Affective Computing Model With Impulse Control in Internet of Things Based on Affective Robotics

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***Abstract*—The combination of Internet of Things (IoT) and artificial intelligence (AI) technology plays an important role in many fields, especially in the field of psychology and medical treatment. This work is mainly to study an affective robotics that can serve humans emotionally based on the IoT and AI technology. The design of affective robotics is important to understand the underlying mechanisms of human behaviors in real life. These mechanisms mainly include human nonverbal behaviors and affective states, which are important but difficult to be precisely modeled. To address this challenge, we introduce a human–robot interaction (HRI) architecture, including emotion recognition, affective computing, emotion diagnosis, and emotion control. First, we propose a system model based on HRI between affective robotics and human, in order to enhance the emotional service. Then, we develop a dynamical model with affective computing and control, where we provide a mathematical formulation method based on stochastic differential equations to quantify the emotional state. Furthermore, we perform the dynamic behavior analysis of the existence, boundedness, and stability of the model solution comprehensively. Numerical results are provided to demonstrate the validity and feasibility of the proposed design techniques.**

***Index Terms*—Affective computing, affective robotics, emotion control, human–robot interaction (HRI), Internet of Things (IoT).**

Manuscript received 31 August 2021; revised 12 March 2022; accepted 13 May 2022. Date of publication 19 May 2022; date of current version 24 October 2022. This work was supported in part by the National Science Foundation Project of China under Grant 61931001; and in part by the Fundamental Research Funds for the Central Universities under Grant QNXM20210038; in part by NSF under Grant IIS-1838024, Grant CNS-1950485, Grant CNS-2128368, and Grant CNS-2107216; and in part by the Scientific and Technological Innovation Foundation of Foshan, University of Science and Technology Beijing (USTB) under Grant BK20AF003. *(Corresponding author: Fuhong Lin.)*

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Digital Object Identifier 10.1109/JIOT.2022.3176323

I. Introduction

The Internet of Things (IoT) is the transmission and control of information between things and between people and things, allowing mutual collection and exchange of data [1], [2]. With the development of the IoT technology, many smart devices play an important role in people's production and life. In medicine, the IoT devices can be used for patient's perception, capture, measurement and transmission, analyze the patient's health status, and establish a continuous monitoring system [4]. In addition, artificial intelligence (AI) can quickly analyze patient behavior and discover profound insights while "learning" large amounts of IoT data. The IoT provides a large amount of data needed for AI learning, and AI transforms these data into meaningful real-time insights. The joint collaboration of AI and the IoT creates new value for organizations in a wide range of industries in health care and other industries. Medical interdisciplinary science based on AI plays an essential role in the development of human health diagnosis and treatment [5], [6]. It is predicted that by 2025, the AI-based medical industry will occupy one fifth of the market scale, reaching 127 billion dollars [7].

Compared with physiology, psychology is a more essential factor for human health and happiness in life. Affective robotics based on the AI technology will have a very broad application prospect in protecting human mental health and assisting human psychotherapy. Marino *et al.* [8] studied a human-assisted social robot, which mainly intervenes and regulates the emotions of children with autism spectrum disorders. Chen *et al.* [7] researched a wearable affective robotics for a wide population, which can improve human health on a spiritual level and give users a friendly emotional experience. Affective robotics are of great significance in providing services to special groups (such as patient with depression, and children with autism spectrum disorders). Based on the estimates by the World Health Organization (WHO), there are about 350 million patients with depression worldwide, which is becoming the second largest killer of humans after cancer [9]. With the development of affective robotics and AI, it is promising to let machines assist human beings in regulating emotions.

Many researchers have studied the problem of incorporating emotions into robots [10], where affective computing is the key technology of affective robotics. Emotion recognition in robots is a research field that uses affective computing to recognize, understand, and simulate human emotions, so that robots can

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recognize and synthesize human-like emotions and interact with humans naturally. The introduction of affective computing technology gives robots the ability to simulate emotions similar to humans, and can interact with people emotionally in human–robot interaction (HRI), thereby making the communication between humans and robots more natural [11]. The research of affective computing originated from [12], and then it has achieved some successful applications in the fields of HRI and affective robotics. However, current research results are far from satisfaction. These robots still have not reached the level of strong AI with perception and self-awareness. In other words, an affective robot can have strong computational and reasoning capabilities, but it is still unsatisfactory in terms of emotion.

For example, although the eye-catching Alphago has strong reasoning ability [13], it still feels like a cold machine, dull and monotonous. The root cause is that these studies understand emotions from a psychological level, do not understand the philosophical nature of emotions, and have not established mathematical models of emotion, nor does it know the change laws of emotion. Obviously, in order to realize the digitalization of emotions, a mathematical model of emotion must be established first [14], [15]. Therefore, establishing a mathematical model that can quantify emotions is a key technical challenge for the development of affective robotics. Some scholars have made preliminary explorations on the emotion modeling. Ta *et al.* [16] emphasized that the emotion modeling is necessary and basic if we want to understand emotions as natural as possible. They established an emotional contagion model for an emotion (fear), and studied the influence of three factors on the dynamics of this emotion.

Our work aims to study the essential laws of emotional evolution, use mathematical models to quantitatively describe the characteristics of the ups and downs of various emotions, and induce individuals' emotions to tend to a healthy and stable state through external therapy. This will be the core function of the affective robotics we designed, and also an important breakthrough in realizing the service provided by the robot for human emotions.

In this article, we propose an HRI architecture, including emotion recognition, affective computing, emotion diagnosis, and emotion control. Specially, we develop an affective computing model with impulse control to model the dynamic change between four emotions (i.e., happiness, anger, anxiety, and fear). Furthermore, the mathematical model of affective computing is endowed with affective robotics, so that it can simulate and demonstrate the change of the human emotional state, and then intervene and regulate human emotion, so as to realize the concept of the machine serving human emotion. Our main contributions are summarized as follows.

1) We proposed HRI emotional service architecture functions include emotion recognition, affective computing, emotion diagnosis, and emotion control, is given as follows.
   a) *Emotion Recognition:* Based on the emotion recognition technology of AI, affective robotics can monitor the host through IoT devices or directly interact with the host, and then identify the current host's emotional state.
   b) *Affective Computing:* After emotion recognition, affective robotics obtain emotional information of human. Then, the proposed an affective computing model with stochastic perturbations will simulate the law of human emotional changes. Using this model, we can predict how the human emotion changes and when the human emotion will reach a stable state.
   c) *Emotion Diagnosis:* Based on the emotion stable state obtained in the step of affective computing, emotion diagnosis could be used to judge whether this emotion state is healthily, which would further generate a health discrimination index to indicate whether medical treatment is needed. Specifically, each HRI is a psychological diagnosis, and the affective computing plays the role of a psychologist, giving the tester's emotional state good or bad judgment. In the HRI service proposed in this article, affective robotics can not only diagnose emotions, but also adopt appropriate strategies to control emotions.
   d) *Emotion Control:* If the affective robotics predict an unhealthy stable state that the host will achieve in a short or long term, then they will perform the control and adjustment of the predictive model, unless achieving a healthy and satisfactory stable state. Then the affective robotics converts the control strategy and intensity of the system into the suggested strategy, which will feed back to human for emotional adjustment. Ultimately, it can realize the AI serve for human as well as the human emotional essentially.
2) This model can achieve three goals. First of all, it can dynamically describe the evolution of the emotional state, and accurately calculate and predict the emotional trend based on the current emotional state. We divide the four emotions into positive emotion (happiness) and negative emotions (anger, anxiety, and fear), and we describe the transformation relationship between the four emotions with four differential equations. Second, considering that the individual's emotional fluctuations will inevitably be interfered by random factors, it is of great significance to use a quantitative calculation method to evaluate these random disturbances to mood fluctuations. We use stochastic differential equations to quantify this phenomenon, which can ensure emotional stability under random factors. Third, the model we built can perform proactive control of the trend of emotional state. In other words, different impulse controls are adopted to realize the effect of regulating and controlling emotions for positive emotions and negative emotions, respectively.

The remainder of this article is organized as follows. Section II reviews previous works in the area of affective robotics. Model development and dynamic analysis are given in Sections III and IV, respectively. The numerical simulations

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are carried out in Section V. Finally, conclusions are given in Section VI.

II. Related Work

With the development of IoT, AI, and HRI technologies, intelligent robots have achieved many successful applications in many fields, including healthcare, education, military, etc. [17]–[19]. The current level of development of robots has strong application value, with strong reasoning ability and the ability to perform simple tasks. But robots are still in their infancy in terms of emotions. The core AI technology of intelligent robots is still at the level of weak AI [20]. The strong AI view is that it is possible to create intelligent machines that can truly reason and solve problems. Such machines are considered to be sentient and self-conscious [21]. Understanding human social-emotional signals to enhance HRI is the core of affective robotics research, which is a challenging research topic [22], [23].

For special groups of people, the affective robotics plays the role of emotional service and emotional intervention, reflecting its value of serving human emotions. An affective robot, iCat, a cat-like robot that plays chess with children on an electronic chessboard, has shown a human-like emotion. They care about winning and losing as much as a human does, which is a typical example of human–computer interaction (HCI) experience [24]. Recent research studies have shown that robots with simple emotional expression capabilities are more popular with customers, and can bring higher participation and more pleasant HCI [25], [26]. Communication between people is emotional, so in the process of interacting with other human-like robots, people truly expect to achieve natural and emotional interactions [27]. Churamani *et al.* [28] emphasized that traditional machine learning cannot adapt well to the dynamic nature of this real-world interaction, because they need to obtain samples from a stable data distribution. Human social activities and emotional states are a continuous dynamic process, not static. The author proposes that this problem can be solved through incremental learning, and believes that continual learning is an effective way to create an affective robot that can promote the development of affective robotics. Fang *et al.* [29] proposed a model of personality and emotion; however, the author did not further formulate the evolution of the emotional state.

Emotion recognition and affective computing, as the key technology of affective robotics, has caused extensive research in academia in the past few years. Emotion recognition is the process of studying the information interaction between humans and machines. To achieve successful HRI, face recognition is a very important part of it, and the hidden information must be tapped. From the perspective of machines serving humans, affective computing studies whether the information technology can determine customer emotions, and embeds emotional elements into the information technology to make decisions from the customer's perspective [30]. Provide corresponding services according to different emotions of customers to change the traditional information technology decision-making method [12].

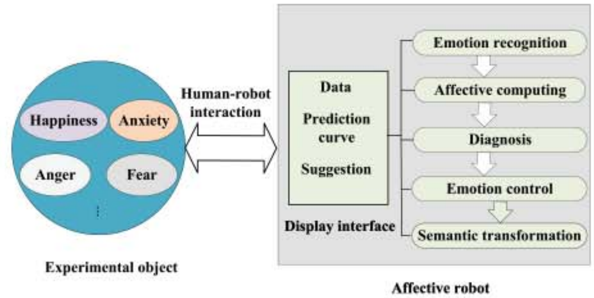

Fig. 1. System-model of HRI.

To break through the technical bottleneck of affective robotics, it is necessary to establish a mathematical model that can describe the changing laws between emotional states. Since the qualitative analysis of psychology cannot meet the needs of strong AI, it is necessary to introduce a quantitative emotional mathematical model. Human beings will be subconsciously affected by the emotions of other individuals in social activities, and ultimately affect their own emotional experience. The field of psychology calls this process emotional infection [31]. Some scholars draw on the classic theories of epidemic models [32] to establish the emotional contagion model. Ta *et al.* [16] established an emotional contagion model and studied the dynamic behavior of the affective model, which provides a new idea for the mathematical modeling of emotions. The author's main concern is an emotion related to fear and its dynamic behavior in several specific scenarios under the influence of three external factors. Their conclusion is based on the simulation platform experiment, not on the result of theoretical analysis of the model.

Analyzing and understanding human nonverbal behaviors, and making appropriate responses are the main goals of affective robot research [28]. Existing works rarely involve emotional control, due to the lack of mathematical models for affective computing and control. In this article, we establish a dynamic model of the evolution of the four emotions, including happiness, anxiety, anger, and fear, in order to find a more universally applicable theoretical method of affective computing and control, and then complete a new and complete HRI process. The ultimate goal of affective robotics is to realize the idea that machines serve human emotions. This work aims to establish an affective computing of the evolution of a single individual's emotional state from a micro perspective, and provide a formalized mathematically method for studying the evolution of emotions. Give affective robotics the theoretical calculation method of emotional changes and control to realize the HRI in which the machine serves human emotions.

III. Problem Description and Affective Computing Model

To achieve a more natural HRI model and improve the emotional service capabilities of affective robotics, a key challenge is to establish a quantifiable affective computing model, and design a complete HRI process that can complete emotional services. In this article, we propose a complete HRI process, as shown in Fig. 1. In the process of HRI, affective

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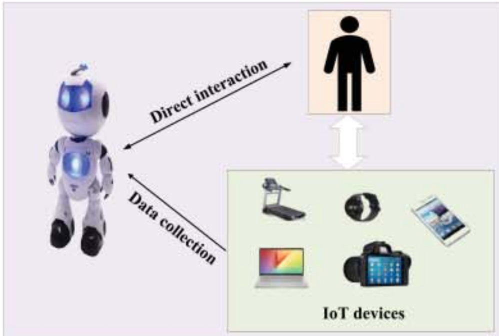

Fig. 2. Data collection based on IoT devices.

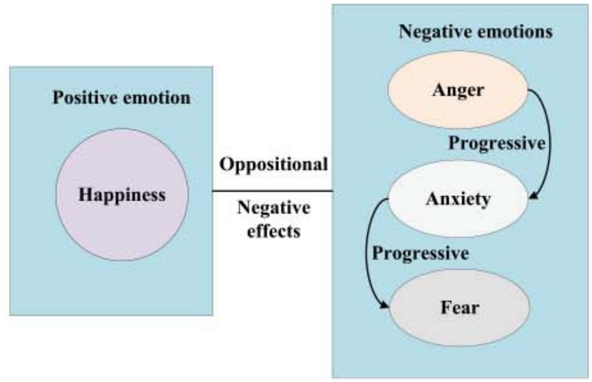

Fig. 3. Transformation of the four basic human emotions.

robotics first recognize the current emotional state of human by emotion recognition. In order to accurately identify the experiment subject's current emotional state, some auxiliary IoT devices need to be deployed for comprehensive data collection, such as smart watches, smart treadmills, smart phones, etc., which is show in Fig. 2. The affective robotics analyzes and learns the experiment subject's massive data from the IoT device and obtains its current emotional state based on a specific AI algorithm. The current emotional state is input into the affective computing module as an initial condition. This module uses an affective computing model to calculate the four emotional components and predict the emotional state in a future period of time. Afterward, the emotional diagnosis module gives a comprehensive evaluation of the emotional health status based on the results of the affective computing. The emotional control module is an impulsive intervention to the affective computing model to achieve the purpose of controlling the trend of emotional changes. Without impulse intervention, the affective computing model can calculate and predict only the subject's emotional change trend, but does not have the function of changing the emotional change, even if the subject's emotion is developing in an unhealthy or even dangerous state. Therefore, an important functionality of the emotion control module is to correct the changing trend of emotions to achieve a healthy and stable state through appropriate impulse control.

Finally, the system inputs the control strategy into the semantic conversion module. In fact, the control strategy involved in the emotion control module is the mathematical method of impulse control for the affective computing model. In the transformation module, it is necessary to transform this quantitative impulse control into an operable psychology-based emotion control strategy. The semantic transformation module is mainly to transform abstract impulse control strategies into concrete operable emotion regulation suggestions. For example, a positive pulse adjustment can be translated into suggestions that the subject listen to a song and exercise for half an hour, etc. The calculation results of the four modules will be shown in the display interface in terms of data, graph, and policy semantics. During this process, affective robotics can be used to recognize, calculate, diagnose, control, and suggest strategies for human emotions. In HRI, it plays the role of psychological doctor, fulfilling the idea that robots serve human emotions.

Next, we will introduce the affective computing model [Section III-A] and emotional control model [Section III-B] established in this article, which are the most important design in the HRI process.

A. Affective Computing Model

In detail, according to modern emotion theory [33], [34], the human emotional state includes four basic emotions: 1) happiness; 2) anger; 3) anxiety; and 4) fear, as other complex emotional examinations are the result of the integration of the four basic emotions. The four emotional states are caused by the stimulus of an external event. The affective robot collects data about facial expressions, posture expressions, intonation expressions, behaviors, etc., and then uses a specific measurement method to calculate the individual's active energy for each emotion at the moment. Suppose the affective robot measures the emotional state of individual at t through emotion recognition as $X(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^T$, where $x_1(t)$, $x_2(t)$, $x_3(t)$, and $x_4(t)$ represent the evaluation of the emotional state of the four basic emotions.

According to the famous theory of the 2-D Arousal-Valence model, that affective states are not independent of one another, but are related to each other in a highly systematic fashion [35]. In order to establish an accurate model to describe the evolution of these four emotions, we first analyze their inherent attributes according to modern emotion theory. We classify happiness as a positive emotional state, which is good for our health. On the contrary, we define anger, anxiety, and fear as negative emotional states, which are bad for our health. Among them, happiness is antagonistic to the other three emotions (similar to competition between species) and has a negative effect on each other. Anger, anxiety, and fear are three emotions that are increasingly harmful to the human body. The transformation is from anger to anxiety, and from anxiety to fear, as shown in Fig. 3.

These relationships above are analogous to the interactions between different populations in the biological world. In the population dynamics model, the differential equations are used to describe the relationship between the multispecies (cooperation, competition, and predation). Based on the mathematical modeling principle of population ecology, and according to the relationship between the generation and transformation of four

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basic affective states, we establish the emotional model of the affective robot for describing the transformation mechanism among the four basic affective states quantitatively.

Specifically, we establish the following model for the emotional evolution of individual:

$$\begin{cases} \mathrm{d}x_1(t) = x_1(t)[b_1 - a_{11}x_1(t) - a_{12}x_2(t) \\ \qquad -a_{13}x_3(t) - a_{14}x_4(t)]\,\mathrm{d}t \\ \mathrm{d}x_2(t) = x_2(t)[b_2 - a_{21}x_1(t) - a_{22}x_2(t) - a_{23}x_3(t)]\,\mathrm{d}t \\ \mathrm{d}x_3(t) = x_3(t)[b_3 - a_{31}x_1(t) + a_{32}x_2(t) \\ \qquad -a_{33}x_3(t) - a_{34}x_4(t)]\,\mathrm{d}t \\ \mathrm{d}x_4(t) = x_4(t)[b_4 - a_{41}x_1(t) + a_{43}x_3(t) - a_{44}x_4(t)]\,\mathrm{d}t \end{cases} \tag{1}$$

where b_i represent the self-production coefficient of four basic emotion valuation $x_i(t)$ $(i = 1, 2, 3, 4)$, which means the self-increasing intensity of the discrete emotions in the current environment. a_{ij} $(i, j = 1, 2, 3, 4)$, and $i \neq j$), respectively represent the interaction coefficient of emotion i to emotion j as well as emotion j to emotion i. a_{12}, a_{13}, a_{14}, a_{31}, a_{31}, and a_{41} represent the antagonistic relationship between the positive emotion, and the three negative emotions. a_{ii} $(1 \leq i \leq 4)$ represents the self-inhibition coefficient of emotion i, which means the individual has self-inhibition to any emotion in a certain range. a_{23}, a_{32}, a_{34}, and a_{43} denote the progressive transfer coefficient of three negative emotions, which are transferred in one direction from small to large according to their harmfulness (hence, $a_{24} = a_{42} = 0$). All the coefficients in the model are positive and normalized. Based on this model, we can calculate that there is a positive equilibrium $X^* = (x_1^*, x_2^*, x_3^*, x_4^*)$ of the model in (1), where

$$x_1^* = \frac{D_1}{D}, \quad x_2^* = \frac{D_2}{D}, \quad x_3^* = \frac{D_3}{D}, \quad x_4^* = \frac{D_4}{D}, \quad (D \neq 0) \tag{2}$$

and

$$\begin{aligned} D = &-a_{12}a_{23}a_{34}a_{41} - a_{12}a_{21}a_{34}a_{43} + a_{11}a_{22}a_{34}a_{43} \\ &- a_{14}[a_{23}a_{32}a_{41} + a_{21}a_{32}a_{43} + a_{22}(a_{33}a_{41} + a_{31}a_{43})] \\ &+ a_{12}a_{23}a_{31}a_{44} + a_{11}a_{23}a_{32}a_{44} \\ &- a_{12}a_{21}a_{33}a_{44} + a_{11}a_{22}a_{33}a_{44} \\ &+ a_{13}[-a_{21}a_{32}a_{44} + a_{22}(a_{34}a_{41} - a_{31}a_{44})] \\ D_1 = &-[a_{14}a_{32}a_{43} + a_{13}a_{32}a_{44} + a_{12}(a_{34}a_{43} + a_{33}a_{44})] \\ &\cdot b_2 + a_{22}[-(a_{14}a_{43} + a_{13}a_{44})b_3 \\ &\quad + a_{34}(a_{43}b_1 + a_{13}b_4) + a_{33}(a_{44}b_1 - a_{14}b_4)] \\ &+ a_{23}[a_{32}(a_{44}b_1 - a_{14}b_4) + a_{12}(a_{44}b_3 - a_{34}b_4)] \\ D_2 = &-[a_{14}(a_{33}a_{41} + a_{31}a_{43}) + a_{13}(a_{34}a_{41} - a_{31}a_{44}) \\ &\quad + a_{11}(a_{34}a_{43} + a_{33}a_{44})]b_2 \\ &+ a_{23}[(a_{14}a_{41} - a_{11}a_{44})b_3 + a_{34}(-a_{41}b_1 + a_{11}b_4) \\ &\quad + a_{31}(a_{44}b_1 - a_{14}b_4)] + a_{21}[(a_{14}a_{43} + a_{13}a_{44})b_3 \\ &\quad - a_{34}(a_{43}b_1 + a_{13}b_4) + a_{33}(-a_{44}b_1 + a_{14}b_4)] \\ D_3 = &-[a_{14}a_{32}a_{41} + a_{11}a_{32}a_{44} + a_{12}(-a_{34}a_{41} - a_{31}a_{44})] \\ &\cdot b_2 + a_{22}[(-a_{14}a_{41} + a_{11}a_{44})b_3 + a_{34}(a_{41}b_1 - a_{11}b_4) \\ &\quad + a_{31}(-a_{44}b_1 - a_{14}b_4)] + a_{21}[a_{32}(-a_{44}b_1 - a_{14}b_4) \\ &\quad + a_{12}(-a_{44}b_3 + a_{34}b_4)] \\ D_4 = &-a_{21}a_{32}a_{43}b_1 + a_{13}a_{32}a_{41}b_2 + a_{12}a_{31}a_{43}b_2 \\ &+ a_{11}a_{32}a_{43}b_2 - a_{12}a_{21}a_{43}b_3 - a_{12}a_{21}a_{43}b_3 \\ &- a_{13}a_{21}a_{32}b_4 - a_{12}a_{21}a_{33}b_4 \\ &+ a_{22}[(a_{13}a_{41} + a_{11}a_{43})b_3 \\ &\quad - a_{33}(a_{41}b_1 + a_{11}b_4) - a_{31}(a_{43}b_1 - a_{13}b_4)] \\ &+ a_{23}[a_{32}(-a_{41}b_1 + a_{11}b_4) + (-a_{41}b_3 - a_{31}b_4)]. \end{aligned} \tag{3}$$

The individual's emotion is inevitably affected by the external stochastic environment, which includes elation, disappointment, prosperity, and adversity. In this article, we use the White Gaussian noise to quantify stochastic perturbations, and assume that stochastic perturbations are mainly represented by the self motivation coefficient b_i of basic emotions, namely

$$b_i \to b_i + \sigma_i \mathrm{d}B_i(t), \quad i = 1, 2, 3, 4 \tag{4}$$

where $B_i(t)$ are independent standard Brownian motions with $B_i(0) = 0$, σ_i indicates the intensity of the stochastic perturbation. Throughout this article, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). Therefore, the evolution rule of affective robot's emotion with stochastic perturbation can be expressed as the following stochastic differential equation model (SDE), i.e.,

$$\begin{cases} \mathrm{d}x_1(t) = x_1(t)[b_1 - a_{11}x_1(t) - a_{12}x_2(t) - a_{13}x_3(t) \\ \qquad -a_{14}x_4(t)]\,\mathrm{d}t + \sigma_1 x_1(t)\,\mathrm{d}B_1(t) \\ \mathrm{d}x_2(t) = x_2(t)[b_2 - a_{21}x_1(t) - a_{22}x_2(t) - a_{23}x_3(t)]\,\mathrm{d}t \\ \qquad +\sigma_2 x_1(t)\,\mathrm{d}B_2(t) \\ \mathrm{d}x_3(t) = x_3(t)[b_3 - a_{31}x_1(t) + a_{32}x_2(t) - a_{33}x_3(t) \\ \qquad -a_{34}x_4(t)]\,\mathrm{d}t + \sigma_3 x_1(t)\,\mathrm{d}B_3(t) \\ \mathrm{d}x_4(t) = x_4(t)[b_4 - a_{41}x_1(t) + a_{43}x_3(t) - a_{44}x_4(t)]\,\mathrm{d}t \\ \qquad +\sigma_4 x_4(t)\,\mathrm{d}B_4(t). \end{cases} \tag{5}$$

B. Emotional Control Model

Psychological research shows that: both positive emotion and negative emotion can be controlled, not only can increase a certain emotion, but also can reduce a certain emotion, as well as maintain a certain emotion [36]. In order to maintain the emotional stability of the individual, it is necessary to monitor and control the emotional state of the system at any time as well as control the overload when the individual is faced with collapse. It includes active stimulation for positive emotion and inhibition for negative emotion. By adding impulse control to the emotion model, we get a controlled emotion model, which can be expressed as model (6), shown at the bottom of the next page. Among them, α_k, β_k, γ_k, and λ_k, respectively, represent the regulation and control proportion of four basic emotions. Each component of a vector $Y = (\alpha_k, \beta_k, \gamma_k, \lambda_k)$ is a nonnegative number, and $Y = (\alpha_k, 0, 0, 0)$ means we only regulate positive emotions $x_1(t)$. Obviously, for positive emotions we use positive control of stimulation (music regulation and food stimulation), for negative emotions we use negative control of inhibition (exercise and distraction). Assign the model (6) to emotional control module of the affective robot.

In the next, we will focus on the dynamic behavior of the model (6) to describe the emotional evolution of individual, as well as the effect of external stochastic perturbation and

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control on the emotional evolution of individual. In fact, if the model (6) is not controlled, when $Y = (0, 0, 0, 0)$, the stochastic impulse model (6) will evolve into a stochastic model (5). Furthermore, it is assumed that in an ideal environment, the emotional state of the affective robot would not be affected by any stochastic perturbation, i.e., $\sigma_i = 0$, and that the stochastic impulse model (6) would evolve into a deterministic model (1). Accordingly, the model (1) and (5) is only a special case of the model (6). For the model (1), there must be a gradually stable state. Based on the above analysis, we need only the comprehensive analysis model (6). In the phase of numerical simulation, the dynamic behaviors of the three models are compared comprehensively.

IV. Model Analysis

In the previous section, we have established an HRI process architecture and a core affective computing model. The function of this model is to combine the current emotional state of the tested individual to predict and calculate the subsequent emotional trend. Before the affective robot completes an emotional service, the following three problems need to be discussed.

1) For a tested individual, given his/her initial emotional state, is it possible to calculate the unique emotional trend?
2) When we perform impulse control on the emotional trend of the tester, can we ensure that the emotional state has an upper bound and a lower bound?
3) Whether there is a stable trend in the fluctuation of emotional state, showing regularity, and periodicity?

These three problems are important attributes of the affective computing model, which we will discuss in detail in the following. Next, we will mainly analyze the dynamic behavior of the model, including the existence and stochastic boundedness [Section IV-A], strong persistence [Section IV-B], and global attractivity of solutions [Section IV-C].

A. Global Positive Solutions and Stochastic Boundedness

Considering the practical significance of individual emotion, before studying the dynamic behavior of the model (6), we first consider the existence of the global positive solution of the model in (6). Note that the model in (6) are system of impulsive stochastic differential equations (ISDEs), for completeness, we refer to the literature [37] for the definition of solution of ISDE, and we first give the definition of the solution for the model in (6).

Definition 1: A stochastic process $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^T$, $t \in [0, +\infty)$ is said to be a solution of ISDE (6) with the initial value $x(0) = x_0 \in \mathbb{R}_+^4$, if $x(t)$ satisfies the following.

1) $x(t)$ is $\mathcal{F}_t$ adapted and is continuous on $(0, t_1)$ and each interval $(t_k, t_{k+1}) \subset \mathbb{R}_+$, $k \in N$ and $F(t, x(t)) \in L^1(\mathbb{R}_+, \mathbb{R}^n)$, $G(t, x(t)) \in L^2(\mathbb{R}_+, \mathbb{R}^n)$.
2) For each t_k, $k \in N$, $x(t_k^+) = \lim_{t_k \to t_k^+} x(t)$ and $x(t_k^-) = \lim_{t_k \to t_k^-} x(t)$ and $x(t_k = x(t_k^-)$ a.s.
3) $x(t)$ obeys the equivalent integral equation of (6) for almost every $t \in \mathbb{R}_+/t_k$ and satisfies the impulsive conditions at each $t = t_k, k \in N$ a.s.

The following theorem gives the existence and uniqueness of global positive solutions for the model in (6).

Theorem 1: For any given initial value $X(0) \in \mathbb{R}_+^4$, there exists a unique positive solution $X(t)$ to model (6) for all $t \geq 0$ a.s.

Proof: See Appendix A. ■

It is not easy to directly prove the existence and uniqueness of its solution since the model in (6) are system of ISDE. Through the relation in (15) we will get a nonimpulsive continuous SDE and the corresponding relationship of the solutions of the two models. In fact, through the relation in (15), we establish the equivalence between a ISDE and a nonimpulse SDE. It also provides an important method for studying the solutions of ISDE. Through this proof process, we can also draw the conclusion that the solution of the model in (6) is continuous, and the explicit expression of the solution can be obtained by the relation in (15) and (18).

From the Theorem 1, we obtain the existence and uniqueness of solution of the model in (6). Furthermore, we want to investigate whether the solution of the model is bounded, even if there is an upper bound for the solution of the model as time approaches infinity. We give the following definition by referring to the definition of stochastically ultimately bounded for the solution of the model [38].

Definition 2: The solution $X(t) = (x_1(t), x_2(t), x_3(t), x_4(t))$ of model (6) is said to be stochastically ultimately bounded if for any ε, there is a positive constant χ_ε such that for any initial value $(x_1(0), x_2(0), x_3(0), x_4(0)) \in \mathbb{R}_+^4$, the solution of the model in (6) has the property that

$$\limsup_{t\to\infty} \mathbb{P}\{|X(t)| > \chi\} < \varepsilon. \tag{7}$$

The impulse control proposed in this article is an active emotional control method, so there must be a reasonable control intensity. Based on this fact, we give the following assumptions.

$$\begin{cases} \left.\begin{array}{l} dx_1(t) = x_1(t)[b_1 - a_{11}x_1(t) - a_{12}x_2(t) - a_{13}x_3(t) - a_{14}x_4(t)]\,dt + \sigma_1 x_1(t)\,dB_1(t) \\ dx_2(t) = x_2(t)[b_2 - a_{21}x_1(t) - a_{22}x_2(t) - a_{23}x_3(t)]\,dt + \sigma_2 x_1(t)\,dB_2(t) \\ dx_3(t) = x_3(t)[b_3 - a_{31}x_1(t) + a_{32}x_2(t) - a_{33}x_3(t) - a_{34}x_4(t)]\,dt + \sigma_3 x_1(t)\,dB_3(t) \\ dx_4(t) = x_4(t)[b_4 - a_{41}x_1(t) + a_{43}x_3(t) - a_{44}x_4(t)]\,dt + \sigma_4 x_4(t)\,dB_4(t) \end{array}\right\} t \neq t_k \\ \left.\begin{array}{l} x_1\left(t_k^+\right) = (1+\alpha_k)x_1(t_k) \\ x_2\left(t_k^+\right) = (1-\beta_k)x_2(t_k) \\ x_3\left(t_k^+\right) = (1-\gamma_k)x_3(t_k) \\ x_4\left(t_k^+\right) = (1-\lambda_k)x_4(t_k) \end{array}\right\} t = t_k \end{cases} \tag{6}$$

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Assumption 1: For all $t > 0$

1) there exists positive constants m_i and M_i such that $m_i < \prod_{0<t_k<t} J_{ki} < M_i$;
2) there exists positive constants n_i and N_i such that

$$\begin{aligned} n_i &< \liminf_{t\to\infty} \frac{1}{t} \sum_{0<t_k<t} \ln J_{ki} \\ &\le \limsup_{t\to\infty} \frac{1}{t} \sum_{0<t_k<t} \ln J_{ki} < N_i \end{aligned} \tag{8}$$

where $i = 1, 2, 3, 4$.

The later research in this article is always based on this assumption. The essence of this hypothesis is to control the intensity of impulse, that is to say, we take impulse control to regulate emotion, and the intensity of control needs to be within a certain range. This is also consistent with our physical therapy and psychotherapy of the basic principles. We will give the stochastic ultimately boundedness of the solution for the model the following theorem.

Theorem 2: Under Assumption 1, then for any initial value $(x_1(0), x_2(0), x_3(0), x_4(0)) \in \mathbb{R}_+^4$, the solution of the model in (6) are stochastic ultimately bounded.

Proof: See Appendix B. ∎

Boundedness is very important for the model in (6), which ensures that the solution of the model does not tend to infinity. It means that there are four basic emotions: 1) happiness; 2) anger; 3) anxiety; and 4) fear. Each emotion $x_i(t)$ has an upper bound, so that $x_i(t)$ always fluctuates within the maximum upper bound of M. Is also the most basic safeguard of individual's physical and mental health. Whichever emotion, whether positive or negative, can not be allowed to go to extremes.

B. Strong Persistence

In the previous section, we have obtained the existence and uniqueness of the solution and stochastic boundedness of the model. On the other hand, given that the model in (6) adds impulse control, we also hope $x_i(t)$ to exist a lower bound ensuring the individual emotion $X(t)$ remains active and not deactivated. More precisely, we want to get the magnitude of each emotion on average. We calculate whether the limit $\liminf_{t\to\infty}\langle x_i(s)\rangle_t$ as well as $\limsup_{t\to\infty}\langle x(s)\rangle_t$ exist and the conditions for its existence. If both limits exist, denoted as k_i and K_i, then they can be asserted that the emotion $x(t)$ will always fluctuate within $[k_i, K_i]$. The length of the interval is the maximum fluctuation of the $x_i(t)$ of the emotion, we give this conclusion in the following theorem.

Theorem 3: The solution of the model in (6) with initial value $(x_1(0), x_2(0), x_3(0), x_4(0)) \in \mathbb{R}_+^4$ satisfies

$$0 < k_i \le \liminf_{t\to\infty}\langle x_i(s)\rangle_t \le \limsup_{t\to\infty}\langle x(s)\rangle_t \le K_i \tag{9}$$

where k_i and K_i $(i = 1, 2, 3, 4)$ as follows:

$$\begin{aligned} K_1 &= \frac{h_1 + N_1}{a_{11}}, \quad K_2 = \frac{h_2 + N_2}{a_{22}} \\ K_3 &= \frac{h_3 + N_3 + a_{32}\frac{h_2+N_2}{a_{22}}}{a_{33}} \\ K_4 &= \frac{h_4 + N_4 + a_{43}\frac{h_3+N_3+a_{32}\frac{h_2+N_2}{a_{22}}}{a_{33}}}{a_{44}} \\ k_1 &= \frac{h_1 + n_1 - a_{12}K_2 - a_{13}K_3 - a_{14}K_4}{a_{11}} \\ k_2 &= \frac{h_2 + n_2 - a_{21}K_1 - a_{23}K_3}{a_{22}} \\ k_3 &= \frac{h_3 + n_3 + a_{32}k_2 - a_{31}K_1 - a_{34}K_4}{a_{33}} \\ k_4 &= \frac{h_4 + n_4 + a_{43}k_3 - a_{41}K_1}{a_{44}}. \end{aligned} \tag{10}$$

Proof: See Appendix C. ∎

From the conclusion of the theorem and the expressions of k_i and K_i, we can also conclude that the intensity of stochastic perturbation will directly affect the fluctuation of each emotion. In particular, when the intensity of stochastic perturbation exceeds a certain threshold, a certain emotion will tend to be inactivated. Properly reducing the intensity of stochastic perturbation is beneficial to emotional stability. However, this is not easy to achieve in reality, because stochasticness is inevitable. Of course, this theoretical result can inspire us to consider more stochastic factors when studying individual emotions, making individual emotions more robust and anti-perturbation ability. At the same time, it also reflects the necessity, flexibility, and controllability of the impulse we added. When an emotion reaches the boundary value, we can intervene appropriately through the impulse to make it fluctuate only within the expected range.

C. Global Attractivity

In the previous section, we have obtained the fluctuation range $[k_i, K_i]$ of each emotion $x_i(t)$. The emotion $x_i(t)$ may fluctuate stochasticly in the interval $[k_i, K_i]$, or it may be attracted to a fixed normal state in the interval $[k_i, K_i]$. If there is a fixed state, it will help to study the overall trend of sentiment and macroscopic forecast and control. In particular, we expect that this fixed state can also be controlled by impulse. The following theorem we will give the attractiveness of the solution for the model in (6).

Definition 3: Let $X(t)$ and $\overline{X}(t)$ be two arbitrary solutions of the model in (6) with initial values $X(0) \in \mathbb{R}_+^4$ and $\overline{X}(0) \in \mathbb{R}_+^4$, respectively. If

$$\lim_{t\to\infty} |X(t) - \overline{X}(t)| = 0 \quad a.s \tag{11}$$

and then the model in (6) is globally attractive.

Before giving the global attractiveness theorem of the model, we first prove the following lemma based on Assumption 2 as following, which is necessary in mathematics.

Assumption 2: Set $\alpha_1 = a_{11} - a_{21} - a_{31} - a_{41} > 0$, $\alpha_2 = a_{22} - a_{21} - a_{23} > 0$, $\alpha_3 = a_{33} - a_{13} - a_{23} - a_{43} > 0$, and $\alpha_4 = a_{44} - a_{14} - a_{34} > 0$.

Lemma 1: Under Assumption 2, the solution of the model in (6) has the property

$$|X(t) - \overline{X}(t)| \in L^1[0, \infty) \tag{12}$$

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where $X(t)$ and $\overline{X}(t)$ be two arbitrary solutions of model (6) with initial values $X(0) \in \mathbb{R}_+^4$ and $\overline{X}(0) \in \mathbb{R}_+^4$, respectively.

Proof: See Appendix D. ■

We introduce the global attractiveness theorem of the model in (6) as following.

Theorem 4: Model in (6) is globally attractive, if Assumption 2 holds.

Proof: See Appendix E. ■

Theorem 4 yields that the solution for the model in (6) tend to a stable state in the end. This stable state is different from the deterministic model, because model (6) receives the perturbation of stochastic factors, which will cause the solution of this model to exhibit oscillating behavior, but the general trend is certain converge the equilibrium of the deterministic model. In addition, since the model in (6) adopts impulse control, the long-term behavior of the model solution will also exhibit periodic phenomena. These results will be demonstrated in the numerical simulation section.

In view of the Theorem 4, the solution of the model in (6) must be convergent. Therefore, we can discuss the computational cost of the method proposed in this article. Assuming that the solution converges to a stable state for the first time in time T, we use a numerical iterative method with a step size of h, and a impulse period of Δt_k, then the following conclusion is true.

Theorem 5: The computational cost of the solution of the model in (6) converges to a steady state is $\mathcal{O}[4 \cdot (T - t_0/\Delta t_k) \cdot (\Delta t_k/h)^4]$.

Proof: Note that the model in (6) consists of four interrelated equations, so the computational cost in each impulse interval is $\mathcal{O}[4 \cdot (\Delta t_k/h)^4]$. There are a total of $(T - t_0/\Delta t_k)$ impulse controls in the interval (t_0, T). Therefore, the conclusion of the theorem is correct. ■

In Theorem 5, the impulse termination time T is a key factor in the impulse control. First, we need to define an emotion diagnostic function $f(X(t))$ to determine the impulse termination time. Impulse control can be stopped when the affective state is diagnosed as healthy. An important basis for judging the validity of the system-model of HRI proposed in this article is the accuracy of the diagnostic function. There are two types of diagnostic errors, the false acceptance rate (FAR) referred to treatment a healthy state (Type I error), and the false rejection rate (FRR) related to the incorrect rejection of a nonhealthy state (Type II error). We will repeatedly adjust the accuracy of the diagnostic function through a large number of experimental sample tests, reduce FAR and FRR, and continuously improve the effectiveness and practicability of the HRI system.

V. Numerical Simulation

In this section, we use numerical simulation to verify the validity of the theoretical results. In view of the uncertainty of emotion, we define that it can be measured according to the probabilities of its appearance as follows:

$$p_i(t) = \frac{x_i(t)}{x_1(t) + x_2(t) + x_3(t) + x_4(t)}, \quad (i = 1, 2, 3, 4). \tag{13}$$

Obviously, the probabilities $p_i(t) \in (0, 1)$. According to definition (14), we calculate the entropy of emotions as follows:

$$H(X(t)) = -\sum_{i=1}^{4} p_i(t)\log_2 p_i(t). \tag{14}$$

The key step to verify the rationality of the model proposed in this article is to set the initial state and parameters. The tester interacts with the affective robot through a series of IoT devices designed by the system, and then samples are taken during the whole process of the system evaluation interaction, and a stable average value is taken as the initial value condition of the tester's current emotional state. Since there will be different initial values for different testers, given an initial value arbitrarily will not affect the generality of the function of our verification model.

Based on the Gross's conceptual framework of emotion regulation, the generation, and inhibition of one emotion is stronger than its transformation into other emotions, i.e., the self-motivation coefficient b_i and self-inhibition coefficient a_{ii} should be greater than interaction coefficient a_{ij} [40]. Therefore, we select the basic parameters of the model in (1) as $b_1 = 0.4$, $b_2 = 0.3$, $b_3 = 0.2$, $b_4 = 0.2$, $a_{ii} = 0.35$, and $a_{ij} = 0.1$. Each parameter does not represent a specific physical scenario by itself, but only represents the ratio coefficient of transforming from one emotion to another, it has been made sure that the parameters follow the conditional constraints, derived in this article. All the parameters together determine the model in (1) as a basic mechanism of emotional transformation. The affective computing model with impulse control is the core design of the affective robotic in the HRI process. In order to analyze the influence of impulse control and random disturbance on the entropy of the affective computing model, we use the controlled variable method. For two variable factors, we divide the model in (6) to four cases. Below we will discuss in detail in four cases.

Case 1: Model in (6) without stochastic perturbation and impulse control.

First of all, we consider the self-growth law of the model in (6) in an ideal environment, that is, when the stochastic perturbation is zero, and impulse control is not added. As an example, we set the initial value condition as $P(t_0) = (p_1(0), p_2(0), p_3(0), p_4(0)) = (0.6, 0.2, 0.1, 0.1)$, where four of the components represent the probabilities of occurrence for the four emotions at time t_0, respectively. The dynamic behavior of the model in (6) is shown in Fig. 4. The four emotions will converges to a stationary probability distribution after long-term evolution [see in Fig. 4(a)], and the entropy of emotions will tend to a fixed value [see in Fig. 4(b)]. However, it is observed the probability $p_1(t)$ of positive emotion in the stationary probability distribution has decreased, but the probabilities of negative emotions (anger, anxiety, and fear) has increased, especially the probability of anxiety will be close to 0.3, which is not a healthy and satisfactory state. This is the necessity of impulse control proposed in this article.

Different individuals have different abilities on the self-production and self-inhibition of a certain emotion, especially in patients with mental illness. Clinical experiments have shown that mental patients (such as psoriasis patients,

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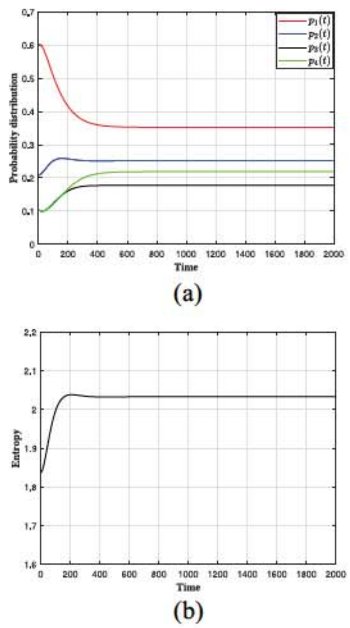

Fig. 4. (a) Probability distribution of emotions. (b) Entropy of emotions.

depressed patients, etc.) are more likely to have negative emotions and even show emotional dysregulation [41]. We set up two types of comparative experiments to explore the effects of different intensities of self-production and self-inhibition on the entropy of emotions, and take the positive emotion $x_1(t)$ and the negative emotion $x_2(t)$ as examples. Fig. 5(a) and (c) show the changing law of the entropy of emotions under different intensities of self-inhibition. Fig. 5(b) and (d) show the changing law of the entropy of emotions under different intensities of self-production. Fig. 5 have a common changing law, i.e., as a coefficient gradually increases, whether it is a_{11}, a_{22}, b_1, or b_2, the entropy of emotions shows a trend of first increasing and then decreasing. This law gives us enlightenment: Set other parameters unchanged, adjust a certain parameter, there must be a threshold to make the entropy of emotions reach the maximum. This will have important guiding significance for the emotion regulation. For example, we can increase the self-generation coefficient of positive emotions, which has achieved the effect of positively regulating emotions. However, there must be a threshold for optimal adjustment and exceeding this threshold will cause some opposite effects.

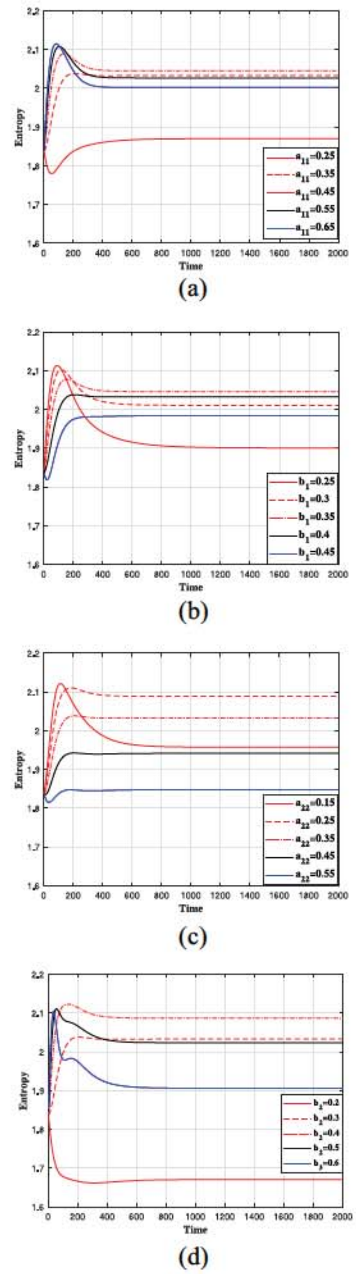

Fig. 5. Affects for different intensities of self-production and self-inhibition on the entropy of emotions. (a) Affects of a_{11} on the entropy. (b) Affects of b_1 on the entropy. (c) Affects of a_{22} on the entropy. (d) Affects of b_2 on the entropy.

Case 2: Model in (6) with impulse control.

In this case, we consider the role of impulse control for the model in (6). In particular, we assume that the stochastic perturbation is 0, i.e., $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_1 = 0$, and select the impulse intensities as $\alpha_k = \alpha = 0.6$, $\beta_k = \beta = 0.6$, $\gamma_k = \gamma = 0.6$, $\lambda_k = \lambda = 0.6$. In Fig. 6(a), we choose to perform impulse control every 200 time periods, in Fig. 6(b), we choose to perform impulse control every 120 time periods, and assume that the intensities of impulse is the same.

In Fig. 6(a), it is observed that the entropy of emotions will drop instantaneously at each impulse moment, and then return to the level of the entropy of emotions in deterministic model (1), until the next pulse control is executed. By contrast, when the impulse frequency is increased and adjusted to be executed every 120 time periods, the entropy of emotions will decrease, and then a stable periodic phenomenon will be formed below the black line [the entropy of emotions of deterministic model (1)], which is shown in Fig. 6(b).

The impulse control we proposed is the positive regulation of positive emotions and the negative regulation of negative emotions. Obviously, such control is beneficial to the individual's emotional health. Corresponding to reality, when an individual is depressed, he can regularly listen to a piece of music and watch a movie to adjust his mood to a good direction. In the long run, this adjustment method will also show periodicity and regularity. In Fig. 6 reveals that impulse control

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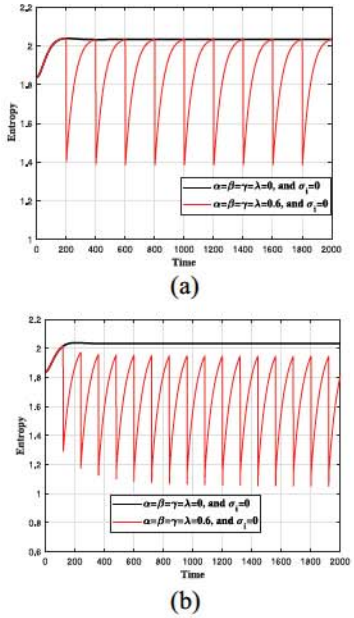

Fig. 6. Affects for different frequencies of impulse control on the entropy of emotions. Affects for impulse with every (a) 200 time periods on entropy and (b) 120 time periods on entropy.

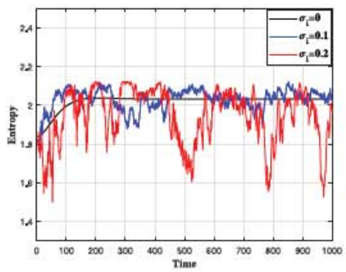

Fig. 7. Affects for different intensities of the stochastic perturbation on the entropy of emotions.

reduces entropy of emotions, and it is related to the impulse frequency. The conclusion in Fig. 6 enlightens us: even if it is a good way of emotional regulation, avoid too frequent.

Case 3: Model in (6) with stochastic perturbation.

In this case, we mainly discuss the influence of stochastic perturbation on the emotional changes. In particular, we assume that the impulse control is 0, and two different sets of stochastic perturbation are taken as $\sigma_i = 0.1$ and $\sigma_1 = 0.2$. The entropy of emotions when the model has stochastic perturbation is shown in Fig. 7. When the model in (6) with stochastic perturbation, the entropy of emotions will exhibit oscillating behavior. It is observed that the amplitude of the entropy of emotions is positively correlated with the intensities of the stochastic perturbation received by the model in (6). As the intensities of stochastic increases, the entropy of emotions will move down, and it will stay below the black line most of the time. In the actual environment, when people are disturbed by the stochastic environment of the outside world, emotions will inevitably have ups and downs. Therefore, appropriate control of stochastic perturbation in real life, which will help us achieve a stable emotional state.

Case 4: Model (6) with stochastic perturbation and impulse control

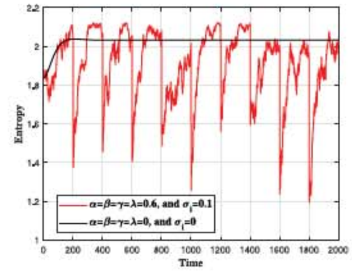

Fig. 8. Affects for stochastic perturbation and impulse control on the entropy of emotions.

TABLE I
FUNCTIONALITY FEATURES OF MODEL IN (6)

Parameters	Functionality features
a_{ii}	The coefficient indicates the ability on the self-production of a certain emotion, which we can appropriately increase (positive emotions) or decrease (negative emotions) to achieve the effect of regulating emotions
b_i	The coefficient indicates the ability on the self-production of a certain emotion, which we can appropriately increase (negative emotions) or decrease (positive emotions) to achieve the effect of regulating emotions
σ_i	This coefficient represents the strength of stochastic perturbation, and controlling this coefficient plays an important role in emotional stability
α, β, γ, and λ	These coefficients represent the external control strength of emotions, which have important guiding significance for the macro-control of emotions

With the conclusions of the previous three cases, in this case, we will focus on entropy of emotions when the model in (6) has both stochastic perturbation and impulse control. In fact, stochastic perturbation always exists in the real environment. In the previous case, we have discussed the necessity of adding impulse control. We select the intensities $\sigma_i = 0.1$ of stochastic perturbation and impulse control intensities $\alpha = \beta = \gamma = \lambda = 0.6$. Compared with Fig. 6, it can be concluded that when the model has both stochastic perturbation and impulse control, its dynamic behavior of entropy is more complicated. In Fig. 8, the entropy of emotions will show oscillating behavior due to the influence of stochastic perturbation, and it will drop instantaneously at each impulse moment. On the whole, under the periodic impulse control, even if there is the influence of stochastic perturbation, the trend of entropy will still show periodic regularity.

The affective computing and control model we propose is an innovative method. The numerical simulation uses the entropy of emotions as a research index, which verifies the correctness of the mathematical theory of the model. Based on the theoretical results and numerical experimental results in this article, we summarize a table of functional characteristics about the main parameters in the model (6), which is show in Table I. This table conveniently summarizes the main meanings and functionality features of the key parameters. The

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work of [16] mainly focuses on fear-related emotions, and the emotional contagion model is mainly implemented and tested on the simulation platform. Our affective computing model mainly studies the mutual evolution law of four emotions, adds the active regulation method of impulse control, and gives strict theoretical derivation. In future applications, we will combine theory with experiment, carry out continuous experimental debugging, add adaptive parameter tuning function, and improve the accuracy of affective robot in providing emotional services to different individuals.

VI. Conclusion

In this article, we propose a mathematical model with stochastic perturbation and impulse control to quantitatively calculate the law of emotional evolution under the control strategy. The model describes the dynamic transformation relationship between the four basic emotions of individuals. The existence, boundedness, and stability of the solution for model are mathematically proved. A meaningful conclusion is that choosing an appropriate impulse control strategy can induce individual negative emotions to positive emotions. That is to say, the affective computing model can also achieve effective emotional control. Furthermore, based on the emotion recognition technology of AI and the emotion diagnosis method of psychology, we propose an affective robot with emotion recognition, affective computing, emotion control, and can propose suggestion strategies. Our work is mainly to propose a mathematical method of affective computing and emotional control, and demonstrate the idea of robots serving human emotions.

Appendix A
Proof of Theorem 1

Proof: First we introduce variable substitution as follows:

$$
\begin{aligned}
y_1(t) &= \prod_{0<t_k<t} (1+\alpha_k)^{-1} x_1(t)\\
y_2(t) &= \prod_{0<t_k<t} (1-\beta_k)^{-1} x_2(t)\\
y_3(t) &= \prod_{0<t_k<t} (1-\gamma_k)^{-1} x_3(t)\\
y_4(t) &= \prod_{0<t_k<t} (1-\lambda_k)^{-1} x_4(t).
\end{aligned} \tag{15}
$$

Then we consider the model without impulses [see model (16), shown at the bottom of the page] with initial value $Y(0) = (y_1(0), y_2(0), y_3(0), y_4(0)) = X(0)$, where $J_{k1} = (1+\alpha_k)$, $J_{k2} = (1-\beta_k)$, $J_{k1} = (1-\gamma_k)$, and $J_{k1} = (1-\lambda_k)$. First, we prove that the model in (16) has a unique global positive solution. Since the coefficient of model (16) are locally Lipschitz continuous (see [42]), then for any given initial value $Y(0) \in \mathbb{R}_+^4$, there is a unique local solution $Y(t)$ on $t \in [0, \tau_e)$, where τ_e is t the explosion time [43], which is represented by

$$
\begin{aligned}
y_1(t) &= \frac{\prod_{0<t_k<t} J_{k1} \exp[F_1(t) + h_1 t + \sigma_1 B_1(t)]}{\frac{1}{y_1(0)} + a_{11}\int_0^t \prod_{0<t_k<s} J_{k1} \exp[F_1(s) + h_1 s + \sigma_1 B_1(s)]\mathrm{d}s}\\
y_2(t) &= \frac{\prod_{0<t_k<t} J_{k2} \exp[F_2(t) + h_2 t + \sigma_2 B_2(t)]}{\frac{1}{y_2(0)} + a_{22}\int_0^t \prod_{0<t_k<s} J_{k2} \exp[F_2(s) + h_2 s + \sigma_2 B_2(s)]\mathrm{d}s}\\
y_3(t) &= \frac{\prod_{0<t_k<t} J_{k3} \exp[F_3(t) + h_3 t + \sigma_3 B_3(t)]}{\frac{1}{y_3(0)} + a_{33}\int_0^t \prod_{0<t_k<s} J_{k3} \exp[F_3(s) + h_3 s + \sigma_3 B_3(s)]\mathrm{d}s}
\end{aligned} \tag{17}
$$

$$
y_4(t) = \frac{\prod_{0<t_k<t} J_{k4} \exp[F_4(t) + h_4 t + \sigma_4 B_4(t)]}{\frac{1}{y_4(0)} + a_{44}\int_0^t \prod_{0<t_k<s} J_{k4} \exp[F_4(s) + h_4 s + \sigma_4 B_4(s)]\mathrm{d}s} \tag{18}
$$

where

$$
\begin{aligned}
F_1(t) &= -a_{12}\int_0^t \prod_{0<t_k<t} J_{k2} y_2(s)\mathrm{d}s - a_{13}\int_0^t \prod_{0<t_k<t} J_{k3} y_3(s)\mathrm{d}s\\
&\quad - a_{14}\int_0^t \prod_{0<t_k<t} J_{k4} y_4(s)\mathrm{d}s\\
F_2(t) &= -a_{21}\int_0^t \prod_{0<t_k<t} J_{k1} y_1(s)\mathrm{d}s - a_{23}\int_0^t \prod_{0<t_k<t} J_{k3} y_3(s)\mathrm{d}s\\
F_3(t) &= -a_{31}\int_0^t \prod_{0<t_k<t} J_{k1} y_1(s)\mathrm{d}s\\
&\quad + a_{32}\int_0^t \prod_{0<t_k<t} J_{k2} y_2(s)\mathrm{d}s - a_{34}\int_0^t \prod_{0<t_k<t} J_{k3} y_3(s)\mathrm{d}s\\
F_4(t) &= a_{41}\int_0^t \prod_{0<t_k<t} J_{k1} y_1(s)\mathrm{d}s + a_{43}\int_0^t \prod_{0<t_k<t} J_{k3} y_3(s)\mathrm{d}s
\end{aligned} \tag{19}
$$

and $h_i = b_i - 1/2\sigma_i^2$ $(1 \le i \le 4)$. Since the solutions of the model in (16) are all in exponential form, it is easy to get $y_i(t) > 0$ $(1 \le i \le 4)$ for all $t \in$. To show the solutions are global, consider the following comparison model (20), shown at the bottom of the next page.

$$
\begin{cases}
\mathrm{d}y_1(t) = y_1(t)\Big[b_1 - a_{11}\prod_{0<t_k<t} J_{k1} y_1(t) - a_{12}\prod_{0<t_k<t} J_{k2} y_2(t) - a_{13}\prod_{0<t_k<t} J_{k3} y_3(t) - a_{14}\prod_{0<t_k<t} J_{k4} y_4(t)\Big]\mathrm{d}t\\
\qquad +\sigma_1 y_1(t)\mathrm{d}B_1(t)\\
\mathrm{d}y_2(t) = y_2(t)\Big[b_2 - a_{21}\prod_{0<t_k<t} J_{k1} y_1(t) - a_{22}\prod_{0<t_k<t} J_{k2} y_2(t) - a_{23}\prod_{0<t_k<t} J_{k3} y_3(t)\Big]\mathrm{d}t\\
\qquad +\sigma_2 y_2(t)\mathrm{d}B_2(t)\\
\mathrm{d}y_3(t) = y_3(t)\Big[b_3 - a_{31}\prod_{0<t_k<t} J_{k1} y_1(t) + a_{32}\prod_{0<t_k<t} J_{k2} y_2(t) - a_{33}\prod_{0<t_k<t} J_{k3} y_3(t) - a_{34}\prod_{0<t_k<t} J_{k4} y_4(t)\Big]\mathrm{d}t\\
\qquad +\sigma_3 y_3(t)\mathrm{d}B_3(t)\\
\mathrm{d}y_4(t) = y_4(t)\Big[b_4 - a_{41}\prod_{0<t_k<t} J_{k1} y_1(t) + a_{43}\prod_{0<t_k<t} J_{k3} y_3(t) - a_{44}\prod_{0<t_k<t} J_{k4} y_4(t)\Big]\mathrm{d}t\\
\qquad +\sigma_4 y_4(t)\mathrm{d}B_4(t)
\end{cases} \tag{16}
$$

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By the same method as above we can get the explicit solutions of the first equation of model (20)

$$y_1(t) = \frac{\prod_{0<t_k<t} J_{k1} \exp[h_1 t + \sigma_1 B_1(t)]}{\frac{1}{y_1(0)} + a_{11} \int_0^t \prod_{0<t_k<s} J_{k1} \exp[h_1 s + \sigma_1 B_1(s)] \mathrm{d}s}. \quad (21)$$

Then, it is easy to see the solution (21) will not explode in any finite time. Similarly, we can conclude that solution $\hat{y}_2(t)$, $\hat{y}_3(t)$ and $\hat{y}_2(t)$ also not explode in any finite time. By the comparison theorem of the stochastic differential equation [44], we have $y_i(t) \leq \hat{y}_i(t)$ $(1 \leq i \leq 4)$ for all t, a.s. Thus, the solutions of model (16) will not also explode in finite time. That is, $\tau_e = \infty$, we get the (16) has a unique global positive solution.

In the next, we prove that $x_i(t)$ $(1 \leq i \leq 4)$ satisfying relation (15) are the solutions of the model in (6). In view of relation (15), it is easy to check that $(x_1(t), x_2(t), x_3(t), x_4(t))$ is continuous on each interval $(t_k, t_{k+1}) \subset \mathbb{R}_+$, $k \in N$, then for all $t \neq t_k$

$$\begin{aligned} \mathrm{d}x_1(t) = x_1(t)[&b_1 - a_{11}x_1(t) - a_{12}x_2(t) - a_{13}x_3(t) \\ &- a_{14}x_4(t)]\,\mathrm{d}t + \sigma_1(t)x_1(t)\,\mathrm{d}B_1(t). \quad (22) \end{aligned}$$

On the other hand, for $k \in N$ and $t_k \in [0, \infty)$

$$\begin{aligned} x_1\left(t_k^+\right) &= \lim_{t \to t_k^+} \prod_{0<t_k<t} J_{k1} y_1(t) = \prod_{0<t_i \leq t_k} J_{k1} y_1\left(t_k^+\right) \\ &= (1 + \alpha_k) \prod_{0<t_i<t_k} J_{i1} y_1(t_k) = (1 + \alpha_k) x_1(t_k) \quad (23) \end{aligned}$$

at the same time

$$\begin{aligned} x_1\left(t_k^-\right) &= \lim_{t \to t_k^-} \prod_{0<t_k<t} J_{k1} y_1(t) = \prod_{0<t_i<t_k} J_{k1} y_1\left(t_k^-\right) \\ &= \prod_{0<t_i<t_k} J_{i1} y_1(t_k) = x_1(t_k). \quad (24) \end{aligned}$$

Similarly, we can verify that the solutions $x_2(t)$, $x_3(t)$, and $x_4(t)$ satisfie model (6). In view of Definition 1, we can get that $x_i(t)$ $(1 \leq i \leq 4)$ satisfying relation in (15) are the solutions of the model in (6), which implies that if $(y_1(t), y_2(t), y_3(t), y_4(t))$ is a solution of model (16), then $(x_1(t), x_2(t), x_3(t), x_4(t))$ is a solution of model (6) on $[0, +\infty)$, and satisfy relation (15).

Next, we will prove the uniqueness of solution of model (6). For $t \in (0, t_1]$ and for any give initial value $(x_1(0), x_2(0), x_3(0), x_4(0)) \in \mathbb{R}_+^4$, since the coefficients of model (6) satisfy the Lipschitz condition, then the solution of model (6) is unique on $(0, t_1]$. For $t \in (t_1, t_2]$, we consider the initial value $(y(t_1^+), y(t_2^+), y(t_3^+), y(t_4^+)) \in \mathbb{R}_+^4$, by the same discussion we can get there is a unique solution on $(t_1, t_2]$. Consequently, the solution of model (6) is unique. The proof is completed. ■

APPENDIX B
PROOF OF THEOREM 2

Proof: First we proof the follows inequality about the solutions of the model in (6) is truth:

$$\begin{aligned} &\left|\mathbb{E}\left[y_1^p(t)\right]\right| \leq K_1(p), \quad \left|\mathbb{E}\left[y_2^p(t)\right]\right| \leq K_2(p) \\ &\left|\mathbb{E}\left[y_3^p(t)\right]\right| \leq K_3(p), \quad \left|\mathbb{E}\left[y_4^p(t)\right]\right| \leq K_4(p), \qquad t \in (0, +\infty] \end{aligned} \quad (25)$$

where $K_i(p)$ $(1 \leq i \leq 4)$ is about the p positive constant and $1 < p$. Applying Itô's formula (see [42, Th. 6.4]) for the model in (16), we can get (26), shown at the bottom of the next page.

For the first equation of (26), we can obtained that

$$\begin{aligned} \mathrm{d}y_1^p(t) \leq p y_1^p(t)\left[b_1 - a_{11} m y_1(t) + \frac{1}{2}p\sigma_1^2\right]\mathrm{d}t \\ + p\sigma_1 y_1^p(t)\mathrm{d}B_1(t). \quad (27) \end{aligned}$$

Taking the exception of both sides on the above inequality we get

$$\frac{\mathrm{d}\mathbb{E}\left[y_1^p(t)\right]}{\mathrm{d}t} \leq p\mathbb{E}\left[y_1^p(t)\right]\left[b_1 + \frac{1}{2}p\sigma_1^2 - a_{11} m\left(\mathbb{E}\left[y_1^p(t)\right]\right)^{\frac{1}{p}}\right]. \quad (28)$$

The comparison equation is constructed as follows:

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = pW(t)\left[b_1 + \frac{1}{2}p\sigma_1^2 - a_{11} m[W(t)]^{\frac{1}{p}}\right]. \quad (29)$$

From the basic formula of the Bernoulli equation we can get

$$\begin{aligned} [W(t)]^{-\frac{1}{p}} = \Bigg(\int_0^t a_{11} \exp\left\{\left[b_1 + \frac{1}{2}p\sigma_1^2\right]s\right\}\mathrm{d}s \\ + W(0)\Bigg) \cdot \exp\left\{\left[-b_1 - \frac{1}{2}p\sigma_1^2\right]t\right\}. \quad (30) \end{aligned}$$

Applying L'Hospital's rule yields that

$$\lim_{t \to \infty} W(t) = \left(\frac{b_1 + \frac{1}{2}p\sigma_1^2}{a_{11}}\right)^p. \quad (31)$$

By virtue of the comparison theorem of the stochastic differential equation [44], we get

$$\limsup_{t \to \infty} E\left[y_1^p(t)\right] \leq \lim_{t \to \infty} W(t) = \left(\frac{b_1 + \frac{1}{2}p\sigma_1^2}{a_{11}}\right)^p \equiv \overline{K}_1(p). \quad (32)$$

According to the definition of sublimit, for any given $\varepsilon > 0$, there is a constant $T > 0$ such that for all $t > T$

$$\mathbb{E}\left[y_1^p(t)\right] \leq \overline{K}_1(p) + \varepsilon. \quad (33)$$

$$\begin{cases} \mathrm{d}\hat{y}_1(t) = \hat{y}_1(t)\left[b_1 - a_{11}\prod_{0<t_k<t} J_{k1}\hat{y}_1(t)\right]\mathrm{d}t + \sigma_1(t)\hat{y}_1(t)\mathrm{d}B_1(t) \\ \mathrm{d}\hat{y}_2(t) = \hat{y}_2(t)\left[b_2 - a_{21}\prod_{0<t_k<t} J_{k1}\hat{y}_1(t) - a_{22}\prod_{0<t_k<t} J_{k2}\hat{y}_2(t)\right]\mathrm{d}t + \sigma_2(t)\hat{y}_2(t)\mathrm{d}B_2(t) \\ \mathrm{d}\hat{y}_3(t) = \hat{y}_3(t)\left[b_3 - a_{31}\prod_{0<t_k<t} J_{k1}\hat{y}_1(t) + a_{32}\prod_{0<t_k<t} J_{k2}\hat{y}_2(t) - a_{33}\prod_{0<t_k<t} J_{k3}\hat{y}_3(t)\right]\mathrm{d}t + \sigma_3(t)\hat{y}_3(t)\mathrm{d}B_3(t) \\ \mathrm{d}\hat{y}_4(t) = \hat{y}_4(t)\left[b_4 - a_{41}\prod_{0<t_k<t} J_{k1}\hat{y}_1(t) + a_{43}\prod_{0<t_k<t} J_{k3}\hat{y}_3(t) - a_{44}\prod_{0<t_k<t} J_{k4}\hat{y}_4(t)\right]\mathrm{d}t + \sigma_4(t)\hat{y}_4(t)\mathrm{d}B_4(t) \end{cases} \quad (20)$$

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On the other hand, from the continuity of $\mathbb{E}[x_1(t)^p]$, there exists $\widehat{K}_1(p) > 0$ such that $\mathbb{E}[y_1^p(t)] < \widehat{K}_1(p)$ for $t \le T$. Set

$$K_1(p) = \max\{\overline{K}_1(p), \widehat{K}_1(p)\} \tag{34}$$

then, for all $t \in \mathbb{R}_+$, $E[y_1(t)^p] \le K_1(p)$. In the same method, we can get similarly $\mathbb{E}[y_i^p(t)] \le K_i(p)$ $(i = 2, 3, 4)$ for all $t \in \mathbb{R}_+$. Substituting (15) and into $E[y_1(t)^p]$ we can get

$$\mathbb{E}[x_1^p(t)] = \mathbb{E}\left[\left(\prod_{0<t_k<t} J_{k1} y_1(t)\right)^p\right] \le M^p \mathbb{E}[y_1^p(t)] \equiv N_1(p). \tag{35}$$

Similarly, we can get

$$\mathbb{E}[x_2^p(t)] \le N_2(p), \quad \mathbb{E}[x_3^p(t)] \le N_3(p), \quad \mathbb{E}[x_4^p(t)] \le N_4(p) \tag{36}$$

where $N_2(p)$, $N_3(p)$, and $N_4(p)$ are three constant related to p. Define the norm of $X(t)$ as $|X(t)| = (x_1^2, x_2^2, x_3^2, x_4^2,)^{\frac{1}{2}}$, then

$$\mathbb{E}[X(t)^p] \le 4^{\frac{p}{2}}[|N_1(p)|^p + |N_2(p)|^p + |N_3(p)|^p + |N_4(p)|^p] \equiv N(p). \tag{37}$$

Furthermore, in view of Chebyshev's inequality, it is easy to get the required assertion that the solution of the model in (6) is stochastic ultimately bounded. This completes the proof. ■

Appendix C
Proof of Theorem 3

Proof: Applying Itô's formula for the first equation of the model in (16)

$$\begin{aligned}\mathrm{d}\ln y_1(t) = \Bigg[& b_1 - \frac{1}{2}\sigma_1^2 - a_{11}\prod_{0<t_k<t} J_{k1}y_1(t) \\ & - a_{12}\prod_{0<t_k<t} J_{k2}y_2(t) - a_{13}\prod_{0<t_k<t} J_{k3}y_3(t) \\ & - a_{14}\prod_{0<t_k<t} J_{k4}y_4(t)\Bigg]\mathrm{d}t + \sigma_1 \mathrm{d}B_1(t).\end{aligned} \tag{38}$$

Integrating both sides of the above equation from 0 to t

$$\begin{aligned}\ln y_1(t) = &= \ln y_1(0) + h_1 t - a_{11}t\langle x_1(s)\rangle_t - a_{12}t\langle x_2(s)\rangle_t \\ & - a_{13}t\langle x_3(s)\rangle_t - a_{14}t\langle x_4(s)\rangle_t + \sigma_1 B_1(t).\end{aligned} \tag{39}$$

In view of relation (15) we have

$$\ln x_1(t) = \ln\left[\prod_{0<t_k<t} J_{k1}y_1(t)\right] = \ln y_1(t) + \sum_{0<t_k<t} \ln J_{k1} \tag{40}$$

substituting (40) into (39)

$$\begin{aligned}\ln x_1(t) = & \ln y_1(0) + \sum_{0<t_k<t} \ln J_{k1} + h_1 t - a_{11}t\langle x_1(s)\rangle_t \\ & - a_{12}t\langle x_2(s)\rangle_t - a_{13}t\langle x_3(s)\rangle_t - a_{14}t\langle x_4(s)\rangle_t + \sigma_1 B_1(t).\end{aligned} \tag{41}$$

Similarly, we can get

$$\begin{aligned}\ln x_2(t) = & \ln y_2(0) + \sum_{0<t_k<t} \ln J_{k2} + h_2 t - a_{21}t\langle x_1(s)\rangle_t \\ & - a_{22}t\langle x_2(s)\rangle_t - a_{23}t\langle x_3(s)\rangle_t + \sigma_2 B_2(t) \\ \ln x_3(t) = & \ln y_3(0) + \sum_{0<t_k<t} \ln J_{k3} + h_3 t - a_{31}t\langle x_1(s)\rangle_t \\ & + a_{32}t\langle x_2(s)\rangle_t - a_{33}t\langle x_3(s)\rangle_t - a_{34}t\langle x_4(s)\rangle_t + \sigma_3 B_3(t) \\ \ln x_4(t) = & \ln y_4(0) + \sum_{0<t_k<t} \ln J_{k4} + h_4 t - a_{41}t\langle x_1(s)\rangle_t \\ & + a_{43}t\langle x_3(s)\rangle_t - a_{44}t\langle x_4(s)\rangle_t + \sigma_4 B_4(t).\end{aligned} \tag{42}$$

Obviously, from (41) we get

$$\ln x_1(t) \le \ln y_1(0) + \sum_{0<t_k<t} \ln J_{k1} + h_1 t - a_{11}t\langle x_1(s)\rangle_t + \sigma_1 B_1(t) \tag{43}$$

$$\begin{aligned}\mathrm{d}y_1^p(t) = & py_1^p\left[b_1 - a_{11}\prod_{0<t_k<t} J_{k1}y_1(t) - a_{12}\prod_{0<t_k<t} J_{k2}y_2(t) - a_{13}\prod_{0<t_k<t} J_{k3}y_3(t) - a_{14}\prod_{0<t_k<t} J_{k4}y_4(t)\right]\mathrm{d}t \\ & + \frac{1}{2}p(p-1)\sigma_1^2\,\mathrm{d}t + p\sigma_1 y_1^p \mathrm{d}B_1(t) \\ \mathrm{d}y_2^p(t) = & py_2^p\left[b_2 - a_{21}\prod_{0<t_k<t} J_{k1}y_1(t) - a_{22}\prod_{0<t_k<t} J_{k2}y_2(t) - a_{23}\prod_{0<t_k<t} J_{k3}y_3(t)\right]\mathrm{d}t \\ & + \frac{1}{2}p(p-1)\sigma_2^2\,\mathrm{d}t + p\sigma_2 y_2^p \mathrm{d}B_2(t) \\ \mathrm{d}y_3^p(t) = & py_3^p\left[b_3 - a_{31}\prod_{0<t_k<t} J_{k1}y_3(t) + a_{32}\prod_{0<t_k<t} J_{k2}y_2(t) - a_{33}\prod_{0<t_k<t} J_{k3}y_3(t) - a_{34}\prod_{0<t_k<t} J_{k4}y_4(t)\right]\mathrm{d}t \\ & + \frac{1}{2}p(p-1)\sigma_3^2\,\mathrm{d}t + p\sigma_3 y_3^p \mathrm{d}B_3(t) \\ \mathrm{d}y_4^p(t) = & py_1^p\left[b_4 - a_{41}\prod_{0<t_k<t} J_{k1}y_1(t) - a_{43}\prod_{0<t_k<t} J_{k3}y_3(t) - a_{44}\prod_{0<t_k<t} J_{k4}y_4(t)\right]\mathrm{d}t \\ & + \frac{1}{2}p(p-1)\sigma_4^2\,\mathrm{d}t + p\sigma_4 y_4^p \mathrm{d}B_4(t)\end{aligned} \tag{26}$$

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and dividing both sides by t on above inequation, and then making use of the strong law of large numbers for martingale, we get $\lim_{t\to\infty}\sigma_1 B_1(t)/t = 0$. Then from lemma in Liu *et al.* [39], we can obtain

$$\limsup_{t\to\infty}\langle x_1(s)\rangle_t \le \frac{h_1+N_1}{a_{11}} \equiv K_1. \tag{44}$$

Similarly, we obtain

$$\limsup_{t\to\infty}\langle x_2(s)\rangle_t \le \frac{h_2+N_2}{a_{22}} \equiv K_2. \tag{45}$$

Then follows from the definition of superior limit, for arbitrary $\varepsilon > 0$, there exists a positive $T(\varepsilon) > 0$ such that for all $t > T$, we have $\langle x_2(s)\rangle_t \le (h_2+M_2)/a_{22}+\varepsilon$. Then, for sufficiently large t to (42)

$$\begin{aligned}\ln x_3(t) = \ln y_3(0) + \sum_{0<t_k<t}\ln J_{k3} + a_{32}t\left(\frac{h_2+N_2}{a_{22}}+\varepsilon\right)\\ +\ h_3 t - a_{33}t\langle x_3(s)\rangle_t + \sigma_3 B_3(t).\end{aligned} \tag{46}$$

Hence, by Lemma in Liu *et al.* [39] again

$$\limsup_{t\to\infty}\langle x_3(s)\rangle_t \le \frac{h_3+N_3+a_{32}\left(\frac{h_2+N_2}{a_{22}}+\varepsilon\right)}{a_{33}}. \tag{47}$$

According to the arbitrariness of ε, we get

$$\limsup_{t\to\infty}\langle x_3(s)\rangle_t \le \frac{h_3+N_3+a_{32}\frac{h_2+N_2}{a_{22}}}{a_{33}} \equiv K_3. \tag{48}$$

In the same method, it can be calculate

$$\limsup_{t\to\infty}\langle x_4(s)\rangle_t \le \frac{h_4+N_4+a_{43}\frac{h_3+N_3+a_{32}\frac{h_2+N_2}{a_{22}}}{a_{33}}}{a_{44}} \equiv K_4. \tag{49}$$

On the other hand, from (41), (45), (48), and (49), for sufficiently large t, one can get

$$\begin{aligned}\ln x_1(t) \ge \ln y_1(0) + \sum_{0<t_k<t}\ln J_{k1} + h_1 t - a_{11}t\langle x_1(s)\rangle_t\\ -\ a_{12}t\left(K_2+\frac{\varepsilon}{3}\right) - a_{13}t\left(K_3+\frac{\varepsilon}{3}\right) - a_{14}t\left(K_4+\frac{\varepsilon}{3}\right).\end{aligned} \tag{50}$$

In view of Lemma in Liu *et al.* [39], and the arbitrariness of ε

$$\liminf_{t\to\infty}\langle x_1(s)\rangle_t \ge \frac{h_1+n_1-a_{12}K_2-a_{13}K_3-a_{14}K_4}{a_{11}} \equiv k_1. \tag{51}$$

Similarly, we obtain that

$$\liminf_{t\to\infty}\langle x_2(s)\rangle_t \ge \frac{h_2+n_2-a_{21}K_1-a_{23}K_3}{a_{22}} \equiv k_2. \tag{52}$$

From the definition of inferior limit for arbitrary $\varepsilon > 0$, there exists a positive $T(\varepsilon) > 0$ such that for all $t > T$, we have $\langle x_2(s)\rangle_t \ge k_2+\varepsilon/3$. Then, for sufficiently large t we get

$$\begin{aligned}\ln x_3(t) \ge \ln y_3(0) + \sum_{0<t_k<t}\ln J_{k3} + h_3 t - a_{31}t\left(K_1+\frac{\varepsilon}{3}\right)\\ +\ a_{32}t\left(k_2+\frac{\varepsilon}{3}\right) - a_{33}t\langle x_1(s)\rangle_t - a_{34}t\left(K_4+\frac{\varepsilon}{3}\right).\end{aligned} \tag{53}$$

Using lemma in Liu *et al.* [39] again, we have

$$\liminf_{t\to\infty}\langle x_3(s)\rangle_t \ge \frac{h_3+n_3+a_{32}k_2-a_{31}K_1-a_{34}K_4}{a_{33}} \equiv k_3. \tag{54}$$

Similarly, we get

$$\liminf_{t\to\infty}\langle x_3(s)\rangle_t \ge \frac{h_4+n_4+a_{43}k_3-a_{41}K_1}{a_{44}} \equiv k_4. \tag{55}$$

The proof is completed. ∎

Appendix D
Proof of Lemma 2

Proof: Define a positive definite Lyapunov function as follows:

$$V(t) = \sum_{i=1}^{i=4}|\ln y_i(t) - \ln\overline{y}_i(t)|, \quad t \ge 0 \tag{56}$$

where $Y(t)$ and $\overline{Y}(t)$ be two arbitrary solutions of model (16) with initial values $Y(0) \in \mathbb{R}_+^4$ and $\overline{Y}(0) \in \mathbb{R}_+^4$, respectively. A direct calculation of the right differential $\mathrm{d}V^+(t)$ of $\mathrm{d}V(t)$ along the model in (16), we have

$$\mathrm{d}V^+(t) = \operatorname{sgn}\sum_{i=1}^{i=4}(y_1-\overline{y}_1)\mathrm{d}\big|\ln y_i(t)-\ln\overline{y}_i(t)\big|. \tag{57}$$

By Itô·s formula, we compute

$$\begin{aligned}\mathrm{d}\ln y_1 = \Bigg[& b_1 - \frac{1}{2}\sigma^2 - a_{11}\prod_{0<t_k<t}J_{k1}y_1 - a_{12}\prod_{0<t_k<t}J_{k2}y_2\\ &- a_{13}\prod_{0<t_k<t}J_{k3}y_3 - a_{14}\prod_{0<t_k<t}J_{k4}y_4\Bigg]\mathrm{d}t + \sigma_1\mathrm{d}B_1(t).\\ \mathrm{d}\ln\overline{y}_1 = \Bigg[& b_1 - \frac{1}{2}\sigma^2 - a_{11}\prod_{0<t_k<t}J_{k1}\overline{y}_1 - a_{12}\prod_{0<t_k<t}J_{k2}\overline{y}_2\\ &- a_{13}\prod_{0<t_k<t}J_{k3}\overline{y}_3 - a_{14}\prod_{0<t_k<t}J_{k4}\overline{y}_4\Bigg]\mathrm{d}t + \sigma_1\mathrm{d}B_1.\end{aligned} \tag{58}$$

Therefore

$$\begin{aligned}&\mathrm{d}|\ln y_1 - \ln\overline{y}_1|\\ &= \Bigg[-a_{11}\prod_{0<t_k<t}J_{k1}(y_1-\overline{y}_1) - a_{12}\prod_{0<t_k<t}J_{k2}(y_2-\overline{y}_2)\\ &\quad - a_{13}\prod_{0<t_k<t}J_{k3}(y_3-\overline{y}_3) - a_{14}\prod_{0<t_k<t}J_{k4}(y_4-\overline{y}_4)\Bigg]\mathrm{d}t.\end{aligned} \tag{59}$$

And similarly, we can calculate that $\mathrm{d}|\ln y_i - \ln\overline{y}_i|$ ($i = 2, 3, 4$). Substituting $\mathrm{d}|\ln y_i - \ln\overline{y}_i|$ ($1 \le i \le 4$) into (57), we have (60), shown at the bottom of the next page.

In view of the fact $\operatorname{sgn}(y_i-\overline{y}_i)\cdot(y_i-\overline{y}_i) = |y_i-\overline{y}_i|$ and $-\operatorname{sgn}(y_i-\overline{y}_i)\cdot(y_j-\overline{y}_j) \le |y_j-\overline{y}_j|$, $(i \ne j)$, we get

$$\mathrm{d}V^+(t) \le -(a_{11}-a_{21}-a_{31}-a_{41})\prod_{0<t_k<t}J_{k1}|y_1-\overline{y}_1|\mathrm{d}t$$

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$$- (a_{22} - a_{21} - a_{23}) \prod_{0<t_k<t} J_{k2}|y_2 - \overline{y}_2|\mathrm{d}t$$
$$- (a_{33} - a_{13} - a_{23} - a_{43}) \prod_{0<t_k<t} J_{k1}|y_3 - \overline{y}_3|\mathrm{d}t$$
$$- (a_{44} - a_{14} - a_{34}) \prod_{0<t_k<t} J_{k2}|y_4 - \overline{y}_4|\mathrm{d}t. \quad (61)$$

Integrating above inequality both sides from 0 to t leads to

$$V(t) - V(0) \le \int_0^t \{-\alpha_1 m_1|y_1 - \overline{y}_1| - \alpha_2 m_2|y_2 - \overline{y}_2| - \alpha_3 m_3|y_3 - \overline{y}_3| - \alpha_4 m_4|y_4 - \overline{y}_4|\}\mathrm{d}t. \quad (62)$$

Therefore

$$V(t) + \int_0^t \{\alpha_1 m_1|y_1 - \overline{y}_1| + \alpha_2 m_2|y_2 - \overline{y}_2| + \alpha_2 m_3|y_3 - \overline{y}_3| + \alpha_3 m_4|y_4 - \overline{y}_4|\}\,\mathrm{d}t \le V(0) < \infty. \quad (63)$$

Set $\theta = \min\{\alpha_1 m_1, \alpha_2 m_2, \alpha_3 m_3, \alpha_4 m_4\}$ and let $t \to \infty$, we have

$$\int_0^\infty |Y(t) - \overline{Y}(t)|\,\mathrm{d}t \le \int_0^\infty \sum_{i=1}^4 |y_4 - \overline{y}_4|\,\mathrm{d}t \le \frac{V(0)}{\theta} < \infty. \quad (64)$$

Noting that $\alpha_i > 0$ and $\theta \ge 0$, we obtain that $|y_i(t) - \overline{y}_i(t)| \in L^1[0,\infty)$. Therefore, we get $|Y(t) - \overline{Y}(t)| \in L^1[0,\infty)$. Moreover, in view of $|x_i(t) - \overline{x}_i(t)| = \prod_{0<t_k<t} J_{ki}|y_i(t) - \overline{y}_i(t)| \le M_i|y_i(t) - \overline{y}_i(t)|$, we have that

$$|X(t) - \overline{X}(t)| \in L^1[0,\infty). \quad (65)$$

This completes the proof of Lemma 1. ■

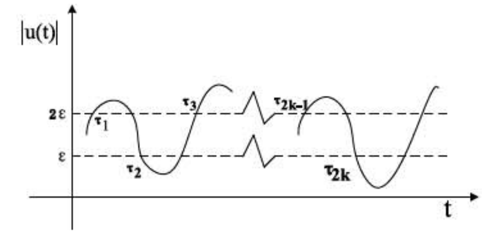

Fig. 9. Stopping time.

Appendix E
Proof of Theorem 4

Proof: Set $z(t) = Y_i(t) - \overline{Y}_i(t)$. Because both $Y_i(t)$ and $\overline{Y}_i(t)$ are continuous functions, $z(t)$ is also a continuous function. It easy to see from (64) that $\liminf_{t\to\infty}|z(t)| = 0$ a.s. Now, we only need to show

$$\limsup_{t\to\infty}|z(t)| = 0 \ a.s. \quad (66)$$

If this statement is false, then

$$\mathbb{P}\left\{\limsup_{t\to\infty}|z(t)| > 0\right\} > 0. \quad (67)$$

This means that for any given a real number $\varepsilon > 0$, there exists a set $\Omega_1 = \{\limsup_{t\to\infty}|z(t)| > 2\varepsilon\}$ such that $\mathbb{P}(\Omega_1) > 2\varepsilon$. Define a sequence stopping times as follows, which is shown in Fig. 9:

$$\begin{aligned} \tau_1 &= \inf\{t \ge 0 : |z(t)| \ge 2\varepsilon\} \\ \tau_2 &= \inf\{t \ge \tau_1 : |z(t)| \le \varepsilon\} \\ \tau_{2k-1} &= \inf\{t \ge \tau_{2(k-1)} : |z(t)| \ge 2\varepsilon\} \\ \tau_{2k} &= \inf\{t \ge \tau_{2k-1} : |z(t)| \le \varepsilon\}, \quad k = \in N^+. \end{aligned} \quad (68)$$

In the light of $\liminf_{t\to\infty}|z(t)| = 0$ and the definition of Ω_1, we get that $\tau_k < \infty$ for any $k \in N^+$ and $\omega \in \Omega_1$. In view

$$\begin{aligned} \mathrm{d}V^+(t) = {} & \mathrm{sgn}(y_1 - \overline{y}_1) \\ & \times \left[-a_{11}\prod_{0<t_k<t} J_{k1}(y_1 - \overline{y}_1) - a_{12}\prod_{0<t_k<t} J_{k2}(y_2 - \overline{y}_2) - a_{13}\prod_{0<t_k<t} J_{k3}(y_3 - \overline{y}_3) - a_{14}\prod_{0<t_k<t} J_{k4}(y_4 - \overline{y}_4)\right]\mathrm{d}t \\ & + \mathrm{sgn}(y_2 - \overline{y}_2) \\ & \times \left[-a_{21}\prod_{0<t_k<t} J_{k1}(y_1 - \overline{y}_1) - a_{22}\prod_{0<t_k<t} J_{k2}(y_2 - \overline{y}_2) - a_{23}\prod_{0<t_k<t} J_{k3}(y_3 - \overline{y}_3)\right]\mathrm{d}t \\ & + \mathrm{sgn}(y_3 - \overline{y}_3) \\ & \times \left[-a_{31}\prod_{0<t_k<t} J_{k1}(y_1 - \overline{y}_1) + a_{32}\prod_{0<t_k<t} J_{k2}(y_2 - \overline{y}_2) - a_{33}\prod_{0<t_k<t} J_{k3}(y_3 - \overline{y}_3) - a_{34}\prod_{0<t_k<t} J_{k4}(y_4 - \overline{y}_4)\right]\mathrm{d}t \\ & + \mathrm{sgn}(y_4 - \overline{y}_4) \\ & \times \left[-a_{41}\prod_{0<t_k<t} J_{k1}(y_1 - \overline{y}_1) + a_{43}\prod_{0<t_k<t} J_{k3}(y_3 - \overline{y}_3) - a_{44}\prod_{0<t_k<t} J_{k4}(y_4 - \overline{y}_4)\right]\mathrm{d}t \end{aligned} \quad (60)$$

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of (64) and Lemma 1, we obtain that

$$\infty > \mathbb{E}\int_0^{\infty} |z(t)|\,\mathrm{d}t > \sum_{k=1}^{\infty}\mathbb{E}\left[I_{\{\tau_{2k-1}<\infty\}}\int_{\tau_{2k-1}}^{\tau_{2k}} |z(t)|\,\mathrm{d}t\right]$$
$$\geq \varepsilon\sum_{k=1}^{\infty}\mathbb{E}\big[I_{\{\tau_{2k-1}<\infty\}}(\tau_{2k-1}-\tau_{2k})\big]. \tag{69}$$

where I_A is the indicator function of set A. We rewrite the model in (16) to the integral form as follows:

$$y_i(t) = y_i(0) + \int_0^t f_i(s, Y(s))\,\mathrm{d}s + \int_0^t \sigma_i y_i(s)\,\mathrm{d}B(s) \tag{70}$$

where $f_i(t, x(t))$ represents the terms included in dt on the right side of each equation of model (16). Applying the conclusion $\mathbb{E}[y_i^p(t)] \leq K_i(p)$ $(i = 2, 3, 4)$ of Theorem 2, we have

$$\mathbb{E}\Big(|f_i(t, Y(t))|^2\Big) \leq F_1(2, Y(0))$$
$$\mathbb{E}\Big(|\sigma_i y_i(s)|^2\Big) \leq G_i(2, Y(0)) \tag{71}$$

where $F_i(2, Y(0))$ and $G_i(2, Y(0))$ are positive constants related to the initial value $Y(0) \in \mathbb{R}_4^+$ and p. Using the Burkholder–Davis–Gundy inequality [42, Th. 7.2, p. 40], we obtain that

$$\mathbb{E}\left[I_{\{\tau_{2k-1}<\infty\}}\sup_{0\leq t\leq T}|y_i(\tau_{2k-1}+t) - y_i(\tau_{2k-1})|^2\right]$$
$$\leq 2T[TF_i(2, Y(0)) + 4G_i(2, Y(0))]. \tag{72}$$

Moreover

$$\mathbb{E}\left[I_{\{\tau_{2k-1}<\infty\}}\sup_{0\leq t\leq T}|Y(\tau_{2k-1}+t) - Y(\tau_{2k-1})|^2\right]$$
$$\leq 2T\sum_{i=1}^{4}[TF_i(2, Y(0)) + 4G_i(2, Y(0))]. \tag{73}$$

Similarly, we get

$$\mathbb{E}\left[I_{\{\tau_{2k-1}<\infty\}}\sup_{0\leq t\leq T}|\overline{Y}(\tau_{2k-1}+t) - \overline{Y}(\tau_{2k-1})|^2\right]$$
$$\leq 2T\sum_{i=1}^{4}\big[TF_i(2, \overline{Y}(0)) + 4G_i(2, \overline{Y}(0))\big]. \tag{74}$$

For the above given ε, choose a sufficiently small T such that

$$16T\sum_{i=1}^{4}\big[TF_i(2, \overline{Y}(0)) + 4G_i(2, \overline{Y}(0))\big] < \varepsilon^3. \tag{75}$$

Set

$$\Omega_k^1 = \left[\sup_{0\leq t\leq T}|Y(\tau_{2k-1}+t) - Y(\tau_{2k-1})| > \frac{\varepsilon}{2}\right]$$
$$\Omega_k^2 = \left[\sup_{0\leq t\leq T}|\overline{Y}(\tau_{2k-1}+t) - \overline{Y}(\tau_{2k-1})| > \frac{\varepsilon}{2}\right] \tag{76}$$

then follows from (73) and (74) that:

$$\mathbb{P}\Big(\{\tau_{2k-1}<\infty\}\cap\Omega_k^1\Big)$$
$$\leq \frac{2T\sum_{i=1}^4[TF_i(2, Y(0)) + 4G_i(2, Y(0))]}{\varepsilon^2/4} \leq \frac{\varepsilon}{2}$$
$$\mathbb{P}\Big(\{\tau_{2k-1}<\infty\}\cap\Omega_k^2\Big)$$
$$\leq \frac{2T\sum_{i=1}^4[TF_i(2, Y(0)) + 4G_i(2, Y(0))]}{\varepsilon^2/4} \leq \frac{\varepsilon}{2}. \tag{77}$$

Combining the above two inequalities can be obtain

$$\mathbb{P}\Big(\{\tau_{2k-1}<\infty\}\cap(\Omega_k^1\cup\Omega_k^2)\Big) \leq \varepsilon. \tag{78}$$

From the definition of Ω_1, we compute

$$\mathbb{P}\Big(\{\tau_{2k-1}<\infty\}\cap(\overline{\Omega_k^1}\cap\overline{\Omega_k^2})\Big)$$
$$= \mathbb{P}(\{\tau_{2k-1}<\infty\}) - \mathbb{P}\Big(\{\tau_{2k-1}<\infty\}\cap(\Omega_k^1\cup\Omega_k^2)\Big) \geq \varepsilon. \tag{79}$$

Therefore

$$\mathbb{P}(\{\tau_{2k-1}<\infty\})\cap\left\{\sup_{0\leq t\leq T}|u(\tau_{2k-1}+t) - u(\tau_{2k-1})| < \varepsilon\right\}$$
$$\geq \mathbb{P}\Big(\{\tau_{2k-1}<\infty\}\cap(\overline{\Omega_k^1}\cap\overline{\Omega_k^2})\Big) \geq \varepsilon. \tag{80}$$

Noting that $\tau_{2k}(\omega) - \tau_{2k-1}(\omega) \geq T$ for $\omega \in \{\tau_{2k-1}<\infty\}\cap\Omega_k^3$, we get

$$\infty > \varepsilon\sum_{k=1}^{\infty}\mathbb{E}\big[I_{\{\tau_{2k-1}<\infty\}}(\tau_{2k-1}-\tau_{2k})\big]$$
$$\geq \varepsilon T\sum_{k=1}^{\infty}\mathbb{P}\Big(\tau_{2k-1}<\infty\}\cap\Omega_k^3\Big) \geq \varepsilon T\sum_{k=1}^{\infty}\varepsilon = \infty \tag{81}$$

this is a contraction. Therefore, (66) must hold. In view of (66) and $\liminf_{t\to\infty}|z(t)| = 0$, we have $\lim_{t\to\infty}|z(t)| = 0$. Moreover

$$\lim_{t\to\infty}|x_i(t) - \overline{x}_i(t)| = \lim_{t\to\infty}\prod_{0<t_k<t}J_{ki}|y_i(t) - \overline{y}_i(t)|$$
$$\leq \lim_{t\to\infty}M_i|y_i(t) - \overline{y}_i(t)| = 0. \tag{82}$$

This completes the proof. ∎

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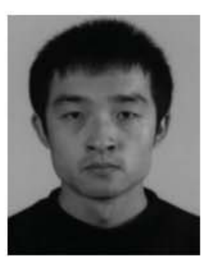

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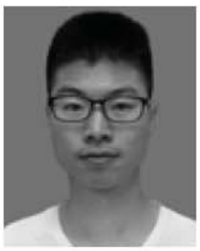

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