Data-Driven Identification of Dissipative Linear Models for Nonlinear Systems

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***Abstract*—We consider the problem of identifying a dissipative linear model of an unknown nonlinear system that is known to be dissipative, from time-domain input–output data. We first learn an approximate linear model of the nonlinear system using standard system identification techniques and then perturb the system matrices of the linear model to enforce dissipativity, while closely approximating the dynamical behavior of the nonlinear system. Further, we provide an analytical relationship between the size of the perturbation and the radius in which the dissipativity of the linear model guarantees local dissipativity of the unknown nonlinear system. We demonstrate the application of this identification technique through two examples.**

***Index Terms*—Dissipativity, identification, learning, nonlinear systems, passivity.**

I. Introduction

The fields of system identification and control design initially developed in isolation [1]. However, two systems that are "close" to each other in terms of the input–output response in the open loop may yield very different performance when put in feedback with the same controller. This realization led to the development of the area of identification for control, where the goal is to identify models such that controllers designed based on these models provide specific performance guarantees on the true system (see [1] for a comprehensive survey of this area). Many such methods were developed over the last few decades, the most popular of which are iterative development of the system model and the controller [2]–[7], and the development of data-based uncertainty sets for robust control [8]–[11]. With the recent emergence of learning-based controller design, this field has seen a resurgence of interest as well. An important challenge that still remains open in this area is that of ensuring analytical guarantees on the stability and performance of the closed-loop system, with controllers that are designed based on models that are learned from data.

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In this article, we consider the following problem. Assume that we have access to some information about the true system satisfying a structural property that makes it easy to design a controller and obtain a desired performance or stability guarantee on the closed-loop system. Can we identify a system model that satisfies this property? In particular, here, we consider the property to be that of dissipativity. Dissipativity is an important input–output property, which encompasses many important special cases such as $\mathcal{L}_2$ stability, passivity, and conicity. Dissipativity finds application in various domains ranging from robotics [12], electromechanical systems [13], and aerospace systems [14], to process control [15], [16], networked control and cyberphysical systems [17]–[19], and energy networks [20]–[23]. Dissipative systems possess several desirable properties like stability and compositionality [17]. Hence, if the original system is known to be dissipative, and we could exploit this fact to learn dissipative models, these models can then be used to design controllers that provide desired stability and performance guarantees on the original system. Note that existing identification methods may not yield a dissipative model even if the system is known to be dissipative. Furthermore, even if the model is dissipative, the dissipativity properties of the model do not, in general, yield any guarantees on the dissipativity properties of the true system, which are crucial to guarantee stability with closed-loop control.

We solve this problem of identifying a dissipative linear model of an unknown dissipative nonlinear dynamical system from given time-domain input-output data. Inspired by passive macromodeling approaches from RF circuit theory [24], we propose a two-stage approach. First, we learn an approximate linear model of the system, referred to as a baseline model, either using standard system identification techniques or using physics-based knowledge of the system. Next, we perturb the system matrices of this baseline linear model to enforce quadratic (QSR) dissipativity. We show that this perturbation can be chosen to ensure that the input–output behavior of the dissipative linear approximation closely approximates that of the original nonlinear system. Further, we provide an analytical condition relating the size of the perturbation to the radius in which local quadratic dissipativity properties of the nonlinear system can be guaranteed by the dissipative linear model. This relationship formalizes the intuition that larger perturbations lead to poorer approximations; in other words, the radius of local dissipativity of the nonlinear system decreases as the size of the perturbation is increased. Finally, we demonstrate the application of this approach to the problem of learning a dissipative model toward control of a switching circuit and a microgrid with high penetration of renewable energy sources.

We remark that if the main objective is simply to learn a linearization of the nonlinear system from data, then a technique like subspace identification can be employed [25]. Alternatively, if the goal is to learn the passivity index of the system, which can be considered a specific dissipativity property, recently developed allied approaches can be utilized to directly learn the index from input-output data [26]–[28]. In contrast to these works, our approach can be used to learn a broader class of dissipative models, encompassing properties like passivity, sector boundedness and $\mathcal{L}_2$ stability. In addition, our approach yields a model with guarantees on the dissipativity and the input–output response of the original system. Learning such a dissipative model provides two advantages. First, a dissipative linear model, as opposed to just a linearization learned from data, allows for a wide variety of control designs that specifically exploit dissipativity for applications like distributed control synthesis [15], [18], [23], [29]. Second, a dissipative model allows for control designs satisfying design specifications such as rise time and overshoot that may not be achievable purely using dissipativity indices learned from data. We illustrate one such application in Section IV. Further, there is work that relates the passivity of a system to its linear approximation [30], [31]; however, in that stream, a (dissipative) model of the system is assumed to be present, and is also assumed to be the first-order Taylor approximation of the nonlinear system, which may not be the case for models identified from data. Another closely related stream of work deals with estimating the range of inputs for which the dissipativity of a linearized model can be guaranteed, assuming that the dynamics of the nonlinear system are known [32]; however, this work does not provide an approach to enforce dissipativity on models of unknown nonlinear systems learned from data, or more importantly, provide any guarantees on the dissipativity of the original unknown system based on the linearized model.

We also note that constrained subspace identification techniques have been developed for the identification of specific subclasses of dissipative systems like positive systems [33], [34]. First, such constrained subspace identification approaches are typically only applicable to linear systems. Further, these approaches typically add constraints like positivity directly to specific system identification optimization problems, generally resulting in nonconvex formulations that may be difficult to solve. In contrast, our proposed approach is independent of the system identification technique used, and therefore does not add to the complexity of the identification problem with additional constraints that may impact its feasibility. Further, our approach also allows the designer to use any system identification technique of choice, not restricted to subspace identification. It also affords the designer the flexibility of choosing from a wide variety of highly efficient off-the-shelf or commercial toolboxes like the MATLAB System Identification toolbox, without requiring the development of a new tool simply for the identification of dissipative models.

Finally, we note that our approach is inspired by similar perturbation approaches used to obtain passive macromodels in RF electronics literature (see [24] and the references therein for a comprehensive survey of this area). Specifically, these perturbation approaches enforce passivity on linearized circuit models by enforcing the bounded real lemma on state-space models [35], [36], perturbing the eigenvalues of the Hamiltonian matrix of the macromodel [37]–[39], or by correcting passivity violations at specific frequency ranges using pole perturbation and/or linear and quadratic programming approaches [40]–[43]. While these approaches enforce passivity or positive realness on the identified model, there is limited literature on the problem of enforcing general quadratic dissipativity, encompassing several desirable properties such as $\mathcal{L}_2$ stability, conicity, and sector-boundedness in this setting. Further, existing perturbation approaches do not provide any guarantees on or estimates of the passivity properties of the original nonlinear system based on the passivity of the linearized model. In contrast, the formulation in our approach allows us to provide analytical guarantees on the dissipativity of the original nonlinear system based on the identified linear model. To the best of our knowledge, such guarantees on dissipativity of nonlinear systems based on linear models identified from data have not been provided thus far in literature. Furthermore, as previously mentioned, we also analytically quantify the tradeoff between the size of the perturbation and the region where dissipativity of the original nonlinear system can be guaranteed, allowing the designer to choose an appropriate perturbation based on application-specific requirements. These dissipativity guarantees on the original system are critical in ensuring stability and performance when these models are used for closed-loop control.

Notation: We denote the sets of real numbers, positive real numbers including zero, and n-dimensional real vectors by $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{R}^n$, respectively. Given a matrix $A \in \mathbb{R}^{m\times n}$, $A' \in \mathbb{R}^{n\times m}$ represents its transpose. A symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}$ is represented as $P > 0$ (and as $P \geq 0$, if it is positive semidefinite). The standard identity matrix is denoted by I, and a matrix with all elements equal to 1 is denoted by $\mathbf{1}$, with dimensions clear from the context. Given a function f, dom f represents its domain.

II. Problem Formulation

We consider an unknown nonlinear dynamical system

$$S_{nl}: \quad \dot{x}(t) = f(x(t), u(t)),\ y(t) = g(x(t), u(t)) \tag{1}$$

where f and g are differentiable functions defined on bounded domains dom$f$ and dom$g$, respectively, and $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^p$ represent the state, input, and output of the system at time $t \in \mathbb{R}_+$ respectively.

Assumption 1: The functions f and g are Lipschitz continuous, that is,

$$\begin{aligned} &||f(a_1) - f(a_2)|| \leq L_f||a_1 - a_2|| \quad \forall a_1, a_2 \in \text{dom}f \subset \mathbb{R}^n \times \mathbb{R}^m \\ &||g(a_1) - g(a_2)|| \leq L_g||a_1 - a_2|| \quad \forall a_1, a_2 \in \text{dom}g \subset \mathbb{R}^n \times \mathbb{R}^m \end{aligned} \tag{2}$$

where L_f and L_g are the Lipschitz constants of the functions f and g, respectively.

Assumption 2: There exists an equilibrium point $(x^*, u^*) = (0, 0)$ for system (1) so that $f(x^*, u^*) = 0$. Further, we assume $g(x^*, u^*) = 0$ at the equilibrium point.

The following definition of dissipativity is standard for such systems; however, we also define the notion of strict dissipativity as follows.

Definition 1 (Dissipativity and Strict Dissipativity): Let $\mathcal{X} \times \mathcal{U}$ be a neighborhood of the equilibrium (origin) $(x^*, u^*) = 0$. The nonlinear system S_{nl} is said to be (locally) *dissipative* with dissipativity matrices $Q = Q'$, S and $R = R'$, if $\forall x \in \mathcal{X}$ and control inputs $u \in \mathcal{U}$

$$y'(t)Qy(t) + u'(t)Ru(t) + 2y'(t)Su(t) \geq 0 \tag{3}$$

holds pointwise at every time $t \in \mathbb{R}_+$. $\forall t \in \mathbb{R}_+$. Further, the system S_{nl} is said to be (locally) *strictly dissipative* (SD) with dissipativity matrices $Q = Q'$, S and $R = R'$, if there exist constants $\rho > 0$ and $\nu > 0$, referred to as dissipativity indices, such that $\forall x \in \mathcal{X}$ and control inputs $u \in \mathcal{U}$,

$$\begin{aligned} &y'(t)Qy(t) + u'(t)Ru(t) + 2y'(t)Su(t) \\ &\geq \rho x'(t)x(t) + \nu u'(t)u(t) \end{aligned} \tag{4}$$

holds pointwise at every time $t \in \mathbb{R}_+$.

We ignore the qualifier "locally" in front of dissipativity properties for pedagogical ease. Similarly, we also drop the dependence of all vectors on time for the simplicity of notation. Dissipativity is an input–output property that generalizes the notion of passive circuit elements used in electrical and electronic circuit theory to a more generalized notion of energy (not necessarily a physical quantity) that is applicable to nonlinear dynamical systems. Definition 1 represents the property of quadratic dissipativity, commonly referred to as QSR-dissipativity.

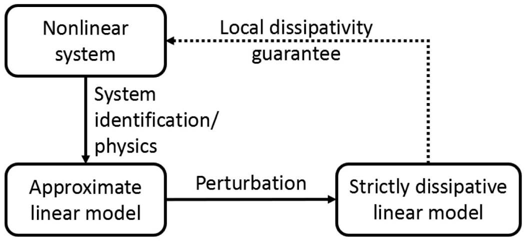

Fig. 1. Two-stage approach for identification of dissipative linear models.

In particular, (3) [and its strict form (4)] corresponds to a differential (pointwise) version of "weak"' dissipativity, and has been shown to be equivalent to several standard integral definitions of dissipativity under mild assumptions [44, Ch. 4.4]. We specifically consider QSR-dissipativity, since it can be used to capture several useful system properties through appropriate choice of the dissipativity matrices Q, S, and R in (3) such as

i) *passivity*, with $Q = 0$, $S = \frac{1}{2}I$ and $R = 0$;
ii) *strict passivity*, with $Q = -aI$, $S = \frac{1}{2}I$, and $R = -bI$, where $a, b \in \mathbb{R}^+ \backslash \{0\}$;
iii) $\mathcal{L}_2$ *stability*, with $Q = -\frac{1}{\gamma}I$, $S = 0$, and $R = \gamma I$ where $\gamma \in \mathbb{R}^+$ is an $\mathcal{L}_2$ gain of the system;
iv) *conicity*, with $Q = -I$, $S = cI$, and $R = (r^2 - c^2)I$, where $c \in \mathbb{R}$ and $r \in \mathbb{R}^+ \backslash \{0\}$, and;
v) *sector-boundedness*, with $Q = -I$, $S = (a + b)I$, and $R = -abI$, where $a, b \in \mathbb{R}$.

Note that any SD system is also dissipative and satisfies (3). We now formally state the problem addressed in this article.

Problem $\mathcal{P}$: Given a set of N time-domain input–output measurements $\{(\hat{y}, \hat{u})\} = \{(\hat{y}_1, \hat{u}_1), (\hat{y}_2, \hat{u}_2), \ldots, (\hat{y}_N, \hat{u}_N)\}$, $u \in \mathcal{U}$ from a dissipative nonlinear system S_{nl} satisfying (3), the aim of this article is to obtain a linear model

$$S_l: \quad \dot{\tilde{x}} = A\tilde{x} + Bu, \ \tilde{y} = C\tilde{x} + Du \tag{5}$$

such that (i) S_l approximates S_{nl} with an error bound $\tilde{\delta}_y$ in the neighborhood $\mathcal{X} \times \mathcal{U}$ of the origin in which the system is locally dissipative, that is, $||\tilde{y} - y||^2 < \tilde{\delta}_y$, for all inputs $u \in \mathcal{U}$ and initial conditions $x_0 \in \mathcal{X}$, and (ii) S_l is SD.

We will address this problem in two stages, as shown in Fig. 1. We will begin by assuming that an approximate linear model of S_{nl} can be estimated either through standard regression-based or subspace system identification, and/or from the physics of the system. If this linear approximation is not SD, we will then introduce a bounded perturbation into the system matrices such that the resulting perturbed model is SD, while closely approximating the behavior of the nonlinear system S_{nl}. We require the linear model S_l to be strictly dissipative rather than just dissipative, since this allows us to provide guarantees on the local dissipativity of S_{nl}. We conclude this section by stating a matrix inequality that can be used to verify if the linear model S_l is SD.

Theorem 1 ([44]): The linear system (5) is strictly dissipative if there exists a symmetric matrix $P = P'$, and constants $\nu > 0$ and $\rho > 0$ satisfying

$$\begin{bmatrix} A'P + PA - C'QC + \rho I & PB - \hat{S} \\ B'P - \hat{S}' & -\hat{R} + \nu I \end{bmatrix} < 0 \tag{6}$$

where $\hat{S} = C'S + C'QD$ and $\hat{R} = R + D'S + S'D + D'QD$. Further, if this condition is satisfied, the linear system is *globally* dissipative.

III. Identification of Dissipative Models

In this section, we describe a two-stage approach to identify a dissipative linear model S_l that closely approximates the nonlinear system S_{nl}.

A. Baseline linear model

Given a set of N time domain input–output measurements $\{(\hat{y}, \hat{u})\}=\{(\hat{y}_1, \hat{u}_1), (\hat{y}_2, \hat{u}_2), \ldots, (\hat{y}_N, \hat{u}_N)\}, \hat{u} \subset \mathcal{U}$ from system S_{nl} in the vicinity of the equilibrium, we begin by assuming that a standard system identification technique, such as subspace or regression-based identification [25] can be used to identify an approximate *baseline linear model*

$$S_b: \quad \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}u, \ \bar{y} = \bar{C}\bar{x} + \bar{D}u \tag{7}$$

such that $||\bar{y} - y||^2 < \bar{\delta}_y$, for all inputs $u \in \mathcal{U}$ and initial conditions $x_0 \in \mathcal{X}$. For the identification of dissipative models, it is recommended that the baseline model is obtained by directly identifying a continuous-time linear system, since the conversion of discrete-time models to continuous-time may result in system zeros that affect the dissipativity of the model. We note that it is fairly straightforward to identify continuous-time models from data using standard software like the MATLAB System Identification Toolbox [25]. We can also estimate the Lipschitz constant L_g from the input–output data $\{(\hat{y}, \hat{u})\}$ as

$$L_g \approx \max_{\hat{u}_i, \hat{u}_j \in \hat{u}, \hat{u}_i \neq \hat{u}_j} ||\hat{y}_i - \hat{y}_j||/||\hat{u}_i - \hat{u}_j||, i, j \in \{1, 2, \ldots, N\}. \tag{8}$$

Alternatively, (8) can be applied to the approximate linear system S_b to easily obtain an estimate of the Lipschitz constant L_g. Note that it has been observed that (8) provides a good estimate of the Lipschitz constant if the dataset $\{(\hat{y}, \hat{u})]\}$ is sufficiently rich [26].

B. Perturbed linear model

If the linear model S_b is not SD, that is, it does not satisfy (4) with y replaced by $\bar{y}$, then, we would like to introduce a bounded perturbation ΔC into the output matrix of S_b to obtain a SD perturbed linear model

$$S_l: \quad \dot{\tilde{x}} = A\tilde{x} + Bu, \ \tilde{y} = C\tilde{x} + Du \tag{9}$$

where $A = \bar{A}$, $B = \bar{B}$, $C = \bar{C} + \Delta C$, and $D = \bar{D}$. We have chosen to perturb the output matrix $\bar{C}$ to obtain the perturbed linear model S_l. However, the dissipativity of the system depends on all the system matrices as seen in (6). In this context, we make the following comments:

1) The input matrix $\bar{B}$ or the feedforward matrix $\bar{D}$ can be perturbed instead of the output matrix $\bar{C}$, depending on system specific requirements.
2) Any perturbation on the system matrix $\bar{A}$ is not preferable, since we would like the perturbed linear model to preserve any information about the dominant modes of the nonlinear system that is embedded in the baseline linear model, thereby allowing the perturbed model to closely approximate the original nonlinear system.
3) If the baseline model S_b has $\bar{D} = 0$ and it is required to ensure $D > 0$ in the linear model S_l to meet strict dissipativity or other desired system properties, then the feedforward matrix $\bar{D}$ can be perturbed to enforce the positive definiteness of D.

We would like to minimize the size of the perturbation $||\Delta C||_2^2$, in order to ensure that the linear model S_l closely approximates the original nonlinear system S_{nl}. Further, we would like to relate the strict dissipativity of S_l to local dissipativity of the nonlinear system S_{nl}. We have the following result on the choice of the perturbation ΔC, and its relationship to the strict dissipativity of S_l and S_{nl}.

Theorem 2: Given the linear model (9), if the problem

$$\mathcal{P}_1: \quad \min_{\nu>0,\rho>0,P>0,\Delta C} \alpha = ||\Delta C||_2^2 \tag{10a}$$

$$\text{s.t.} \begin{bmatrix} A'P + PA - C'QC + \rho I & PB - \hat{S} \\ B'P - \hat{S}' & -\hat{R} + \nu I \end{bmatrix} < 0,$$

$$\hat{S} = C'S + C'QD,\ \hat{R} = R + D'S + S'D + D'QD$$

$$P = P' > 0 \tag{10b}$$

is feasible, then

1) S_l is SD with dissipativity matrices Q, S, and R,
2) S_{nl} is locally dissipative with dissipativity matrices Q, S and R in a neighborhood $\mathcal{X} \times \mathcal{U}$ around the origin, and
3) S_l approximates S_{nl} with an error bound δ_y, that is, $\|\tilde{y} - y\|^2 < \tilde{\delta}_y$, for all inputs $u \in \mathcal{U}$ and initial conditions $x_0 \in \mathcal{X}$, where $\tilde{\delta}_y$ is a linear function of the perturbation α and the error of the baseline model $\bar{\delta}_y$.

Proof: We separately prove each part of Theorem 2.

1) This follows from Theorem 1, (10b) and (4).
2) Define the error in the input–output response between the linear model S_l and the nonlinear system S_{nl} as

$$\epsilon_g = C\tilde{x} + Du - g(x, u). \tag{11}$$

Then, we have $y = \tilde{y} - \epsilon_g$. Now, if $\mathcal{P}_1$ is feasible, S_l is SD and satisfies (4) pointwise at each time $t \in \mathbb{R}_+$. Therefore, since (4) holds for any $\tilde{x}$, it must also hold for $\tilde{x} = x$. Therefore, we have

$$\tilde{y}'Q\tilde{y} + u'Ru + 2\tilde{y}'Su \geq \rho||x||^2 + \nu||u||^2. \tag{12}$$

Also, from (11), (2), and Assumption 2, using the triangle inequality, we can write

$$\begin{aligned} ||\epsilon_g|| &\leq L_g||x|| + L_g||u|| + ||C||||x|| + ||D||||u|| \\ &= (L_g + ||C||)\,||x|| + (L_g + ||D||)\,||u||. \end{aligned} \tag{13}$$

Using Jensen's inequality in (13) gives

$$||\epsilon_g||^2 \leq 2(L_g + ||C||)^2||x||^2 + 2(L_g + ||D||)^2||u||^2. \tag{14}$$

Now consider

$$I = y'Qy + 2y'Su + u'Ru = \phi - 2\epsilon_g'Q\tilde{y} - 2\epsilon_g'Su \tag{15}$$

where $\phi = \tilde{y}'Q\tilde{y} + u'Ru + 2\tilde{y}'Su - \epsilon_g'Q\epsilon_g$. Then, from (12) and (14), we have

$$\begin{aligned} \phi &\geq \left(\rho - 2||Q||^2(L_g + ||C||)^2\right)||x||^2 \\ &\quad + \left(\nu - 2||Q||^2(L_g + ||D||)^2\right)||u||^2. \end{aligned} \tag{16}$$

We also have

$$2\epsilon_g'Q\tilde{y} + 2\epsilon_g'Su = 2\epsilon_g'QCx + 2\epsilon_g'(S + QD)u \tag{17}$$

$$\begin{aligned} 2\epsilon_g'QCx &\leq ||\epsilon_g||^2 + ||Q||^2||C||^2||x||^2 \\ 2\epsilon_g'(S + QD)u &\leq ||\epsilon_g||^2 + ||(S + QD)||^2||u||^2. \end{aligned} \tag{18}$$

If $\mathcal{P}_1$ is feasible, then (10c) and (10d) hold. Then, using (10c), (10d,) and (16)–(18) in (15), we have

$$\begin{aligned} I &\geq \hat{\rho}||x||^2 + \hat{\rho}||u||^2 \geq 0, \quad \hat{\nu} > 0,\ \hat{\rho} > 0 \\ \hat{\rho} &= \rho - ||Q||^2||\bar{C} + \Delta C||^2 \\ &\quad - 2\left(||Q||^2 + 1\right)\left(L_g + ||\bar{C} + \Delta C)||\right)^2 \\ \hat{\nu} &= \nu - 2\left(||Q||^2 + 1\right)\left(L_g + ||D||\right)^2 - ||S + QD||^2. \end{aligned} \tag{19}$$

Note that all of the above inequalities hold pointwise for every time $t \in \mathbb{R}_+$. Using (19) in Definition 1, S_{nl} is locally dissipative in a neighborhood $\mathcal{X} \times \mathcal{U}$ of the origin if $\mathcal{P}_1$ is feasible, where $\mathcal{X} \times \mathcal{U}$ is an ϵ-ball around the origin, with

$$\epsilon = \min\left(\frac{\epsilon_g}{\sqrt{2}(L_g + ||\bar{C} + \Delta C||)}, \frac{\epsilon_g}{\sqrt{2}(L_g + ||\bar{D}||)}\right). \tag{20}$$

3) If $\mathcal{P}_1$ is feasible, then, for the baseline model S_b with $\bar{u} = u \in \mathcal{U}$, we have $||\bar{y} - y||_2^2 < \bar{\delta}_y$. Then,

$$\begin{aligned} ||\tilde{y} - y||_2^2 &= ||\tilde{y} - y + \bar{y} - \bar{y}||_2^2 \\ &\leq ||\tilde{y} - \bar{y}||_2^2 + ||\bar{y} - \hat{y}||_2^2 \\ &\leq ||\tilde{y} - \bar{y}||_2^2 + \bar{\delta}_y \leq \alpha||x||^2 + \bar{\delta}_y = \tilde{\delta}_y. \end{aligned} \tag{21}$$

Theorem 2 provides conditions that can be used to choose the perturbation such that the linear model obtained closely approximates the original nonlinear system. Further, if $\mathcal{P}_1$ is feasible, then the nonlinear system S_{nl} is strictly dissipative in a neighborhood around the origin. Algorithm 1 provides the procedure to identify a linear model S_l that solves $\mathcal{P}$. We make the following observations regarding Theorem 2.

1) As the size of the perturbation $||\Delta C||_2^2$ increases, the constraint (10c) becomes harder to satisfy, that is, the model will require higher dissipativity indices.
2) Equation (20) provides a condition relating the size of the perturbation to the radius in which local strict dissipativity of S_{nl} can be guaranteed by strict dissipativity of S_l. The ϵ-neighborhood in which the local dissipativity of the nonlinear system is guaranteed shrinks with the size of the perturbation. Therefore, while large perturbation may be used to obtain a dissipative linear model of a nonlinear system, the radius of validity of this model would be extremely small. From (21), we also observe that the error bound $\tilde{\delta}_y$ of S_l grows linearly with the size of the perturbation α. Similarly, from (20), a poor estimation (overestimation) of the Lipschitz constant would result in a small radius of validity of the learned model.
3) While the constraint (10c) is nonconvex, in practice, it is easy to solve $\mathcal{P}_1$ in two steps. First, we find some $\nu > 0$ and $\rho > 0$ such that $\mathcal{P}_1$ is feasible with constraints (10b) and (10d). Then, we check if (10c) is feasible. If not, we increase the value of ρ and re-solve $\mathcal{P}_1$. It is also possible to further simplify the solution of the problem $\mathcal{P}_1$ by choosing a fixed perturbation $\Delta C = \gamma\mathbf{1}$.
4) For a given input–output dataset, it is possible to identify a set of models that closely approximate the system. However, selection of a model depends not only on the model output error but also other application specific requirements. The same is true for the identification of a perturbed dissipative model satisfying $\mathcal{P}_1$.

vi) The dissipativity matrices, or their parameters mentioned in the definition, can be optimization variables in $\mathcal{P}_1$.

IV. Case Studies

In this section, we provide two numerical examples to illustrate the identification approach proposed in Section III.

Example 1 - Tunnel Diode Switching Circuit: As a simple numerical example, we consider a tunnel diode switching circuit as shown in Fig. 2(d)-Inset. Such circuits have been widely used as high-speed switches for several decades [45] and have recently found utility in microwave photonics applications [46]. The switching circuit in Fig. 2(d)-Inset is modeled by the nonlinear system

$$\begin{aligned} \dot{x}_1 &= (-x_1^2 + x_2)/C,\ \dot{x}_2 = (-x_1 - Rx_2 + (0.5x_1 + 1)u)/L \\ y &= x_1 + x_2 + (0.5x_1 + 1)u \end{aligned} \tag{22}$$

where $x_1 = v_C$, $x_2 = i_L$, $y = v_R$, and $R = L = C = 1$ p.u.

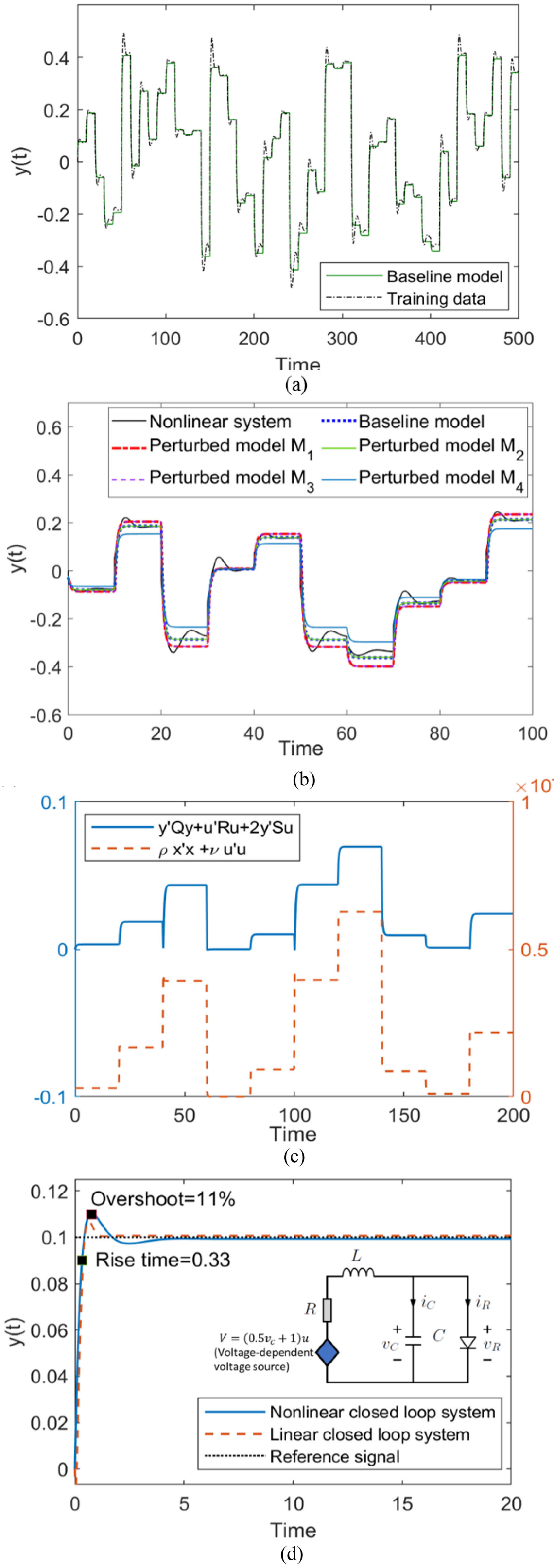

Fig. 2. (a) Baseline model and training data for system. (b) Input–output performance of the strictly passive linear models $\mathbf{M_1}, \mathbf{M_2}, \mathbf{M_3}$, and $\mathbf{M_4}$, the baseline model, and the nonlinear system. (c) Visualization of dissipativity inequality (4) for the learned passive linear model $\mathbf{M_1}$ with the same input as Fig. 2(b). (d) Performance of controller satisfying rise-time and overshoot specifications, applied to the linear model $\mathbf{M_1}$ and the original nonlinear system. Inset: Tunnel diode switching circuit.

Algorithm 1: Identification of Dissipative Model.

Input Measurement vectors $\{(\hat{y}, \hat{u})\}$.
Output A, B, C, D and α.

1: **Estimate baseline model:** Use standard subspace or regression-based identification techniques [25] to estimate $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ of S_b such that $||\bar{y} - y||_2^2$ is minimized.
2: Check if $\mathcal{P}_2$ is feasible, where

$$\mathcal{P}_2: \quad \text{Find:} \quad \nu > 0, \rho > 0, P > 0$$

$$\text{s.t.} \quad \begin{bmatrix} \bar{A}'P + P\bar{A} - \bar{C}'Q\bar{C} + \rho I & P\bar{B} - \hat{S} \\ \bar{B}'P - \hat{S}' + & -\hat{R} + \nu I \end{bmatrix} \leq 0$$

$$\hat{S} = \bar{C}'S + \bar{C}'Q\bar{D}, \ \hat{R} = R + \bar{D}'S + S'\bar{D} + \bar{D}'Q\bar{D}.$$

3: **if** $\mathcal{P}_2$ is feasible, **then**
4: Set $A = \bar{A}$, $B = \bar{B}$, $C = \bar{C}$, $D = \bar{D}$.
5: **else**
6: **Perturbation model:** Set $C_i \leftarrow \bar{C} + \Delta C$, $\Delta C = \gamma \mathbf{1}$.
7: Find $\nu > 0, \rho > 0, P > 0$ and $\gamma > 0$ solving $\mathcal{P}_1$ with constraints (10b) and (10d).
8: **if** ρ from Step 7 satisfies constraint (10c) **then**
9: Set $A = \bar{A}$, $B = \bar{B}$, $C = \bar{C} + \Delta C$, $D = \bar{D}$.
10: Compute α.
11: **else**
12: Increase $\rho \mapsto \rho + d$, where $d > 0$. Go to Step 7.
13: **end if**
14: **end if**

It can be verified that (22) is dissipative, and more specifically, strictly passive. While the dynamics of Fig. 2(d)-Inset are simple to write, it is not so straightforward to compute the same for more complicated circuits with interacting switching components. Therefore, we would like to learn a dissipative linear model of such a system from data. In this application, the aim is to design a feedback controller such that the output y tracks a reference step (switching) signal $s(t)$ with a specified fast rise time (<0.5 p.u.), and small overshoot ($<15\%$). Furthermore, the closed-loop system is required to be passive to allow for easy physical implementation. Since the nonlinear system is strictly passive, simply ensuring strict passivity of the feedback controller to be designed is sufficient to preserve the passivity of the closed-loop system. While learning the passivity indices of the system is sufficient to design such a controller, it is not possible to design a controller to meet rise time and overshoot specifications on the step response simply using the passivity indices. Therefore, we proceed to learn a linear dissipative model of the system toward synthesizing a tracking controller. Following the procedure in Algorithm 1, we first learn a baseline linear model of this system with

$$\bar{A} = \begin{bmatrix} 0 & 1 \\ -46.24 & -22.31 \end{bmatrix}, \bar{B} = [0 \quad 1]',$$
$$\bar{C} = [95.61 \quad -4.78], \quad \bar{D} = 0.1 \tag{23}$$

using the MATLAB System Identification Toolbox. The response of the baseline model and the training data used to obtain the model are shown in Fig. 2(a). We then verify that $\mathcal{P}_2$ in Step 2 of Algorithm 1 is not feasible with the baseline model (23). Therefore, we employ the proposed approach to learn passive linear models of the system as follows.

1) *Perturbed linear model* $\mathbf{M_1}$*:* We first follow the procedure outlined in Algorithm 1 to obtain the perturbed linear model with

$\Delta C = \gamma\mathbf{1} = 9.53 \times [1 \;\; 1]$. As seen in Fig. 2(c), this perturbed linear model is strictly passive, and satisfies inequality (6) with the appropriate dissipativity matrices. We also observe that the perturbed linear model closely approximates the nonlinear system by validating its input–output response for a test dataset [see Fig. 2(b)].

2) *Perturbed linear model* $\mathbf{M_2}$*:* We chose a fixed perturbation $\Delta C = \gamma\mathbf{1}$ in order to simplify the solution of problem $\mathcal{P}_1$. We now examine the conservativeness of this relaxation as follows. We follow the procedure in Algorithm 1, albeit without placing any restrictions on the structure of the perturbation ΔC in Step 6, to obtain another perturbed model $\mathbf{M_2}$, with $\Delta C = [8.75 \;\; -7.84]$. We observe that model $\mathbf{M_2}$ is passive with only a slightly smaller perturbation than model $\mathbf{M_1}$. Further, as seen in Fig. 2(b), the model $\mathbf{M_1}$ is very close to model $\mathbf{M_2}$ in its input–output response, indicating that the choice of the fixed perturbation may simplify the problem $\mathcal{P}_1$ without introducing too much conservatism. Nevertheless, choosing the same perturbation γ to be applied to all elements of the C matrix is by far not optimal, and it is expected that—with the proper implementation of the baseline optimization scheme—using other system norms will provide much better results.

3) *Perturbed linear model* $\mathbf{M_3}$*:* While we have chosen to minimize the norm $||\Delta C||_2^2$ in problem $\mathcal{P}_1$ to enforce closeness between the input-output behavior of the baseline model and the perturbed model, it is possible to consider other system norms for this purpose. As an example, we choose a commonly used metric in the model reduction and macromodeling literature [24], namely, $\alpha = \mathrm{trace}(\Delta C W_c \Delta C^T)$, where W_c is the controllability Gramian of the baseline model, as the objective function in problem $\mathcal{P}_1$. With this setup, we follow the procedure in Algorithm 1 with a fixed perturbation $\Delta C = \gamma\mathbf{1}$ in Step 6, to obtain the perturbed linear model $\mathbf{M_3}$ with $\gamma = 9.51$, which turns out to be extremely close to the perturbation in model $\mathbf{M_1}$. As seen in Fig. 2(b), the input–output response of model $\mathbf{M_3}$ is also virtually indistinguishable from that of model $\mathbf{M_1}$. These results indicate that the choice of the norm $||\Delta C||_2^2$ in obtaining model $\mathbf{M_1}$ is not too conservative in this regard.

4) *Perturbed linear model* $\mathbf{M_4}$*:* Finally, we consider the objective function $\alpha = \mathrm{trace}(\Delta C W_c \Delta C^T)$ in the problem $\mathcal{P}_1$, and solve Algorithm 1 without placing any restrictions on the structure of the perturbation ΔC in Step 6. We thus obtain the perturbed model $\mathbf{M_4}$ with $\Delta C = [8.33 \;\; -17.26]$. Interestingly, this approach results in a much larger perturbation and a more conservative model as compared to models $\mathbf{M_1}$, $\mathbf{M_2}$, and $\mathbf{M_3}$. Further, we observe a larger deviation between the input–output responses of model $\mathbf{M_4}$ and the original nonlinear system in Fig. 2(b). One possible explanation for this is as follows. Due to the nonconvexity of the optimization problem, the solution obtained here may be a locally optimal one whose performance may be inferior to other solutions like those obtained in models $\mathbf{M_1}$, $\mathbf{M_2}$, and $\mathbf{M_3}$. This study indicates that, contrary to intuition, a lack of restriction on the structure of the perturbation may not always yield less conservative models as compared to the fixed perturbation $\Delta C = \gamma\mathbf{1}$.

We note that the proposed approach can be extended to consider norms other than the ones described above based on application-specific requirements. For example, bandlimited norms which measure the system perturbation only over a particular frequency band of interest may be considered if the bands are known *a priori*.

We now use the perturbed linear model $\mathbf{M_1}$ to design a controller

$$\dot{x}_c = -0.07x_c + y,\; y_c = 4.59x_c + 0.02y,\; u = s - y \qquad (24)$$

that satisfies the given rise time and overshoot specifications. The tracking performance of this controller with the nonlinear system (22) is shown in Fig. 2(d) and can be verified to meet the design specifications. The controller (24) is strictly passive and satisfies (6) with $\rho = 0.61$ and $\nu = 0.01$. Since the feedback interconnection of two strictly passive systems is also passive, the closed-loop interconnection of (22) and (24) is guaranteed to be passive.

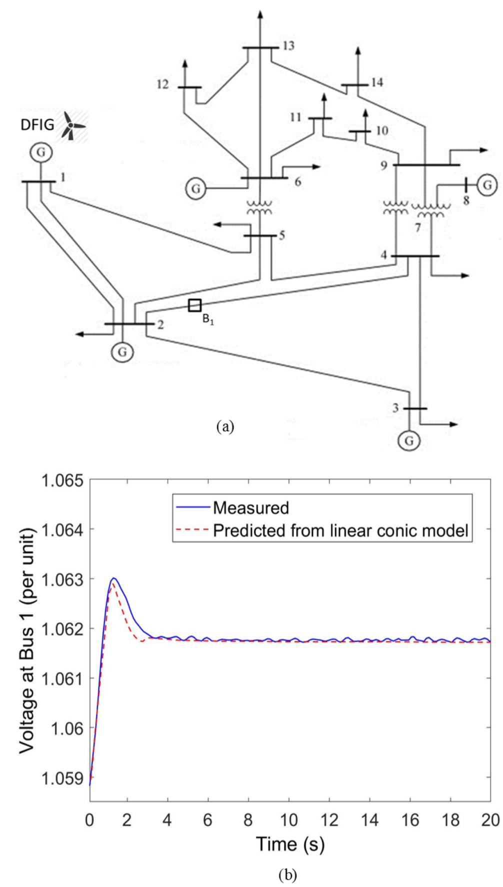

Fig. 3. 14-bus microgrid: (a) schematic, and (b) comparison of linear dissipative (conic) model and nonlinear system.

Example II—Microgrid: We now consider the application of the proposed approach to obtain a dissipative model of the 14-bus microgrid system shown in Fig. 3(a), in the vicinity of a specific power flow operating point (equilibrium). The system shown in Fig. 3(a) is obtained as a modification the standard IEEE 14-bus test system by replacing the largest generators in the system at buses 1, 2, and 3 with equivalent DFIG wind, photovoltaic, and solid oxide fuel cell plants of 600 kVA, 60 kVA, and 60 kVA, respectively. The synchronous generators at buses 6 and 8 are rated 25 kVA each (see [47] detailed state space models of the system). Therefore, 93.5% of the generation in this system is attributed to renewable generators, making this system challenging to control. However, the system is known to be conic, and this property can be exploited to design controllers that enhance the performance and stability of this system, even with the variability introduced by the renewable energy generators [20], [21]. Therefore, we would like to obtain a linear conic model of this system.

We note that a baseline model for this system can be readily obtained since the structure of the nonlinear differential equations, as well as estimates of the system parameters are well known from the system physics [47]. We obtain a baseline model with $A \in \mathbb{R}^{35\times35}$, $B \in \mathbb{R}^{35\times5}$, $C \in \mathbb{R}^{5\times35}$, and $D \in \mathbb{R}^{5\times5}$ around the power flow operating point (equilibrium) where the DFIG real and reactive power outputs are 350 MW and -28 MVAr, respectively. The 35 system states comprise of the speed, pitch angle, and d-q axes currents of the DFIG, the partial pressures of the reactants (hydrogen, oxygen, and water), molar flow of hydrogen, current, and voltage of the SOFC, d-q axes inverter currents and voltage output of the solar cell, the angle, speed, and d-q axes voltages of the synchronous generators, four states corresponding to internal voltages of the automatic voltage regulator (AVR) of the synchronous generators, and three states corresponding to the turbine governor of the synchronous generators. The inputs of the system comprise of the reference pitch angle and speed of the DFIG, the reference signals of the AVRs, and the modulation index of the solar cell.

Using the procedure described in Algorithm 1, we obtain a linear perturbed model from the baseline model. This linear perturbed model is conic with $\rho = 0.07$, $\nu = 0.21$, conic sector radius $r = 2.575$ and cone center $c = 5$; c.f. the definition of conic systems mentioned earlier. Fig. 3(b) shows the comparison between the measured voltage outputs and those generated by the conic model at bus 1 (wind generator) for a load change (disturbance) where all loads in the network are decreased by 2%. These results indicate that models with suitable dissipativity properties can be constructed to closely approximate the dynamics of complex nonlinear networked systems around specific operating points.

V. Conclusion

We considered the problem of identifying a dissipative linear model of an unknown nonlinear system from time-domain input–output data, when a baseline linear model of the system can be easily obtained using the physics of the system and/or standard system identification techniques. We propose a technique to perturb the system matrices of the baseline model to obtain a strictly dissipative linear model that closely approximates the original nonlinear system. While the proposed approach is offline, it is promising to extend the perturbation approach to quickly identify dissipative models in an online setting, where a baseline model is typically already available. In scenarios where the baseline model also needs to be identified online, further investigation into the computational complexity introduced by this additional step will be necessary, which is an interesting direction for future research.

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