E I N F O

Keywords:
Crowdsourcing
Emerging markets
Multi-player game
Event management
Pricing
Equilibrium

A B S T R A C T

An event-based parking pricing problem, the Crowdsourced Event Parking Market Pricing Problem, is proposed wherein parking lot owners and others who are willing to rent out privately owned spaces compete to attract drivers who are looking for available parking spaces. Each parking location owner's problem is modeled as a bi-level program, where the upper-level parking garages, individuals and consolidators (the players) compete for customers, setting their prices to maximize revenue given the response of the lower-level followers. The lower-level followers choose their parking locations based on the utilities they derive from the spaces, which is a function of the proximity of the spaces to their destinations, parking fees and crowdedness. Prices are set as a function of the time of reservation. Reservation-time based pricing enables price differentiation for late comers either in the form of lower prices to attract last-minute customers when excess spaces are anticipated or higher-pricing if few spaces are expected to remain empty. A multi-period, stochastic Equilibrium Problem with Equilibrium Constraints (EPEC) formulation of the Crowdsourced Event Parking Market Pricing Problem is presented, and a diagonalization method with embedded gradient ascent approach for solution of individual player Mathematical Programs with Equilibrium Constraints (MPECs) is proposed for its solution. Both supply- and demand-side uncertainties are explicitly modeled. Solutions provide competitive parking prices set by reservation time for each parking facility, whether the facility involves a large parking garage or single parking space owner. Numerical experiments were conducted to illustrate the proposed concepts and assess the potential impact of crowdsourced parking spaces on the parking market. The results show that social welfare increases by more than 5% when crowdsourced parking locations account for 7% of the parking market. Results from additional numerical experiments show that ignoring stochasticity results in revenue loss for all parking owners. The developed techniques aim to facilitate existing and new parking facility owners to participate in crowdsourced event-parking markets.

1. Introduction

Whether commuting to work, leaving from an airport, heading to an appointment or attending an event, finding parking in dense urban areas can be problematic. For many cities, providing additional parking spaces may not be physically or economically feasible. It also may be inconsistent with community ideals, which may aim to increase public transportation ridership and reduce land claimed

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https://doi.org/10.1016/j.trc.2021.103409
Received 24 September 2020; Received in revised form 20 August 2021; Accepted 17 September 2021
Available online 25 September 2021

for automobile use (Plumer, 2016). According to a report by the Research Institute for Housing America (Scharnhorst, 2018), in Philadelphia there are 25.3 parking spaces per acre or 3.7 parking spaces per household. Plumer (2016) states that there are a billion parking spots across the U.S., four for every existing car. Similar numbers have been found for San Francisco and other U.S. cities (e.g. Smart City Challenge; San Francisco, 2016; Scharnhorst, 2018). Despite this parking space excess, finding parking in desired locations is often very difficult. Henao (2017) found that parking difficulty or cost is the second (only to avoiding driving while intoxicated) reason individuals who ordinarily drive their own cars switch to ridesharing services, such as Lyft and Uber. While these and additional autonomous services of the future (Nourinejad et al., 2018) will reduce single-owner car usage and likely car ownership, as cities aim to reclaim land used for vehicles for greener, more social uses, the number of parking spaces is expected to diminish in coming years. Thus, parking space availability in key areas will remain elusive and space allocated to parking will need to be more efficiently utilized.

In support of more efficient parking space usage are on-line applications that can be used for reserving a parking space in advance of one's arrival at a destination. Recently, such applications have enabled non-parking businesses, such as hotels and individuals, to join this parking market, selling the use of their excess or residential parking spaces through consolidators, such as SpotHero, ParkWhiz and Just Park, during hours of the day when they are not using them. While owners can choose any price, they may not know what price the market will bear given both the current demand for and supply of spaces. Cooperation through consolidation of these individual assets, along with existing parking facilities, together form the parking market.

In this paper, an event-based parking pricing problem, the Crowdsourced Event Parking Market Pricing Problem, is proposed wherein parking lot owners and others who are willing to rent privately owned spaces compete to attract drivers who are looking for available parking spaces. Each parking owner's problem is modeled as a bi-level program, where the upper-level parking garages, individuals and consolidators (the players) compete for customers, setting their prices to maximize revenue given the response of the lower-level followers. The lower-level followers choose their parking locations based on the utilities they derive from the spaces, a function of the proximity of the space to their destinations, parking fees and crowdedness. Together, the players' pricing problems, with common lower-level parking choice problem, create an Equilibrium Problem with Equilibrium Constraints (EPEC). Prices upon solution are obtained at a generalized Nash equilibrium (Nash, 1951) between parking providers who serve a common set of drivers and a Stackelberg equilibrium between parking providers (leaders) and drivers (followers). In the upper level, no player can unilaterally change the price setting for her parking space offering and improve her profit; whereas in the lower level, no driver can unilaterally change his parking choice and improve his utility for the given set of spaces and prices. Finally, with this modeling concept, a multi-period stochastic EPEC formulation that captures uncertainties in both supply and demand and time-differentiated pricing is presented. Solution provides competitive parking prices set by reservation time for each parking facility, whether a large parking garage or single parking space owner. Reservation-time based pricing enables price differentiation for late comers either in the form of lower prices to attract last-minute customers when excess spaces are anticipated or higher-pricing if few spaces are expected to remain empty. Developed techniques aim to facilitate existing and new parking facility owners to participate in crowdsourced event-parking markets.

The contributions of this paper are fourfold: (1) a multi-regime, stochastic parking pricing problem (EPEC) formulation that incorporates both public (privately or publicly owned) and crowdsourced parking locations with differentiated pricing and elastic demand; (2) an exact solution approach based on concepts of diagonalization and gradient ascent; (3) a pricing mechanism to support the inclusion of crowdsourced parking space owners within the larger parking market; and (4) insights from numerical experiments related to equilibrium parking pricing, the value of explicitly recognizing demand uncertainties in pricing to total revenue, and potential market impacts of facilitating crowdsourced parking options.

A review of related literature is provided in the next section that helps to establish these contributions. In Section 3, the multi-period, stochastic EPEC formulation of the Crowdsourced Event Parking Market Pricing Problem is presented. Section 4 describes exact and approximate solution methods and discusses their advantages and disadvantages. This is followed by a proposed solution procedure that builds on concepts from diagonalization and gradient ascent methods. Problem conceptualization, formulation and solution are illustrated on a hypothetical event parking problem. Experimental details, results from numerical experiments and results analysis are provided in Section 5, while conclusions are drawn in Section 6.

2. Literature review

The first work to model competition between parking lots was (Froeb et al., 2003). This work sought to analyze the effects of mergers among parking lots. They studied the competitive behavior of a set of firms with differentiated products through a Bertrand competition between firms facing capacity constraints. An algorithm is introduced to compute the Nash equilibrium solution. In a parking policy analysis, Arnott (2006) modeled the competition between downtown parking garages by considering a two-stage sequential entry, spatial competition. In the first stage of the game, the location and capacity of the garages are determined, and in the second stage garages set their parking fees for the foreseeable future. Demand is assumed to be inelastic and uniformly distributed over space, and parking garages are assumed to work at full capacity. At a Nash equilibrium, parking garage operators in the second stage are provided with an identical optimal parking price assuming spatial uniformity of options. The model is further extended to consider curbside parking, as well as competition from a mass transportation system. Arnott and Rowse (2009) expand on this work, integrating privately owned parking facilities, along with curbside parking. They studied cruising time for finding curbside parking and the effects of curbside pricing policy on traffic congestion. Travel demand is, again, assumed to be inelastic. This inelastic demand assumption is relaxed in (Arnott et al., 2015), where the effect of curbside parking capacity needed to achieve a social optimum is examined. Also taking a long-term planning perspective, spatial competition in parking pricing and related policies are studied in (Anderson and de Palma, 2004; Anderson and de Palma, 2007; Calthrop and Proost, 2006; Inci and Lindsey, 2015). Inci (2015) provides a comprehensive review of these and other articles through an economics lens.

The work herein learns from and expands on concepts from these earlier works that consider competition between parking owners. Here, the long-term planning (e.g. focus on garage construction) and policy perspectives are redirected toward immediate-term, event-based applications. Crowdsourced parking capacity is enabled through new technologies. Inherent uncertainties and demand elasticity are included.

For more immediate-term application as is considered in this study, several works investigate dynamic parking pricing. Qian and Rajagopal (2014a) studied how dynamic parking pricing can be used to manage recurrent parking demand under the assumption of complete real-time knowledge of occupancy and parking pricing. The prices are assumed to be known to the driver either through online information provision or the traveler's day-to-day experience. In their study, the traveler parking choice problem is studied under both user equilibrium and system optimal conditions for travelers differentiated by their origin–destination pairs, departure times, and preferred parking areas. Parking space desirability is a function of a generalized cost function of travel, walk and parking search times, along with parking fees. Parking demand is assumed to be fixed (i.e. inelastic). The number of parking spaces in any location is limited. They found that the best system performance is achieved at final parking lot occupancy rates of approximately 85–95%. The same authors (Qian and Rajagopal, 2014b) investigated a dynamic parking pricing problem under demand uncertainty accounting for traveler heterogeneity. The parking pricing problem is formulated as a stochastic control problem, where the objective is to minimize total generalized travel costs. These works provide useful insights on dynamic pricing under real-time occupancy and pricing information. The works presume a single decision-maker, however.

There are several recent works that study parking management through dynamic pricing. Nourinejad and Roorda (2017) showed that hourly parking pricing can complement or substitute roadway pricing to manage travel demand if parking dwell time elasticity is explicitly considered. Jakob et al. (2016) studied the impact of dynamic parking pricing on search-for-parking times. How parking pricing and parking permit management can be used to minimize total system cost at park-and-ride facilities was investigated (Wang et al., 2020)

Dynamic parking pricing strategies that exploit technological advancements in smart parking management and reservation systems are the subject of several recent works. Sayarshad et al. (2020) proposed a parking game wherein drivers compete with each other over parking spaces to obtain their lowest generalized costs, including parking price, walking distance, and cruising time. A socially optimal dynamic pricing policy is presented. Using data for San Francisco, the authors found that their approach can lead to an increase in social welfare by 54% per vehicle compared to other parking pricing policies.

An idea of predicting occupancy of nearby parking locations to find optimal parking prices to achieve an occupancy goal was proposed by Simhon et al. (2017). This approach eliminates the need for knowledge of a price-responsive demand function, but requires historical parking price and occupancy data. Tian et al. (2018) studied dynamic parking pricing for a reservation-based system from a revenue management perspective wherein parking prices are set to maximize the revenue obtained from parking. Parking reservation requests are assumed to arrive randomly and are responsive to price. The authors claim that this pricing scheme is not only beneficial for the parking manager, but significantly reduces cruising times. Wang and Wang (2019) proposed a flexible reservation mechanism that accounts for uncertainty in late departures of customers. A bilevel Stackelberg model is proposed to model the parking agency's decision-making process. The model involves a multi-stage, stochastic program that incorporates the dynamics in short-term reservations.

A recent area of study includes parking pricing for a future environment with autonomous vehicles (Liu, 2018; Fulman and Benenson, 2019; Nourinejad and Amirgholy, 2018; Millard-Ball, 2019). In such environments, the parking paradigm and costs, including walking and cruising times, differ from existing environments.

While also considering only a single parking location owner, yet most relevant to the work herein, Mackowski et al. (2015) introduced a dynamic parking pricing problem. They embed a bi-level formulation in a rolling horizon framework. The upper level seeks optimal pricing of spaces assuming a single owner and expected user demand. A user equilibrium solution that responds to upper-level prices is obtained from solution of the lower level. This dynamic pricing problem is extended in (Lei and Ouyang, 2017) to include parking-space reservations. An approximate dynamic programming approach is used for setting prices. Building on the work by Mackowski et al. (2015), Mirheli and Hajibabai (2020) developed a dynamic parking pricing and parking utilization model in which uncertainty in future demand and occupancy at each facility is addressed. To solve their model, a Monte Carlo tree search heuristic is used.

With the exception of the few earlier works that address long-term parking facility design and pricing, these prior works presume the existence of a single authority who sets parking prices or a single parking owner of all parking spaces. However, in reality, parking facilities are mostly privately owned and managed, and the objective of their owners is to maximize their own revenue gained by attracting drivers to park in their facilities. The proposed stochastic EPEC formulation given herein extends the parking pricing problem of these earlier works to account for competition among parking facilities. Moreover, it creates a conceptual framework for incorporating individuals and others with excess parking in the market, enabling a crowdsourced event parking market. If only one owner exists, or a single decision maker sets the prices for all parking locations, for a single realization of random problem parameters, the Crowdsourced Event Parking Market Pricing Problem reduces to a mathematical problem with equilibrium constraints (MPEC), i.e., the problem modeled by Mackowski et al. (2015). That is, this work extends their single-owner problem to a multi-owner and competitive problem setting. This extension necessitates not only the development of the more complicated EPEC formulation and explicit treatment of demand- and supply-side uncertainties, but an alternative solution technique that generates both a Stackelberg equilibrium solution between a leader and a set of followers and a Nash equilibrium across leaders.

If all parking options have unlimited capacity, solution to the Crowdsourced Event Parking Market Pricing Problem can be obtained by solving for equilibrium prices assuming the existence of a horizontally differentiated Bertrand competition (Bertrand, 1883). Froeb et al. (2003) modeled their long-term parking pricing problem as a Bertrand competition with capacity limitations presuming that customers who do not obtain their first choice (lowest-disutility) parking spaces will choose a second or third choice option. Mathematically, they note that this choice introduces a kink in the profit function. Here, choice of parking space limitations is modeled through solution of a capacitated UE. This creates the need for the proposed EPEC conceptualization and allows for the inclusion of uncertainty in formulation, as well as time-differentiated solutions.

3. The crowdsourced event parking market pricing problem

Formulation of the Crowdsourced Event Parking Market Pricing Problem takes the form of an EPEC involving multiple, competing players (parking space/lot owners) at the upper level who seek an optimal price for the parking spaces they offer and, at the lower level, drivers who seek parking spaces with attractive pricing in a limited market, but with the option to switch to an alternative mode of transportation with no parking requirements. That is, demand is elastic to disutility. The parameters of the demand function are known only with uncertainty. Unlike public parking spaces, the number of crowdsourced parking locations that will be placed on the market in any time period cannot be known deterministically in advance, because owners may opt to remove their spaces from the market at any time during the day. Thus, there is uncertainty in both demand and supply sides of the market. The customers pay a one-time parking fee for the duration of the event, or any portion thereof, based on the time they reserve the parking space. The formulation of this proposed multi-period (time-differentiated), stochastic EPEC follows. It is preceded by notational definitions and a problem overview.

3.1. Notation

Notation used in the formulation follows that given in (Mackowski et al., 2015) as closely as possible to aid the reader in seeing how the prior parking formulation in the literature is subsumed within this multi-leader, multi-follower framework. The notation is defined in terms of parking locations containing parking spaces, including single crowdsourced parking spaces.

Sets and indices

O: set of all origins $o \in O$

J: set of all parking locations $j \in J$

T: set of time periods in the planning horizon

Variables

π_j: revenue received by parking location owner $j \in J$

$p_{t,j}$: price of parking in a space in parking location $j \in J$ at time$t \in T$

$u_{t,o}(\omega)$: equilibrium disutility of users traveling from origin $o \in O$ who reserve a parking space at time $t \in T$ under scenario $\omega \in \Omega$

$\phi_{t,o,j}(\omega)$: disutility for parking at parking location $j \in J$ for user traveling from $o \in O$ at time $t \in T$ under scenario $\omega \in \Omega$

$h_{t,o,j}(\omega)$: number of users traveling from origin $o \in O$ to parking location$j \in J$ at time $t \in T$ under scenario $\omega \in \Omega$

$f_{t,j}(\omega)$: parking occupancy (reserved) in terms of number of spaces at parking location $j \in J$ at time $t \in T$ under scenario $\omega \in \Omega$

$D_{t,o}(\omega)$: parking demand from origin $o \in O$ at time $t \in T$ under scenario $\omega \in \Omega$

$d_{t,j}(\omega)$: crowdedness delay at parking location $j \in J$ at time $t \in T$ under scenario $\omega \in \Omega$

Parameters

p^U: upper limit for unit parking price

p^L: lower limit for unit parking price

$C_j(\omega)$: capacity in terms of number of spaces for parking location $j \in J$ under scenario $\omega \in \Omega$

P_ω: probability of scenario $\omega \in \Omega$

$t_{o,j}$: driving cost from origin $o \in O$ to parking location $j \in J$

w_j: walking cost from parking location $j \in J$ to the event venue

$H_{t,o}$: demand function for origin $o \in O$ at time $t \in T$

$a_{t,o}(\omega)$ and $b_{t,o}(\omega)$: parameters of the demand function for origin $o \in O$ at time $t \in T$ under scenario $\omega \in \Omega$

3.2. Problem statement

Consider an upcoming event (e.g. a game, performance, conference or show) in an urban area in which there are several parking locations available for people who are planning to attend the event. These parking locations may be of different forms, including: on-street parking, garages, small lots or even personally owned driveways. Each of these parking locations is priced by its owner who seeks to maximize his/her own revenue. Parking options differ in terms of their distances to the event venue, resulting in different levels of desirability and, therefore, different parking fees they can attract.

Several assumptions were made in developing the EPEC formulation. A reservation system is presumed to facilitate the process of reserving parking spaces for users. Users who come from different origins can reserve their parking spaces prior to the event. Demand for parking from each origin in each time period is elastic with regard to total travel cost, consisting of driving time, parking price, delay at the parking location due to crowdedness, and walking distance. Thus, higher total travel cost results in fewer people who are willing to park. They may instead use alternative modes of transportation, such as carpooling, ride-sharing, an event-based shuttle from remote parking, or public transportation, for example, to travel to the event venue. Additionally, users will pay a one-time parking fee for the duration of the event. Prices are differentiated by the time-of-reservation; thus, the price users pay depends on the time they make the reservation.

The bi-level structure of the EPEC formulation is depicted in Fig. 1. Upper-level parking owners (the leaders) set the parking prices for each reservation-time period to maximize their own revenues and the lower-level users (the followers) select their parking locations to minimize their individual travel disutilities. If a user's disutility is too high, the user may forgo his trip (elastic demand).

Within this structure, each owner possesses a set of parking locations with different capacities. The owners set their prices competitively in solution of their upper-level problems, while simultaneously considering user response to the offerings in solution to the common lower-level problem. The potential for differing market shares by the parking owners is illustrated with hatched rectangles in the figure. In place of maximizing a measure of social welfare, as is typical of centralized (single-owner) approaches, each owner in this competitive framework seeks to maximize her own revenue. The outcome of the EPEC formulation is a set of equilibrium prices at which point no parking owner can unilaterally change his/her parking price and enhance her revenue.

The lower-level problem, which reflects the parking decisions of the users given the set of parking prices from the upper-level, has a similar structure to a capacitated user equilibrium (UE) traffic assignment. Users traveling from each origin $o \in O$ have a set of parking locations $j \in J$ through which to reach the final destination. Each route from origin to the venue location consists of three parts (similar to links in UE traffic assignment): driving, parking, and walking (Fig. 2). Their weighted sum constitutes the route's disutility. While the choice of parking location does not impact travel and walking times, it does impact the crowdedness (a function of the number of reservations) at parking locations which affects the time required for accessing (i.e., entering or exiting) the facility. Crowdedness differs from the cruising for parking time used in parking studies to show the wasted time of users searching for a vacant parking spot. Crowdedness is, thus, captured in the disutility of each parking location. The parking locations contain a limited number of spaces.

For the simpler case with undifferentiated pricing, a generalized disutility of parking locations $j \in J$ for users traveling from origins $o \in O$, i.e. $\overline{\phi}_{o,j}$, will be identical and equal to parking disutility for that origin, i.e. u_o, if any user uses that parking location, i.e. $h_{o,j} > 0$, and it will be greater than u_o, otherwise. That is, at an equilibrium, all chosen paths from the same origin o to the event will have the same disutility, u_o. If a parking location is not used, its disutility is higher than, u_o. Mathematically speaking, these conditions can be stated following the Wardrop equilibrium principle:

$$h_{o,j} > 0 \Rightarrow \overline{\phi}_{o,j} = u_o, \forall o \in O, j \in J \tag{1}$$

$$h_{o,j} = 0 \Rightarrow \overline{\phi}_{o,j} \geq u_o, \forall o \in O, j \in J \tag{2}$$

The underlying deterministic problem formulation along with accompanying optimality conditions yielding conditions (1) and (2) for the time-differentiated case are presented in the Appendix. Presented next is the more general case where parking owners can set differentiated parking space prices by reservation time period and both supply and demand parameters are uncertain.

Without loss of generality, for the following formulation, each owner is assumed to own one parking location. In this formulation, users at the lower level (the parking demand) are identified by their origins o. Consider a single parking location owner's perspective. The owner of parking location j seeks to maximize her revenue by filling the parking spaces in that location at a competitive price, i.e. parking owners compete with each other to attract users who have multiple parking location options. Each parking owner j sets the parking price, $p_{t,j}$, for customers who reserve a space in the parking facility at time $t \in T$. Since this is an event parking pricing problem, the permitted start and end time for parking is fixed. Each user, thus, pays a one-time parking fee that depends on the time the

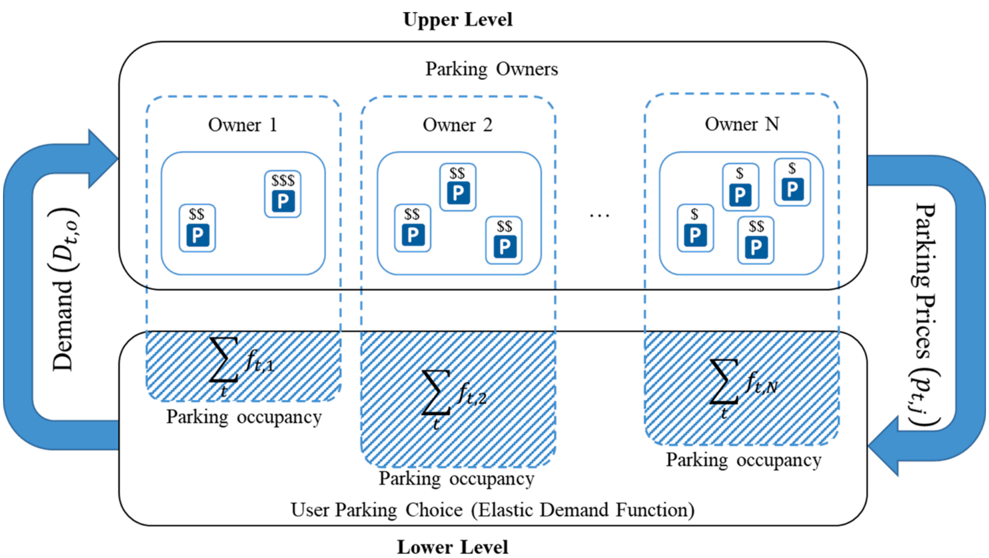

Fig. 1. Bi-level structure of proposed parking-pricing problem as an EPEC.

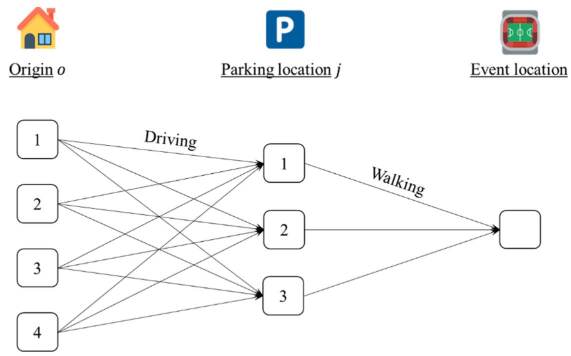

Fig. 2. Schematic picture of parking location choices.

reservation is made and not on the duration of use of the parking space. That is, the parking duration or dwell time is immaterial.

Customers from origin $o \in O$ who reserve their parking space at time $t \in T$ will respond to offered prices by choosing the parking location with least disutility (a function of price and other factors). The demand function is assumed to be linear in form: $H(u_{t,o}) = a_{t,o} - b_{t,o} \cdot u_{t,o}$, where $u_{t,o}$ is the equilibrium disutility of users from origin $o \in O$ reserving their space at time $t \in T$. $H(u_{t,o})$ is elastic with respect to the equilibrium disutility $u_{t,o}$. Parameters $a_{t,o}$ and $b_{t,o}$ correspond to the function's intercept and slope. These parameters can be estimated from historical parking price and occupancy data. For each origin o and reservation time t, $a_{t,o}$ and $b_{t,o}$ can be estimated from samples taken from historical data associated with the number of reservations $H(u_{t,o})$ and related disutilities $(u_{t,o})$, calibrating the linear demand function. The estimated parameters will be at best known with uncertainty, and thus, are random variables with probability distributions. A price vector is evaluated in terms of the expected revenue, which is a function of both space offerings and realized demand.

To capture uncertainty of the parameters of demand and capacity (C_j), a finite number of possible scenarios, $\omega \in \Omega$, is generated. Each scenario is a realization of a sequence of random variables over time $t \in \mathrm{T}$, i.e. $\left\{\left((a_{1,o}, b_{1,o})_{\forall o \in O}, (C_j)_{\forall j \in J}\right), \cdots, \left((a_{t,o}, b_{t,o})_{\forall o \in O}, (C_j)_{\forall j \in J}\right), \cdots, \left((a_{|T|,o}, b_{|T|,o})_{\forall o \in O}, (C_j)_{\forall j \in J}\right)\right\}$. The probability associated with each scenario $\omega \in \Omega$ is given by P_ω. The scenarios can be generated by sampling from the underlying probability distribution associated with each of the unknown random variables.

Together, this results in a stochastic EPEC formulation. Stochastic EPECs have been primarily used to model competition in the energy markets (e.g. Henrion and Römisch, 2007, and Pozo and Contreras, 2011), and recently have been used in other application areas, e.g., to model port competition and collaboration among ports in disaster (Asadabadi and Miller-Hooks, 2020).

3.3. Mathematical Formulation: A time-differentiated, stochastic EPEC

The stochastic EPEC is composed of as many stochastic MPECs as there are parking location owners (where the owners of personal spaces can be clustered as discussed in Section 5). These MPECs share decision variables and equilibrium constraints (Su, 2005). Thus, the stochastic EPEC consists of a set of stochastic MPECs, where the stochastic MPEC of any player can be given as formulated in this section.

Given Ω, the upper-level problem is formulated as:

$$\max \pi_j = \sum_{\omega \in \Omega} P_\omega \sum_{t \in T} \sum_{o \in O} p_{t,j} h_{t,o,j}(\omega) = \sum_{\omega \in \Omega} P_\omega \sum_{t \in T} p_{t,j} \sum_{o \in O} h_{t,o,j}(\omega) \tag{3}$$

s.t.

$$p^L \leq p_{t,j} \leq p^U, \forall t \in T, j \in J. \tag{4}$$

Objective function (3) seeks to maximize the expected total revenue of the parking owner or leader j. If a single entity owns more than one parking garage, that owner's objective will be summed over all her locations. Constraints (4) guarantee that parking prices fall within their upper and lower bounds. The upper bound may be regulated by city authorities and the lower bound is the least parking fee that parking owner j is willing to charge users.

The upper-level problem formulated through (3) and (4) is constrained by the lower-level parking choice problem given next that resembles a multi-period, capacitated traffic user equilibrium problem with elastic demand. The lower-level problem for a given scenario $\omega \in \Omega$ is presented next.

$$\min \sum_{t \in T} \sum_{o \in O} \sum_{j \in J} \int_0^{h_{t,o,j}} t_{o,j}(s) ds + \sum_{t \in T} \sum_{j \in J} \int_0^{f_{t,j}} \left(p_{t,j}(s) + d_{t,j}(s)\right) ds + \sum_{t \in T} \sum_{j \in J} \int_0^{f_{t,j}} w_j(s) ds - \sum_{t \in T} \sum_{o \in O} \int_0^{D_{t,o}} H_{t,o}^{-1}(s) ds, \tag{5}$$

s.t.

$$\sum_{j\in J} h_{t,o,j} = D_{t,o}, \forall t \in T, o \in O, \tag{6}$$

$$\sum_{t'}^{t} f_{t',j} \le C_j, \forall t \in T, j \in J, \tag{7}$$

$$\sum_{o\in O} h_{t,o,j} = f_{t,j}, \forall t \in T, j \in J, \tag{8}$$

$$h_{t,o,j} \ge 0, \forall t \in T, o \in O, j \in J,$$

$$D_{t,o} \ge 0, \forall t \in T, o \in O$$

Let $u_{t,o}$ and $\rho_{t,j}$be the Lagrangean multipliers of the constraints (6) and (7), respectively. The Lagrangean of the problem for a given scenario $\omega \in \Omega$ can be stated as follows:

$$\begin{aligned} L(\boldsymbol{h},\boldsymbol{D},\boldsymbol{u},\boldsymbol{\rho}) = & \sum_{t\in T}\sum_{o\in O}\sum_{j\in J}\int_0^{h_{t,o,j}} t_{o,j}(s)ds + \sum_{t\in T}\sum_{j\in J}\int_0^{f_{t,j}} \left(p_{t,j}(s) + d_{t,j}(s)\right)ds + \sum_{t\in T}\sum_{j\in J}\int_0^{f_{t,j}} w_j(s)ds - \sum_{t\in T}\sum_{o\in O}\int_0^{D_{t,o}} H_{t,o}^{-1}(s,)ds \\ & + \sum_{t\in T}\sum_{o\in O} u_{t,o}\left(D_{t,o} - \sum_{j\in J} h_{t,o,j}\right) + \sum_{t\in T}\sum_{j\in J}\rho_{t,j}\left(\sum_{t'}^{t} f_{t',j} - C_j\right) \end{aligned} \tag{9}$$

The Lagrangean function is constrained by non-negativity constraints on $h_{t,o,j}$ and $D_{t,o}$ and constraints (8). Thus, the Karush-Kuhn-Tucker (KKT) conditions can be given by:

$$0 \le h_{t,o,j} \perp \frac{\partial L(\boldsymbol{h},\boldsymbol{D},\boldsymbol{u},\boldsymbol{\rho})}{\partial h_{t,o,j}} \ge 0, \forall t \in T, o \in O, j \in J, \tag{10}$$

$$0 \le D_{t,o} \perp \frac{\partial L(\boldsymbol{h},\boldsymbol{D},\boldsymbol{u},\boldsymbol{\rho})}{\partial D_{t,o}} \ge 0, \forall t \in T, o \in O, \tag{11}$$

$$D_{t,o} - \sum_{j\in J} h_{t,o,j} = 0, \forall t \in T, o \in O, \tag{12}$$

$$0 \le \rho_{t,j} \perp \left(C_j - \sum_{t'}^{t} f_{t',j}\right) \ge 0, \forall t \in T, j \in J. \tag{13}$$

Equations (10) and (11) can be rewritten as in constraints (14) and (15):

$$0 \le h_{t,o,j} \perp \left(\phi_{t,o,j} - u_{t,o} + \rho_{t,j}\right) \ge 0, \forall t \in T, \forall o \in O, j \in J, \tag{14}$$

$$0 \le D_{t,o} \perp \left(-H_{t,o}^{-1}(D_{t,o}) + u_{t,o}\right) \ge 0, \forall t \in T, \forall o \in O, \tag{15}$$

where

$$\phi_{t,o,j} = t_{o,j} + p_j + d_{t,j} + w_j \forall t \in T, o \in O, j \in J. \tag{16}$$

$d_{t,j}$ is a function of occupancy of parking location j, i.e. $d_{t,j} = e_j \cdot \sum_{t'=1}^{t}\sum_{o\in O} h_{t,o,j}$, where e_j is the parking location-specific constant. For simplicity, delays at crowdsourced parking locations are assumed to be zero, $d_{t,j} = 0$.

Note that the convexity, and hence global optimality, of this lower-level program is guaranteed for simpler crowdedness functions, e.g. $d_{t,j}= e_j \cdot f_{t,j}$, as shown in the Appendix. For more complicated functions, it may provide only a local optimum.

The lower-level problem given parking prices from the upper-level, for a given scenario $\omega \in \Omega$, can be stated as in (17)-(20):

$$0 \le h_{t,o,j} \perp \left(\phi_{t,o,j} - u_{t,o} + \rho_{t,j}\right) \ge 0, \forall t \in T, o \in O, j \in J \tag{17}$$

$$0 \le D_{t,o} \perp \left(D_{t,o} - \left(a_{t,o} - b_{t,o}u_{t,o}\right)\right) \ge 0, \forall t \in T, o \in O \tag{18}$$

$$D_{t,o} - \sum_{j\in J} h_{t,o,j} = 0, \forall t \in T, o \in O \tag{19}$$

$$0 \le \rho_{t,j} \perp \left(C_j - \sum_{t'=1}^{t}\sum_{o\in O} h_{t',o,j}\right) \ge 0, \forall t \in T, j \in J. \tag{20}$$

The complementarity constraints can be linearized using disjunctive constraints (Fortuny-Amat and McCarl, 1981), and will result in a mixed-integer program that can be solved using a commercial solver. This is explained in detailed in section 4. Alternatively, the lower-level problem formulated by (5)-(8) might be solved directly with a commercial solver if rewritten as an equivalent convex program (e.g. Yang and Wang, 2011). Such an approach has computational advantages and obviates the need for linearization. Recent advancements (e.g. Wei et al., 2019) offer alternative, computationally efficient methods that are aimed at large-scale problem instances.

Constraints (3), (4) and (17)-(20) together create an individual MPEC related to a single parking location owner. The collection of all MPECs associated with all owners can be combined to create the overarching EPEC formulation of the Crowdsourced Event Parking Market Pricing Problem. The EPEC is, thus, the problem of finding a Nash equilibrium among parking location owners, consistent with the more general EPEC formulation in which a Nash equilibrium is sought among players (Gabriel et al., 2012). Mathematically, the problem is to find $\left\{\boldsymbol{p}_1\boldsymbol{p}_2,\cdots,\boldsymbol{p}_{|J|};\boldsymbol{h},\boldsymbol{\phi},\boldsymbol{u},\boldsymbol{\rho},\boldsymbol{D}\right\}$ such that for $j\in J$, the MPEC ((3), (4) and (17)-(20)) is solved given $\boldsymbol{p}_j = \left[p_{t,j}\right]_{t\in T}$, $\boldsymbol{h} = \left[h_{t,o,j}\right]_{t\in T,o\in O,j\in J}$, $\boldsymbol{\phi} = \left[\phi_{t,o,j}\right]_{t\in T,o\in O,j\in J}$, $\boldsymbol{u} = \left[u_{t,o}\right]_{t\in T,o\in O}$, $\boldsymbol{\rho} = \left[\rho_{t,j}\right]_{t\in T,j\in J}$, and $\boldsymbol{D} = \left[D_{t,o}\right]_{t\in T,o\in O}$.

An individual MPEC for a given scenario realization corresponds closely to that formulated in (Mackowski et al., 2015) for a centralized parking decision problem involving one owner and deterministically known problem characteristics. The formulation derived here, however, excludes dummy lots, which were introduced in (Mackowski et al., 2015) to absorb excess demand for cases where demand outpaces supply. Their approach involving dummy lots also requires settings for disutility values for the dummy lots and the addition of complicating constraints. The formulation proposed herein via equations (3), (4) and (17)-(20) obviates the need for these additions. Instead, the approach herein prevents cases where demand is higher than the capacity. Consider complementarity constraints (20). $\rho_{t,j}$ (the first term) can only take non-zero values at time t and later if parking location j is occupied up to that time (i.e., the second term is zero). If all parking locations are fully reserved at time t, i.e. the second term in constraints (20) is zero, $\rho_{t,j}$ can take non-zero values for all $j\in J$ from time t forward. To simultaneously enforce constraints (17), the equilibrium disutility $u_{t,o}$ will take a high value, pushing the demand toward zero. This is possible, because $\rho_{t,j}$ can take any positive, non-zero value. In other words, if demand might exceed supply, the disutility will be excessive and demand will diminish to zero.

Further complicating the problem posed herein is that considering multiple parking owners results in a non-convex objective function (3) that cannot be reformulated into a series of linear and quadratic terms as was the case in (Mackowski et al., 2015), where only one decision-maker (one owner) is modeled. This extension to multiple owners significantly increases the complexity of solution.

4. Solution methodology

EPECs are computationally difficult to solve and exact solution may be formidable. Several methods, including both exact and heuristic approaches, were implemented in seeking to tackle this computationally challenging stochastic EPEC. These are discussed first, before proceeding to a description of the proposed solution methodology. Inclusion of these approaches provides deeper insights into the nature of this problem.

In the previous section, first-order optimality conditions, i.e. the KKT conditions, of the lower-level problem common to all parking owners were derived and a single EPEC formulation containing the objectives and constraints of each owner plus the common KKT conditions was developed. This stochastic EPEC can be solved equivalently by simultaneously solving each owner's individual problem (an MPEC). That is, the EPEC is a collection of MPECs that are interrelated, here, through a common lower-level problem. The KKT conditions of each owner's MPEC creates a system of equations that when combined across players creates a single grand problem of all owners' equations. Unfortunately, this approach fails due to the nonlinearity of the complementarity conditions that arise from reducing each owner's bi-level program to a single-level MPEC. Consequently, resulting KKT conditions for each player will not meet constraint qualifications, and therefore may not be necessary for optimality (Gabriel et al., 2012).

An alternative approach to tackling this multi-player problem could be to replace the lower level of each owner's problem by its primal and dual constraints along with strong duality theorem equality to create a single-level problem for each player. This approach obviates the need to include complementarity conditions in reducing the bi-level problem to a single level. The KKT conditions of each owner's problem, thus, can be taken and combined to create one grand problem. This requires that for the lower-level problem, the primal (dual) problem has a finite optimal solution, which infers that the dual (primal) problem does, as well. Allevi et al. (2018) used this approach in solving a cement production problem formulated as an EPEC. A similar approach was employed by Ruiz et al. (2012) and Shahmohammadi et al. (2018) in the context of power generation. Unfortunately, the strong duality theorem equality includes bilinear terms, and thus, again constraint qualifications are not necessarily satisfied to have meaningful KKT conditions. In other words, the KKT conditions will provide only stationary solutions of the EPEC, and among these solutions one may find no, a single, or multiple equilibria. This issue is recognized in prior works by Allevi et al. (2018), Ruiz et al. (2012) and Shahmohammadi et al. (2018). Thus, solving this EPEC by transforming it into a single grand problem, while academically interesting, is not practical.

An alternative is to use a heuristic. Koh (2012) proposed a differential evolution algorithm for obtaining solutions for general EPECs. This heuristic makes use of an earlier proposed method by Lung and Dumitrescu (2008) for obtaining Nash equilibria of multi-player normal form games. Taking the game theoretic perspective, the EPECs can be considered as generalized non-cooperative Nash equilibrium problems (Facchinei and Kanzow, 2007). Koh (2012) used this idea to find the solution to an EPEC by finding the Nash equilibrium of the equivalent non-cooperative game employing the concept of Nash dominance previously introduced by Lung and Dumitrescu (2008). He implemented this approach using a differential evolutionary algorithm and tested the proposed algorithm on small examples for transportation and electricity generation problems. Herein, this heuristic was implemented on a very small, deterministic hypothetical application of the Crowdsourced Event Parking Market Pricing Problem. Convergence to equilibrium prices was not observed, however. This is likely due to the heuristic nature of evolutionary algorithms, which like other heuristics are designed to find good, but not necessarily optimal solutions. Achieving Nash equilibrium within the steps of the algorithm highly depends on the selection of the initial population, diversity, and creation of children to possibly move to the next generation. That is, a non-dominated strategy profile (vector of prices) for a population does not necessarily meet the criteria of a Nash equilibrium. The technique was ultimately abandoned as it did not perform much better than "searching in the dark."

Another approach to solving EPECs is to use the diagonalization method (see Gabriel et al., 2013 for additional background). In a single iteration, this technique cycles over owners who determine unilaterally their optimal prices assuming fixed prices set by all other owners. The technique iterates over owners until convergence is achieved. If successful, an equilibrium across owners will be reached where no owner can unilaterally change her parking prices and obtain a higher revenue.

Alas, the diagonalization method using a gradient ascent methodology to solve the individual time-differentiated, stochastic MPEC associated with each owner within each iteration is adopted. A Monte Carlo sampling method is employed to create a set of scenarios in a preprocessing step. This approach not only successfully converges for realistic size problem instances, but does so while accounting for supply- and demand-side uncertainties, as well as time differentiated pricing options. Earlier versions of this proposed methodology involved the use of a genetic algorithm for solution of each owner's problem; however, this technique did not converge, because exact solution is required to obtain an equilibrium set of prices over all owners' problems. The proposed method via scenario sampling, gradient ascent and overarching diagonalization is presented next.

4.1. Diagonalization technique

The proposed diagonalization technique simultaneously, although in an iterative methodology, solves each parking owner's stochastic MPEC given fixed prices for all other owners, iterating until convergence in the prices is achieved. To solve the stochastic MPEC of each owner, a set of samples, $\omega \in \Omega$, is generated, each representing a possible realization of the random supply (number of available spaces) and demand parameters, and each with equal occurrence probability, P_ω. Each stochastic MPEC is solved with the use of a gradient ascent method. Steps of the proposed algorithm are described in detail next.

Algorithm: Diagonalization

1. Initialization:

a. Set convergence criterion ϵ, maximum number of iterations N and relaxation parameter θ.
b. Randomly generate each leader's pricing decisions $p^0_{t,j}$ for all $t \in T, j \in J$.
c. Set $n = 1$

2. Solve the set of stochastic MPECs:

For all $j \in J$ do:

a. Use **algorithm gradient ascent** to solve the stochastic MPEC associated with each owner j and obtain the optimal solution $\left(p^*_{t,j}\right)_{\forall t \in T}$ given current optimal decisions of all other owners, $\left(p^n_{t,l}\right)_{t \in T, l \in J, l \neq j}$.
b. Update owner j's current best pricing decisions:

$$p^n_{t,j} = \theta \cdot p^*_{t,j} + (1-\theta) \cdot p^{n-1}_{t,j} \forall t \in T$$

3. Convergence:

If $\left|p^n_{t,j} - p^{n-1}_j\right| < \epsilon \; \forall t \in T, j \in J$ or $n = N$:

stop and report $\left(p^n_{t,j}\right)_{\forall t \in T, j \in J}$ as the set of optimal parking prices.

Else:

$n = n+1$ and go to step 2.

In this algorithm, the best solution is updated using relaxation parameter θ (step 2.b). It was found that including θ can help algorithm convergence.

The stochastic MPEC of each player, derived from equations (3), (4), and (17)-(20), is nonconvex. Thus, an off-the-shelf solver cannot be used to solve it directly. An option could be to solve it by a technique such as manifold suboptimization as in (Lawphongpanich and Yin, 2010). Here an iterative solution technique based on the gradient ascent algorithm (Cauchy, 1847) is proposed, details of which are included in the steps of algorithm gradient ascent. The algorithm starts from an initial solution and moves iteratively toward an improved solution using gradient information and a decaying learning rate, λ, until convergence is observed. Solution of equations (3) and (4) (prices set in the upper-level problem) requires knowledge of customer response to price settings obtained from solution of equations (17)-(20) (demand for spaces at the lower level), and demand for spaces in the lower level depend on prices set in the upper level. An iterative approach between decisions in upper- and lower-levels is taken. In each step, solution of the lower-level problem takes as input the most recent values of upper-level decisions. Convergence is achieved when demand and price settings stabilize. To prevent finding only locally optimal solutions, multiple starting points were employed. While convergence is not guaranteed, it is consistently achieved in the numerical runs. Multiple equilibria may exist and can be identified through runs with varying starting points.

Consider the stochastic MPEC of owner j. Beginning from an initial starting point $\left(p_{1,j},\cdots p_{t,j},\cdots,p_{|T|,j}\right)$, a price vector, and all other owner prices, the response (the demand) of lower-level users is obtained by solving (17)-(20). The optimal solution of the lower-level gives the demand for each parking location during each time period under each scenario. This demand is used to calculate expected revenue obtained from each parking location. It is also employed in numerical computation of the gradient needed for determining the direction of the next move (step 3a). That is, following a methodology of a gradient ascent method, the gradient of the upper-level objective function, π_j , is needed to determine the direction of the next move. π_j is computed by increasing price values by a small amount for the various time periods, t, for the target owner, and assessing the response of demand through solution of the lower-level problem. The lower-level problem is reduced to a set of stochastic (scenario-based), mixed-integer constraints, where constraints are applied under all scenarios $\omega\in\Omega$. This creates $|\Omega|$ mixed-integer programs (MIPs). For improved computational speed, these constraints can be solved in parallel.

After solving the lower-level MIPs, the numerical gradient can be calculated. The gradient, along with the updated learning rate, λ, are used to compute the direction and size of the next move. This is repeated until either termination criterion is met (step 4): (1) the maximum number of iterations is reached or (2) price changes (weighted by a diminishing learning rate) are trivially small. Note that solution of the lower-level problem is complicated by complementarity constraints. The problem, however, can be reformulated as an equivalent mixed-integer program enabling exact solution. This reformulation is provided in Section 4.3.

Algorithm: Gradient Ascent

1. **Initialization**:
   a. Set learning rate λ_1, convergence criterion ϵ, maximum number of iterations K, small neighborhood δ
   b. Take current $\left(p_{t,j}\right)_{\forall t\in T}$ for owner j and $\left(p_{t,l}\right)_{\forall t\in T,l\in J,l\neq j}$ for all other owners
   c. Set $k=1$ and $\left(p^k_{t,j}\right)_{\forall t\in T}=\left(p_{t,j}\right)_{\forall t\in T}$
2. **Gradient computation**:
   a. Solve the lower-level problem given upper-level prices $\left(p^k_{t,j}\right)_{\forall t\in T}$ and $\left(p_{t,l}\right)_{\forall t\in T,l\in J,l\neq j}$
   b. For each time $t\in T$ do:
      i. Solve the lower-level MIPs, each associated with a scenario, given $\left(p^k_{1,j},\cdots,p^k_{t,j}+\delta,\cdots,p^k_{|T|,j}\right)$ and $\left(p_{t,l}\right)_{\forall t\in T,l\in J,l\neq j}$ to obtain$h^*_{t,o,j}(\omega)$
      ii. Calculate revenues: $\pi_j\left(\left(p^k_{1,j},\cdots,p^k_{t,j}+\delta,\cdots,p^k_{|T|,j}\right)\right)=\sum_{\omega\in\Omega}P_\omega\sum_{t\in T}p^k_{t,j}\sum_{o\in O}h^*_{t,o,j}(\omega)$
   c. $\nabla\pi^k_j\left(\left(p^k_{1,j},\cdots,p^k_{|T|,j}\right)\right)=\left(\cdots,\frac{\pi_j\left(\left(p^k_{1,j},\cdots,p^k_{t,j}+\delta,\cdots,p^k_{|T|,j}\right)\right)-\pi_j\left(\left(p^k_{1,j},\cdots,p^k_{|T|,j}\right)\right)}{\varepsilon},\cdots\right)$
3. **Update prices**:
   a. $\boldsymbol{p}^{k+1}_j=\boldsymbol{p}^k_j+\lambda_k\cdot\nabla\pi^k_j$, where $\boldsymbol{p}^k_j=\left(p^k_{1,j},\cdots,p^k_{|T|,j}\right)$
   b. $\forall t\in T$: if $p^k_{t,j}<p^L$, then $p^k_{t,j}=p^L$ and if $p^k_{t,j}>p^U$, then $p^k_{t,j}=p^U$
4. **Termination**:

   If $\lambda_k\cdot\nabla\pi^k_j<\epsilon\forall \mathrm{t}\in \mathrm{T}$ or $k=K$:
   Stop and report $\left(p^k_{t,j}\right)_{\forall t\in T}$ as the optimal parking prices.
   Else:
   $\lambda_{k+1}=\frac{\lambda_k}{k+1}$, $k=k+1$, and go to step 2.

4.2. Transformation of Lower-Level problem

Complementarity constraints (17), (18) and (20) can be transformed into a set of mixed-integer, linear constraints using a well-known disjunctive constraints approach (Fortuny-Amat and McCarl, 1981). Through this approach, these equations are replaced by Eqs. (21)-(26):

$$0\leq h_{t,o,j}\leq M\cdot q_{t,o,j},\forall t\in T,o\in O,j\in J \tag{21}$$

$$0\leq \phi_{t,o,j}-u_{t,o}+\rho_{t,j}\leq M\left(1-q_{t,o,j}\right),\forall t\in T,o\in O,j\in J \tag{22}$$

$$0 \leq D_{t,o} \leq M \cdot k_{t,o}, \forall t \in T, o \in O \tag{23}$$

$$0 \leq D_{t,o} - \left(a_{t,o} - b_{t,o}u_{t,o}\right) \leq M\left(1 - k_{t,o}\right), \forall t \in T, o \in O \tag{24}$$

$$0 \leq \rho_j \leq M \cdot r_j, \forall t \in T, j \in J \tag{25}$$

$$0 \leq C_j - \sum_{t^{'}=1}^{t} \sum_{o \in O} h_{t^{'},o,j} \leq M\left(1 - r_j\right), \forall t \in T, j \in J \tag{26}$$

where $q_{t,o,j}$, $k_{t,o}$ and r_j are binary variables and M is a large positive number.

The collection of (21)-(26) with (19) constructs the lower-level problem of an owner *j*.

4.3. Time differentiated pricing and gradient ascent performance

A toy problem is constructed here to illustrate the steps of the gradient ascent methodology and how it permits time-differentiated pricing.

Consider Fig. 3 in which the revenue surface for owner 3 is depicted assuming the prices of all other parking locations of 10 are randomly generated and fixed. This graph is produced assuming two time periods for price differentiation. As shown in this graph, while the revenue function is not concave, the globally optimal solution can be obtained using several starting points.

Fig. 4 shows the contour map associated with this surface, as well as the outcome of moves taken in the steps of the gradient ascent method as indicated by arrows. Multiple starting points were used to avoid reaching only a local optimum or saddle point. $\left(p_1^L, p_2^L\right)$ serves as the initial starting point for the first iteration of the diagonalization method. All other starting points are randomly selected in this iteration. For the second iteration forward, the starting points will include the optimal solution from the prior iteration. Fig. 5 shows that there are instances where starting from the optimal solution from the previous iteration suffices for obtaining the optimal solution for that iteration. Thus, this strategy is taken in the implementation of the case study described in the next section.

Fig. 4 also illustrates the concept of time-differentiated pricing as employed herein. If an owner offers prices at reservation times 1 and 2 of \$5 and \$15, respectively, then many users will prefer this location to that of the competitors at time period 1. However, this parking location will have few empty spaces for customers to consider at reservation time 2. Since revenue is low for time period 1 due to the low price offering, and there are few remaining parking spaces to offer to customers at time period 2, the overall revenue for this owner will be small at \$1,433. If instead the parking owner were to charge \$15 and \$5 for reservation times 1 and 2, respectively, the total revenue will be much higher at \$3,015. Thus, the price set in period 1 affects opportunities for period 2. The proposed formulation and solution methodology explicitly account for this connection in optimal pricing across periods.

Fig. 5 provides the contour map of parking owner 3′s revenue over the 4 iterations of the diagonalization technique needed until convergence is observed. As shown in this figure, the optimal prices for use by owner 3 indicates stability from iteration 4, i.e. the revenue surface remains identical from recent prior iterations. The figure also illustrates that using the optimal solution from the previous iteration as the starting point in future iterations sufficed in terms of reaching the optimal solution, and despite that the function is not concave, the globally optimal solution is obtained.

5. Illustrative example

The proposed event parking problem along with the solution technique was tested on an illustrative example. The example consists

Fig. 3. Revenue of parking owner 3 given fixed prices for all other parking locations.

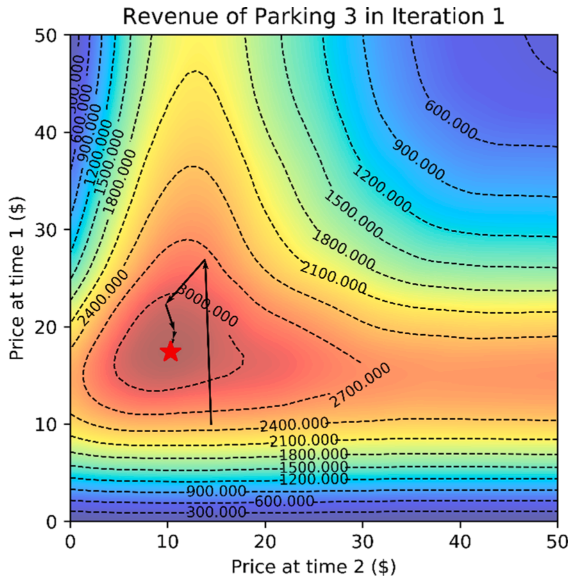

Fig. 4. Contour map of the revenue for parking owner 3.

of users who are planning to travel to an event from four different origin zones. They will park at one of 10 parking options. Parking locations 1 to 7 are public and include garages with capacities of between 100 and 500 vehicles, while parking locations 8 to 10 are crowdsourced parking spaces, each consisting of 30 or 50 parking spaces. Spaces at the same facility are priced identically. The available public parking, as well as personally owned parking locations, of this example are depicted in Fig. 6. The price of a parking space is a function of the space's disutility and the total reward the owner or group of owners represented by a consolidator can achieve. Personally-owned parking spaces that are geographically proximate to one another, with common driving and walking times to the venue, thus, can be treated as if they belong to a common parking location. This can be completed using any clustering algorithm during preprocessing. All spaces included in a cluster will have identical prices. Where it is desirable, the personally owned parking spaces can be further clustered by owner; however, this increases the number of players and thus the dimension of the problem. Since there are only one or two spaces available at each crowdsourced location, it is assumed that there is no delay due to crowdedness; that is, access times are common for each space within a group.

Table 1 shows the walking distances from each parking location to the event venue, as well as the parking capacities. The crowdsourced parking accounts for 7.1% of the zone's total parking capacity. Also, the driving distance from all 4 origins are shown in Table 2. Without loss of generality, it is assumed that driving distances from all origins to all parking locations are the same.

The demand function for each origin at each time is assumed to follow linear function:

$$H_{t,o} = a_{t,o} - b_{t,o} \cdot u_{t,o}, \forall t \in T, o \in O$$

$a_{t,o}$ and $b_{t,o}$ are assumed to follow a normal distribution with means 1,500 and 20 and standard deviations 200 and 3, respectively. Note that for illustration purposes, two time periods are considered here. Total demand for parking from all origins can be at most 12,000 on average (this is higher than total capacity which is 1,830). As demand is elastic, actualized demand is expected to be much lower than this maximum.

The proposed diagonalization was coded in Gurobi 8.1.1 using the Python interface. Numerical experiments were conducted on a compute node of a high-performance computing cluster named ARGO. A 2 GHz Intel Xeon Processor and 24 GB of RAM were used in the runs.

5.1. Numerical results

In this section, first the effectiveness of the proposed algorithm in finding equilibrium prices and achieving convergence is investigated. Then, numerical results are presented to discuss several aspects of the problem including sensitivity to changes in demand function parameters, value of considering uncertainty, and impact of crowdsourced parking options on the parking market.

5.1.1. Finding equilibrium prices and achieving convergence

Figs. 7 and 8 show the prices to be charged and revenue, respectively, at each parking location and time of reservation after completing 10 iterations of the diagonalization technique. These results for each parking location are given individually along their own lines. As shown in the figures, convergence is achieved after 7 iterations. These results are summarized in Table 3.

Note that since crowdsourced spaces are assumed to have identical disutilities and are priced here with the help of a consolidator, users will be indifferent between the individual (non-public) spaces.

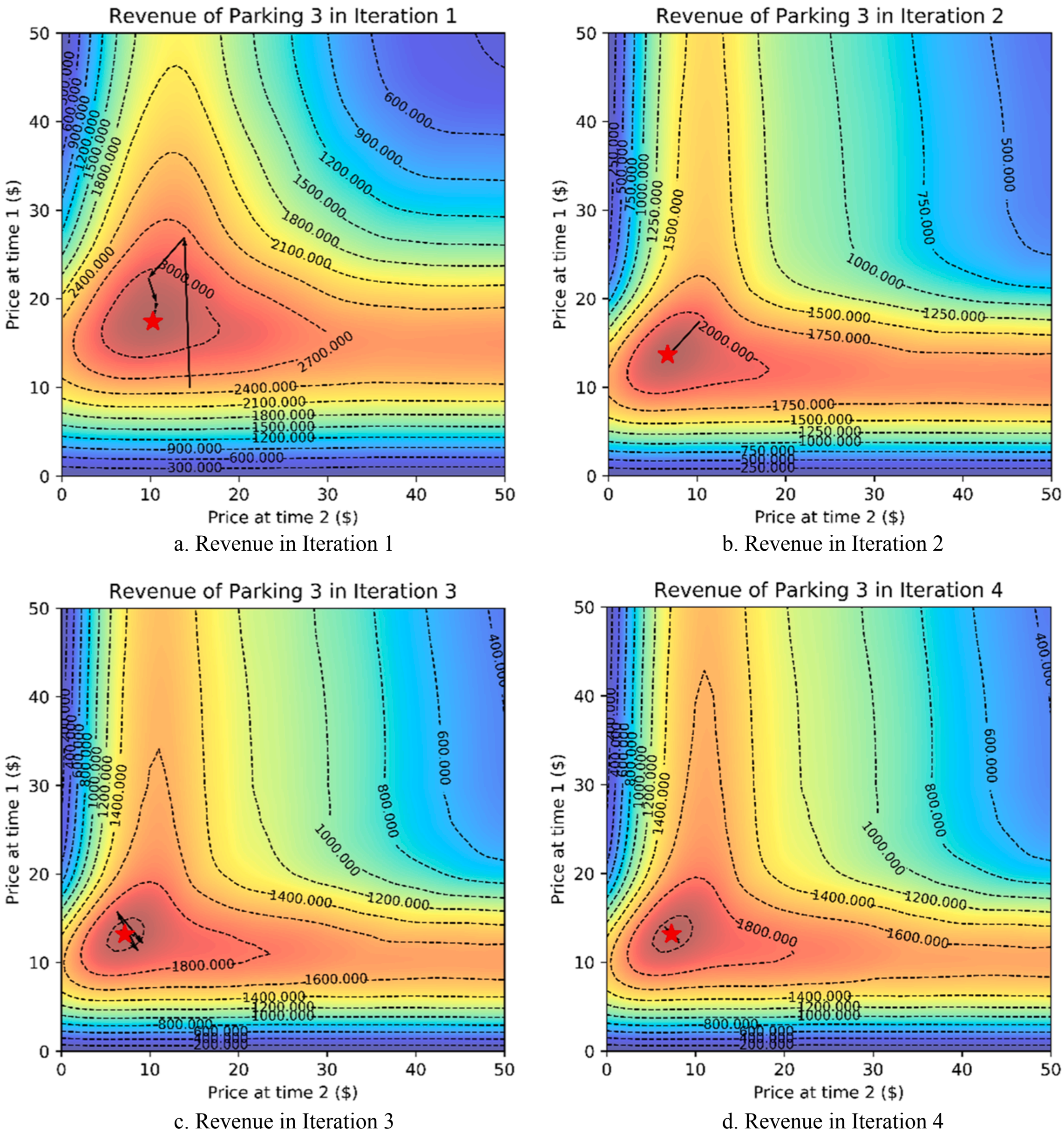

Fig. 5. a. Revenue in Iteration 1. Fig. 5b. Revenue in Iteration 2. Fig. 5c. Revenue in Iteration 3. Fig. 5d. Revenue in Iteration 4.

Fig. 9 shows the reservations made at time periods 1 and 2 and the empty spaces remaining at each parking location.

To check whether the parking prices as shown in Table 3 are the equilibrium prices, i.e. no parking owner can benefit by unilaterally deviating from his/her current price, the price of each parking location at each time is changed and the resulting revenue is calculated. Tables 4 and 5 show the resulting revenue for each parking space when the prices at each reservation time for that location is increased and decreased by 5%, respectively, while prices of all other parking locations remain fixed. The results show that any parking owner will lose revenue by changing his/her current prices, whether by increasing or decreasing the price. Moreover, the impact of increasing the price is more detrimental to revenue than is decreasing the price. Thus, Tables 4 and 5 show the reduction in revenue for each parking location if that owner charges a price that deviates from the equilibrium price by 5% when all other locations maintain their prices at the equilibrium price.

5.1.2. Sensitivity to changes in demand function parameters

Sensitivity analysis was performed to study the impact of parameters in the demand function, i.e. $a_{t,o}$ and $b_{t,o}$, on the prices and resulting revenue. Note that a higher value for $a_{t,o}$ means the potential demand (i.e. demand when disutility is 0) is higher, while a higher value of $b_{t,o}$ allows customers to be more sensitive to parking location disutility. Figs. 10 and 11 portray how the revenue of each

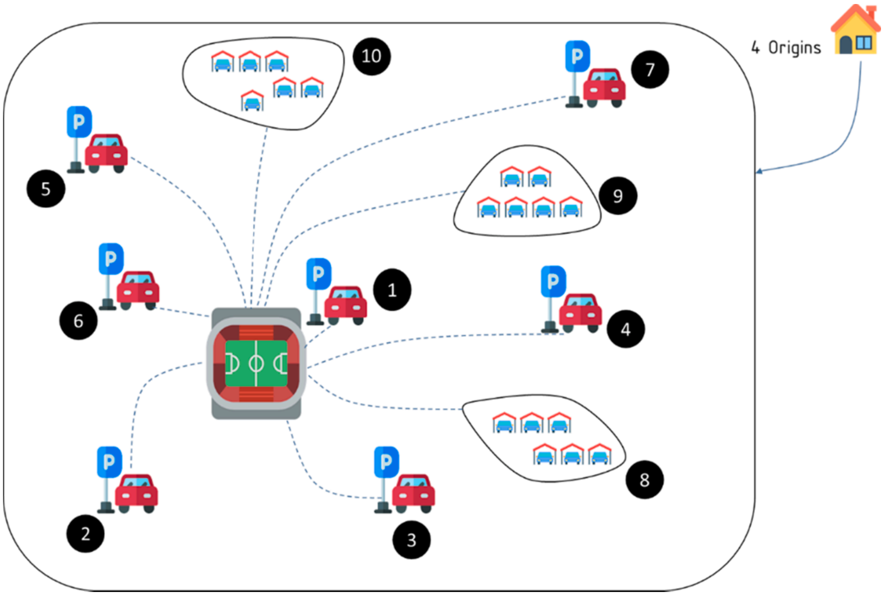

Fig. 6. Available parking locations.

Table 1
Characteristic of the parking locations.

	Parking number	Walking distance ($)	Capacity
Public parking	1	10	500
	2	10	300
	3	20	300
	4	20	200
	5	25	200
	6	25	100
	7	30	100
Crowdsourced parking	8	25	50
	9	30	50
	10	35	30

Table 2
Driving distance from each origin.

Origin	Driving distance ($)
1	20
2	15
3	25
4	30

parking location changes as the mean value of random variables $a_{t,o}$ and $b_{t,o}$, shown as $\overline{a}_{t,o}$ and $\overline{b}_{t,o}$, change, respectively. The results indicate that as $\overline{a}_{t,o}$ increases or $\overline{b}_{t,o}$ decreases (for all times t (two here) and origins o), the total market revenue increases substantially. This is expected, since at higher levels of demand, all locations can charge higher prices without losing many customers.

5.1.3. Value of considering uncertainty

Furthermore, to show the value of considering uncertainty, specifically randomness in demand-function parameters $a_{t,o}$ and $b_{t,o}$, results in terms of revenue are compared with additional runs that employ the average value of these parameters in a deterministic problem version. While presuming an average value may reduce the complexity of the problem, the numerical experiments show that ignoring this uncertainty results in an average of 9.3% and 22.9% reduction in revenue for public parking and crowdsourced parking locations, respectively. The impact is greatest for the crowdsourced parking locations, because the public parking locations can push the crowdsourced parking owners out of the market through reduced price offerings, since the crowdsourced parking locations are less attractive to customers due to longer walking times to the event locale.

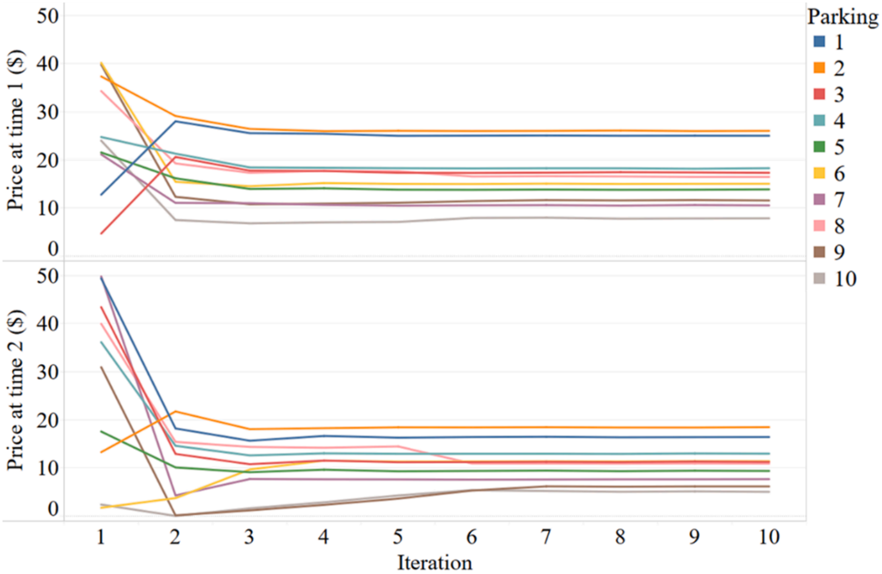

Fig. 7. Price of each parking location at each iteration.

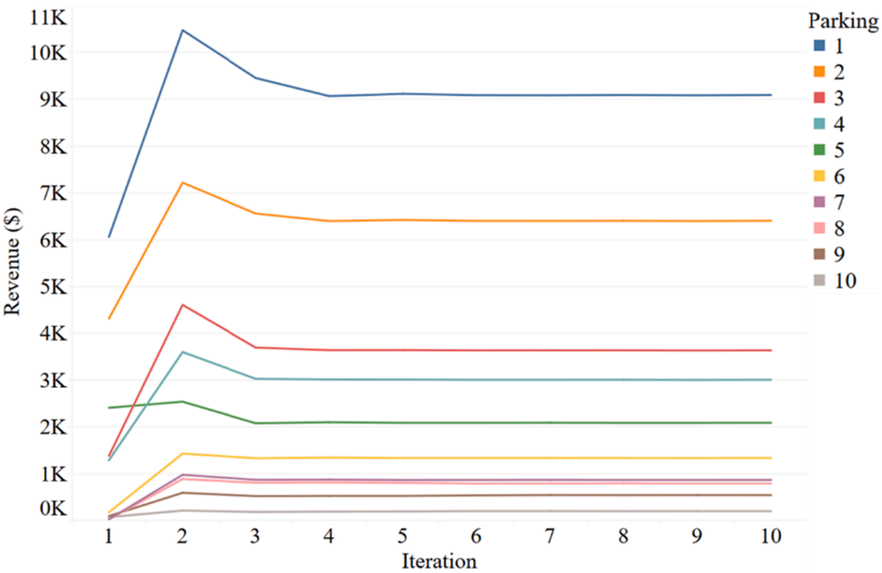

Fig. 8. Revenue at each parking location at each iteration.

Table 3
Parking prices at each time and revenue.

Parking	Price at Time 1 ($)	Price at Time 2 ($)	Total Revenue ($)
1	25.02	16.44	9,093.7
2	26.03	18.49	6,404.8
3	17.32	11.27	3,633.9
4	18.27	12.99	3,006.0
5	13.88	9.40	2,086.0
6	15.04	11.43	1,332.3
7	10.55	7.70	865.0
8	16.46	10.93	789.8
9	11.57	6.19	541.0
10	7.85	5.06	198.5

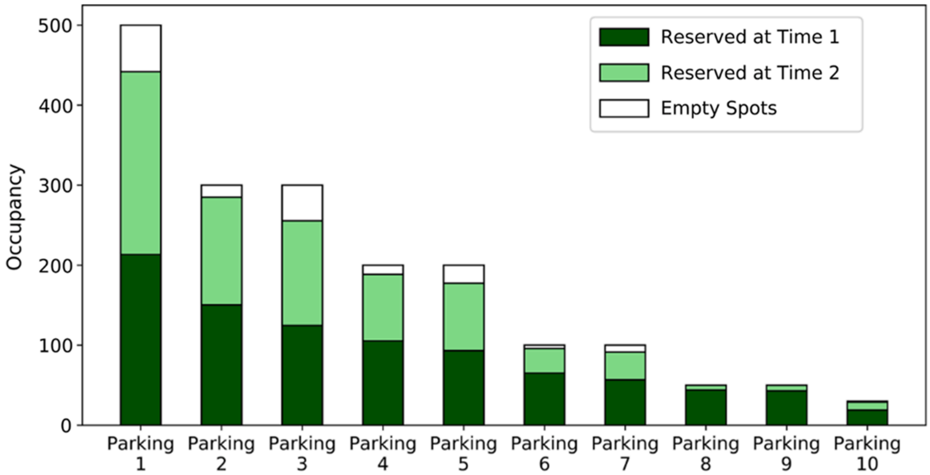

Fig. 9. Reservations made at time periods 1 and 2, and the remaining empty spaces.

Table 4
Revenue of each parking space when prices are increased unilaterally by 5%

Parking	Revenue at Equilibrium ($)	New Revenue ($)	Percentage Change (%)
1	9,093.7	7,461.1	−18.0%
2	6,404.8	4,881.1	–23.8%
3	3,633.9	3,082.1	−15.2%
4	3,006.0	2,423.7	−19.4%
5	2,086.0	1,788.5	−14.3%
6	1,332.3	1,128.0	−15.3%
7	865.0	770.5	−10.9%
8	789.8	761.7	−3.6%
9	541.0	540.0	−0.2%
10	198.5	187.1	−5.8%

Table 5
Revenue of each parking when prices are decreased unilaterally by 5%

Parking	Revenue at Equilibrium ($)	New Revenue ($)	Percentage Change (%)
1	9,093.7	8,286.5	−8.9%
2	6,404.8	5,772.2	−9.9%
3	3,633.9	3,354.7	−7.7%
4	3,006.0	2,784.7	−7.4%
5	2,086.0	1,951.8	−6.4%
6	1,332.3	1,283.4	−3.7%
7	865.0	836.0	−3.4%
8	789.8	771.0	−2.4%
9	541.0	529.9	−2.1%
10	198.5	189.7	−4.5%

5.1.4. *Impact of crowdsourced parking options on the parking market*

To investigate and quantify the impact of crowdsourced parking on the parking market, the number of crowdsourced parking spaces were considered at their current number and were also increased from zero to 650 in numerical experiments. Consumer surplus, total revenues and social welfare obtained for each such scenario are provided.

Consumer surplus is an economic measure of the difference between the highest price that consumers are willing to pay for a product (here disutility of parking including parking fees) and the price that they actually pay. Greater consumer surplus is beneficial to consumers, because they receive the same reserved parking space for less than they are willing to pay. The consumer surplus for users from origin o and reservation time t under scenario ω is depicted in Fig. 12. Given the probability of each scenario P_ω, the average total consumer surplus over all customers can be calculated as follows:

$$\text{Consumer Surplus} = \frac{1}{2}\sum_{\omega\in\Omega}P_\omega\sum_{t\in T}\sum_{o\in O}\left(u_{t,o}^{\max}(\omega) - u_{t,o}(\omega)\right)\cdot\sum_{j\in J}h_{t,o,j}(\omega) = \frac{1}{2}\sum_{\omega\in\Omega}P_\omega\sum_{t\in T}\sum_{o\in O}\left(\frac{a_{t,o}(\omega)}{b_{t,o}(\omega)} - u_{t,o}(\omega)\right)\cdot D_{t,o}(\omega)$$

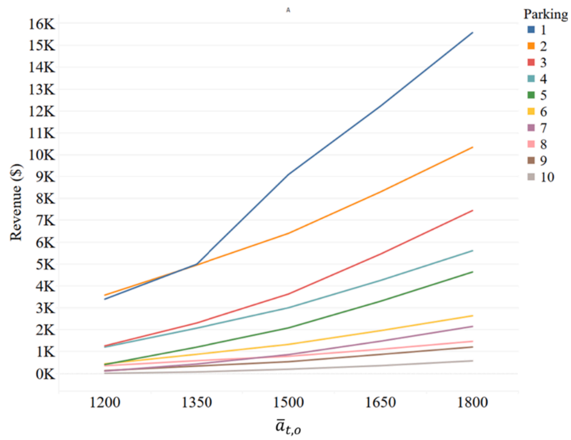

Fig. 10. Impact of $\overline{a}_{t,o}$ on the total revenue of the parking market.

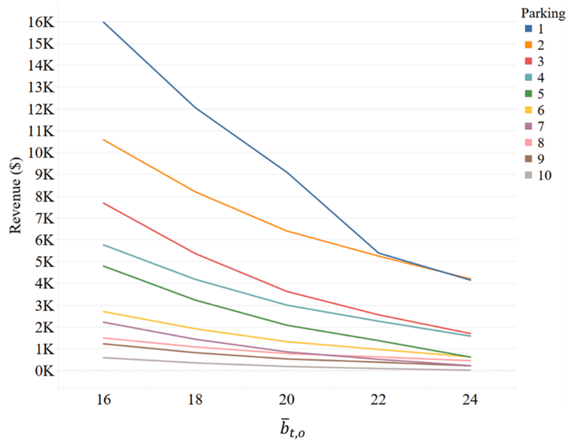

Fig. 11. Impact of $\overline{b}_{t,o}$ on the total revenue of the parking market.

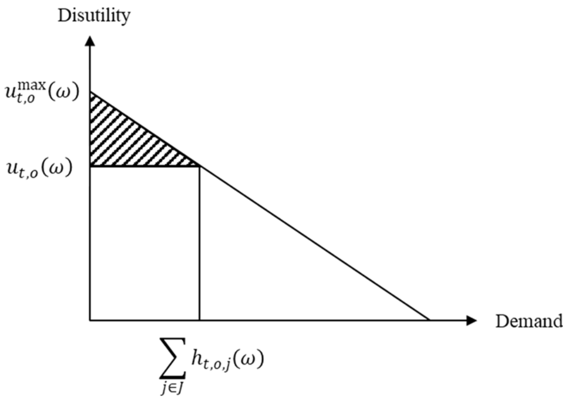

Fig. 12. Consumer surplus for users with origin o and reservation time t under a given scenario ω, where the hatched section indicates consumer surplus.

If the cost of parking supply to the owners is negligible, then the total revenue of the market will be equivalent to the producer surplus (revenue). Social welfare is the sum of consumer and producer surpluses.

As shown in Table 6, by increasing the number of crowdsourced parking spaces, total revenue (total producer surplus) for public parking owners decreases. However, average consumer surplus and social welfare increase, inferring that encouraging individuals to

Table 6
Consumer and producer surplus and social welfare for different penetration levels of crowdsourced parking spaces.

Number of Crowdsourced Parking Spaces	Percent Marketshare as function of number of spaces (%)	Total Consumer Surplus ($)	Total Public Parking Revenue ($)	Total Parking Market Revenue (producer surplus) ($)	Social Welfare ($)
0	0	17,517	27,886	27,886	45,403
130	7.1	19,300	26,422	27,951	47,251
260	13.3	19,901	24,528	27,672	47,573
390	18.7	21,622	23,117	27,286	48,908
520	23.4	23,289	21,928	26,932	50,221
650	27.7	24,321	21,877	26,669	50,990

Table 7
Total revenue of single decision maker vs. multiple decision makers to set parking prices.

Model	Consumer Surplus ($)	Total Parking Market Revenue (producer surplus) ($)	Social Welfare ($)
Multiple decision makers seeking to maximize own revenue	19,300.08	27,951.00	47,251.08
Single decision maker seeking to maximize total revenue	13,994.40	30,836.04	44,830.44

enter the event parking market as suggested herein has a positive overall impact. The potential to benefit society may have policy implications for local jurisdictions.

The results also show that by adding the crowdsourced parking locations to the proposed parking market, which at 130 spaces comprises 7.1% of the total parking capacity in the study zone, average parking prices drop by 15%, total revenue and users served increase by 0.23% and 7.2%, but revenue obtained by public parking locations decreases by 5.3%. Thus, the crowdsourced parking spaces have a positive impact on the parking market from a customer's perspective, and simultaneously, unsurprisingly, a negative impact on the revenue of public parking locations.

Table 7 shows what would happen if all the parking locations, including 130 crowdsourced parking spaces, are owned by a single company aiming to maximize its total revenue. As shown in the table, when competition is removed and the prices are set by a single company to maximize its total revenue, the total revenue of the parking market increases by 10.3% while consumer surplus and social welfare decrease by 27.5% and 5.1%, respectively. Thus, for the case study, competition benefits the whole market with increased total revenue.

6. Discussion and conclusions

In this paper, a parking pricing problem is proposed wherein the owners of small, crowdsourced parking locations with one or two parking spaces can participate in a parking market with larger, public facilities, whether privately or publicly owned. This problem reflects recent advancements in parking management technologies and consolidation opportunities. A stochastic EPEC formulation and exact approach for its solution are proposed. These methodologies were applied in numerical experiments on an illustrative example. Results of the experiments show that increasing the number of crowdsourced parking spaces and pricing them competitively can benefit the whole parking market as measured by social welfare. Ignoring stochasticity in demand parameters was found to work against all parking facility types through lost revenue. The results also show that competition between parking owners benefits the overall parking market in terms of social welfare. Social welfare was found to be 5.1% higher when parking owners compete for demand.

There are several future directions that can be pursued to extend the current study. Heterogeneous customers with differing utility functions can be modeled through a multi-user class extension. Additional disutility components may be included to capture distaste for parking at an individual's home and to reflect other personal preferences. Increasing the number of time periods for differentiating prices both nonlinearly increases the computational complexity of the problem and may create a likelihood of multiple equilibria and the possibility of choosing a suboptimal solution. Where more than two time-based price offerings are desired, it may be useful to develop an alternative solution approach. The model can be extended for dynamic pricing over multiple stages for real-time application, where occupancy and price are updated over time by embedding the proposed technique in a rolling horizon framework. A fast solution approach for this purpose, however, would likely require approximate equilibria solutions. Machine learning approaches, such as reinforcement learning, may be useful for this purpose.

CRediT authorship contribution statement

Hossein Fotouhi: Conceptualization, Formal analysis, Methodology, Software, Visualization. **Elise Miller-Hooks:** Conceptualization, Formal analysis, Methodology.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported by the U.S. National Science Foundation Grant 1823474. This support is gratefully acknowledged, but implies no endorsement of the findings.

Appendix

Derivation of KKT conditions

The upper-level problem formulated through equations (3) and (4) is constrained by the lower-level parking choice problem, which resembles a capacitated traffic user equilibrium problem with elastic demand. This formulation is presented next.

$$\min\sum_{o\in O}\sum_{j\in J}\int_0^{h_{o,j}} t_{o,j}(s)ds+\sum_{j\in J}\int_0^{f_j}\left(p_j(s)+d_j(s)\right)ds+\sum_{j\in J}\int_0^{f_j} w_j(s)ds-\sum_{o\in O}\int_0^{D_o} H_o^{-1}(s)ds, \tag{27}$$

s.t.

$$\sum_{j\in J}h_{o,j}=D_o,\forall o\in O, \tag{28}$$

$$f_j\leq C_j,\forall j\in J, \tag{29}$$

$$h_{o,j}\geq 0,\forall o\in O,j\in J$$

$$D_o\geq 0,\forall o\in O$$

This program is subject to the following definitional constraints:

$$\sum_{o\in O}h_{o,j}=f_j,\forall j\in J. \tag{30}$$

In this formulation, $t_{o,j}$ is the driving cost from origin o to parking j, p_j is the price of parking at location j, d_j is the crowdedness delay at location j, H_o is the demand function for origin $o\in O$.

Let u_o and ρ_j be the Lagrangean multipliers of the constraints (28) and (29), respectively. The Lagrangean of the problem can be stated as follows: $L(\boldsymbol{h},\boldsymbol{D},\boldsymbol{u},\boldsymbol{\rho})=\sum_{o\in O}\sum_{j\in J}\int_0^{h_{o,j}}t_{o,j}(s)ds+\sum_{j\in J}\int_0^{f_j}\left(p_j(s)+d_j(s)\right)ds+\sum_{j\in J}\int_0^{f_j}w_j(s)ds-\sum_{o\in O}\int_0^{D_o}H_o^{-1}(s)ds+\sum_{o\in O}u_o\left(D_o-\sum_{j\in J}h_{o,j}\right)+\sum_{j\in J}\rho_j\left(f_j-C_j\right)$ (31)

The Lagrangean function is constrained by non-negativity constraints on $h_{o,j}$ and D_o and constraints (30). Thus, the KKT conditions can be stated as follows:

$$0\leq h_{o,j}\perp\frac{\partial L(\boldsymbol{h},\boldsymbol{D},\boldsymbol{u},\boldsymbol{\rho})}{\partial h_{o,j}}\geq 0,\forall o\in O,j\in J, \tag{32}$$

$$0\leq D_o\perp\frac{\partial L(\boldsymbol{h},\boldsymbol{D},\boldsymbol{u},\boldsymbol{\rho})}{\partial D_o}\geq 0,\forall o\in O, \tag{33}$$

$$D_o-\sum_{j\in J}h_{o,j}=0,\forall o\in O, \tag{34}$$

$$0\leq\rho_j\perp\left(C_j-f_j\right)\geq 0,\forall j\in J. \tag{35}$$

Equations (32) and (32) can be further stated as constraints (36) and (37).

$$0\leq h_{o,j}\perp\left(\phi_{o,j}-u_o+\rho_j\right)\geq 0,\forall o\in O,j\in J, \tag{36}$$

$$0\leq D_o\perp\left(-H_o^{-1}(D_o)+u_o\right)\geq 0,\forall o\in O,j\in J, \tag{37}$$

where

$$\phi_{o,j} = t_{o,j} + p_j + d_j + w_j \forall o \in O, j \in J. \tag{38}$$

$t_{o,j}$ is the travel time from origin o to parking location j, and w_j is the walking time from the parking location j to the event location. d_j is the delay function that captures delay due to crowdedness in parking location j. d_j is a function of occupancy of parking location j, i.e. $d_j = e_j \cdot \sum_{o \in O} h_{o,j}$, where e_j is the parking location-specific constant. For simplicity, delays at crowdsourced parking locations are assumed to be zero, $d_j = 0$..

The demand function is assumed to be linear in form: $H(u_o) = a_o - b_o \cdot u_o$. $H(u_o)$is elastic with respect to the equilibrium disutility u_o. Parameters a_o and b_o correspond to the function's intercept and slope. These parameters can be estimated from parking price and occupancy data.

The lower-level problem, given parking prices from the upper-level, is equivalently (39)-(42).

$$0 \le h_{o,j} \perp \left(\phi_{o,j} - u_o + \rho_j\right) \ge 0, \forall o \in O, j \in J, \tag{39}$$

$$0 \le D_o \perp (D_o - (a_o - b_o u_o)) \ge 0, \forall o \in O, \tag{40}$$

$$D_o - \sum_{j \in J} h_{o,j} = 0, \forall o \in O, \tag{41}$$

$$0 \le \rho_j \perp \left(C_j - \sum_{o \in O} h_{oj}\right) \ge 0, \forall j \in J. \tag{42}$$

Establishing the convexity of the lower level program

To establish the convexity of the lower-level problem, a similar logic as applied in (Sheffi, 1985) for the establishing the convexity of a related UE formulation is applied here. For simplicity, scenario index ω is dropped.

Equality constraints (6) and (8) are affine, and the function in the inequality constraints (7), i.e. $\sum_{t'}^{t} f_{t',j} - C_j, \forall t \in T, j \in J$, is convex. Thus, the feasible region described by (6)-(8) is convex. To prove the convexity of problem (5)-(8), then, it need only be shown that the objective function (5) is convex. The objective function, $z(\boldsymbol{f}, \boldsymbol{D})$, is decomposed into two terms:

$$z(\boldsymbol{f}, \boldsymbol{D}) = z_1(\boldsymbol{f}) + z_2(\boldsymbol{D})$$

where, $z_1(\boldsymbol{f})$ and $z_2(\boldsymbol{D})$ are as follows:

$$z_1(\boldsymbol{f}) = \sum_{t \in T} \sum_{o \in O} \sum_{j \in J} \int_0^{h_{t,o,j}} t_{o,j}(s) ds + \sum_{t \in T} \sum_{j \in J} \int_0^{f_{t,j}} \left(p_{t,j}(s) + d_{t,j}(s)\right) ds + \sum_{t \in T} \sum_{j \in J} \int_0^{f_{t,j}} w_j(s) ds$$

$$z_2(\boldsymbol{D}) = -\sum_{t \in T} \sum_{o \in O} \int_0^{D_{t,o}} H_{t,o}^{-1}(s) ds$$

Assuming the crowdedness term $d_{t,j}$ is equal to $e_j \cdot f_{t,j}$. The first derivatives of $z_1(\boldsymbol{f})$ with respect to $f_{t,j}$ can be determined as follows:

$$\frac{\partial z_1(\boldsymbol{f})}{\partial f_{t,j}} = p_{t,j} + d_{t,j} + w_j$$

Thus, the second derivatives of the objective function are calculated as follows:

$$\frac{\partial^2 z_1(\boldsymbol{f})}{\partial f_{t,j} \partial f_{t',j'}} = \begin{cases} e_j & \text{for} t = t', j = j' \\ 0 & \text{otherwise} \end{cases}$$

The matrix of second derivatives, or the Hessian, with respect to $f_{t,j}$ has the following form:

$$\nabla^2 z_1(\boldsymbol{f}) = \begin{bmatrix} e_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e_j \end{bmatrix}$$

where $e_j \ge 0, \forall j \in J$. This Hessian matrix is positive semi-definite. Thus, $z_1(\boldsymbol{f})$ is convex with respect to $f_{t,j}$.

Next, consider $z_2(\boldsymbol{D}) = -\sum_{t \in T} \sum_{o \in O} \int_0^{D_{t,o}} H_{t,o}^{-1}(s) ds$. The demand function $H_{t,o}(u_{t,o})$ is monotonically decreasing in its argument. Thus, its inverse $H_{t,o}^{-1}(\cdot)$ is also a decreasing function. The integral of a decreasing function is also a strictly concave function, and the sum of several strictly concave functions is a strictly concave function. The negative of a strictly concave function is a strictly convex function. Thus, $z_2(\boldsymbol{D})$ is strictly convex with respect to $D_{t,o}$.

The sum of a convex function, i.e. $z_1(\boldsymbol{f})$, and a strictly convex function, $z_2(\boldsymbol{D})$, is strictly convex. Thus, $z(\boldsymbol{f}, \boldsymbol{D})$ is strictly convex with respect to $f_{t,j}$.

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