# EXIT Analysis for Belief Propagation in Degree-Correlated Stochastic Block Models

Hussein Saad, Ahmed Abotabl and Aria Nosratinia
Department of Electrical Engineering,
The University of Texas at Dallas, Richardson, TX 75083-0688, USA,
E-mail: hussein.saad@utdallas.edu; ahmed.abotabl@utdallas.edu; aria@utdallas.edu.

***Abstract*—This paper proposes the extrinsic information transfer (EXIT) method for the analysis of belief propagation in community detection on random graphs, specifically under the degree correlated stochastic block model. Belief propagation in community detection has been studied under density evolution; this work for the first time brings EXIT analysis to community detection on random graphs, which has certain advantages that are well documented in the parallel context of error control coding. We show using simulations that in the case of equally-sized communities, when the probability of connectivity in the communities are different, there is only one intersection point, hence belief propagation is optimal. When the probability of connectivity in the communities are the same, we show that belief propagation is equivalent to random guessing and the curves intersect at the trivial zero-zero point. For the roughly equal-sized communities, we show that there is always only one intersection point, suggesting that belief propagation is optimal. Finally, for the communities with disparate size, we show that there are multiple intersection points, hence belief propagation is likely to be sub-optimal.**

***Index Terms*—Community detection, Stochastic block model, Belief Propagation, Exit charts.**

## I. Introduction

The problem of learning community structures in random graphs has been intensely studied in various fields: statistics [1], [2], computer science [3], [4] and theoretical statistical physics [5]. In these problems, a collection of vertices are divided into several communities or clusters and then a random graph is drawn in a way that is dependent on the community assignment. The goal is to recover the underlying communities from the observation of the graph. The problem of detecting communities has many applications: finding like-minded people in social networks [6], improving recommendation systems [7], detecting protein complexes [8].

Several models are now being studied for random graphs that exhibit a community structure; a survey can be found in [9]. In this paper, we are interested in the binary degree correlated stochastic block model, which assumes that n vertices are partitioned into two communities with edge probability $\frac{a}{n}$ in the first community, $\frac{c}{n}$ in the second community and $\frac{b}{n}$ between the two communities. If $a \neq c$, the vertex degrees are stochastically correlated with the community structure, and hence, the name of degree correlated stochastic block model.

This work was supported in part by the grant CIF1219065 from the National Science Foundation.

Different recovery regimes have been studied depending on the scaling behavior of the parameters a, b and c with n. We present here two:

- Weak recovery: if the average degree is $\Theta(1)$, the resulting graph will have at least a constant fraction of isolated nodes, and hence one can only hope to find communities that are positively correlated with the true one. In the binary symmetric stochastic block model with $a = c$, it was proved in [10] that weak recovery is possible if and only if $\frac{(a-b)^2}{(a+b)} > 2$.
- Exact recovery: it is possible to recover all the communities (up to a global flip) with high probability, if the average degree is $\Omega(\log n)$. For results about exact recovery phase transitions, see [11], [12].

Recently, a different question of what is the minimum fraction of misclassified vertices on average was addressed in [13] under the binary degree correlated stochastic block model. Specifically, consider the following regime:

$$a = b + \sqrt{b}\mu, \quad c = b + \sqrt{b}\nu, \quad b \to \infty, \quad b = n^{o(1)} \tag{1}$$

for fixed constants $\mu$ and $\nu$. Under this regime, the authors showed that for the case of equally sized communities and $\mu \neq \nu$, the minimum fraction of misclassified on average is $Q(\sqrt{v^*})$, where $Q(x)$ is the Q-function of the standard normal, $v^*$ is the unique fixed point of $v = \frac{(\mu-\nu)^2}{16} + \frac{(\mu+\nu)^2}{16}\,\mathbb{E}[\tanh(v + \sqrt{v}Z)]$, and $Z$ is standard normal. They also showed that this minimum value can be attained by a local algorithm, namely belief propagation.

Density evolution, however, is complex and does not easily offer a great deal of insight into the operation of message passing algorithms, e.g. belief propagation [14]. This is due to the fact that calculating and tracking the densities in general can be complicated. An alternative approach is to track the mutual information at each iteration. This is done via extrinsic information transfer (EXIT) charts. Not only EXIT charts provide reduction in complexity, but also is more insightful. In fact, by observing the EXIT chart, one can predict: whether the decoder will fail or not, approximation of the number of iterations needed to decode, approximation of the error probability. Moreover, EXIT charts have the advantage of the information theoretic interpretation [14].

Based on the above observations, in this paper we propose EXIT analysis of the degree correlated stochastic block model.

To the extent of our knowledge, this is the first time EXIT charts are used in the context of community detection under the stochastic block model. To show the effectiveness of the approximations done using EXIT charts, we corroborate the results of [13], where we show through simulations that:

- For the case of equal sized (balanced) communities and $\mu \neq \nu$, there is only one intersection point in the EXIT chart and that the curves are concave, and hence belief propagation is optimal. We also show that as the difference between $\mu$ and $\nu$ increases, the intersection point approaches the maximum of the mutual information, i.e. fraction of misclassified nodes decreases. Moreover, we calculate an approximation of the fraction of misclassified nodes from the EXIT charts, and show that it is a good approximation by comparing it to the exact values calculated in [13].
- For the case of balanced communities and $\mu = \nu$, we show that the local belief propagation will always fail, since the EXIT chart will have the trivial $(0,0)$ intersection point. However, we show from the figures the phase transition threshold for the weak recovery case, where we show that if $\mu = \nu > 2$, there is always another intersection point other than the $(0,0)$ point, which can not be achieved via a local algorithm, though.
- For the roughly balanced case, we show that local belief propagation can still be optimal, i.e. only one intersection point. On the other hand, if the communities are significantly unbalanced, local belief propagation can be strictly sub-optimal, i.e. multiple intersection points.

The paper is organized as follows: In section II, the system model is presented. In section III, the belief propagation algorithm introduced in [13] is reviewed and the EXIT analysis are presented. The simulation results are presented in section IV. Finally, we conclude in section V.

## II. System Model

In this paper, we consider the binary degree correlated stochastic block model. We have n nodes partitioned into two communities. Each node is assigned independently to the first community with probability $\rho \in (0,1)$ and to the second community with probability $1-\rho$. After the assignment of nodes, each two nodes are connected independently with probability $\frac{a}{n}$ if the two nodes are assigned to the first community, with probability $\frac{c}{n}$ if they are assigned to the second community, and $\frac{b}{n}$ if they are assigned to different communities.

We denote the observed graph by $G = (V, E)$, and the vector of nodes' assignment by $\boldsymbol{x}$, where $x_i = 1$ if node $i$ belongs to the first community and $x_i = -1$ if it belongs to the second community, for $i \in \{1, \cdots, n\}$. The goal is to recover the node assignment $\boldsymbol{x}$ from the observation of $G$. Finally, we assume that $\rho$ is fixed and that a, b and c follow regime (1). Note that since we are working in regime (1), exact recovery is not possible [13], and hence, one could ask what is the minimum fraction of misclassified vertices on average.

## III. EXIT Chart Analysis

Before we begin characterizing the mutual information, we need to briefly review Algorithm 1 presented in [13]. This algorithm is a local iterative belief propagation algorithm, where after t iterations, the belief of node $i \in G$ is defined as:

$$R_i^t = \frac{(\rho b + (1-\rho)c) - (\rho a + (1-\rho)b)}{2} + \sum_{j \in N(i)} F(R_{j\to i}^{t-1}) \tag{2}$$

where $N(i)$ is the set of neighbors of node i, $F(x) = \frac{1}{2}\log(\frac{e^{2x}\rho a + (1-\rho)b}{e^{2x}\rho b + (1-\rho)c})$, and $R_{j\to i}^{t-1}$ are the messages transmitted from node j to node i in the $(t-1)$-th iteration and are defined as:

$$R_{j\to i}^{t-1} = \frac{(\rho b + (1-\rho)c) - (\rho a + (1-\rho)b)}{2} + \sum_{k \in N(j)\backslash i} F(R_{k\to j}^{t-2}) \tag{3}$$

After t iterations, each node i uses its belief $R_i^t$ to estimate its community assignment $\hat{x}_i^t$ using:

$$\hat{x}_i^t = 2 \times \mathbf{1}_{\{\mathbf{R_i^t} \geq -\phi\}} - 1 \tag{4}$$

where $\phi = \frac{1}{2}\log\frac{\rho}{1-\rho}$. Note that $R_i^t$ is not the exact log-likelihood ratio of node i. However, the authors proved that in regime (1), the observed graph is locally tree-like [10]. Thus, the community detection problem can be replaced by the reconstruction problem on trees, for which belief propagation is known to be optimal, i.e. achieves the maximum likelihood performance [13]. Then, the authors used density evolution [15] to analyze the performance of belief propagation on trees, where they showed that conditioned on the label of node $x_i = \pm 1$, the belief at iteration t follows a normal distribution with mean $\pm v^t$ and variance $v^t$, where:

$$v^t = \theta + \lambda \mathbb{E}[\tanh(v^{t-1} + \sqrt{v^{t-1}}Z + \phi)] \tag{5}$$

where $Z \sim \mathcal{N}(0,1)$, $v^0 = 0$, $\theta = \frac{\rho(\mu-\nu)^2}{8} + \frac{(1-2\rho)\nu^2}{4}$, and $\lambda = \frac{\rho(\mu+\nu)^2}{8}$.

Based on the above results, we introduce EXIT analysis for the degree correlated stochastic block model. To exploit EXIT charts, we calculate the mutual information between the label $x_i$ and the belief $R_i^t$ as follows:

$$\begin{aligned} I(x_i, R_i^t) &= H(x_i) - H(x_i|R_i^t) \\ &= H(\rho) - \rho \int_{-\infty}^{\infty} \frac{e^{\frac{-(y-v^t)^2}{2v^t}}}{\sqrt{2\pi v^t}} \log_2(1 + \frac{(1-\rho)e^{-2y}}{\rho})dy \\ &\quad - (1-\rho)\int_{-\infty}^{\infty} \frac{e^{\frac{-(y-v^t)^2}{2v^t}}}{\sqrt{2\pi v^t}} \log_2(1 + \frac{\rho e^{-2y}}{(1-\rho)})dy \end{aligned} \tag{6}$$

where the last equation follows from the fact that $x_i \sim Bern(\rho)$ and that conditioned on $x_i = \pm 1$, $R_i^t$ is $\sim$

$\mathcal{N}(\pm v^t, v^t)$. Note that for a fixed value of $\rho$, $I(x_i, R_i^t)$ is function of $v^t$ only. Hence, we will denote it by $J(v^t)$.

For a given node i, at iteration t, it receives the beliefs of all nodes $j \in N(i)$ calculated at iteration (t-1). We define the input information to node i from node j as $I_{in}$. Then, node i computes the new information it has at iteration t. We define this information as $I_{out}$. Note that both $I_{in}$ and $I_{out}$ can be calculated using( 6) as $J(v^{t-1})$ and $J(v^t)$, respectively. Since $J(v^t)$ is monotonically increasing in $v^t$ [16], $J(v^t)$ is reversible. Thus, $v^t = J^{-1}(I(x_i, R_i^t))$. Moreover, since $v^{t-1}$ and $v^t$ are related by (5), we can define the relation between $I_{in}$ and $I_{out}$ for node i as:

$$I_{out} = J(\theta + \lambda \mathbb{E}[\tanh(J^{-1}(I_{in}) + \sqrt{J^{-1}(I_{in})}Z + \phi)] \quad (7)$$

Note that there is a fundamental difference between using EXIT charts in our context of community detection in stochastic block models and the context of coding theory. In coding theory, lets take Low Density Parity Checlk (LDPC) Codes as an example, there is a variable node i connected to a check node j. What variable node $i$ actually receives from a check node $j$ is how much the check node believes the value of the variable node is one or zero (assuming binary transmission). Thus, the input log-likelihood ratio received by variable node i is actually calculated conditioned on the value of the variable node i. In our context, the case is different. Each node i receives the belief of node j. However, the belief of node j is calculated conditioned on the value of node j, not the value of node i. Hence, in our case, for a node i at iteration t, we define $I_{in} = I(x_j, R_j^{t-1})$, and $I_{out} = I(x_i, R_i^t)$. Note that one can define $I_{in} = I(x_i, R_j^{t-1})$, which can be easily calculated based on the facts that $R_j^{t-1}$, conditioned on the label of node $x_j = \pm 1$, follows a normal distribution with mean $\pm v^{t-1}$ and variance $v^{t-1}$ and that $x_j \sim 2 * Bern(\frac{a\rho}{a\rho + b(1-\rho)}) - 1(2 * Bern(\frac{b\rho}{b\rho + c(1-\rho)}) - 1)$ conditioned on $x_i = 1(-1)$. However, since $R_j^{t-1}$ is calculated at node j conditional on $x_j$, not $x_i$, it is more intuitive to use $I_{in} = I(x_j, R_j^{t-1})$. In other words, by using $I_{in} = I(x_j, R_j^{t-1})$, each point on the curve of the EXIT chart reflects how much the information the belief of node j carries about its value $x_j$ affects the information belief of node i carries about its value $x_i$. Another important difference is that in coding theory, the intersection point $(H(\rho), H(\rho))$ always exists on the exit chart. This is because in coding theory, both the incoming and outgoing beliefs are calculated with respect to the same node. Thus, if one belief reaches its maximum of $H(\rho)$, the other will reach its maximum too, since both of them are beliefs about the value of the same node. In our case, the incoming and outgoing beliefs are calculated with respect to different nodes. Hence, we do not expect the EXIT chart curves to intersect at $(H(\rho), H(\rho))$. This is also expected, since exact recovery of all nodes in regime (1) is not possible [13], and hence, the maximum of the mutual information can't be attained.

To compute $J$ and $J^{-1}$, we apply curve fitting using the Levenberg Marquardt algorithm [17] to obtain:

$$J(v) \approx \begin{cases} H(\rho)(a_{J,1}v^3 + b_{J,1}v^2 + c_{J,1}v), & 0 \le v \le v_1 \\ H(\rho)(1 - e^{a_{J,2}v^3 + b_{J,2}v^2 + c_{J,2}v + d_{J,2}}), & v_1 < v \le v_2 \\ 1, & v > v_2 \end{cases} \quad (8)$$

$$J^{-1}(I) \approx \begin{cases} H(\rho)(a_{v,1}I^2 + b_{v,1}I + c_{v,1}\sqrt{I}), & 0 \le I \le I_1 \\ -a_{v,2}log[b_{v,2}(H(\rho) - I)] - c_{v,2}I, & I_1 < I < H(\rho) \end{cases} \quad (9)$$

where all the variables in the last two equations will be defined in the simulations based on the value of $\rho$.

## IV. Simulation Results

In this section we use EXIT chart to characterize the performance of the belief propagation algorithm for different values of $\rho$, $\mu$, and $\nu$.

### A. $\rho = 0.5$

In this case (and the case of $\rho = 0.3$), we use the following values for the parameters of equations (8) and (9):

$$\begin{aligned} &a_{J,1} = 0.1405, \quad b_{J,1} = -0.3458, \quad c_{J,1} = 0.7206, \\ &v_1 = 0.3, \quad a_{J,2} = 0.0032, \quad b_{J,2} = -0.5541, \\ &c_{J,2} = -0.1620, \quad d_{J,2} = 0.0472, \quad v_2 = 10, \\ &a_{v,1} = 1.5657, \quad b_{v,1} = 1.1958, \quad c_{v,1} = 0.0403, \\ &I_1 = 0.4, \quad a_{v,2} = 1.831, \quad b_{v,2} = 0.9484, \\ &c_{v,2} = 0.6196 \end{aligned} \quad (10)$$

Figure 1 shows the EXIT curves at $\rho = 0.5$ for two cases: $\mu = 3, \nu = 2$ and $\mu = 5, \nu = 2$. From the figure, we can conclude the following:

- The EXIT chart is symmetric. This is expected since the exchange of beliefs is symmetric between the nodes. In other words, if we reverse the role of nodes i and j, the curves will not change.
- For both cases, the maximum point of the input and output mutual information $(1, 1)$ is not met. As discussed before, this was expected, since for node i, the incoming belief to it and the outgoing beliefs from it, are calculated with respect to different nodes. Besides, in regime (1) exact recovery is not possible anyway, and hence, the mutual information can not reach its maximum.
- For both cases, from the figure we can see that $I_{out}$ as a function of $I_{in}$ is concave. Furthermore, there is only one intersection point between the two curves showing the relation between $I_{in}$, $I_{out}$ and between $I_{out}$, $I_{in}$, respectively. This result suggests that the local belief propagation algorithm is optimal in such case [13].
- From the figure, we can see that as the difference between $\mu$ and $\nu$ increases, the intersection point approaches the maximum of the mutual information, which is one in this case. This makes sense, since as the difference increases,

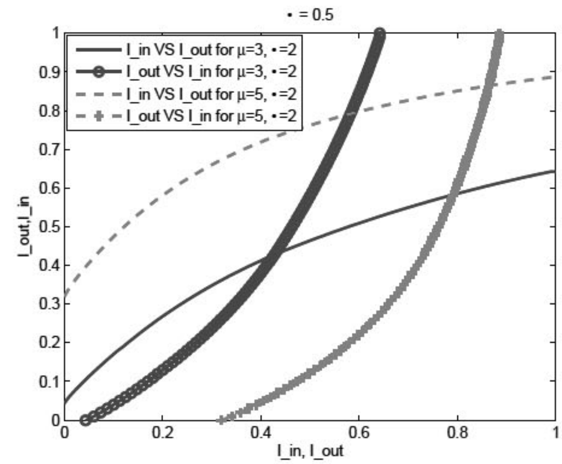


Fig. 1. EXIT Chart at $\rho = 0.5$ for two cases: $\mu = 3, \nu = 2$ and $\mu = 5, \nu = 2$.

| $(\mu,\nu)$ | Exact | Approximation |
|---|---|---|
| (3,2) | 0.1805 | 0.1787 |
| (5,2) | 0.0345 | 0.0346 |
| (7,2) | 0.0053 | 0.0054 |

TABLE I
COMPARISON BETWEEN THE EXACT AND THE APPROXIMATED USING EXIT CHARTS FRACTION OF MISCLASSIFIED NODES.

the two communities become statistically more different, and hence, the recovery becomes easier.

- Finally, from the figure we calculate an approximation of the fraction of misclassified nodes. To do so, we use the result proved in [13] that the minimum fraction of misclassified nodes is given by $Q(\sqrt{v^*})$, where $v^*$ is defined in section I. We approximate $v^*$ by $J^{-1}(I^*)$, where $I^*$ is the intersection point of the EXIT chart. Table I shows the approximation for several values of $\mu$ and $\nu$, where it can be seen that EXIT chart provide a good approximation of the performance of the belief propagation algorithm.

Figure 2 shows the EXIT curves at $\rho = 0.5$ for two cases: $\mu = 2, \nu = 2$ and $\mu = 3, \nu = 3$. From the figure, we can conclude the following:

- For both cases, the local belief propagation algorithm will fail, since it will get stuck at the $(0, 0)$ intersection point. Hence, belief propagation can not do better than random guessing, since in such case the fraction of misclassified nodes will be $Q(0) = 0.5$.
- Although, in both cases the belief propagation algorithm fails, in the case where $\mu = \nu = 3$, there is another intersection point. This corroborates the weak recovery threshold proved in [10], which states that when $\rho = 0.5$, and $a = c$, weak recovery is possible if and only if $\frac{(a-b)^2}{(a+b)} > 2$. Note again, that this intersection point can not be achieved via the local belief propagation algorithm.

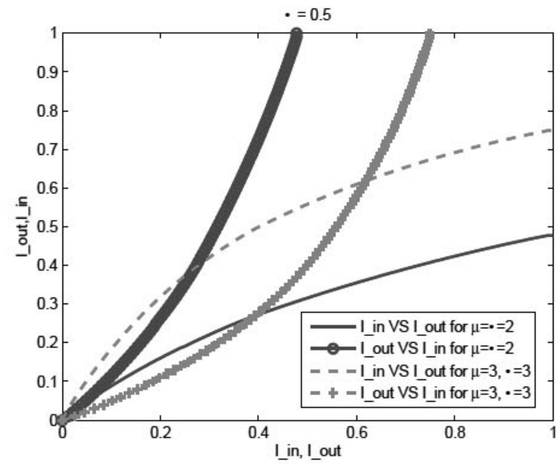


Fig. 2. EXIT Chart at $\rho = 0.5$ for two cases: $\mu = 2, \nu = 2$ and $\mu = 3, \nu = 3$.

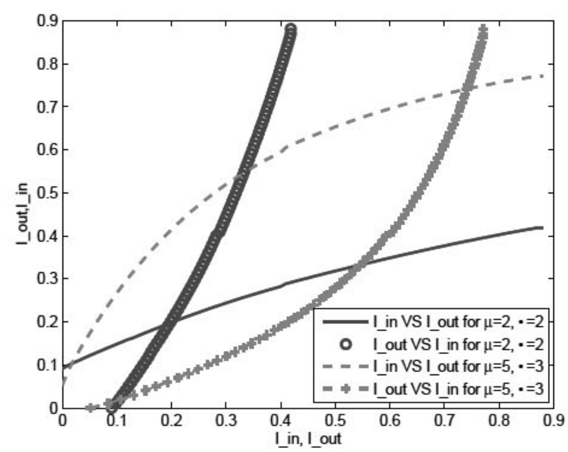


Fig. 3. EXIT Chart at $\rho = 0.3$ for two cases: $\mu = 5, \nu = 3$ and $\mu = 2, \nu = 2$.

### B. $\rho = 0.3$

Figure 3 shows the EXIT curves at $\rho = 0.3$ for two cases: $\mu = 2, \nu = 2$ and $\mu = 5, \nu = 3$. From the figure, we can conclude that unlike the $\rho = 0.5$ case, here for both cases, when $\mu = \nu$ and when $\mu \neq \nu$, there is only one intersection point. Since from the figure, the curves are concave, this suggests that the belief propagation algorithm is still optimal, hence corroborating the conjecture in [13], that for $\rho \geq 0.2$, belief propagation algorithm is still optimal.

### C. $\rho = 0.01$

In this case, we use the following values for the parameters of equations (8) and (9):

$$a_{J,1} = -0.0224, \quad b_{J,1} = -0.0025, \quad c_{J,1} = 0.3531,$$
$$v_1 = 0.3, \quad a_{J,2} = -1.4022e-05, \quad b_{J,2} = -0.5834,$$

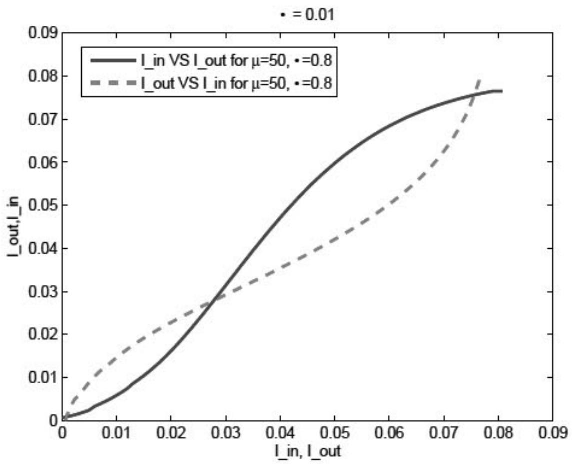


Fig. 4. EXIT Chart at $\rho = 0.01$ for $\mu = 50, \nu = 0.8$.

$$
\begin{aligned}
&c_{J,2} = 0.2637, \quad d_{J,2} = -0.0846, \quad v_2 = 10,\\
&a_{v,1} = 420.9558, \quad b_{v,1} = 432.5107, \quad c_{v,1} = 0.00361,\\
&I_1 = 0.005, \quad a_{v,2} = 1.8795, \quad b_{v,2} = 11.8123,\\
&c_{v,2} = -5.2027
\end{aligned} \tag{11}
$$

Figure 4 shows the EXIT curves at $\rho = 0.01$ for $\mu = 50, \nu = 0.8$. From the figure, we can conclude that in such case, the belief propagation algorithm can be strictly suboptimal, since the curves are no longer concave and there are multiple intersection points.

## V. Conclusion

In this paper, EXIT chart analysis is provided for the community detection problem under the degree correlated stochastic block model. More precisely, we use EXIT charts to analyze the performance of a belief propagation algorithm introduced in the literature. We show through simulations that the approximations done using EXIT charts matches some of the results and conjectures in the literature. We show that in the case of equally-sized communities, when the probability of connectivity in the communities are different, there is only one intersection point, hence belief propagation is optimal. When the probability of connectivity in the communities are the same, we show that belief propagation is equivalent to random guessing and the curves intersect at the trivial zero-zero point. For the roughly equally sized communities, we show that there is always only one intersection point, suggesting that belief propagation is still optimal. Finally, for the significantly unequally-sized communities, we show that there are multiple intersection points, hence, belief propagation can be strictly sub-optimal. We believe this is the first time EXIT charts are introduced in such context and this could lead to further connections between the problem of community detection in random graphs and tools, like EXIT charts, that are extensively used in coding theory.

## References


[1] P. Holland, K. Laskey, and S. Leinhardt, "Stochastic blockmodels: First steps," *Social Networks*, vol. 5, no. 2, pp. 109–137, Jun. 1983.

[2] A. Zhang and H. Zhou, "Minimax rates of community detection in stochastic block models," *arXiv:1507.05313*, Nov. 2015.

[3] S. Jafar, "Statistical-computational tradeoffs in planted problems and submatrix localization with a growing number of clusters and submatrices," *ICML, 2014 In proceedings of*, Feb. 2014.

[4] Y. Chen, S. Sanghavi, and H. Xu, "Improved graph clustering," in *Information Theory, IEEE Transactions on*, vol. 60, no. 10, Oct. 2014, pp. 6440–6455.

[5] A. Decelle, F. Krzakala, C. Moore, and L. Zdeborová, "Asymptotic analysis of the stochastic block model for modular networks and its algorithmic applications," *Phys. Rev. E*, vol. 84, p. 066106, Dec. 2011.

[6] M. Girvan and M. E. J. Newman, "Community structure in social and biological networks," *Proceedings of the National Academy of Sciences*, vol. 99, no. 12, pp. 7821–7826, 2002.

[7] J. Xu, R. Wu, K. Zhu, B. Hajek, R. Srikant, and L. Ying, "Jointly clustering rows and columns of binary matrices: Algorithms and trade-offs," *arXiv:1310.0512*, Feb. 2014.

[8] J. Chen and B. Yuan, "Detecting functional modules in the yeast proteinprotein interaction network," *Bioinformatics*, vol. 22, no. 18, pp. 2283–2290, Sep. 2006.

[9] A. Lancichinetti and S. Fortunato, "Community detection algorithms: A comparative analysis," *Phys. Rev. E*, vol. 80, p. 056117, Nov 2009.

[10] E. Mossel, J. Neeman, and A. Sly, "Reconstruction and estimation in the planted partition model," *Probability Theory and Related Fields*, vol. 162, no. 3, pp. 431–461, 2014.

[11] E. Abbe, A. Bandeira, and G. Hall, "Exact recovery in the stochastic block model," *IEEE Trans. Inf. Theory*, vol. 62, no. 1, pp. 471–487, Jan 2016.

[12] ——, "Community detection in general stochastic block models: fundamental limits and efficient recovery algorithms," *arXiv:1503.00609*, March 2015.

[13] E. Mossel and J. Xu, "Density evolution in the degree-correlated stochastic block model." *arXiv:1509.03281*, 2015.

[14] S. Johnson, *Iterative error correction: turbo, low-density parity-check and repeat-accumulate codes*. Cambridge University Press, 2010.

[15] T. Richardson and R. Urbanke, *Modern Coding Theory*. New York, NY, USA: Cambridge University Press, 2008.

[16] S. ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes." *IEEE Transactions on Communications*, vol. 49, no. 10, pp. 1727–1737, 2001.

[17] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in C (2Nd Ed.): The Art of Scientific Computing*. New York, NY, USA: Cambridge University Press, 1992.