Inter-platform competition in a regulated ride-hail market with pooling

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Abstract

This paper studies an aggregate ride-hail market in which two platforms compete with each other, as well as with transit, under different supply and regulatory conditions. The duopoly is built on a general market equilibrium model that explicitly characterizes the physical matching process, including pairing two passengers for a pooling ride. Depending on whether drivers' work affiliation with a platform is exclusive or not, the duopoly is said to have a single- or multi-homing supply mode. We describe the outcome of the duopoly pricing game as a Nash Equilibrium (NE) and solve it by transforming it into a variational inequality problem (VIP). When a regulatory constraint is imposed, the duopoly equilibrium becomes a generalized NE, which corresponds to a quasi VIP. We show that multi-homing may lead to disastrous outcomes in an unregulated duopoly and demonstrate it through numerical experiments constructed using data from Chicago. Specifically, passenger and driver surplus, as well as platform profits, are all significantly lower in a multi-homing duopoly than in a single-homing counterpart. This disaster arises because (i) the multi-homing duopoly is locked in a self-destructive pricing war analogous to *the tragedy of the commons*; and (ii) the competition among passengers limits economy of scale in trip production. We show that the negative consequences of this tragedy can be mitigated by (i) discouraging multi-homing behavior; (ii) imposing a minimum wage on both platforms; and (iii) encouraging the platforms to specialize in different services. The results also show the efficiency in matching passengers and drivers is a crucial asset for a platform's competitiveness, more so in a multi-homing duopoly. In general, the platform with a higher matching efficiency ends up making more money and providing a better level of service.

Keywords: ride-hail; pooling; duopoly; Nash Equilibrium; tragedy of the commons

1 Introduction

In 2011, Uber first launched its service in San Francisco. Since then, transportation network companies (TNCs) have been growing rapidly and in that process reshaped the ride-hail industry

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and beyond. Many TNCs, Uber included, have marketed themselves as "ride-sharing" service providers that promise to reduce traffic congestion. However, critics (e.g., Rayle et al., 2014; Nie, 2017) often point out that the e-hail service provided by TNCs is not a form of "sharing", as the drivers join the service exclusively to make money. Also, since the majority of TNC trips are not in fact shared, their entry into the ride-hail market has had an overall negative impact on congestion, especially in large metropolitan areas (Schaller, 2017, 2018; Erhardt et al., 2019). To be fair, most TNCs nowadays do provide rides that are shared, if only partially, by passengers. Such a service is called ride-pooling, or simply *pooling*, in this paper. When requesting a ride, a passenger could choose between a regular e-hail ride (referred to as *solo ride* hereafter) and a pooling ride. Since pooling passengers do not necessarily share the same origin and destination, detours often occur in the pooling rides and thus the platform has to offer a discounted trip fare to encourage pooling. The question is, *driven by their own profit-making objective and constrained by an increasingly hostile regulatory environment, to what extent would TNCs find pooling an attractive option*? Besides, *how would TNCs' pooling decision impact drivers, passengers and the society at large*?

Zhang and Nie (2019) address precisely the above questions. They consider a single TNC platform that offers both solo rides and pooling rides while competing with transit. Based on a spatial matching model that captures the key features of pooling, they formulate the equilibrium problem for an aggregate ride-hail market and various optimal pricing problems with and without regulatory constraints. Their analyses provide managerial insights for both service providers and regulators. For example, they found the platform is *always* better off—in terms of both market share and profit—by providing both solo and pooling services. Another notable finding is that the minimum wage policy, which has been widely discussed in the public discourse (e.g., Parrott and Reich, 2018; Li et al., 2019), could be counterproductive in the long run, even though it improves the social welfare in the short term at the expense of the platform.

One of the most restrictive assumptions in the analysis of Zhang and Nie (2019) is that the market is dominated by a single platform. In reality, inter-platform competition is commonplace, with the duopoly of Uber and Lyft being a well-known example. In this study, we set out to model the inter-platform competition and examine how it affects the platforms' pricing decisions and market equilibrium. We consider a ride-hail duopoly in which two platforms compete for both passengers and drivers in a shared aggregated market. The competition is formulated as a simultaneous pricing game. That is, the platforms set the price for passengers and the compensation rate for drivers in order to maximize their own profits. The interactions between passengers, drivers and the platforms are characterized by extending the aforementioned market equilibrium model developed in Zhang and Nie (2019). We employ the concept of Nash equilibrium to analyze the duopoly in both regulated and unregulated environments. Since the regulatory constraints introduce interdependence between the platforms' strategy sets, the regulated duopoly equilibrium must be characterized as a generalized Nash equilibrium (Arrow and Debreu, 1954; Ichiishi, 1983).

Other than enriching the service options available to passengers, the competition also enables different supply modes, since a driver could now choose to join either or both platforms. The former is referred to as *single-homing*, and the latter *multi-homing*. A multi-homing driver would register on both platforms, and switch back and forth, sometimes in real time, depending on job availability. To be sure, most drivers are likely to have a strong preference for one platform over the other. Also, TNCs often penalize multi-homing practice, e.g., by rewarding loyal drivers who work longer hours. Thus, frequently switching between platforms to maximize one's income is not necessarily a winning strategy for drivers. The question is whether multi-homing, be detrimental to the platforms it may seem, actually serves the overall system better. If it does, then perhaps regulations could be introduced to encourage such practice. To this end, we consider two extreme supply modes. On the one extreme, all drivers are single-homing and hence each platform has an exclusive fleet whose size is determined by the pricing game. On the other extreme, the platforms share all drivers, who are committed to serving both platforms based on the overall expected earning potential. Although the real world likely lies somewhere in the middle, examining these corner cases could shed light on the fundamental differences between single- and multi-homing supply modes.

Our numerical experiments will show, among other findings, that multi-homing may lead to disastrous outcomes in an unregulated duopoly. Specifically, passenger and driver surplus, as well as platform profits, are all significantly lower in a multi-homing duopoly than in a single-homing counterpart. A close look into the game evolution reveals that this disaster results from a self-destructive pricing war that is analogous to the *tragedy of commons* (Ostrom et al., 1994). To give a preview of the mechanism, note that multi-homing drivers make the entry decision based on the average wage rate of working "simultaneously" for both platforms. Thus, it is better for each platform to lower the compensation rate paid to the drivers (hence save its own operating cost) than raise it to attract more drivers (which would be shared by its rival). While the strategy makes senses individually, collectively it causes the ruin of the business for all. One naturally wonders whether multi-homing could, in addition to condemning the platforms to the pricing war, also have a positive effect on efficiency, since it tends to increase the scale of the market. The benefit would indeed materialize if trip production displays increasing returns to scale. However, Zhang et al. (2019) show that, while cruising taxis do benefit from a thicker market, that is generally not true for e-hail. In an e-hail market, passengers are subjected to much more intense competition than in a cruising market, leading to a nearly constant returns to scale. As the matching model developed in this study conforms to this observation, the effect of multi-homing is largely dominated by the tragedy of commons. However, we will explore a range of options that could be used to mitigate the negative consequences of this tragedy.

The remainder of this paper is organized as follows. Section 2 briefly reviews the related studies. Section 3 presents the spatial matching model employed in this study. A general aggregate market equilibrium is developed in Section 4, along with the specifications of single- and multi-homing supply modes. Section 5 formulates the duopoly pricing game with and without regulatory constraints. Results from the numerical experiments are reported and discussed in Section 6. Finally, Section 7 summarizes the main findings and comments on possible directions for future research.

2 Related work

There is a large and rapidly expanding literature on modeling and analyzing ride-hail services. No attempt is made here to provide a comprehensive review. The reader interested in reading one is referred to Wang and Yang (2019). Instead, we shall focus on the *equilibrium analysis of* *aggregate e-hail markets*. This line of research, motivated by the rise of TNCs in the past decade, is largely built on the studies of taxi markets since 1970s (e.g., Douglas, 1972; De Vany, 1975; Beesley and Glaister, 1983; Cairns and Liston-Heyes, 1996; Arnott, 1996). Centered at the ride-hail equilibrium models is the interaction between passengers and drivers, the so-called *matching process*. Most existing studies simplify the matching process using either aggregate matching functions (e.g., Yang et al., 2010b; Buchholz, 2019; Frechette et al., 2019) or queuing systems (e.g., Banerjee et al., 2015; Afeche et al., 2018; Besbes et al., 2018; Xu et al., 2019). Others develop physical models (e.g., Chen et al., 2018; Zha et al., 2018b; Zhang and Nie, 2019) to capture more details in the process. In this study, we employ the matching model developed in Zhang and Nie (2019) because it not only differentiates the physical features of solo and pooling rides, but also is analytically tractable.

Below, we first review aggregate equilibrium models of e-hail service (Section 2.1), and then discuss the inter-platform competition in e-hail and related service markets (Section 2.2). Finally, the issue of regulations is addressed in Section 2.3.

2.1 Aggregate equilibrium of e-hail market

The radio-dispatching service considered in Arnott (1996) may be viewed as a primitive form of e-hail. These two types of services do share some essential features: (i) passengers and drivers are matched in the space with an almost infinite matching radius, (ii) the pickup time dominates passenger's wait time, and (iii) a driver assigned to a passenger is no longer available to other passengers. Using a simplified spatial matching model, Arnott (1996) shows the expected passenger wait time is inversely proportional to the square root of the spatial density of idle drivers. This matching mechanism implies economies of density, which in turn is used to justify subsidizing taxi operation.

Wang et al. (2016) consider an aggregate taxi market where passengers can either hail taxis on street or use e-hail. The matching process is described using the Cobb-Douglas function (Cobb and Douglas, 1928) for both street-hail and e-hail. Assuming a fixed fleet, they analyze how profit and social welfare vary with pricing strategies. Based on a similar matching model, Zha et al. (2016) formulate the optimal pricing problem in an e-hail market. They show that, in a monopoly, the first-best pricing leads to deficit and thus is unsustainable, which confirms the finding by Arnott (1996). They suggest that regulating the commission rate (i.e., the fraction of trip fare collected by the platform) could guarantee a second-best solution. A follow-up study by Zha et al. (2018a) adds the time dimension into the aggregate model to analyze surge pricing. They find surge pricing generally benefit the platform and drivers, but it could hurt passengers.

Much attention has been paid to surge pricing, or more generally dynamic pricing. Banerjee et al. (2015) study dynamic pricing using a queuing model. They find it is more robust against system fluctuations, even though it does not necessarily outperform the optimal static pricing strategy in terms of trip production and profit. Based on the model developed in Arnott (1996), Castillo et al. (2018) demonstrate and analyze an intriguing phenomenon called *Wild Goose Chase* (WGC). WGC manifests when a disproportionately high percentage of vacant vehicles are stuck in the pickup phase, causing a catastrophic shortage of idle vehicles ready to take new orders. The authors then argue that surge pricing is an effective tool to keep the system out of WGC. Xu et al. (2019) investigate the relationship between trip production and passenger wait time under different operational strategies and market conditions. They find WGC always exists as an "inefficient state" (similar to the congested state on a highway) and suggest that some operational strategies could mitigate its impact, such as limiting the matching radius,.

With a couple of exceptions to be discussed below, most studies of aggregated e-hail markets have not considered pooling, which is a focal point of the present study. Yan et al. (2019) propose an idealized matching mechanism for pooling rides called *dynamic waiting*. With this mechanism, each passenger waits for certain time to be matched with another passenger. If a pool-match is found, the two passengers are picked up and dropped off at the midpoint of their origins and destinations, respectively. The authors show that a joint implementation of dynamic pricing and dynamic waiting could improve vehicle utilization, trip production and social welfare. Zhang and Nie (2019) allow the two passengers of a pooling ride to be picked up and dropped off one at a time. This implies additional detours will incur in both pickup and delivery phases. The present paper is built on the work by Zhang and Nie (2019).

2.2 Inter-platform competition

Many studies have addressed the issue of inter-platform competition in the broad context of two-sided service market, of which e-hail may be viewed as an example. On the one hand, these markets enjoy the positive *cross-side network effect*, that is, a larger number of users on one side of the market makes the service more appealing to users on the other side (Rysman, 2009; Tucker and Zhang, 2010). On the other hand, they may suffer from the negative *same-side network effect*, that is, the system efficiency degrades as the number of users on the same side grows out of control (e.g., Belleflamme and Toulemonde, 2009; Aloui and Jebsi, 2011; Bernstein et al., 2019). Both effects are observed in e-hail (Castillo et al., 2018), though the latter seems more consequential in high density areas (Zhang et al., 2019). Besides the network effects, the underlying supply mode also affects how platforms compete for users and service providers in a two-sided market. A central issue has to do with how "tightly" drivers are affiliated with a platform. When the affiliation is exclusive, it leads to the so-called single-homing supply mode; otherwise, drivers are allowed to be affiliated with multiple platforms, resulting in the multi-homing mode. This seemingly minor difference could result in distinctive pricing strategies (e.g., Rochet and Tirole, 2003; Armstrong, 2006; Böhme et al., 2010).

The aforementioned study by Zha et al. (2016) also analyzes a duopoly with single-homing drivers. Their main finding is that inter-platform competition does not necessarily lower the trip fare or improve social welfare. This finding is confirmed by Nikzad (2017), who adopts a different matching model and a general supply mode (i.e., drivers could be either single- or multi-homing). Nikzad (2017) further shows that the duopoly price would be higher than the monopoly price when the size of potential supply is small. Yet, the driver wage is always higher in the duopoly equilibrium. Bai and Tang (2018) model a duopoly market with symmetric platforms and single-homing drivers, where the passenger wait time is defined as a general function of demand and supply. They show that when the so-called *pooling effect* is present—i.e., wait time decreases when supply and demand both increases by one unit—both platforms break even at equilibrium. Only in the absence of the pooling effect would the platforms be profitable. Bernstein et al. (2019) compares the duopoly equilibria under single-homing and multi-homing in a general two-sided market, where the same-side network effect is described as a function of utilization rate. It is found that, even though individual drivers may prefer multi-homing, all players are worse off when all drivers are multi-homing. In a sequel to Bai and Tang (2018), Wu et al. (2020) explore a sequential driver-passenger game, in which drivers and passengers make decisions sequentially. Unlike the standard simultaneous game, they show no sequential equilibrium exist such that the competing platforms would both possess a positive market share.

Compared to the above, our study features a few innovations. First, the duopoly equilibrium is built on a more realistic matching model that promises managerial insights concerning the choice between pooling and solo rides, for both the platforms and the passengers. Second, by introducing regulatory constraints into duopoly games, the impact of regulations can be analyzed under different supply modes.

2.3 Regulations

The traditional taxi industry is known to be a decreasing-average-cost industry (Douglas, 1972; Beesley and Glaister, 1983). Accordingly, it is subject to natural monopoly, and is likely to produce below the efficient level (Arnott, 1996). Moreover, full competition in a taxi market does not usually maximize social welfare (Douglas, 1972; De Vany, 1975; Cairns and Liston-Heyes, 1996). These observations explain why the taxi market has been tightly regulated in terms of price and entry in most parts of the world (Frankena and Pautler, 1986; Cairns and Liston-Heyes, 1996; Flores-Guri, 2003; Yang et al., 2005, 2010a).

Regulating TNCs has become a subject of debate since the launch of Uber. On the one hand, e-hail appears to share a similar cost structure with taxi that would justify similar regulatory actions (Zha et al., 2016). On the other hand, TNCs consider their drivers as "independent contractors" who neither earn a fixed wage nor commit to a fixed work schedule. While this view of TNC's labor structure has been challenged frequently, it is worth noting that an e-hail operator indeed does not have a full control over its service capacity. Another critical difference brought by e-hail is the importance of pooling rides. Therefore, one might expect conventional regulations for taxi may fail to deliver desired results when directly applied to e-hail.

Gurvich et al. (2019) analyze the service capacity management problem for a service provider relying on a flexible supply model similar to that of e-hail. They argue that imposing a minimum wage will force the provider to restrict the number of agents on the platform during certain time periods, effectively limiting their scheduling flexibility. Parrott and Reich (2018) argue the minimum wage policy proposed by the New York City will only lead to a relatively minor increase in passenger wait time, and a minimal fare adjustment would suffice to absorb the increase in driver pay. Li et al. (2019) show that a cap on the number of vehicles benefits the platform but hurts drivers. Yet, imposing a minimum wage benefits both drivers and passengers because it pushes the platform to hire more drivers. From a different perspective, Yu et al. (2019) analyze the welfare effects of the entry control policy in a market where e-hail and traditional taxi services compete for passengers. They conclude that there exists an optimal capacity cap that can best balance competing objectives of various stake holders. Allowing pooling rides in a monopoly, Zhang and Nie (2019) find the minimum wage leads to an substantial short-term improvement in social welfare at the expense of the platform's profit. However, in the long run, the platform will respond to the policy by restricting the potential supply in a similar manner suggested by Gurvich et al. (2019), which could be detrimental to social good. Furthermore, the minimum wage policy discourages pooling because it forces the platform to promote solo rides in order to consume the extra vehicle supply induced by a higher (minimum) wage.

Our reading of literature above suggests no study has examined the impact of regulations on the inter-platform competition in an aggregate ride-hail market that allows pooling.

3 Matching model

In this section, we briefly review the matching model developed in Zhang and Nie (2019), who introduce the following assumptions to simplify the analysis.

Assumption 1 *1. A pooling ride is always shared by two passengers. It starts when both passengers are picked up and ends when they both arrive at their destinations.*

2. *Vacant vehicles and waiting passengers for solo and pooling rides are uniformly distributed in space with densities Λ, Π_s and Π_p, respectively;*

3. *Vacant vehicles are cruising at the same speed v;*

4. *Passengers keep waiting at the same location prior to pickup; and*

5. *The origins and destinations of all passengers are uniformly distributed in space.*

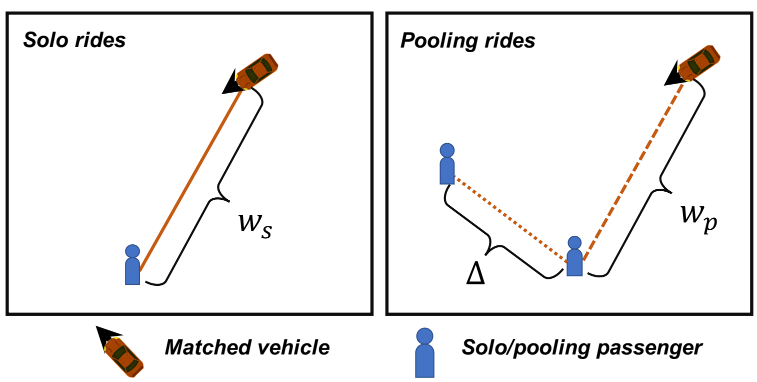


Figure 1: Illustration of the pickup process for solo and pooling rides (revised based on a similar plot in Zhang and Nie (2019)).

Based on the definitions above, the wait time for pooling rides consists of two parts, as illustrated in the right panel of Figure 1. The first part, denoted as w_p, is the pickup time for the passenger closer to the matched vehicle, and the second part, denoted as Δ, is the detour time to pickup the second passenger. In other words, Δ is considered part of the wait time of the first passenger even if she spends that time in the vehicle. In contrast, the wait time for solo rides only has one segment, denoted as w_s, that depends on the distance between the passenger and the matched vehicle (see the left panel of Figure 1).

Zhang and Nie (2019) show w_s, w_p and Δ can be estimated as[1]

[1] Repeating the lengthy derivation of these formula would occupy much space. To avoid repetition, the reader is referred to Zhang and Nie (2019), which is available on SSRN (3497808).

$$w_s = \frac{\delta}{2v}\sqrt{\frac{\Pi_{\text{eff}}}{k\Lambda}}, \tag{1a}$$

$$w_p \simeq w_s\sqrt{\frac{\kappa + 4b\Pi_p}{2\kappa + 4b\Pi_p}}, \tag{1b}$$

$$\Delta = \frac{\delta}{2v\sqrt{b\Pi_p}}, \tag{1c}$$

where

- $\Pi_{\text{eff}} = \Pi_s + \Pi_p/2$ is the effective waiting passenger density;
- δ is the detour ratio of the road network (Boscoe et al., 2012; Yang et al., 2018), defined as the ratio between path distance and line distance between two points in the network;
- k and b are exogenous parameters that measure the matching efficiency between vehicles and passengers, and between pooling passengers, respectively [2];
- κ is an adjustable parameter related to the approximations made when deriving w_p.

Eq. (1) captures several key features in the matching process. First, the nature of spatial matching dictates that both w_s and w_p are proportional to the square root of vacant vehicle density Λ (Larson and Odoni, 1981). Second, the expected solo wait time decreases with the matching efficiency k but increases with the intensity of the inter-passenger competition, measured by the effective waiting passenger density Π_{eff}. Since two pooling passengers share one vehicle, their contribution to the competition for vehicles is half that of a solo passenger. Importantly, the existence of this competition implies the trip production in e-hail displays a nearly constant returns to scale (Zhang et al., 2019). Third, pooling passengers enjoy a collective competing power when matching with vacant vehicles. This power decreases with the distance between the paired passengers, or equivalently, increases with Π_p. As $\Pi_p \to 0_+$, $w_p \to w_s/\sqrt{2}$. When the paired passengers happen to wait at the same location, i.e., Δ reduces to zero, they lose this advantage, leading to $w_p = w_s$.

4 Market equilibrium with multiple platforms

Although we focus on a duopoly in this study, the equilibrium model presented in this section actually applies to markets with any number of platforms. Below, Section 4.1 first presents this model, and in Section 4.2 we specify its supply component based on two different modes: single-homing and multi-homing.

[2] As in Zhang and Nie (2019), we assume k and b as exogenous parameters that are determined by a platform's matching and dispatching algorithms. Effectively, this assumption isolates the platform's pricing strategies from the matching/dispatching strategies. This simplification enables us to focus on the former, allowing it to drive the supply-demand equilibrium, while the latter's effects are accounted for through the two parameters.

4.1 General model

Consider an aggregate ride-hail market where a set of platforms, denoted as $\mathcal{P} = \{A, B, \dots\}$, competing with each other and against transit, by offering solo and/or pooling rides. On the demand side, passengers can choose solo or pooling rides served by any platform, as well as transit, based on the generalized cost. On the supply side, drivers decide whether or not to join platform according to the average wage rate. The interaction between demand and supply is described by the matching model presented in Section 3, which yields the wait time that directly affects a passenger's generalized cost. The aggregate passenger demand for solo and pooling rides determines the occupied vehicle time and thus, in turn, influences the average wage rate.

To focus on the inter-platform competition, we make the following assumptions.

Assumption 2 *Transit is a fallback option for all passengers and is served at a constant general cost. The transit operator always breaks even, i.e., the trip fare equals the marginal cost.*

Assumption 3 *Passengers choose a mode from transit and the e-hail services—solo or pooling—provided by the platforms, based on the generalized cost of each mode.*

Assumption 4 *The impact of e-hail vehicles on traffic congestion, hence on the general cost of all modes, is small and negligible.*

Let s, p and t denote solo ride, pooling ride and transit, respectively. The generalized cost for each mode is defined as follows:

$$\text{Solo:} \qquad u_s^j = f_s^j + \nu(w_s^j + \tau_s^j), \tag{2a}$$

$$\text{Pooling:} \qquad u_p^j = f_p^j + \nu(w_p^j + \Delta^j + \tau_p^j), \tag{2b}$$

$$\text{Transit:} \qquad u_t = f_t + (\nu + \zeta)\tau_t, \tag{2c}$$

where

- ν is the value of time and ζ represents an additional disutility associated with transit trip (e.g., privacy, comfort and crowdedness);
- f_t is the transit fare and f_s^j (f_p^j) is the fare for solo (pooling) rides on platform j;
- w_s^j ($w_p^j + \Delta^j$) denote the wait time for solo (pooling) rides on platform j;
- τ_s^j and τ_p^j give the average trip duration for solo and pooling rides on platform j, respectively.

We require $\tau_p^j \geq \tau_s^j, \forall j$ to account for the extra travel time incurred in the pooling rides due to passengers' different destinations. Note that, among all the variable in Eq. (2), w_s^j, w_p^j, Δ^j are endogenous state variables determined by the equilibrium, while f_s^j and f_p^j are decision variables of platform j, determined from the pricing game discussed in the next section.

Let the total demand rate be D_0 (trip per unit time per unit area) and the potential supply be S_0 (vehicle per unit area). The share of platform j for mode $m \in \mathcal{M} \setminus \{t\}$, is represented as a continuous and differentiable function of the generalized cost u_m^j as well as those of alternative modes (denoted as $\mathbf{u}_{-m,j}$), i.e., $q : \mathbb{R}_+^{|\mathcal{M}|} \to (0,1)$. The functional form of $q(\cdot)$ will be further specified and discussed in Section 6.

The fleet size of each platform j, denoted as N^j, is determined by a general supply function $g : \mathbb{R}_+^{|\mathcal{P}|} \to (0,1)$ of the average wage rate of platform j, denoted by e^j, and that of the other platforms, denoted by $\mathbf{e}_{-j}$, which gives its share of S_0. The specification of e^j varies according to the supply mode, which will be discussed in Section 4.2.

Accordingly, the market equilibrium over a unit time (e.g., an hour) can be described by the following equation system:

$$\text{Demand:} \quad Q_m^j = D_0 q(u_m^j, \mathbf{u}_{-m,j}), \tag{3a}$$

$$\text{Supply:} \quad N^j = S_0 g(e^j, \mathbf{e}_{-j}), \tag{3b}$$

$$\text{Flow conservation:} \quad V^j = l(\mathbf{N}, \mathbf{Q}_s, \mathbf{Q}_p), \tag{3c}$$

$$\text{Solo wait time:} \quad w_s^j = \frac{\delta}{2v}\sqrt{\frac{\Pi_{\text{eff}}^j}{k^j V^j}}, \tag{3d}$$

$$\text{Pooling wait time:} \quad w_p^j = w_s^j \sqrt{\frac{\kappa + 4b^j \Pi_p^j}{2\kappa + 4b^j \Pi_p^j}}, \tag{3e}$$

$$\Delta^j = \frac{\delta}{2v}\frac{1}{\sqrt{b^j \Pi_p^j}}. \tag{3f}$$

Without loss of generality, we assume the network detour ratio δ, cruising speed v and the approximation parameter κ be platform independent, while matching efficiency k and pooling efficiency b be platform specific. k and b will be treated as the key asymmetric features discussed in Section 6.6.

Eqs. (3a) and (3b) describe passengers' mode choice and drivers' platform choice, respectively. Eq. (3c) represents the flow conservation condition, that is, within a unit operation period, the vacant vehicle time on platform j, denoted as V^j, is a function of the fleet size for each platform and the demand for each mode and platform. Here, we represent the variables associated with all platforms as vectors, i.e., $\mathbf{N} := \{N^j, j \in \mathcal{P}\}$, $\mathbf{Q}_s := \{Q_s^j, j \in \mathcal{P}\}$ and $\mathbf{Q}_p := \{Q_p^j, j \in \mathcal{P}\}$. Both the supply function $g(\cdot)$ in Eq. (3b) and the vacant vehicle time function $l(\cdot)$ in Eq. (3c) will be further specified in Section 4.2. Eq. (3d) is obtained by replacing Λ in Eq. (1a) with vacant vehicle time V^j (which equals vacant vehicle density since all variables are defined per unit area). Moreover, as per Little's formula (Little, 1961), the density of the passengers waiting for pooling rides, Π_p^j in Eqs. (3e) and (3f), can be replaced by $\Pi_p^j = Q_p^j(w_p^j + \Delta^j)$. Similarly, Π_{eff}^j can be expressed as a function of $Q_s^j w_s^j$ and $Q_p^j(w_p^j + \Delta^j)$ according to the supply mode (see Section 4.2).

The equation system Eq. (3) can be recast as a fixed point system $\mathbf{x} = F(\mathbf{x})$, where $\mathbf{x} = [\mathbf{w}_s, \mathbf{w}_p, \boldsymbol{\Delta}, \mathbf{N}]^T$, and $\mathbf{w}_s := \{w_s^j, j \in \mathcal{P}\}, \mathbf{w}_p := \{w_p^j, j \in \mathcal{P}\}$ and $\boldsymbol{\Delta} := \{\Delta^j, j \in \mathcal{P}\}$. In other words, the market equilibrium can be fully characterized by the vectors of passenger wait time (solo and pooling) and the fleet size. The existence of such an equilibrium can be established by invoking the fixed point theorem. The reader is referred to Appendix B for more details.

4.2 Specification of supply modes

To fully specify the equilibrium model Eq. (3), we need to elaborate the supply mode that determines the fleet size of each platform. As mentioned in Section 1, we consider two supply modes: single-homing and multi-homing.

The *single-homing* mode assumes each driver only join one platform if he chooses to enter the ride-hail market. Consider drivers working for platform j is paid at a compensation rate η^j per unit occupied time. Then, the average wage rate is given by

$$e^j = \frac{\eta^j}{N^j}\left(Q_s^j\tau_s^j + \frac{1}{2}Q_p^j\tau_p^j\right), \tag{4}$$

the flow conservation condition Eq. (3c) becomes

$$V^j = N^j - Q_s^j\tau_s^j - \frac{1}{2}Q_p^j\tau_p^j, \tag{5}$$

and the effective waiting passenger density is

$$\Pi_{\text{eff}}^j = Q_s^j w_s^j + \frac{1}{2}Q_p^j(w_p^j + \Delta^j). \tag{6}$$

Recall that the effective waiting passenger density Π_{eff} measures the passenger competition for vacant vehicles. In the case of single-homing, solo and pooling passengers from the same platform compete for the vacant vehicles of that platform (see Eq. (6)).

In practice, the flexibility promised by TNCs allows drivers to register and work on multiple platforms. The obvious rationale for a driver is to follow jobs wherever they arise, rather than putting all eggs (in this case his time) in one basket. In Chicago, for example, about 25% drivers are affiliated with more than one platform in September 2019, and a preliminary analysis suggests these drivers do serve more trips on average than others[3].

In this study, we consider an extreme version of *multi-homing* supply mode to simplify the analysis. Namely, we assume all drivers who join the ride-hail market work for every platform, i.e., $N = N^j, j \in \mathcal{P}$. Therefore, the average wage rate is given by

$$\bar{e} = e^j = \frac{1}{N}\sum_j \eta^j\left(Q_s^j\tau_s^j + \frac{1}{2}Q_p^j\tau_p^j\right), \tag{7}$$

and there is only one flow conservation equation Eq. (3c), i.e.,

$$V = V^j = N - \sum_j\left(Q_s^j\tau_s^j + \frac{1}{2}Q_p^j\tau_p^j\right). \tag{8}$$

In addition, as the waiting passengers compete for the same pool of vacant vehicles, the effective waiting passenger density is the sum of all platforms, i.e.,

$$\Pi_{\text{eff}} = \Pi_{\text{eff}}^j = \sum_j\left[Q_s^j w_s^j + \frac{1}{2}Q_p^j(w_p^j + \Delta^j)\right]. \tag{9}$$

[3]Computed from the Chicago TNC data in September 2019 (available at `https://data.cityofchicago.org/Transportation/Transportation-Network-Providers-Drivers/j6wf-834c`)

Here, solo and paired pooling passengers from each platform join the competition for the same pool of vacant vehicles. It is worth emphasizing that, although multi-homing intensifies competition for vacant vehicles, it has nothing to do with the process of pairing passengers. Pooling passengers from the same platform are always paired among themselves and together they compete for vacant vehicles. This is reflected in the fact that the detour Δ^j depends on Π_p^j, which is different among platforms. In other words, our model does not allow passengers on different platforms to be matched in the same pooling ride.

Therefore, multi-homing intensifies the inter-passenger competition even though it expands the pool of vacant vehicles. With single-homing, a passenger on one platform only competes with those who use the same platform. Yet, she would have to compete with every other waiting passenger in the aggregate market under multi-homing mode. Consider a system with two symmetric single-homing platforms. At a stationary state, the passenger wait time of each platform is dictated by the ratio Π_{eff}/Λ, where Π_{eff} and Λ are respectively the effective waiting passenger density and vacant vehicle density on *each* platform. If the system switches from single-homing to multi-homing, the ratio remains the same, because both Π_{eff} and Λ double. As a result, the passenger wait time would not be affected at all. In other words, the benefit of having a greater supply pool is offset by the more intense competition on the demand side.

Eq. (9) also implies that the solo wait times of all platforms are linearly correlated, i.e., $w_s^{j_1}/w_s^{j_2} = \sqrt{k^{j_2}/k^{j_1}}, \ \forall j_1, j_2 \in \mathcal{P}$. Therefore, it suffices to represent the solo wait time vector $\mathbf{w}_s$ using the solo wait time on any reference platform. If we choose $j = A$ as the reference platform, then the solution to the multi-homing equilibrium problem can be represented using a vector $\mathbf{x} = [w_s^A, \mathbf{w}_p, \mathbf{\Delta}, N]^T$.

5 Duopoly pricing game

The previous section establishes the market equilibrium given the pricing strategies of multiple platforms. In this section, we explore how these pricing decisions shape the overall equilibrium of the aggregate market. This process is viewed as a game played by the platforms with the objective to maximize their own profits through choosing an optimal pricing strategy. In Section 5.1, we assume the platforms compete in a regulation-free environment. Then, Section 5.2 discusses the scenario where a regulatory constraint is imposed on both platforms. In this study, we focus on the minimum wage policy, though other policies can be similarly analyzed.

5.1 Unregulated game

5.1.1 Equilibrium conditions

Let $\mathbf{y}^j = \left[f_s^j, f_p^j, \eta^j\right]^T \in \mathbb{R}_+^3$ denote the pricing strategy of platform j. Without regulations, the feasible set of $\mathbf{y}^j$ is not affected by the other platform's strategy. In this case, the equilibrium state can be characterized as a Nash Equilibrium (NE), i.e., no platform can increase its profit by unilaterally changing its own pricing strategy.

Let $R^j(\mathbf{y}^j, \mathbf{y}^{-j})$ be the profit of platform j given the pricing strategies of both platforms ($-j$

denotes $\mathcal{P} \setminus \{j\}$), which is evaluated as

$$R^j(\mathbf{y}^j, \mathbf{y}^{-j}) = f_s^j Q_s^j + f_p^j Q_p^j - \eta^j \left(Q_s^j \tau_s^j + \frac{1}{2} Q_p^j \tau_p^j \right), \tag{10}$$

that is, the total revenue (trip fares collected from both solo and pooling passengers) less the direct expense of trip production (payment to drivers based on their occupied time).

Mathematically, the duopoly equilibrium strategy $\mathbf{y}^*$ satisfies the following conditions:

$$R^j(\mathbf{y}^{j*}, \mathbf{y}^{-j*}) \geq R^j(\mathbf{y}^j, \mathbf{y}^{-j*}), \ \forall \mathbf{y}^j \in \mathbb{R}_+^3, \ j \in \{A, B\}. \tag{11}$$

It is well known that the Nash equilibrium condition can be transformed into a variational inequality problem (VIP) (e.g., Harker, 1991). Consider the optimal pricing problem of platform j

$$\max_{\mathbf{y}^j \in \mathbb{R}_+^3} R^j(\mathbf{y}^j, \mathbf{y}^{-j}). \tag{12}$$

The first-order necessary condition is given by

$$-\nabla_{\mathbf{y}^j} R^j(\mathbf{y}^{j*}, \mathbf{y}^{-j})^T (\mathbf{y}^j - \mathbf{y}^{j*}) \geq 0, \quad \forall \mathbf{y}^j \in \mathbb{R}_+^3, \tag{13}$$

where $\nabla_{\mathbf{y}^j} R^j(\mathbf{y}^{j*}, \mathbf{y}^{-j})$ is the gradient of R^j with respect to platform j's pricing vector and $\mathbf{y}^{j*}$ is the solution to (12) given the other platform's pricing strategy $\mathbf{y}^{-j}$. Note that Eq. (13) is sufficient only if $R^j(\cdot, \mathbf{y}^{-j})$ is concave. Otherwise, the solution to Eq. (13) is either a stationary point (i.e., $\nabla_{\mathbf{y}^j} R^j(\mathbf{y}^{j*}, \mathbf{y}^{-j}) = 0$) or a local maximum (i.e., $R^j(\mathbf{y}^{j^*}, \mathbf{y}^{-j}) \geq R^j(\mathbf{y}^j, \mathbf{y}^{-j})$, $\forall \mathbf{y}^j$ in a neighborhood of $\mathbf{y}^{j^*}$).

Since there is no binding constraints across players (i.e., the feasible set of the game is the full Cartesian product of individual player's strategy set), we can construct the VIP by summing up the above first-order condition of all players. Let $\mathbf{y} = [f_s^A, f_p^A, \eta^A, f_s^B, f_p^B, \eta^B]^T$ be the joint pricing strategy of the game. The VIP is stated as

$$\text{[VIP]} \quad \text{Find } \mathbf{y}^* \in \mathbb{R}_+^6 \text{ such that } \quad -\nabla R(\mathbf{y}^*)^T (\mathbf{y} - \mathbf{y}^*) \geq 0, \ \forall \mathbf{y} \in \mathbb{R}_+^6, \tag{14}$$

where $\nabla R(\mathbf{y}) = [\nabla_{\mathbf{y}^A} R^A(\mathbf{y})^T, \nabla_{\mathbf{y}^B} R^B(\mathbf{y})^T]^T$ and $\nabla_{\mathbf{y}^j} R^j(\mathbf{y}) = \nabla_{\mathbf{y}^j} R^j(\mathbf{y}^j, \mathbf{y}^{-j})$.

5.1.2 Solution method

Due to the complex structure of the market equilibrium problem (Eq. (3)), we cannot obtain a closed-form expression of $R^j(\cdot, \mathbf{y}^{-j})$ or establish its concavity. Hence, in theory a solution to the VIP (14) does not necessarily satisfy the equilibrium condition Eq. (11), though the reverse is evidently true. In what follows, we first establish the solution existence for the VIP, and then propose to solve it using a gradient ascent algorithm. Because a solution to VIP found by such an algorithm must be a local maximum, it must satisfy Eq. (11).

The solution existence for (14) is not obvious because the feasible set of $\mathbf{y}$ is not compact. To prove it, let us first restate a result from Harker and Pang (1990) as follows.

Lemma 1 (Harker and Pang (1990) Corollary 3.1) *Let X be a nonempty, closed and convex subset of $\mathbb{R}^n$ and F be a continuous mapping $\mathbb{R}^n$ to itself. Then, the VIP*

$$F(x^*)^T(x - x^*) \geq 0, \ \forall x \in X$$

has a solution if there exists a nonempty bounded set $D \subset X$ such that for every $x \in X \setminus D$ there is a $x_0 \in D$ with $F(x)^T(x_0 - x) \leq 0$, that is, no point outside D is a solution candidate for the VIP.

Proposition 1 *The VIP defined in Eq.* (14) *has a solution.*

Proof. Let us rewrite the objective function of each platform $j \in \{A, B\}$ as

$$R^j(\mathbf{y}^j, \mathbf{y}^{-j}) = (f_s^j - \eta^j \tau_s^j) Q_s^j + \left(f_p^j - \frac{1}{2}\eta^j \tau_p^j \right) Q_p^j. \tag{15}$$

First, we can see from Eq. (15) that $\lim_{\mathbf{y}^j \to 0} R^j = 0$. When $\mathbf{y}^j \to \infty$, it is obvious $Q_i^j \to 0, \forall i = \{s, p\}$ (extremely high pricing eliminates all demands). Thus, $\lim_{\mathbf{y}^j \to \infty} R^j = 0$. In other words, the profit function approaches zero at both boundaries. Since $R = (R^A, R^B)$ is continuously differentiable, there must exist a point $\mathbf{y}_0 = (\mathbf{y}_0^A, \mathbf{y}_0^B)$ such that $R(\mathbf{y}_0) \geq 0$ and $\nabla_{\mathbf{y}_0^A} R(\mathbf{y}_0) \leq 0$ and $\nabla_{\mathbf{y}_0^B} R(\mathbf{y}_0) \leq 0$. Therefore, for all $\mathbf{y} \geq \mathbf{y}_0$, we have $-\nabla R(\mathbf{y})^T(\mathbf{y}_0 - \mathbf{y}) \leq 0$. Noting $\mathbb{R}_+^6$ is a nonempty, closed and convex set, we prove Eq. (14) must have a solution by invoking Lemma 1. □

Since the platforms always yield zero profit at the boundaries, the equilibrium price must lie inside of the feasible set (i.e., $\mathbf{y}^* > \mathbf{0}$). Hence, it is sufficient to solve $\nabla R(\mathbf{y}) = 0$ for the VIP (14). To search for such an interior solution, we start from a randomly selected initial point, and iteratively ascend along the gradient, which is the "steepest" direction.

Given a price strategy $\mathbf{y}$ and a market equilibrium $\mathbf{x}$, the gradient ∇R in each iteration is evaluated as

$$\nabla R = \frac{\partial R}{\partial \mathbf{y}} + \frac{\partial R}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{y}}. \tag{16}$$

Here, the first term measures the "direct" impact of $\mathbf{y}$ on R, while the second term measures the "indirect" effect through the change in market equilibrium (i.e., wait time, passenger demand and fleet size). The analytical form of $\partial R / \partial \mathbf{y}$ and $\partial R / \partial \mathbf{x}$ are readily available. To derive $\partial \mathbf{x} / \partial \mathbf{y}$, we differentiate the fixed point system $\mathbf{x} = F(\mathbf{x})$, which yields

$$\frac{\partial \mathbf{x}}{\partial \mathbf{y}} = \nabla_{\mathbf{y}} F(\mathbf{x}, \mathbf{y}) + \nabla_{\mathbf{x}} F(\mathbf{x}, \mathbf{y}) \frac{\partial \mathbf{x}}{\partial \mathbf{y}}. \tag{17}$$

Here, $\nabla_{\mathbf{y}} F(\mathbf{x}, \mathbf{y})$ and $\nabla_{\mathbf{x}} F(\mathbf{x}, \mathbf{y})$ can be computed by automatic differentiation (Baydin et al., 2017). Thus, Eq. (17) becomes a linear equation system with unknown variables $\partial \mathbf{x} / \partial \mathbf{y}$. Effectively, $\partial \mathbf{x} / \partial \mathbf{y}$ measures the sensitivity of the market equilibrium to a small perturbation in the pricing strategy. This idea is similar to the sensitivity-analysis-based approach that has been widely used in the literature (e.g., Tobin and Friesz, 1988; Yang, 1995; Patriksson, 2004). The detailed algorithm is included in Appendix C.

5.2 Regulated duopoly equilibrium

5.2.1 Equilibrium condition

In the unregulated game, the platforms only interact through their profit function, and the feasible sets of their pricing strategies are independent. This property implies that the feasible set of the pricing game is the full Cartesian product of each individual player's feasible set (i.e., $\mathbb{R}^3_+ \times \mathbb{R}^3_+ = \mathbb{R}^6_+$). However, with regulatory constraints, the feasible set of one platform is affected by the other. In this case, the feasible set of the regulated pricing game can be viewed as a point-to-set mapping $\Omega(\mathbf{y}) = \prod_{j \in \mathcal{P}} \Omega^j(\mathbf{y}^{-j})$, where $\Omega^j(\mathbf{y}^{-j}) = \{\mathbf{y}^j | h^j(\mathbf{y}^j, \mathbf{y}^{-j}) \leq 0, \mathbf{y}^j \in \mathbb{R}^3_+\}$. For a minimum wage policy, the constraint $h^j(\mathbf{y}^j, \mathbf{y}^{-j}) \leq 0$ can be further specified as

$$e^j = \frac{\eta^j}{N^j} \left(Q_s^j \tau_s^j + \frac{1}{2} Q_p^j \tau_p^j \right) \geq E \tag{18}$$

for single-homing supply mode and

$$\bar{e} = \frac{1}{N} \sum_j \eta^j \left(Q_s^j \tau_s^j + \frac{1}{2} Q_p^j \tau_p^j \right) \geq E \tag{19}$$

for multi-homing, where E is the minimum wage rate imposed by the regulator. Note that, in addition to $\mathbf{y}^j$, $\mathbf{y}^{-j}$ is also involved in Eqs. (18) and (19) through passenger demands Q_s^j, Q_p^j and fleet size N^j.

This interaction through interdependent feasible sets gives rise to the concept of generalized Nash equilibrium (GNE) (Arrow and Debreu, 1954; Ichiishi, 1983). In the context of regulated duopoly pricing game, it is characterized as

$$R^j(\mathbf{y}^{j*}, \mathbf{y}^{-j*}) \geq R^j(\mathbf{y}^j, \mathbf{y}^{-j*}), \ \forall \mathbf{y}^j \in \Omega(\mathbf{y}^*), \ j \in \{A, B\}. \tag{20}$$

Similar to the case of NE, GNE corresponds to a quasi-variational inequality problem (QVIP) (see e.g., Harker, 1991; Facchinei and Kanzow, 2010), which reads

$$\text{[QVIP] Find } \mathbf{y}^* \in \Omega(\mathbf{y}^*) \text{ such that } \quad -\nabla R(\mathbf{y}^*)^T(\mathbf{y} - \mathbf{y}^*) \geq 0, \ \forall \mathbf{y} \in \Omega(\mathbf{y}^*). \tag{21}$$

Again, because we cannot ensure that R is concave, a solution to the QVIP is not necessarily a GNE. However, the reverse statement must be true.

5.2.2 Solution method

To find a solution for the QVIP, we employ the penalty algorithm developed by Facchinei and Kanzow (2010). The basic idea of the algorithm is to convert (21) into a standard VIP by adding a penalty term to the objective function of each platform. i.e.,

$$P^j(\mathbf{y}, \rho^j) = R^j(\mathbf{y}) + \rho^j \left[(h_+^j(\mathbf{y}))^2 + \varepsilon \right]^{1/2}, \tag{22}$$

where ρ^j is a penalty parameter associated with platform j, $h_+^j(\cdot) = \max\{0, h^j(\cdot)\}$ and $\varepsilon \in (0, 1]$ is a scalar parameter that ensures smoothness.

Let $\nabla P(\mathbf{y}) = [\nabla_{\mathbf{y}^A} P^A(\mathbf{y}, \rho)^T, \nabla_{\mathbf{y}^B} P^B(\mathbf{y}, \rho)^T]^T$, then the penalized VIP is defined as

$$\text{[Penalized VIP] Find } \mathbf{y}^* \in \mathbb{R}^6_+ \text{ such that} \quad -\nabla P(\mathbf{y}^*)^T(\mathbf{y} - \mathbf{y}^*) \geq 0, \ \forall \mathbf{y} \in \Omega(\mathbf{y}^*). \tag{23}$$

While the solution existence for QVIP is more difficult to establish, Theorem 2.5 of Facchinei and Kanzow (2010) asserts that a solution must exist if the penalty algorithm terminates in a finite number of iterations. The detailed algorithm is described in Appendix C.

5.3 Single-homing vs multi-homing

As discussed in Section 4.2, switching from single-homing to multi-homing does not affect the passenger wait time if the platforms are symmetric and the inputs fed to the matching process (i.e., densities of vacant vehicles and waiting passengers) remain the same. However, the marginal effect of a platform's pricing strategy on these equilibrium variables does vary between the two supply modes. To elaborate the difference, consider two symmetric platforms $\mathcal{P} = \{A, B\}$ that only serve solo rides. The gradient $\nabla_{\mathbf{y}^A} R^A$ can be derived as

$$\frac{\partial R^A}{\partial f_s^A} = Q_s^A + (f_s^A - \eta^A \tau_s^A)\frac{\partial Q_s^A}{\partial f_s^A}; \tag{24a}$$

$$\frac{\partial R^A}{\partial \eta^A} = -Q_s^A \tau_s^A + (f_s^A - \eta^A \tau_s^A)\frac{\partial Q_s^A}{\partial \eta^A}. \tag{24b}$$

The terms $\partial Q_s^A / \partial f_s^A$ and $\partial Q_s^A / \partial \eta^A$ can be evaluated by

$$\frac{\partial Q_s^A}{\partial f_s^A} = \nabla_{u_s^A} q + \nu \left(\nabla_{u_s^A} q \frac{\partial w_s^A}{\partial f_s^A} + \nabla_{u_s^B} q \frac{\partial w_s^B}{\partial f_s^A} \right), \tag{25}$$

$$\frac{\partial Q_s^A}{\partial \eta^A} = \nu \left(\nabla_{u_s^A} q \frac{\partial w_s^A}{\partial \eta^A} + \nabla_{u_s^B} q \frac{\partial w_s^B}{\partial \eta^A} \right), \tag{26}$$

where $\nabla_{u_s^A} q$ and $\nabla_{u_s^B} q$ are the partial derivatives of the demand function $q(\cdot)$ with respect to u_s^A, u_s^B, respectively.

Without loss of generality, we may assume $\nabla_{u_s^A} q < 0$, $\nabla_{u_s^B} q > 0$ and $|\nabla_{u_s^A} q| > |\nabla_{u_s^B} q|$ (i.e., the general cost associated with one platform has a larger impact on itself than the other platform). Hence, the sign of $\partial Q_s^A / \partial f_s^A$, which indicates whether an increase in trip fare would increase or decrease the platform's market share, depends on $\partial w_s^A / f_s^A$ and $\partial w_s^B / f_s^A$. The same is true for the compensation rate η^A. As per Eq. (1a), the solo wait time increases with passenger demand but decreases with vehicle supply. Since a higher trip fare usually drags down the demand while a higher compensation rate always attracts more drivers to enter the system, we may assume $\partial w_s^A / \partial f_s^A < 0$ and $\partial w_s^A / \partial \eta^A < 0$, regardless of the supply mode.

Under single-homing, an increase in f_s^A would raise the wait time on platform B, i.e., $\partial w_s^B / \partial f_s^A > 0$, because it would increase the demand on platform B, thus intensifying competition there. Similarly, we expect $\partial w_s^B / \partial \eta^A > 0$ because an increase in η_A attracts more drivers to platform A, at the expense of platform B. On the other hand, as shown in Section 4.2, $w_s^A \propto w_s^B$ under multi-homing and the constant ratio is positive. Therefore, we have $\partial w_s^B / \partial f_s^A \propto \partial w_s^A / \partial f_s^A \to \partial w_s^B / \partial f_s^A < 0$. Since the passenger wait time depends on both platforms under multi-homing, the influence of each platform is expected to be less compared to single-homing. Hence,

$$\left(\frac{\partial w_s^B}{\partial f_s^A}\right)_{SH} > 0 > \left(\frac{\partial w_s^B}{\partial f_s^A}\right)_{MH} \propto \left(\frac{\partial w_s^A}{\partial f_s^A}\right)_{MH} > \left(\frac{\partial w_s^A}{\partial f_s^A}\right)_{SH},$$

where subscripts "SH" and "MH" denote single-homing and multi-homing, respectively.

Accordingly, it yields

$$\begin{aligned}(\partial Q_s^A/\partial f_s^A)_{MH} &= \nabla_{u_s^A} q + \nu\left[\nabla_{u_s^A} q\left(\partial w_s^A/\partial f_s^A\right)_{MH} + \nabla_{u_s^B} q\left(\partial w_s^B/\partial f_s^A\right)_{MH}\right] \\ &< \nabla_{u_s^A} q + \nu\left[\nabla_{u_s^A} q\left(\partial w_s^A/\partial f_s^A\right)_{SH} + \nabla_{u_s^B} q\left(\partial w_s^B/\partial f_s^A\right)_{SH}\right] = (\partial Q_s^A/\partial f_s^A)_{SH}.\end{aligned} \tag{27}$$

The inequality holds because $\nabla_{u_s^A} q < 0, \nabla_{u_s^B} q > 0$, as discussed above. The same reasoning can be used to derive $(\partial Q_s^A/\partial \eta^A)_{MH} < (\partial Q_s^A/\partial \eta^A)_{SH}$. Bringing this result to Eq. (24), we can conclude that, under multi-homing, the platforms are less inclined to raise trip fare and compensation rate because their profit tends to rise less when they do so than under single-homing.

To further explore the impact of a shift in compensation rate on the vehicle supply, we derive $\partial N/\partial \eta$ under single-homing and multi-homing modes. First, Eq. (3b) for platform A leads to

$$\frac{\partial N^A}{\partial \eta^A} = S_0 \nabla_{e^A} g \frac{\partial e^A}{\partial \eta^A}, \tag{28}$$

where g is the supply function and $\nabla_e^A g$ is the derivative of g with respect to e^A. It is reasonable to assume $\nabla_{e^A} g > 0$, since the fleet size tends to increase with average wage rate. Hence, in general $\partial N^j/\partial \eta^j$ is positively proportional to $\partial e^j/\partial \eta^j$, for $j = A, B$.

For single-homing, first taking logarithm on both sides of Eq. (4) and then getting derivative with respect to η yield

$$\frac{1}{e^A}\frac{\partial e^A}{\partial \eta^A} = \frac{Q_s^A \tau_s^A + \eta^A \tau_s^A \frac{\partial Q_s^A}{\partial \eta^A}}{\eta^A Q_s^A \tau_s^A} - \frac{1}{N^A}\frac{\partial N^A}{\partial \eta^A}. \tag{29}$$

Plugging Eq.(28) into Eq. (29) gives

$$\left(\frac{\partial e^A}{\partial \eta^A}\right)_{SH} = \left(\frac{1}{e^A} + \frac{S_0^A}{N^A}\nabla_{e^A} g\right)\frac{Q_s^A \tau_s^A + \eta^A \tau_s^A \frac{\partial Q_s^A}{\partial \eta^A}}{\eta^A Q_s^A \tau_s^A}. \tag{30}$$

Following a similar derivation, $\partial e^A/\partial \eta^A$ under multi-homing can be derived form Eq. (7) as

$$\left(\frac{\partial e^A}{\partial \eta^A}\right)_{MH} = \left(\frac{1}{e^A} + \frac{S_0^A}{N^A}\nabla_{e^A} g\right)\frac{Q_s^A \tau_s^A + \eta^A \tau_s^A \frac{\partial Q_s^A}{\partial \eta^A} + \eta^B \tau_s^B \frac{\partial Q_s^B}{\partial \eta^A}}{\eta^A Q_s^A \tau_s^A + \eta^B Q_s^B \tau_s^B}. \tag{31}$$

When platforms are symmetric, we have $\eta^A = \eta^B, Q_s^A = Q_s^B, \tau_s^A = \tau_s^B$. In addition, it can be derived from Eq. (26) that $\partial Q_s^A/\partial \eta^A = \partial Q_s^B/\partial \eta^A > 0$[4]. Hence, we have

$$\left(\frac{\partial e^A}{\partial \eta^A}\right)_{MH} = \left(\frac{1}{e^A} + \frac{S_0^A}{N^A}\nabla_{e^A} g\right)\frac{\frac{1}{2}Q_s^A \tau_s^A + \eta^A \tau_s^A \frac{\partial Q_s^A}{\partial \eta^A}}{\eta^A Q_s^A \tau_s^A} < \left(\frac{\partial e^A}{\partial \eta^A}\right)_{SH}. \tag{32}$$

[4]This is only true under symmetric multi-homing where $w_s^A = w_s^B$

Thus, an increase in compensation rate would induce less supply under multi-homing than under simple-homing. In other words, when a platform attempts to squeeze profit by reducing the compensation to drivers, it only bears part of the cost caused by supply depression. In economics, such a phenomenon is widely known as the tragedy of the commons (Ostrom et al., 1994). Here, it is the drivers that are the "commons" shared (and abused) by profit-driven platforms under the multi-homing mode.

6 Numerical experiments

In this study, numerical experiments are constructed based on the TNC data collected in the City of Chicago[5]. Table A1 in Appendix A reports the default values of the input parameters used in the experiment. The reader is referred to Zhang and Nie (2019) for details about how these parameters are estimated from the data.

In what follows, Section 6.1 specifies the demand and supply models used in all experiments. Section 6.2 introduces the benchmark monopoly model and defines the social welfare function for all studied scenarios. Section 6.3 explores how the equilibrium in a duopoly varies with supply modes and other important inputs, when the pricing strategy is fixed. Sections 6.4 and 6.5 report and analyze the results from the experiments of unregulated and regulated duopoly pricing games, respectively. The experiments conducted in Sections 6.3–6.5 all assume the two platforms are symmetric, i.e., they are essentially identical in every aspect. This restriction is lifted in Section 6.6, in which the pricing games played by asymmetric platforms are analyzed.

6.1 Specification of demand and supply models

We estimate a passenger's mode choice using a nested Logit (NL) model (e.g., Williams, 1977), where the generalized cost defined in Eq. (2) is used as the observed disutility. Since ride-hail services of different platforms and modes have many features in common, we introduce a simple nested structure that encompasses them all: the upper level consists of transit and ride-hail while the lower level has four ride-hail modes of both platforms[6].

Accordingly, the mode share of $i \in \{s, p\}$ for platform j is given by

$$Q_i^j = D_0 \frac{\exp(-\theta_c I_c^r)}{\exp(-\theta_c u_t) + \exp(-\theta_c I_c^r)} \frac{\exp(-\theta_c^r u_i^j)}{\exp(-\theta_c^r I_c^r)}, \tag{33}$$

Here, θ_c is the dispersion parameter that determines the choice uncertainty between transit and ride-hail services. The composite cost of ride-hail services I_c is specified as

$$I_c^r = -\frac{1}{\theta_c^r} \ln \left[\sum_{j \in \mathcal{P}} \sum_{m \in \{s,p\}} \exp(-\theta_c^r u_m^j) \right], \tag{34}$$

where θ_c^r is the dispersion parameter that determines the choice uncertainty among different modes and platforms (a larger value corresponds to larger uncertainty).

[5]Available at `https://data.cityofchicago.org`.

[6]Although more sophisticated nested structures could be accommodated within our framework, we note that testing the suitability of these structures is a task beyond the scope of the present study.

Based on the assumption of NL, we have $\theta_c^r \geq \theta_c$, and the ratio θ_c/θ_c^r measures the magnitude of correlations among the ride-hail modes. $\theta_c/\theta_c^r = 1$ implies that all ride-hail modes are independent alternatives, i.e., the independence of irrelevant alternatives (IIA) assumption holds. On the other hand, if $\theta_c/\theta_c^r \to 0$, all ride-hail alternatives will be viewed together as a single ride-hail mode (with a utility equal to the average of the individual alternatives), and the share of this "composite mode" will be evenly split among the original alternatives.

Since the data available is insufficient to calibrate the dispersion parameters, in most experiments, we simply set $\theta_c^r = \theta_c$ and $\theta_d^r = \theta_d$. Because this setting effectively upholds the IIA assumption, it tends to overestimate the total rid-hail market share in the duopoly case. To gauge this impact, a sensitivity analysis is performed against θ_d^r and θ_c^r in Appendix E.

Similarly, we characterize drivers' decision using an NL model. For single-homing, drivers first decide whether to enter the ride-hail market and then choose which platform to join. Let e_0 be the wage rate of the best alternative opportunity outside the ride-hail market, then the fleet size of platform j (Eq. (3b)) is specified as

$$N^j = S_0 \frac{\exp(\theta_d I_d^r)}{\exp(\theta_d e_0) + \exp(\theta_d I_d^r)} \frac{\exp(\theta_d^r e^j)}{\exp(\theta_d^r I_d^r)}, \tag{35}$$

where the composite utility of ride-hail is

$$I_d^r = \frac{1}{\theta_d^r} \ln \left[\sum_{j \in \mathcal{P}} \exp(\theta_d^r e^j) \right]. \tag{36}$$

Again, θ_d and θ_d^r are the dispersion parameters with respect to each level of the NL model.

For multi-homing, since each driver joins all platforms, the choice model reduces to a multinomial logit (MNL) model with two alternatives and the fleet size of all platforms (Eq. (3b)) is given by

$$N = N^j = S_0 \frac{\exp(\theta_d \bar{e})}{\exp(\theta_d e_0) + \exp(\theta_d \bar{e})}. \tag{37}$$

6.2 Monopoly as a benchmark

For comparison, we also solve the monopoly pricing problem using the same inputs, where the supply model reduces to an MNL with two choices (ride-hail with an average wage rate e and the outside option). We define the social welfare of the aggregate market in the three scenarios as follows:

1. Monopoly

$$\begin{aligned} W_{\text{MO}} = &\frac{D_0}{\theta_c} \ln \left\{ 1 + \left\{ \sum_{m \in \mathcal{M}} \exp[\theta_c^r (u_t - u_m)] \right\}^{\theta_c/\theta_c^r} \right\} \\ &+ \frac{S_0}{\theta_d} \ln \left\{ 1 + \exp[\theta_d (e - e_0)] \right\} \\ &+ \sum_{j \in \mathcal{P}} \left[f_s^j Q_s^j + f_p^j Q_p^j - \eta^j \left(Q_s^j \tau_s^j + \frac{1}{2} Q_p^j \tau_p^j \right) \right] - c_0 N; \end{aligned} \tag{38}$$

2. Duopoly under single-homing

$$W_{\text{single}} = \frac{D_0}{\theta_c} \ln \left\{ 1 + \left\{ \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{P}} \exp[\theta_c^r (u_t - u_m)] \right\}^{\theta_c / \theta_c^r} \right\} \tag{39}$$
$$+ \frac{S_0}{\theta_d} \ln \left\{ 1 + \left\{ \sum_{j \in \mathcal{P}} \exp[\theta_d (e^j - e_0)] \right\}^{\theta_d / \theta_d^r} \right\}$$
$$+ \sum_{j \in \mathcal{P}} \left[f_s^j Q_s^j + f_p^j Q_p^j - \eta^j \left(Q_s^j \tau_s^j + \frac{1}{2} Q_p^j \tau_p^j \right) \right] - c_0 N;$$

3. Duopoly under multi-homing

$$W_{\text{multi}} = \frac{D_0}{\theta_c} \ln \left\{ 1 + \left\{ \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{P}} \exp[\theta_c^r (u_t - u_m)] \right\}^{\theta_c / \theta_c^r} \right\} \tag{40}$$
$$+ \frac{S_0}{\theta_d} \ln \{ 1 + \exp[\theta_d (\bar{e} - e_0)] \}$$
$$+ \sum_{j \in \mathcal{P}} \left[f_s^j Q_s^j + f_p^j Q_p^j - \eta^j \left(Q_s^j \tau_s^j + \frac{1}{2} Q_p^j \tau_p^j \right) \right] - c_0 N.$$

In all three welfare functions, the first term represents passenger surplus based on the logsum term derived from the NL or MNL model (see e.g., Kohli and Daly, 2006; De Jong et al., 2007). Here, the utility of the transit trip is used as a benchmark to measure the extra utility contributed by the e-hail services. The second term similarly measures driver surplus, using the fallback option as a benchmark. The third term quantifies the total profits of all platforms and the last term approximates the congestion cost contributed by all vehicles employed by the e-hail market (c_0 is the marginal congestion cost per vehicle per unit operating time).

6.3 Equilibrium in a duopoly market with fixed pricing strategy

Throughout this section, we assume both platforms, which are identical in their defining features, adopt the same fixed pricing strategies f_s = \$14, f_p =\$10, η = \$20/hr. We examine how passenger wait times, market share and vacant vehicle density at the market equilibrium vary with there key inputs: the total demand D_0, the potential supply S_0 and the en-route detour $\tau_p - \tau_s$. Each market equilibrium problem (3), VIP (14) or QVIP (21) is solved multiple times with randomly selected initial solutions. We include a brief discussion of the existence of multiple equilibria in Appendix D.

As the total demand increases (Figure 2(a)), the passenger wait time tends to increase and the e-hail's market share tends to shrink, primarily due to greater competition. The vacant vehicle density first rises with the demand, reaches a long stretch of plateau and then begins a slow decline. These general trends hold for all supply modes: monopoly, single-homing and multi-homing. A noticeable deviation is the wait times for pooling rides: they first decrease mildly as the demand increases, thanks to the reduced pickup detour Δ. However, when the demand continues to rise, the stronger inter-passenger competition pushes up the pickup time w_p that eventually more than offsets the savings in Δ. This pattern explains why market share of the pooling rides (the middle plot in Figure 2) first increases and then decreases as the demand goes up.

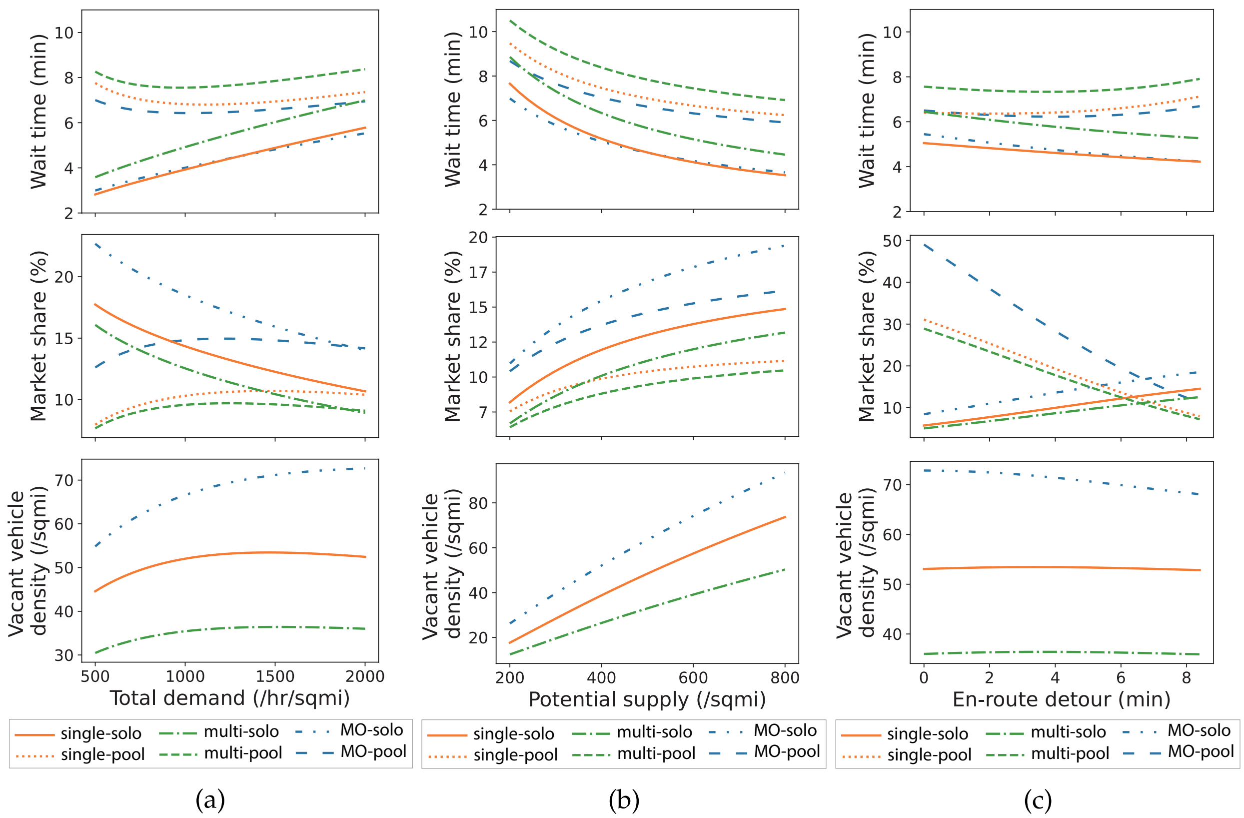


Figure 2: Sensitivity of passenger wait time, market share and vacant vehicle density to (a) total demand D_0, (b) potential supply S_0, and (c) en-route detour $\tau_p - \tau_s$. We only plot the market share of one platform because the other one is the same due to symmetry. The vacant vehicle density under multi-homing is the total vacant vehicle divided by two, i.e., the expected vehicle supply available for one platform.

Figure 2(b) shows that, as expected, when the potential supply increases, the wait time decreases, and both the market share of e-hail and the vacant vehicle density increases. The impact of en-route detour is more nuanced, as shown in Figure 2(c). This detour, which measures how much additional time pooling would add to the trip duration, has a dramatic effect on the market share of pooling rides, regardless of the supply mode. For every two minutes of additional detour, the market share roughly drops by 5%. Solo rides gain market share, but not enough to make up for the loss suffered by pooling rides. Interestingly, neither the wait time nor the vacant vehicle density is much affected by the detour. For solo rides, the wait time slightly decreases, likely because the declining market share eases up competition. Yet, the wait time for the pooling rides eventually turns back up, thanks to the increase in Δ caused by the loss in the market share.

We now examine the differences associated with the supply mode. In all cases, the monopoly has the highest share for both solo and pooling rides, followed by the single-homing and multi-homing duopoly. This is expectable because a monopoly platform enjoys a higher market power.

However, the total market shares in a duopoly market is higher than those in a monopoly. We caution that this result likely overestimates the appeal of the duopoly (for both single- and multi-homing modes) due to the IIA assumption. For the same reason, the fleet size under single-homing could be overestimated, leading to a shorter wait time compared to that under multi-homing.

The bottom row of Figure 2 shows the monopoly enjoys a much higher vacant vehicle density than the other two markets. However, its level of service (LOS) is only marginally better as measured by passenger wait time (the saving is less than 1 minute). This is primarily due to the higher demand rate, which intensifies the competition among waiting passengers and thus drags down the LOS. On the other hand, the wait time under multi-homing is consistently higher than that under single-homing, both for solo and pooling rides. This result could be explained by the much lower vacant vehicle density.

Although the change in market shares against the total demand and supply follows the same pattern under the two duopoly modes, multi-homing shows a higher sensitivity. Specifically, under multi-homing the market share of ride-hail decreases faster as total demand increases, and increases faster as potential supply increases, as shown in the middle subplots of Figures 2(a) and 2(b).

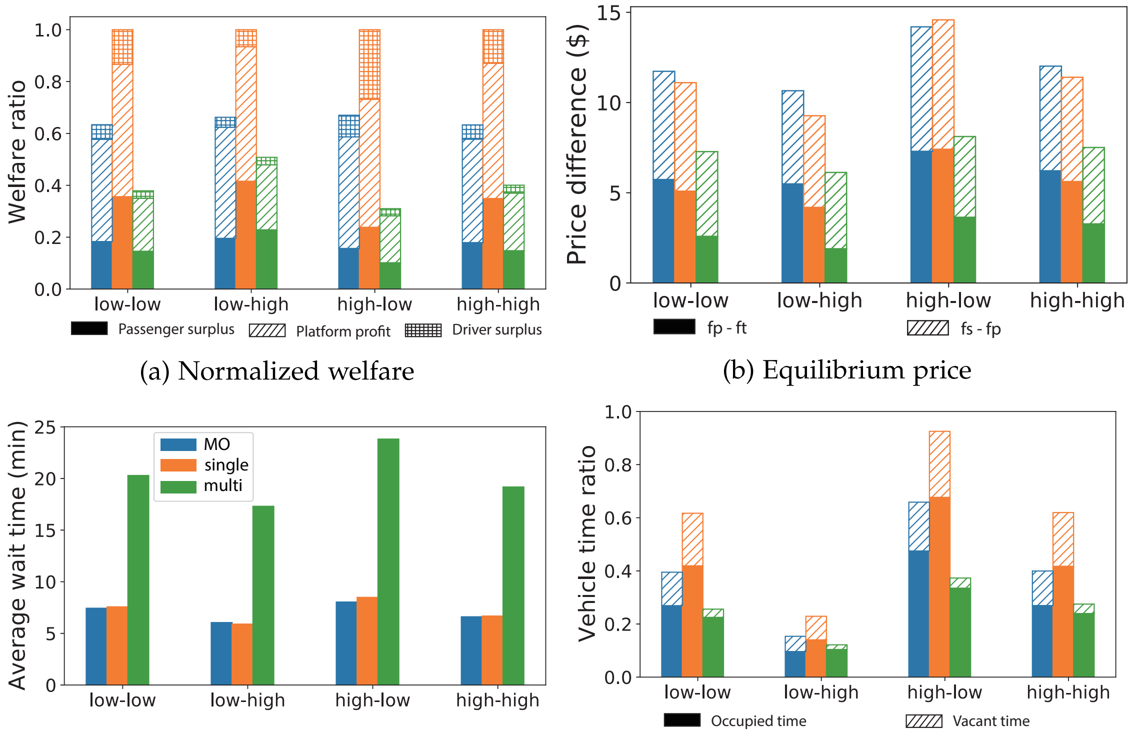
6.4 Performance of unregulated duopoly game

Starting from this section, we focus on four representative demand-supply conditions, dubbed as "low-low" (low-demand-low-supply), "low-high" (low-demand-high-supply), "high-low" (high-demand-low supply) and "high-high" (high-demand-high-supply); see Table A2 in Appendix A.

Figure 3 compares the results of the duopoly pricing game under the two supply modes with those in monopoly. For clarity, the social welfare is normalized against the highest value under each market condition. The normalized total welfare is compared in Figure 3(a), along with its three main components: consumer surplus, driver surplus and platform profit[7]. In all cases, the platform(s) is the winner, taking the lion share of the gains in the social welfare. The single- and multi-homing duopoly respectively achieve the highest and lowest social welfare under all conditions, with the monopoly lies right in the middle. The system performance under multi-homing is surprisingly bad, consistently lagging behind single-homing in all three components of the social welfare.

Figure 3(c) indicates that the lower passenger surplus in the multi-homing duopoly is likely due to the much longer wait time (which triples that in either the monopoly or the single-homing duopoly). It is further confirmed in Figure 3(d), which plots the ratio of vehicle time dedicated to ride-hail services. Even under the most favorable condition (high demand low supply), barely 40% of all potential workforce are working as ride-hail drivers in the multi-homing duopoly, compared to over 90% in the single-homing duopoly and about 70% in the monopoly. The vacant vehicle time is even lower, directly leading to the excessively long wait time in multi-homing, which in turn would depress the demand. Figure 3(b) shows why the platforms also suffer in the multi-homing duopoly: they have to charge a lower price to make up for their overall poorer LOS. Another noteworthy observation is the multi-homing duopoly tends to maintain a

[7] The congestion externality cost is deducted from driver surplus.



(a) Normalized welfare

(b) Equilibrium price

(c) Passenger wait time

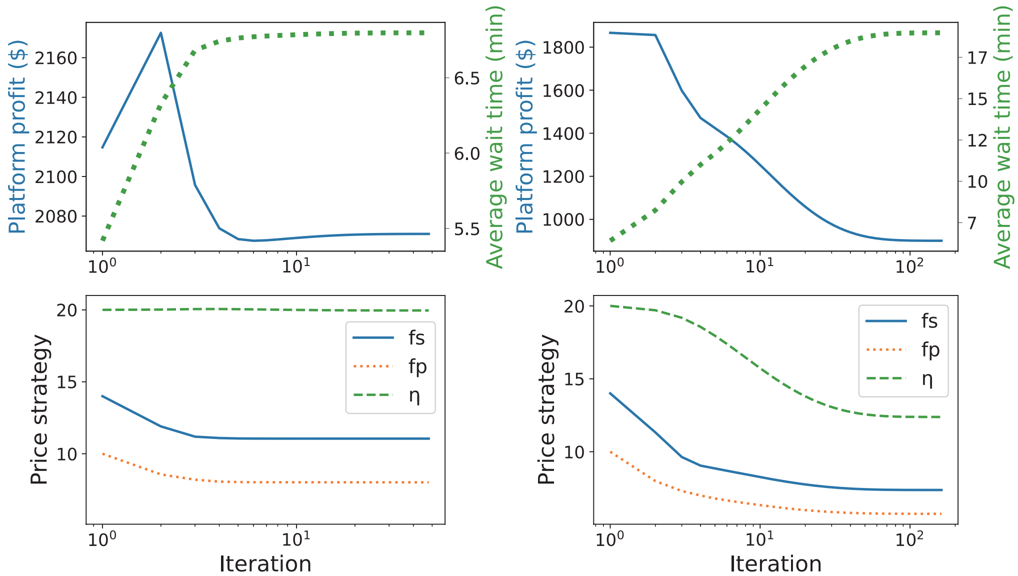
(d) Vehicle time normalized by S_0

Figure 3: System performance without regulatory constraints: duopoly vs. monopoly. "MO" stands for monopoly; "single" stands for single-homing duopoly game; "multi" stands for multi-homing duopoly game.

greater price spread between solo and pooling rides than the spread between pooling and transit, a choice that tends to favor pooling rides. For the monopoly, this difference between two spreads is mostly negligible.

Zha et al. (2016) and Nikzad (2017) pointed out that, despite the competition it introduces, a duopoly could set a higher price than a monopoly under certain demand-supply scenarios. This observation is confirmed in our results. As shown in Figure 3(b), when demand is high and supply is low (high-low), both solo and pooling trip prices are slightly higher in the single-homing duopoly than in the monopoly. This aligns with the finding of Nikzad (2017) especially well, who concludes that the duopoly price could be higher than the monopoly price if the market is not "sufficiently thick" (i.e., when the potential supply is low). Importantly, in most cases, the single-homing duopoly lowers the price for pooling, suggesting *the pressure of competition encourage the platforms to promote pooling*.

Why does the multi-homing duopoly perform so poorly? The discussion in Section 5.3 suggests that the culprit is the tragedy of the commons. To further examine the underlying mechanisms, we solve the duopoly game under the two supply modes with the same initial solution and plot the results over iterations in Figure 4. Specifically, the total demand, potential supply and initial price are set to be the default values in Table A1. The evolution can be seen as a repeated game where the two platforms slightly adjust their strategies along the best direction at each stage. Due to symmetry, the two platforms in each game follow exactly the same evolution path.



(a) Single-homing

(b) Multi-homing

Figure 4: Solving unregulated duopoly equilibrium.

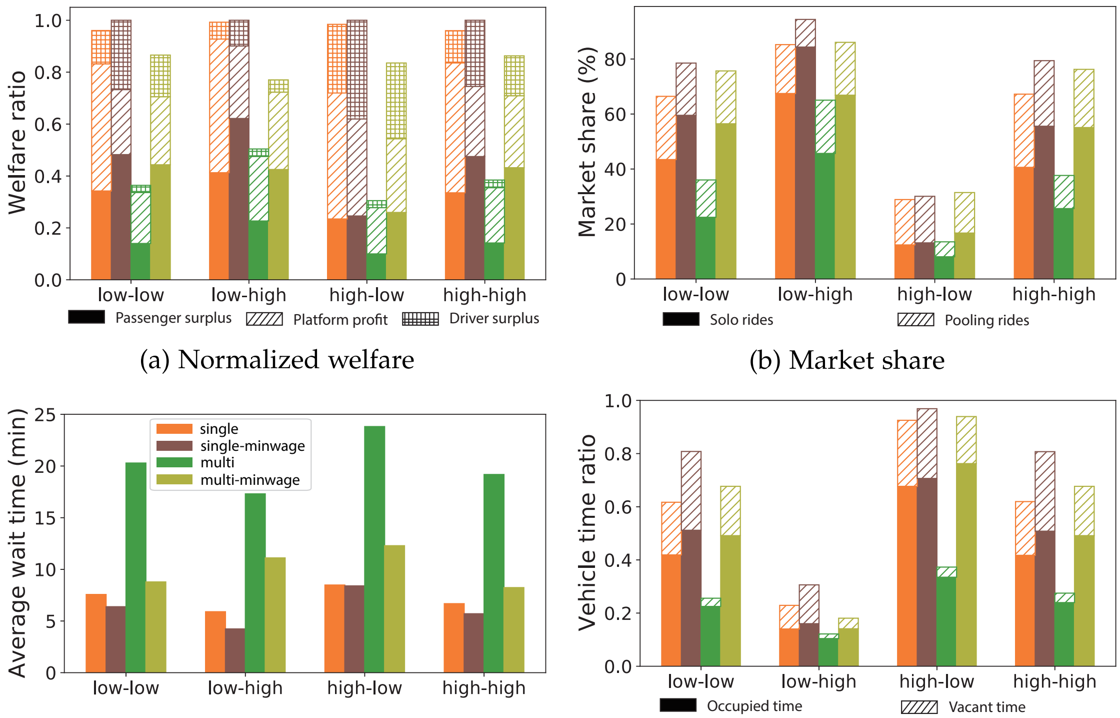
Comparing the bottom panels of Figure 4(a) and (b), one can find the platforms in the multi-homing duopoly tend to attract passengers with lower trip fares, rather than a higher LOS (i.e., shorter wait times). Since a multi-homing driver's decision to enter service hinges on the "market wage rate", it is more difficult for a platform to attract drivers by unilaterally raising the compensation rate η. Instead, the platform is better off by lowering η to reduce its operating cost, in exchange for a supply loss that is shared by its rival. This behavior is consistent with the prediction in Section 5.3. Consequently, *the pricing game in the multi-homing duopoly is stuck in a trap with lower fare, lower compensation and lower LOS*.

Interestingly, in reality, TNC platforms clearly dislike multi-homing——many actively discourage such behavior by implementing loyalty programs[8] or openly imposing penalty. Such a strong preference for an exclusive rather than shared workforce may be explained by the fact that multi-homing is as detrimental to the fundamentals of the platforms as to the efficiency and productivity of the entire system.

6.5 Impact of regulations

In this section we repeat the experiments in the previous section but add a minimum wage constraint. The wage floor E in Eqs. (18) and (19) is selected as follows. We first solve a monopoly optimal pricing problem to maximize the social welfare Eq. (38). The "system optimal" wage rate obtained in this manner is then rounded to the whole dollar to get E, as reported in Table A2. The main results are presented in Figure 5.

[8] https://www.uber.com/newsroom/uberpro/https://www.uber.com/newsroom/uberpro/,https://www.lyft.com/rider/rewards



(a) Normalized welfare

(b) Market share

(c) Passenger wait time

(d) Vehicle time

Figure 5: System performance under the minimum wage policy: single-homing vs. multi-homing duopoly games.

As shown in Figure 5(a), the minimum wage policy does increase the social welfare under single-homing, but the net gain is rather modest. Although both passenger and driver surplus are improved substantially, these are largely gained at the expense of the platforms, and as a result, are likely to be wiped out over time. One option available to the platforms, as discussed in Zhang and Nie (2019), is to boost its profits by reducing the supply pool S_0. Such a response, however, would significantly undermine social welfare.

In the multi-homing duopoly, however, the minimum wage policy more than doubles the social welfare in most cases. Particularly, this improvement is achieved without sacrificing the platforms' profits. On the contrary, the platforms actually *benefit* from the policy, even though their gains are not as large as those of the passengers and drivers. The significant increase in the passenger surplus achieved by the minimum wage policy can be attributed to the dramatic improvement in passenger wait time; see Figure 5(c). Importantly, *the wage floor effectively prevents the market from the self-destructive price competition* found in Figure 4(b). As a result, more drivers are attracted to the ride-hail market by a higher average wage rate. In turn, the improved supply condition increases vacant vehicle time, lowers wait time, and finally grows the market share (Figure 5(b)). In a nutshell, *the minimum wage policy seems much more useful in a multi-homing duopoly than in a single-homing duopoly*.

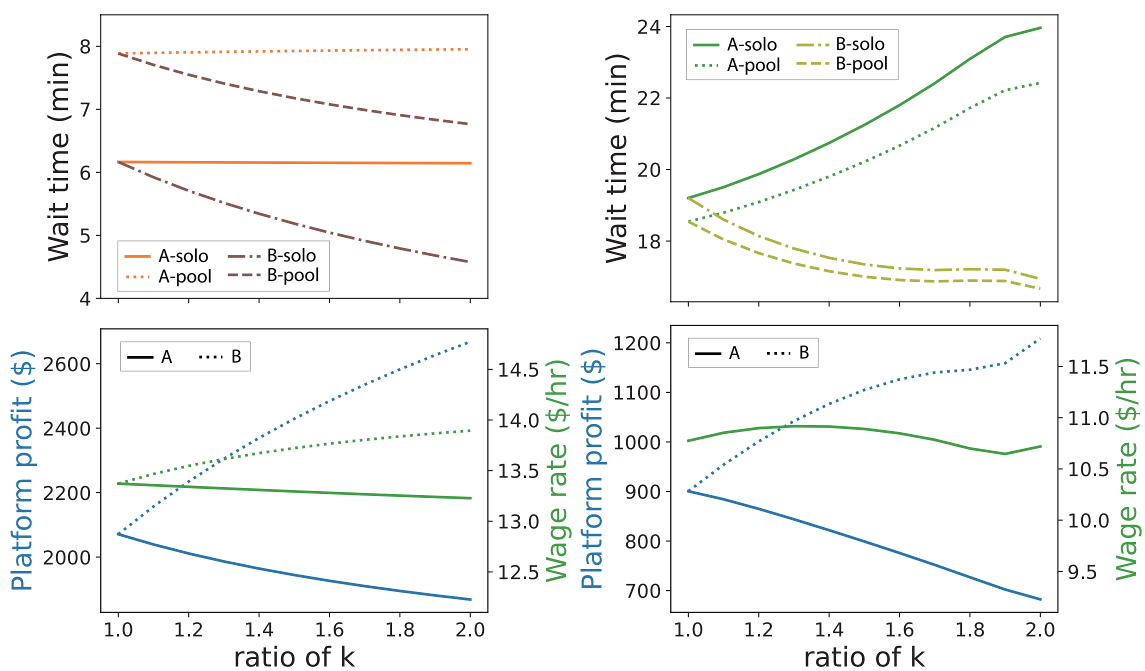
As indicated in Figure 5(b), the effect of the minimum wage policy on pooling is detectable but not significant. For single-homing, the market share for pooling rides has a slight but clear dip in all but the case of high-demand-low-supply. This occurs despite the solo rides gain market share.

In the multi-homing duopoly, both solo and pooling rides gain market share after the minimum wage policy is imposed, though most growth goes to solo rides. In a word, the minimum wage policy seems to favor solo over pooling rides, which echos the finding by (Zhang and Nie, 2019).

We end this section by noting that the above analysis is limited to the short-term effect, when the platforms must cope with the extra supply induced by a minimum wage higher than the unregulated state. In the long term, the platforms could respond to this policy by reducing their driver pools (i.e., the potential supply S_0). The reader is referred to Zhang and Nie (2019) for a more in-depth discussion of the long-term effect.

6.6 Asymmetric platforms

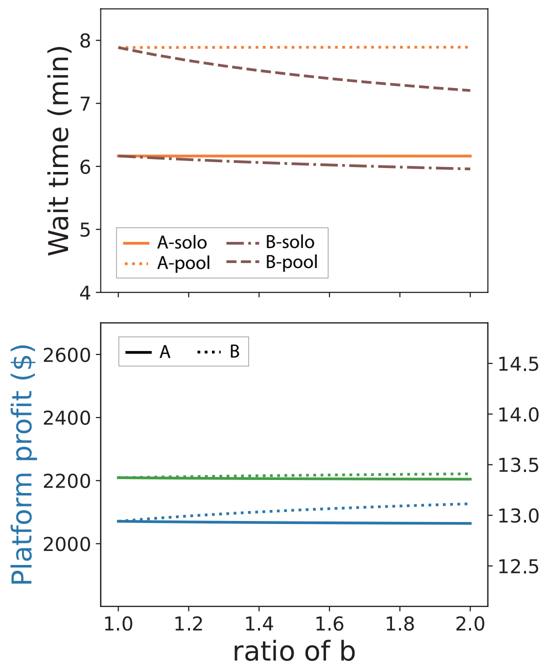
We now introduce asymmetric players into the duopoly game. First, we allow the two platforms in the unregulated pricing game to differ from each other on their competitive features: the matching efficiency k and the pooling efficiency b. For simplicity, we fix the parameter values of platform A while varying k^B and b^B, respectively. Figures 6 and 7 illustrate, respectively, the variation of the duopoly equilibrium as k^B/k^A and b^B/b^A increases from 1 (symmetric game) to 2 (highly asymmetric game).



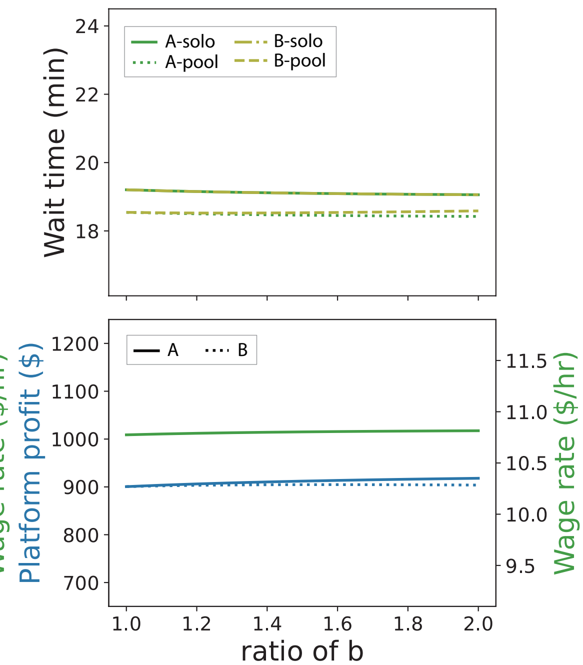
(a) Single-homing (b) Multi-homing

Figure 6: Sensitivity of the duopoly game to asymmetric matching efficiency. The ratio of k denotes k^B/k^A.

As expected, improving the matching efficiency substantially increases the platform's LOS and profit. However, its impact on the competing platforms varies with the supply mode. In the single-homing duopoly, both passenger wait time and driver wage rate on platform A are rather stable as platform B becomes increasingly more efficient. Platform A does gradually loses profits, but its loss amounts to merely 10% even when its rival becomes twice as efficient. On the contrary, a more efficient competitor does far more harm to platform A in the multi-homing duopoly. As shown in Figure 6(b), the passenger wait time increases about 20% while the profit drops more than 20% on platform A when k^B/k^A doubles from 1 to 2. Because of the competitive edge it enjoys, platform B has less incentive to maintain sufficient vacant vehicle time or control inter-passenger congestion. Consequently, platform A, at a competitive disadvantage, suffers much more from the resulting supply-demand imbalance. Although it is similarly impeded by its relative inefficiency, *platform A in the single-homing duopoly manages to mitigate its losses because it has better control on the supply-demand relationship through pricing*.



(a) Single-homing

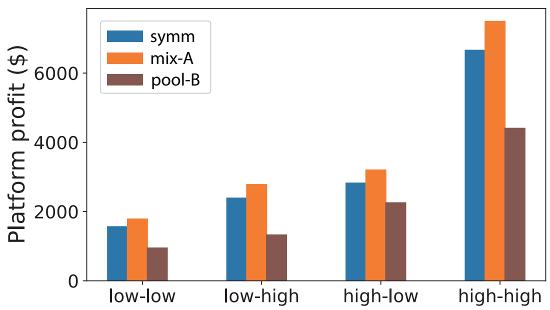


(b) Multi-homing

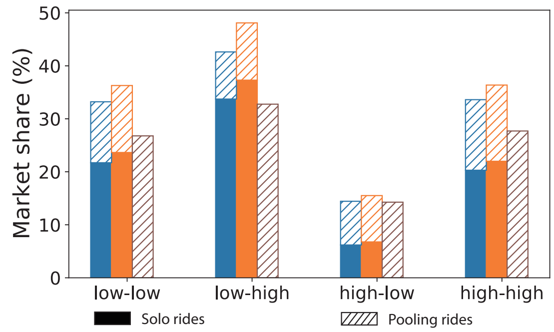
Figure 7: Sensitivity of the duopoly game to asymmetric pooling efficiency b. The ratio of b is b^B/b^A.

Unlike k, the impact of the pooling efficiency b on the outcome of the duopoly game is rather small, as illustrated in Figure 7. For single-homing, when the pooling efficiency increases, the wait time on platform B decreases mildly for both pooling and solo rides, though the pooling rides enjoy a greater improvement. The higher pooling efficiency also slightly drives up profitability and wage rate for platform B. The effect of b in the multi-homing duopoly is almost negligible, likely because the pickup time w_p is much larger than Δ (which decreases as b increases) due to the lower vacant vehicle density.

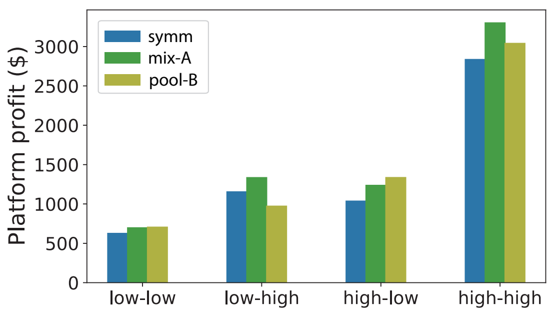
We finally consider asymmetry in terms of operational strategies. Specifically, we assume platform A offers both solo and pooling rides (i.e., a mixed-mode strategy) while platform B only serves pooling rides (a pure-pooling strategy). As some TNCs focus on providing pooling services (e.g., Via in the United States), it is interesting to examine how such a strategy fares against a competitor with a mixed service strategy. Figure 8 shows the performance of the two platforms (Mix-A vs. Pool-B) in an asymmetric duopoly game, and compares it with that of a platform in a symmetric game (with both platforms adopting a mixed strategy) .



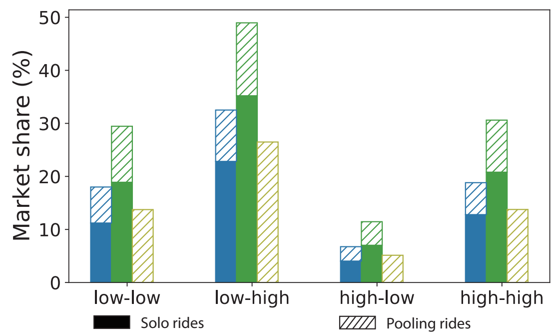
(a) Single-homing: Platform profit



(b) Single-homing: Market share



(c) Multi-homing: Platform profit



(d) Multi-homing: Market share

Figure 8: Duopoly equilibrium with asymmetric operational strategies.

In the single-homing duopoly, platform A always achieves a higher profit and market share, compared to platform B. The performance of the platform in a symmetric game lies right in between the two. This finding is largely expected, as Zhang and Nie (2019) had found the mixed strategy outperforms either the pooling or the solo strategy.

The results in the multi-homing duopoly, however, are far more intriguing. In the case of low-demand-low-supply, platform B achieves almost the same profit as platform A, while in the case of high-demand-low-supply, its profit is even higher than platform A. Moreover, platform B consistently earns a higher profit than the symmetric platform, except for the case of low-demand-high-supply. Interestingly, while platform B has a much lower market share than both platform A and the symmetric platform, the total market share of the asymmetric game is consistently higher than that of the symmetric game.

Because multi-homing effectively means the two platforms share the supply, platform B can afford a compensation rate much lower than that offered by platform A (remember drivers are assumed to focus on the average wage rate, rather than the wage rate of any individual platform). In this way, it can offer a reasonable pooling service at a rather low operating cost. On the other hand, since platform A essentially monopolizes the solo rides, it has a greater incentive to grow the market share, and attract more drivers to improve the LOS.

Therefore, multi-homing forges specialization in this game: platform B specializes in low-cost pooling service whereas platform A draws most revenues from high quality solo riders. This specialization helps explain why the asymmetric game tends to outperform its symmetric counterpart in terms of both market share and profits. It also suggests multi-homing likely encourages specialization in pooling under asymmetric conditions. We note that such specialized platforms do exist. For example, Via—an exclusive pooling service provider in the US—maintains a small but stable market share in big US cities (e.g., New York City[9] and Chicago), where multi-homing is a prevalent phenomenon.

7 Conclusions

In this paper, we study an aggregate ride-hail market in which two platforms compete with each other, as well as with transit, under different supply and regulatory conditions. The duopoly is built on a general market equilibrium model that explicitly characterizes the physical matching process, including pairing two passengers for a pooling ride. To account for the similarities between ride-hail services, a passenger's mode choice among solo, pooling and transit ride is represented using a Nested Logit model. We consider two different supply modes for the duopoly, single-homing or multi-homing, depending on whether drivers' work affiliation with a platform is exclusive or not. The outcome of the duopoly pricing game is described as a Nash Equilibrium (NE) and solved by transforming it into a variational inequality problem (VIP). When a regulatory constraint is imposed, the duopoly equilibrium becomes a generalized NE, which corresponds to a quasi VIP. The main findings from the numerical experiments are summarized below.

1. The level of service (LOS) in a multi-homing duopoly is in general lower than that in a single-homing counterpart. In the multi-homing duopoly, although passengers using either platform have access to all vacant vehicles, they also have to compete with every other passenger in the market. In the end, the greater competition on the demand side offsets the benefits brought by a larger fleet.

2. Without regulations, multi-homing may lead to a disastrous end. Specifically, passenger and driver surplus, as well as the platform profits, are all significantly lower in the multi-homing duopoly. This disaster arises because the platforms are locked in a self-destructive pricing war. Since drivers make decision to enter the market based on the average market (rather than platform) wage rate, the platforms soon discover lowering the payment to drivers to cut spending is better than raising it to attract more drivers. While the strategy makes senses individually, collectively it causes the collapse of the total supply and eventually the ruin of the business for all. This phenomenon may be interpreted as the tragedy of the commons, in which drivers are the "commons" shared and over-exploited by the platforms.

3. While the minimum wage policy has a rather minor impact on the single-homing duopoly, it significantly improves the surplus for both passengers and drivers in the multi-homing duopoly, and does so without compromising platform profits. In essence, a suitable wage floor could resolve the pricing dilemma in the unregulated market, and thereby both platforms are able to attract a sufficient number of drivers to maintain a reasonable LOS. The

[9] https://toddwschneider.com/dashboards/nyc-taxi-ridehailing-uber-lyft-data/

minimum wage policy also tends to discourage pooling rides regardless of the supply mode, albeit the impact observed in our experiments is small.

4. The matching efficiency seems a much more important asset in the competition compared to the pooling efficiency. In general, the platform with a higher matching efficiency ends up making more money and providing better LOS. Yet, the performance of the platform with a lower matching efficiency suffers much more in the multi-homing duopoly than in the single-homing duopoly.

5. Offering both solo and pooling rides is a winning strategy in the monopoly and the single-homing duopoly. In the multi-homing duopoly, however, this winning strategy no longer holds a clear advantage. More surprisingly, having a platform only offer pooling rides in the multi-homing duopoly may improve the collective market share and profit. This counter-intuitive phenomenon can be attributed to the de-escalation of the pricing war achieved by service specialization.

This study only considers two extreme supply modes: either all drivers are multi-homing, or none is. In reality, it is more likely that only a fraction of drivers are multi-homing. In Chicago, for instance, this fraction is about one quarter. Relaxing the definition multi-homing to allow drivers to join either one or both platforms hence constitutes an interesting direction for future research. Such a model not only moves one step closer to reality, but could also reveal a range of phenomena that have not arisen in the extreme cases studied herein.

To focus on the pricing strategies, we assume the platforms adopt a stable matching/dispatching strategy that keeps the parameters k and b in the matching model constant and independent of the pricing game. A future study could endogenize k and b and optimize them jointly with the pricing strategy for a given market condition.

Another limitation of the present study has to do with the Nested Logit models used to represent passenger and driver choices. Despite their critical roles, these choice models have not been properly calibrated to match empirical data. Given the focus of the current paper is modeling and solving duopoly pricing games, it seems reasonable to leave such an endeavor to future research.

Finally, the present study has left out other policies that have been discussed frequently in the literature and by policy makers, such as fleet cap and congestion tax. An interesting direction for future research is to design an "optimal" policy portfolio, similar to the joint implementation of minimum wage and congestion tax proposed in Zhang and Nie (2019).

References

P. Afeche, Z. Liu, and C. Maglaras. Ride-hailing networks with strategic drivers: The impact of platform control capabilities on performance. *Columbia Business School Research Paper*, (18-19):18–19, 2018.

C. Aloui and K. Jebsi. Optimal Pricing of a Duopoly Platform with Two-Sided Congestion Effect. *Journal of Research in Industrial Organization*, 2011:1–10, 2011.

M. Armstrong. Competition in two-sided markets. *The RAND Journal of Economics*, 37(3):668–691, 2006.

R. Arnott. Taxi travel should be subsidized. *Journal of Urban Economics*, 40(3):316–333, 1996.

K. J. Arrow and G. Debreu. Existence of an equilibrium for a competitive economy. *Econometrica: Journal of the Econometric Society*, pages 265–290, 1954.

J. Bai and C. S. Tang. Can two competing on-demand service platforms be both profitable? Available at SSRN 3282395 (Accessed: 2019-10-23), 2018.

S. Banerjee, C. Riquelme, and R. Johari. Pricing in ride-share platforms: A queueing-theoretic approach. Available at: SSRN 2568258 (Accessed: 2018-11-12), 2015.

A. G. Baydin, B. A. Pearlmutter, A. A. Radul, and J. M. Siskind. Automatic differentiation in machine learning: a survey. *The Journal of Machine Learning Research*, 18(1):5595–5637, 2017.

M. E. Beesley and S. Glaister. Information for regulating: the case of taxis. *The Economic Journal*, 93(371): 594–615, 1983.

P. Belleflamme and E. Toulemonde. Negative intra-group externalities in two-sided markets. *International Economic Review*, 50(1):245–272, 2009.

F. Bernstein, G. DeCroix, and N. B. Keskin. Competition between two-sided platforms under demand and supply congestion effects. Available at SSRN 3250224 (Accessed: 2020-03-05), 2019.

O. Besbes, F. Castro, and I. Lobel. Spatial capacity planning. Available at SSRN 3292651 (Accessed: 2019-10-23), 2018.

E. Böhme, C. Müller, and J. W. Goethe-university. Comparing monopoly and duopoly on a two-sided market without product differentiation. Available at `https://mpra.ub.uni-muenchen.de/26938/` (Accessed: 2019-11-11), 2010.

F. P. Boscoe, K. A. Henry, and M. S. Zdeb. A nationwide comparison of driving distance versus straight-line distance to hospitals. *The Professional Geographer*, 64(2):188–196, 2012.

L. E. J. Brouwer. Über abbildung von mannigfaltigkeiten. *Mathematische Annalen*, 71(1):97–115, 1911.

N. Buchholz. Spatial equilibrium, search frictions and efficient regulation in the taxi industry. Available at `https://scholar.princeton.edu/sites/default/files/nbuchholz/files/taxi_draft.pdf` (Accessed: 2020-03-04), 2019.

R. D. Cairns and C. Liston-Heyes. Competition and regulation in the taxi industry. *Journal of Public Economics*, 59(1):1–15, 1996.

J. Castillo, D. T. Knoepfle, and E. G. Weyl. Surge pricing solves the wild goose chase. Available at SSRN 2890666 (Accessed: 2018-5-3), 2018.

H. Chen, K. Zhang, X. Liu, and Y. M. Nie. A physical model of street ride-hail. Available at SSRN 3318557 (Accessed: 2019-1-18), 2018.

C. W. Cobb and P. H. Douglas. A theory of production. *American Economic Review*, 18:139–165, 1928.

G. De Jong, A. Daly, M. Pieters, and T. Van der Hoorn. The logsum as an evaluation measure: Review of the literature and new results. *Transportation Research Part A: Policy and Practice*, 41(9):874–889, 2007.

A. S. De Vany. Capacity utilization under alternative regulatory restraints: an analysis of taxi markets. *The Journal of Political Economy*, pages 83–94, 1975.

G. W. Douglas. Price regulation and optimal service standards: The taxicab industry. *Journal of Transport Economics and Policy*, pages 116–127, 1972.

G. D. Erhardt, S. Roy, D. Cooper, B. Sana, M. Chen, and J. Castiglione. Do transportation network companies decrease or increase congestion? *Science advances*, 5(5):eaau2670, 2019.

F. Facchinei and C. Kanzow. Penalty methods for the solution of generalized nash equilibrium problems. *SIAM Journal on Optimization*, 20(5):2228–2253, 2010.

D. Flores-Guri. An economic analysis of regulated taxicab markets. *Review of industrial organization*, 23(3): 255–266, 2003.

M. W. Frankena and P. A. Pautler. Taxicab regulation: an economic analysis. *Research in Law and Economics*, 9:129–165, 1986.

G. R. Frechette, A. Lizzeri, and T. Salz. Frictions in a competitive, regulated market: Evidence from taxis. *American Economic Review*, 109(8):2954–92, 2019.

I. Gurvich, M. Lariviere, and A. Moreno. Operations in the on-demand economy: Staffing services with self-scheduling capacity. In *Sharing Economy*, pages 249–278. Springer, 2019.

P. T. Harker. Generalized nash games and quasi-variational inequalities. *European journal of Operational research*, 54(1):81–94, 1991.

P. T. Harker and J.-S. Pang. Finite-dimensional variational inequality and nonlinear complementarity problems: a survey of theory, algorithms and applications. *Mathematical programming*, 48(1-3):161–220, 1990.

T. Ichiishi. *Game Theory for Economic Analysis*. Academic Press, 1983.

S. Kohli and A. Daly. The use of logsums in welfare estimation: application in prism. In *Proceedings of the European Transport Conference, Strasbourg*, 2006.

R. C. Larson and A. R. Odoni. *Urban operations research*. 1981.

S. Li, H. Tavafoghi, K. Poolla, and P. Varaiya. Regulating tncs: Should uber and lyft set their own rules? *Transportation Research Part B: Methodological*, 129, 2019.

J. D. Little. A proof for the queuing formula: L= λ w. *Operations research*, 9(3):383–387, 1961.

Y. M. Nie. How can the taxi industry survive the tide of ridesourcing? evidence from shenzhen, china. *Transportation Research Part C: Emerging Technologies*, 79:242–256, 2017.

A. Nikzad. Thickness and competition in ride-sharing markets. Available at SSRN 3065672 (Accessed: 2019-10-15), 2017.

E. Ostrom, R. Gardner, J. Walker, J. M. Walker, and J. Walker. *Rules, games, and common-pool resources*. University of Michigan Press, 1994.

J. A. Parrott and M. Reich. An earning standard for new york city app-based drivers: Economic analysis and policy assessment, 2018.

M. Patriksson. Sensitivity analysis of traffic equilibria. *Transportation Science*, 38(3):258–281, 2004.

L. Rayle, S. A. Shaheen, N. Chan, D. Dai, and R. Cervero. App-based, on-demand ride services: comparing taxi and ridesourcing trips and user characteristics in san francisco. Technical report, Citeseer, 2014.

J.-C. Rochet and J. Tirole. Platform competition in two-sided markets. *Journal of the european economic association*, 1(4):990–1029, 2003.

M. Rysman. The economics of two-sided markets. *Journal of economic perspectives*, 23(3):125–43, 2009.

B. Schaller. Unsustainable? the growth of app-based ride services and traffic, travel and the future of new york city, 2017.

B. Schaller. The new automobility: Lyft, Uber and the future of American cities, 2018.

R. L. Tobin and T. L. Friesz. Sensitivity analysis for equilibrium network flow. *Transportation Science*, 22(4): 242–250, 1988.

C. Tucker and J. Zhang. Growing two-sided networks by advertising the user base: A field experiment. *Marketing Science*, 29(5):805–814, 2010.

H. Wang and H. Yang. Ridesourcing systems: A framework and review. *Transportation Research Part B: Methodological*, 129:122–155, 2019.

X. Wang, F. He, H. Yang, and H. O. Gao. Pricing strategies for a taxi-hailing platform. *Transportation Research Part E: Logistics and Transportation Review*, 93:212–231, 2016.

H. C. Williams. On the formation of travel demand models and economic evaluation measures of user benefit. *Environment and planning A*, 9(3):285–344, 1977.

S. Wu, S. Xiao, and S. Benjaafar. Two-sided competition between on-demand service platforms. Available at SSRN 3525971 (Accessed: 2020-03-04), 2020.

Z. Xu, Y. Yin, and J. Ye. On the supply function of ride-hailing systems. *Transportation Research Part C: Emerging Technologies*, 00, 2019.

C. Yan, H. Zhu, N. Korolko, and D. Woodard. Dynamic pricing and matching in ride-hailing platforms. *Naval Research Logistics (NRL)*, 2019.

H. Yang. Heuristic algorithms for the bilevel origin-destination matrix estimation problem. *Transportation Research Part B: Methodological*, 29(4):231–242, 1995.

H. Yang, M. Ye, W. H. Tang, and S. C. Wong. Regulating taxi services in the presence of congestion externality. *Transportation Research Part A: Policy and Practice*, 39(1):17–40, 2005.

H. Yang, C. Fung, K. Wong, and S. Wong. Nonlinear pricing of taxi services. *Transportation Research Part A: Policy and Practice*, 44(5):337–348, 2010a.

H. Yang, C. W. Leung, S. Wong, and M. G. Bell. Equilibria of bilateral taxi–customer searching and meeting on networks. *Transportation Research Part B: Methodological*, 44(8):1067–1083, 2010b.

H. Yang, J. Ke, and J. Ye. A universal distribution law of network detour ratios. *Transportation Research Part C: Emerging Technologies*, 96:22–37, 2018.

J. J. Yu, C. S. Tang, Z.-J. Max Shen, and X. M. Chen. A balancing act of regulating on-demand ride services. *Management Science*, 2019.

L. Zha, Y. Yin, and H. Yang. Economic analysis of ride-sourcing markets. *Transportation Research Part C: Emerging Technologies*, 71:249–266, 2016.

L. Zha, Y. Yin, and Y. Du. Surge pricing and labor supply in the ride-sourcing market. *Transportation Research Part B: Methodological*, 117(PB):708–722, 2018a.

L. Zha, Y. Yin, and Z. Xu. Geometric matching and spatial pricing in ride-sourcing markets. *Transportation Research Part C: Emerging Technologies*, 92:58–75, 2018b.

K. Zhang and Y. M. Nie. To pool or not to pool: Equilibrium, pricing and regulation. Availabel at: SSRN 3497808 (Accessed: 2019-12-30), 2019.

K. Zhang, H. Chen, S. Yao, L. Xu, J. Ge, X. Liu, and Y. M. Nie. An efficiency paradox of uberization. Availabel at: SSRN 3462912 (Accessed: 2019-10-15), 2019.

A Parameter settings

Table A1: Default value of parameters in numerical experiments.

Parameter		**Unit**	**Value**
Detour ratio of road network	δ		1.3
Cruising speed	v	mph	13.6
Matching efficiency	k	/mi^2	0.16
Pooling efficiency	b		0.05
Approximation parameter	m		4
Average solo trip duration	τ_s	hr	0.28
Average pooling trip duration	τ_p	hr	0.40
Average transit trip duration	τ_t	hr	0.53
Passengers' value of time	ν	$/hr	27.69
Relative disutility rate of transit	ζ	$/hr	6.92
Dispersion parameter for passenger choice	θ_c (θ_c^r)		0.5
Dispersion parameter for drive choice	θ_d (θ_d^r)		0.25
Average reservation wage rate	e_0	$/hr	15
Total demand	D_0	/mi^2/hr	1200
Potential supply	S_0	/mi^2/hr	550
Solo trip fare	f_s	$/ride	14
Pooling trip fare	f_p	$/ride	10
Transit trip fare	f_t	$/ride	2.69
Compensation rate	η	$/hr	20
Congestion cost per vehicle	c_0	$/hr	2.9

Table A2: Representative market conditions.

Name	**Total demand (D_0)**	**Potential supply (S_0)**	**Wage floor (E)**
Low-low	500	200	18
Low-high	500	800	9
High-low	2000	200	26
High-high	2000	800	18

B Existence of market equilibrium

Proposition A1 *Suppose wait times are bounded from above by $\overline{w}_s$, $\overline{w}_p$ and $\overline{\Delta}$. Then, there exists an* $\mathbf{x}^* = \left[\mathbf{w}_s^*, \mathbf{w}_p^*, \mathbf{\Delta}^*, \mathbf{N}^*\right]^T$ *that satisfies Eq.* (3).

Proof. The proof is mostly the same as Proposition 2 in Zhang and Nie (2019), except that we now consider the fleet size $\mathbf{N}$ as part of the solution as well. Essentially, the solution existence is established by invoking Brouwer's fixed-point theorem (Brouwer, 1911), which requires the feasible set to be compact and convex and the mapping $F(\cdot)$ to be continuous. The assumption of bounded wait times ensure the first condition, while the second condition directly follows from the specification of demand and supply function. □

C Algorithms for duopoly equilibrium

Algorithm 1 summarizes the procedure used to identify the Nash equilibria of the unregulated duopoly pricing game. Although the algorithm moves on the ascending direction, it is possible to be stuck at a local optimum due to bad initialization. Hence, the algorithm checks the obtained solution against the equilibrium condition Eq. (11) in the end (lines 12 - 15).

Algorithm 1 Gradient Ascent Algorithm for Unregulated Duopoly Pricing Game

1: **Inputs:** data required to define VIP (14), the step size α, and the convergence criterion ε.
2: **Initialization:**
3: Choose an initial solution $\mathbf{y}^{(0)}$, and set the iteration index $l = 0$ and gap $g = \infty$.
4: **while** $g \geq \varepsilon$ **do**
5:     Solve Eq. (3) with the current pricing strategy $\mathbf{y}^{(l)}$ for the equilibrium solution $\mathbf{x}^{(l)}$.
6:     Evaluate partial derivative $\partial \mathbf{x}^{(l)} / \partial \mathbf{y}^{(l)}$ using Eq. (17).
7:     Compute gradient ∇R using Eq. (16).
8:     Update $\mathbf{y}^{(l+1)}$ using $\mathbf{y}^{(l+1)} = \mathbf{y}^{(l)} + \alpha \nabla R(\mathbf{y}^{(l)})$ and compute gap $g = ||\nabla R||_{\infty}$.
9:     Set $l = l + 1$.
10: **end while**
11: **Outputs:**
12: **if** $\mathbf{y}^{(l)}$ satisfies Eq. (11) **then**
13:     Set $\mathbf{y}^* = \mathbf{y}^{(l)}$ as a Nash equilibrium.
14: **else**
15:     Report no Nash equilibrium is found.
16: **end if**

Algorithm 2 summarizes the procedure of the penalty algorithm. Note that lines 5-11 of the algorithm explain how the penalty parameter ρ^j and the smooth parameter ε are updated based on two predefined constants c_0 and c_1. Again, the solution to QVIP may not satisfy the GNE conditions of the regulated game. Thus, any solution found by the penalty algorithm should also be verified against these conditions (lines 16-19).

D Existence of multiple equilibria

In all numerical experiments, we solve the market equilibrium problem (3), VIP (14) or QVIP (21) multiple times with randomly selected initial solutions. Yet, in all our experiments, we did not encounter a single instance for which more than one equilibrium solution to (3) is found. As for VIP (14) and QVIP (21), we do find multiple equilibria for different initial solutions. To illustrate these equilibria, we solve the unregulated duopoly game with symmetric platforms using default parameter values reported in Table A1 and various combination of initial prices. Specifically, we fix the initial values of f_s and η, while varying the values of f_p for the two platforms.

Figure A2 plots the region of attraction (ROA) under the single-homing supply mode[10]. In each plot, ROA is colored according to the type of solutions. It is worth emphasizing that, *for each type, there is always a unique equilibrium*. Thus, once the initial solution falls into an ROA, the outcome of the game will be "attracted" to that equilibrium associated with that ROA. In all cases, only four ROAs are identified and they can be characterized based on the market share of

[10]The results for multi-homing supply are very similar and thus are omitted here for brevity.

Algorithm 2 Penalty Algorithm for Regulated Duopoly Pricing Game

Inputs: data required to define QVIP (21), parameters $c_0 \in (0,1)$, $c_1 > 1$, initial smooth parameter $\varepsilon \in (0,1]$ and its convergence criterion ε_M.
Initialization:
Set iteration index $l = 0$, $\rho^j = 0, \forall j$. Choose an initial solution $\mathbf{y}^{(0)}$.
while $\mathbf{y}^{(l)} \notin \Omega(\mathbf{y}^{(l)})$ or $\varepsilon \geq \varepsilon_M$ **do**
    Let $I = \{j | h^j(\mathbf{y}^{(l)}) \geq 0\}$
    for $j \in I$ **do**
        if $||\nabla_{\mathbf{y}^j} R^j(\mathbf{y}^{(l)})|| > c_0 \lambda^j ||\nabla_{\mathbf{y}^j} ||h^j_+(\mathbf{y}^{(l)})||_2||$ **then**
            set $\rho^j = \rho^j + c_1 h^j(\mathbf{y}^{(l)})$.
        end if
    end for
    If $h^j(\mathbf{y}^{(l)}) \leq \varepsilon, \forall j$, set $\varepsilon = \varepsilon/2$.
    Solve the penalized VIP (23) by calling Algorithm 1.
    Set $l = l + 1$.
end while
Outputs:
if $\mathbf{y}^{(l)}$ satisfies Eq. (20) **then**
    Set $\mathbf{y}^* = \mathbf{y}^{(l)}$ as a GNE.
else
    Report no GNE solution is found.
end if

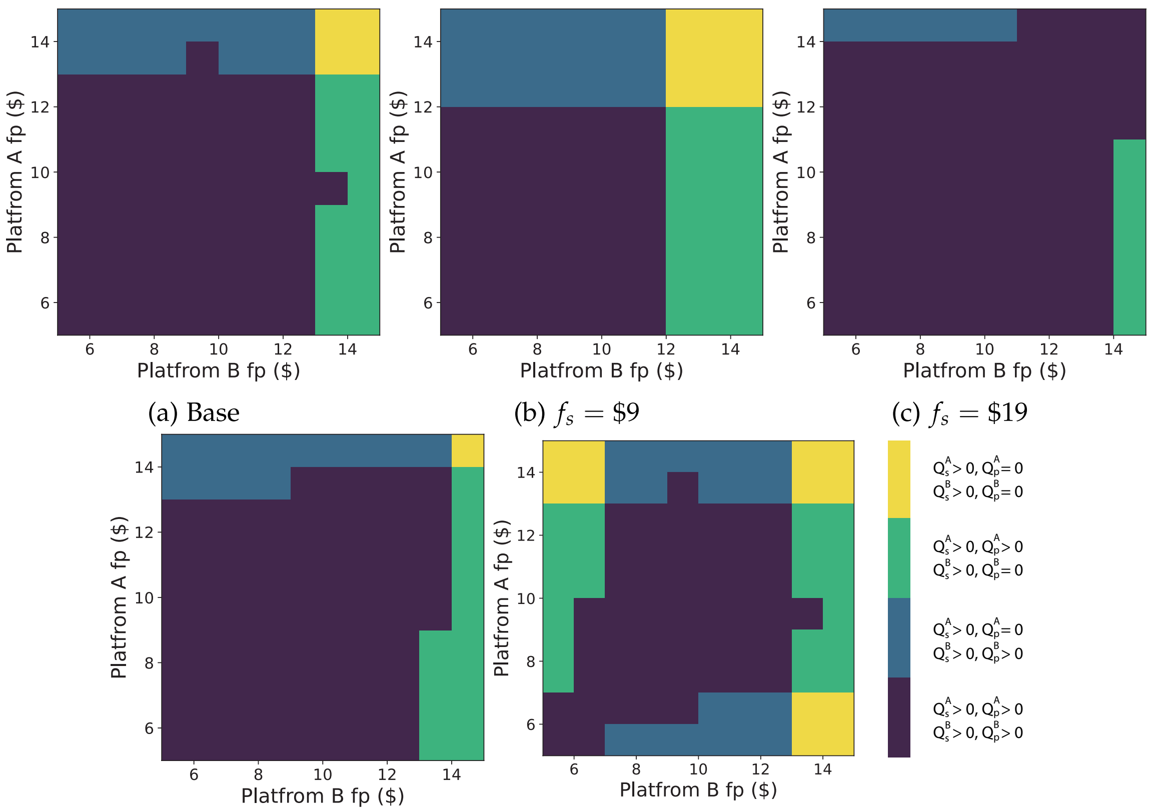
the pooling rides at the corresponding equilibrium: (i) $Q_p^A = Q_p^B = 0$; (ii) $Q_p^A > 0$ and $Q_p^B = 0$; (iii) $Q_p^A = 0$ and $Q_p^B > 0$ and (iv) $Q_p^A > 0$ and $Q_p^B > 0$.

As expected, if a platform initially prices the pooling rides too high (roughly \$13 in the base case; see Figure A2(a)), it risks losing all pooling market share as the game evolves. This threshold varies with the solo fare f_s and the compensation rate η. When f_s is lower than the default value (Figure A2(b)), the threshold drops to \$12; and when f_s increases to \$19 (Figure A2(b)), it rises to \$14. In addition, a high initial compensation rate η attracts a large fleet size and thus the platforms prefer serving more solo rides. Accordingly, in this case (Figure A2(e)), it is much more likely for the game to reach an equilibrium where at least one platform completely eliminates pooling rides. Because pooling is one of our primary interests, we will only focus on the equilibrium pattern (iv) above, i.e. $Q_p^A > 0$ and $Q_p^B > 0$. It is worth emphasizing again that, in all experiments conducted, such a solution is always unique.

E Impact of the IIA assumption

To relax the IIA assumption, we simply allow in our model the ratio of θ_c^r/θ_c to increase from 1 to 2. In the single-homing duopoly, we also similarly vary the ratio θ_d^r/θ_d. For simplicity, we assume the two ratios in this case are the same, referred to as the "ratio of dispersion parameter" in Figure A3.

As expected, when θ_c^r/θ_c increases—meaning the four options are increasingly viewed as good substitutes for each other—the total market share of ride-hail dips. In the case of single-homing, the total share (including both solo and pooling) decreases from about 50% to a little more than 30% as θ_c^r/θ_c doubles. It is the pooling rides, however, that is mostly affected. Indeed,



(a) Base (b) $f_s = \$9$ (c) $f_s = \$19$

(d) $\eta = \$15/\text{hr}$ (e) $\eta = \$25/\text{hr}$

Figure A2: Equilibrium solutions of unregulated duopoly game under single-homing supply mode. Each grid represents an initial solution defined by f_p^A and f_p^B, rounded to full dollar. The default pricing strategy $f_s = \$14, f_p = \$10, \eta = \$20/\text{hr}$ for both platforms. Other parameters are set to be the default values in Table A1.

the vast majority of the losses is inflicted on pooling rides, while the market share of solo rides is largely intact. The drop in the market share slightly improves the wait time for solo rides, thanks to less inter-passenger competition. However, LOS degrades for pooling rides, likely because the collapsing market share significantly prolongs the detour time Δ.

Overall, the above finding seems to indicate that the IIA assumption could result in overly optimistic estimation about the potential of pooling rides. However, it is worth noting that the simple setup of our NL model may treat pooling rides unfairly. Intuitively, pooling and solo rides provided by the same platform should be viewed as much more distinctive than pooling rides provided by two different platforms. Yet, they are all lumped into the same nest in the current setting, without recognizing this distinction. Addressing this issue requires a nested structure with two layers, allowing passengers to first choose a mode before choosing a platform (or vice versa). However, without data needed to calibrate such an NL model, we would be guessing the actual specification of the NL model (hence the results) one way or the other. Thus, it seems appropriate to leave a more sophisticated choice modeling to a future study.

The reader should be aware that the findings made with the IIA assumption in the following sections are subject to the biases that comes with it, especially those related to the comparison of the total market share between the monopoly and the duopoly. Most sensitivity results, however, may not be affected by this bias. In particular, Figure A3 suggests the impact of IIA is insensitive to the supply mode (note that the patterns of single- and multi-homing results are very similar). In other words, most findings about the relative performance between the two supply modes should stand free of the assumption.

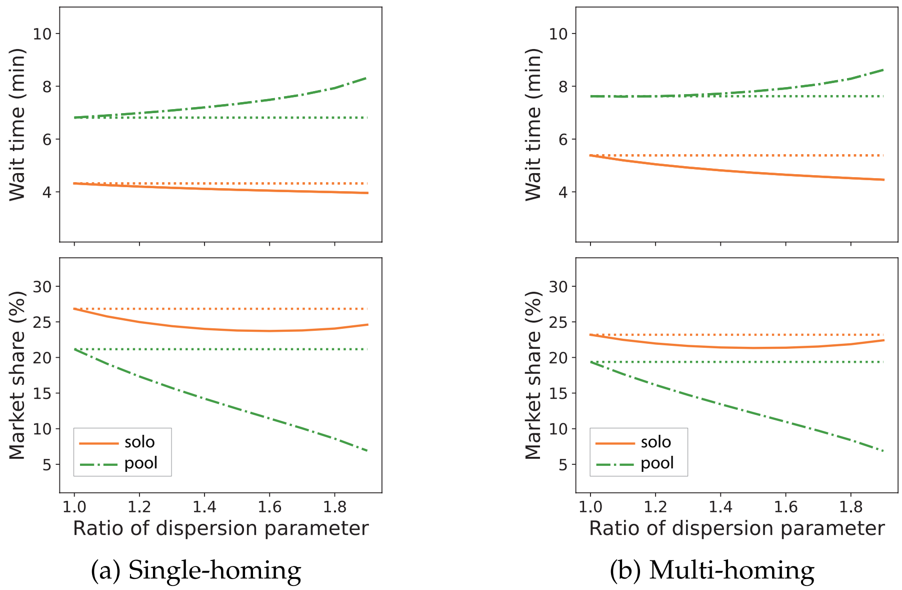


Figure A3: Sensitivity of passenger wait time and market share to the ratio of dispersion parameter ($\theta_c^r/\theta_c = \theta_d^r/\theta_d$). Market shares for single- and multi-homing cases are the sum of the two platforms' market shares. Other parameters are set as the default values in Table A1.