

Potential of carpool for network traffic management

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Abstract

This study examines the impact of carpool on network traffic in a highly idealized futuristic world, where all travelers are willing to participate in carpool arranged by a Transportation Network Company. We build a parsimonious carpool model that focuses on the trade-off between inconvenience costs and travel cost savings. Underlying the model is a non-linear bipartite matching problem that seeks to maximize commuters' welfare. By assuming the congestion effect is negligible, we derive several useful analytical results. When the inconvenience cost is less than the median trip valuation of a rider, the platform could always achieve an *almost* perfect match while maximizing commuters' welfare, which corresponds to a 50% reduction in vehicular traffic flow. In the case of perfect match, if there is an even number of travelers, we propose a pricing policy that possesses all desired properties of the Vickrey-Clark-Groves (VCG) policy—a benchmark truthful policy for achieving socially optimal solution—but runs a lower deficit. Otherwise, we show the VCG policy always generates a profit. If the inconvenience cost is too high, the perfect match is no longer socially optimal, but the VCG policy still yields a positive profit. Solutions from numerical experiments generally agree with the analytical results. They also suggest that matching across O-D pairs occurs only when it has a significantly lower inconvenience cost than matching within, an unlikely event in reality. Moreover, when cross O-D matching does become prevalent, it leads to higher vehicle miles travelled, hence worse congestion. Thus, from the point of view of traffic management, cross O-D carpool should not be encouraged.

Keywords: carpool; perfect match; pricing; VCG policy

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1 Introduction

Ridesharing allows several travelers who have similar itineraries and schedules to share a vehicle for an entire or a portion of a trip. Making such an arrangement necessarily introduces some inconvenience and delays for everyone involved, but it also saves the cost of travel, both for the travelers themselves (fuel, toll, parking fees, vehicle depreciation, etc.) and for the society at large (pollutants, carbon footprint, etc.). Ridesharing may be arranged in advance or in real time by either travelers (e.g. casual carpooling, Kelly, 2007) or an intermediate (e.g. employer vanpool programs, Ferguson, 1990). In the United States (US), the first oil crisis in the 1970s had shifted the focus of transportation planners from expanding highway networks to managing travel demand in multi-modal transportation systems (Boyce and Williams, 2015). Promoted as a form of travel demand management, ridesharing had gathered much enthusiasm in late 1970s and early 1980s. Yet, by 1990, the tide had long receded. The share of carpool in US work trips declined 32% within a decade, from 19% in 1980 to about 13.4% in 1990 (Ferguson, 1997). While this disappointing performance is largely attributed to the rising car ownership and declining gasoline price (Ferguson, 1997), the practical challenge of organizing large-scale ridesharing programs might have also played a role in dampening the interest of busy commuters.

Thanks to the emerging industry of transportation network companies (TNCs), the popularity of ridesharing, as both a mode of travel and a research topic, has grown markedly in recent years (Furuhata et al., 2013; Mourad et al., 2019). The new ridesharing services created by TNCs largely fall into two categories: *carpool* such as Waze carpool¹ and Didi Hitch², and *ridepool* such as Uber Pool and Lyft Line. The carpool services are typically prearranged and provided by a driver who shares the trip with the passengers and whose primary goal is to fulfill his/her own travel need at a reduced cost. In contrast, ridepool is a form of taxi service that dynamically matches multiple passengers with a driver who sells the service exclusively for money. Fueling this renewed interest in ridesharing is the promise to make it flexible, user friendly, efficient and cheap. On a grander scale, ridesharing is seen as integral to the idea of Mobility-as-a-Service (MaaS), which has been touted as the inevitable future of transportation (Goodall et al., 2017), along with autonomous and electric cars (Mahmassani, 2016). It is plausible, in the era of MaaS, ridesharing in various forms would become a mainstream, if not dominating, mode of travel. Given this prospect, this study explores to what extent this trend can be expected to affect vehicular traffic distribution in transportation networks, and what institutional arrangement and policies can facilitate a socially beneficial outcome in the process. We focus on *carpool* in commute trips, because these trips largely shape urban traffic congestion patterns, which in turn drive transportation planning decisions (Vickrey, 1969).

To be sure, the latest round of ridesharing innovations have yet to see any uptake from commuters. Quite the contrary, the 2016 American Community Survey found the carpool share among commuter trips hovers around 9%, and has been slowly declining since 2007 (Tomer, 2017). This is hardly surprising. A transition towards MaaS, even if it does materialize in the end, is likely to take decades to complete. Yet, anticipating and planning for possible future scenarios like this is exactly the mission of long-range urban travel forecasting (Boyce and Williams, 2015). To this end, we envision and analyze a futuristic world, in which commuters rely on a TNC

¹<https://www.waze.com/en-GB/carpool>

²<http://www.didachuxing.com/static/h5/didahome/index.html>

to arrange and price carpool rides. Commuters are free to choose a carpool role (driver or rider) or drive alone to maximize their own utility. Using auction as a price discovery tool, the TNC invites commuters traveling between different origin-destination (O-D) pairs to submit their preferences, and organize them into an “optimal” sharing pattern that not only assigns a role to each commuter but also determines the details of logistics (route, pickup and drop-off sequence etc.).

While many had attempted to understand the effect of carpool on traffic patterns in the long term, few had adopted a similar institutional arrangement as considered herein. In particular, what is missing from most existing analyses is an intermediate that arranges and prices carpool for everyone through a two-sided market. Instead, they rely on indirect policy instruments, such as congestion pricing (e.g. Yang and Huang, 1999; Liu and Li, 2017), parking restriction (e.g. Xiao et al., 2016) and exclusive lanes for high occupancy vehicles (e.g., Qian and Zhang, 2011; Zhong et al., 2020), to influence carpool decisions. We shall show, using a parsimonious model, that the pricing mechanism in a two-sided market offers a powerful tool to not only discover travelers’ preferences, but also maximize the potential of carpool in reducing vehicle miles traveled, even in the absence of these external interventions mentioned above. Under mild assumptions, a carpool ride can be priced for each O-D pair such that almost no traveler would find driving alone an attractive option. Moreover, while directing drivers from one O-D pair to pick up riders from another is possible in theory, it is unlikely that such an arrangement would lead to a mutually beneficial outcome. In other words, our analysis suggests that carpool within the same O-D pair should receive more attention from modellers.

In what follows, Section 2 briefly reviews related studies. Section 3 presents the setting of the model used to conduct the analysis. Based on the model, Sections 4 and 5 examine the matching and pricing problems for single and multiple O-D cases, respectively. Section 6 reports results from numerical experiments. Finally, Section 7 concludes the study with a summary of findings and comments on possible future research directions.

2 Related studies

Our focus is on the carpool problem. The reader is referred to Furuhata et al. (2013) and Mourad et al. (2019) for comprehensive reviews on ridesharing, particularly those pertinent to ridepool. The existing carpool studies either adopt a dynamic or static modeling framework. The vast majority in the former category is based on the bottleneck model (Vickrey, 1969), which simplifies traffic congestion as queuing behind a bottleneck between home and work. These dynamic analyses (e.g. Qian and Zhang, 2011; Xiao et al., 2016; Liu and Li, 2017; Ma and Zhang, 2017; Zhong et al., 2020) allow departure time and ridesharing choice to interact with time-varying traffic conditions, but are often limited in their ability to consider the effect of cross O-D interactions and network topology. Since the model adopted in this paper is static, we will only review the static models in what follows. For those who are interested in dynamic models, Zhong et al. (2020) provide a relatively up-to-date review.

The static carpool studies usually adopt the modeling framework of the classic traffic assignment problem (TAP, Beckmann et al., 1956). Daganzo (1981) proposes to use a multi-class TAP model to consider the carpool effect on traffic flow distribution in a network. Travelers are

classified based on the number of passengers in the car, and everyone sharing the same vehicle, regardless of their role, incurs exactly the same cost. The class-specific demands are assumed to be endogenously given. In other words, the choice of whether and how to carpool is not modelled. Yang and Huang (1999) study how congestion pricing may be used to promote carpool. Their analysis focuses on a two-road (a regular and an HOV road) network, and only considers two-person carpool. Travelers may choose carpool or solo-driving, depending on the tradeoff between travel time, congestion toll and the inconvenience cost associated with carpool. They show that the HOV and regular roads require different tolls in order to decentralize a system optimal solution, and accordingly propose a second-pricing scheme that eliminates the difference. In a sequel to Yang and Huang (1999), Konishi and Mun (2010) assume the carpool inconvenience cost as a random variable that follows a given continuous distribution. Their analysis suggests that HOV lanes are socially beneficial and converting HOV lanes to HOT (high occupancy toll) lanes could further improve system efficiency. Xu et al. (2015a) consider a static carpool problem conceptually similar to that of Yang and Huang (1999), but extend it to a general network. They formulate the problem as an elastic demand TAP, in which the tradeoff between ridesharing price, congestion and the number of commuters willing to participate ridesharing is embedded in an aggregated elastic demand function. Because of the structure of the underlying model, carpool is limited to travelers from the same O-D pair.

None of the above studies differentiates carpool drivers from riders in terms of their behaviors or costs: all carpoolers have exactly the same cost, and the matching process is not modelled at all. Among the first few studies that explicitly model the matching process is Xu et al. (2015b), who group travellers into solo drivers, carpool drivers or carpool riders. The resulting multi-class TAP model is built on the behavioral assumption that all travelers, regardless of their class, must have the same generalized travel cost, which, among other things, includes an *exogenously given price* of the shared ride (or a payment for carpool driver). Since matching is allowed to occur between drivers and riders from different O-D pairs, enforcing the consistency of class-specific flows at link level is a main challenge that requires the introduction of a number of side constraints and asymmetric interactions between class-specific flows. As a result, the model is formulated as a nonlinear complementarity problem (NCP) that is difficult to solve. To address this difficulty, Di et al. (2017) limit carpool to travelers from the same O-D pair. The simplification makes it possible to represent the inconvenience cost, user fee and carpool logistics constraints all at the path level, leading to a path-based nonlinear complementarity problem with side constraints. Like Yang and Huang (1999), they apply their model to analyze pricing strategies on HOT lanes. In a follow-up study, Di et al. (2018) formulate the optimal HOT lane design problem as a bi-level optimization problem, in which the upper-level problem determines the location and magnitude of the tolls. They formulate the lower level problem using a revised model of Xu et al. (2015b). In addition to allowing the revenue earned by carpool drivers to change with the number of passengers they carry, they also propose a node-link formulation that is easier to solve. Li et al. (2019) propose a RUE model that has a structure similar to that of Xu et al. (2015b) but can handle nonadditive path costs. Using the model, they analyze Braess paradox in the context of ridesharing equilibrium. Their numerical results confirm HOT lane can help promote ridesharing.

As seen from the above, few had considered the issue of pricing (of shared rides) in the context of addressing the interaction between network traffic flow and ridesharing. Most studies that do consider pricing in ridesharing are primarily concerned with mechanism design, often

exclusively viewing it through the lens of bipartite matching. The central theme there is how to spilt the savings from ridesharing to ensure fairness and provide sufficient incentives, among other objectives. The reader is referred to Furuhata et al. (2013) and Li et al. (2020) for this line of work, from which our analysis draws some conceptual tools.

3 Model setting

We consider a simple trip-based traffic assignment model with two origin-destination (O-D) pairs, as shown in Figure 1. Let q^w be the number of commuters traveling between O-D pair $w \in \{1, 2\}$, and t^w be the travel time between O-D pair w . We use π to represent the vehicle operating cost per unit time (inclusive of fuel, vehicle acquisition and maintenance, etc.). Hence the monetary cost of a trip is πt^w .

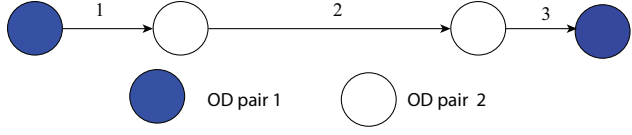


Figure 1: Illustration of the simple network model.

We assume all commuters are interested in ridesharing, and they all have a car and can play one of the three roles: solo driver (*s-driver* for brevity hereafter), carpool driver (*driver* hereafter), or carpool passenger (*rider* hereafter). The commuters participate in ridesharing through a TNC that helps match drivers and riders. Each driver can take at most one rider. The flow of s-drivers, drivers and riders for O-D pair w is denoted as f_s^w , f_d^w and f_r^w . The topology of the network is such that drivers from O-D pair 1 could be matched with riders from O-D pair 2, but it would make no sense to have a driver from O-D pair 2 pick up a rider from O-D pair 1. We denote the flow of the cross O-D drivers as f_c^1 . Thus, the total vehicular flow on each of three links of the network is

$$y_1 = f_d^1 + f_s^1 + f_c^1, y_2 = y_1 + f_s^2 + f_d^2, y_3 = y_1. \quad (1)$$

Taking the conventional assumption that the travel time on each link is a nondecreasing and separable function of the vehicular flow on the link, it is easy to show that the travel time on each link would be minimized when $f_s^w = 0, \forall w, f_c^1 = 0$, i.e., there is neither solo driving nor cross-OD matching. This desired outcome would only be achieved by a *perfect match*, defined as follows.

Definition 1 (Perfect match). *A perfect match is an outcome in which every commuter from an O-D pair is matched with exactly one partner from the same O-D pair.*

The question central to our inquiry here is whether and how such a perfect match can be achieved. To begin, let us assume the TNC matches drivers with riders and discover the pricing strategy through an auction. To participate the auction, commuters must report their private preference to the TNC. In general, commuters preferences might include, among other things, the willingness to pay (WTP) for the trip, the value of time (VOT) and the inconvenience cost related to ridesharing. To simplify the analysis, we estimate the WTP as a commuter's valuation of the trip if s/he has to drive alone, which depends on his/her VOT, the journey time and the operating cost (fuel, equipment, etc.). The inconvenience cost is assumed to be a publicly known constant, denoted as Δ , for any trip paired within the same O-D pair. Further, because each driver

can only take one rider, the rider is not subjected to any detours. As such, we assume only the carpool driver bears the inconvenience cost. Finally, for cross O-D ridesharing, the inconvenience cost is Δ' .

We now discuss VOT, the only preference treated as independent private information in this study. Let $\tilde{\beta}_d$ and $\tilde{\beta}_r$ be the private valuation of driving and riding time, respectively, which are considered random variables, and define $\tilde{\alpha} = \tilde{\beta}_d - \tilde{\beta}_r$. Each commuter has a pair of private valuation (β_d, β_r) , where β_d and β_r can be viewed as a realization of $\tilde{\beta}_d$ and $\tilde{\beta}_r$, respectively, for the commuter; and $\alpha = \beta_d - \beta_r$ is called the commuter's value of *productivity gain from riding* (PGR). It is reasonable to assume that $\alpha \geq 0$ for a rational decision maker. Hence, the random variable $\tilde{\alpha}$ has a nonnegative support. The cumulative distribution function for $\tilde{\alpha}$ is denoted as $F_{\tilde{\alpha}}$, which is assumed to be identical for both O-D pairs.

Prior to their trip, commuters report the PGR values (i.e. α) to the platform. The set of all PGR values received by the platform is denoted as $\mathfrak{A}$. Let $\bar{\alpha}$, $\hat{\alpha}$ and $\underline{\alpha}$ be the maximum, median and minimum in $\mathfrak{A}$, respectively. Using $\mathfrak{A}$, the platform assigns each traveler a role, and determines the price paid by the rider, p_r , and the payment to the driver, p_d , through a double auction. The focus is to show how the platform can design the assignment and pricing strategies to achieve a perfect match. In what follows, we will first analyze the auction problem with a single O-D (Section 4), and then extend it to the case with two O-D pairs (Section 5). The following two assumptions are introduced to further simplify the analysis.

Assumption 1. q^w is an even integer.

Assumption 2. $\mathfrak{A}$ contains no pairs of identical PGR values.

Assumption 1 is introduced because, strictly speaking, a “perfect match” is only possible when q^w is an even number. Assumption 2 ensures the matching result be unique.

4 Single O-D problem

4.1 Formulation of bipartite matching problem

Consider a set of commuters $\mathcal{I}^w = \{1, \dots, q^w\}$ for O-D pair w . Each commuter $i \in \mathcal{I}^w$ is associated with a tuple of VOT (β_d^i, β_r^i) , and $\alpha^i = \beta_d^i - \beta_r^i$. Without loss of generality, we assume that commuters in $\mathcal{I}^w$ are ranked in the descending order of α^i , i.e., $\alpha^1 > \alpha^2 > \dots > \alpha^{q^w}$. Accordingly, we can define the cost of being a driver, a rider and an s-driver, respectively, for commuter i as follows:

$$c_d^i = \beta_d^i t^w + \pi t^w + \Delta; c_r^i = \beta_r^i t^w; c_s^i = \beta_d^i t^w + \pi t^w. \quad (2)$$

Since c_s^i represents the commuter's WTP, the valuation of commuter i for each of the three roles can be represented by

$$v_d^i = c_s^i - c_d^i = -\Delta; v_r^i = c_s^i - c_r^i = (\alpha^i + \pi) t^w; v_s^i = 0. \quad (3)$$

Accordingly, the utilities are

$$u_d^i = p_d - \Delta; u_r^i = (\alpha^i + \pi) t^w - p_r; u_s^i = 0, \quad (4)$$

where p_d and p_r are determined by the platform's pricing policy. To characterize the platform's decision, let x_{ij} be a binary assignment variable: $x_{ij} = 1$ if commuter i is assigned as a driver to a rider j ; $=0$ otherwise. Then the bipartite matching problem can be formulated as

$$\max \sum_{i \in \mathcal{I}} \left(\sum_{j \in \mathcal{J}, j \neq i} (v_r^j + v_d^i) x_{ij} \right) \quad (5a)$$

subject to:

$$\sum_{j \in \mathcal{J}, j \neq i} x_{ij} + \sum_{k \in \mathcal{I}, k \neq i} x_{ki} \leq 1, \forall i \in \mathcal{I}, \quad (5b)$$

$$x_{ij} \in \{0, 1\}, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, i \neq j, \quad (5c)$$

where the objective (5a) is to maximize the total gains of all commuters from ridesharing (referred to as *commuters' welfare* hereafter) and Constraints (5b) ensure the validity of matching. Strictly speaking, Problem (5) is a mixed nonlinear integer program, because v_r^j depends on t^w , which in turn depends on $\mathbf{x} = \{x_{ij}\}$. To avoid the difficulty, we can solve (5) by enumerating all possible values of t^w , each corresponding to a vehicular flow that ranges between $[0.5q^w, q^w]$ ($0.5q^w$ corresponds to the perfect match while q^w corresponds to no match at all). Since q^w is an even number, the number of t^w that needs to be tested is finite.

Given the special structure of the problem, however, the solution to (5), along with its pricing strategy, can be obtained analytically. To facilitate the analysis, we introduce two additional assumptions.

Assumption 3. The O-D travel time t^w is a constant.

Assumption 4. For any O-D pair w , given Assumption 3 and suppose $k = \max\{k \in \mathcal{I}^w | v_r^k > \Delta\}$, then $k \geq 0.5q^w$.

Assumption 3 is necessary, because without it, Problem 5 becomes a nonlinear problem. If the objective is to achieve a perfect match, this assumption is not as restrictive as it appears, since at a perfect match, the travel time of the system is always the same and lowest. Assumption 4 implies at least for half of the commuters, the benefit of being a rider outweighs the inconvenience cost. For most real-world applications, Δ should be smaller than the operating cost $t^w \pi$. Thus, Assumption 4 is easy to satisfy in practice. In the following, Section 4.2 analyzes perfect match solutions and the corresponding pricing strategies under Assumptions 1 - 4. Section 4.3 discusses imperfect match solutions when Assumptions 1 and 4 are relaxed.

4.2 Perfect match and pricing policies

Figure 2 illustrates the utility of all travellers in the ascending order of PGR. Before compensation, a rider's utility is represented by a straight line with a positive slope t_w and an intercept πt^w , whereas a driver's utility is the horizontal line at $-\Delta$. For the purpose of illustration, suppose the compensation received by the driver equals the payment made by the rider, i.e., $p_d = p_r \equiv p$. With the compensation, the horizontal line representing the driver's utility is pushed up by p , cutting through the rider's utility line. The shaded areas highlight the total utility for drivers

and riders. The shaded rectangle indicates that each driver has the identical utility whereas the shaded triangle indicates each rider's utility increases with α .

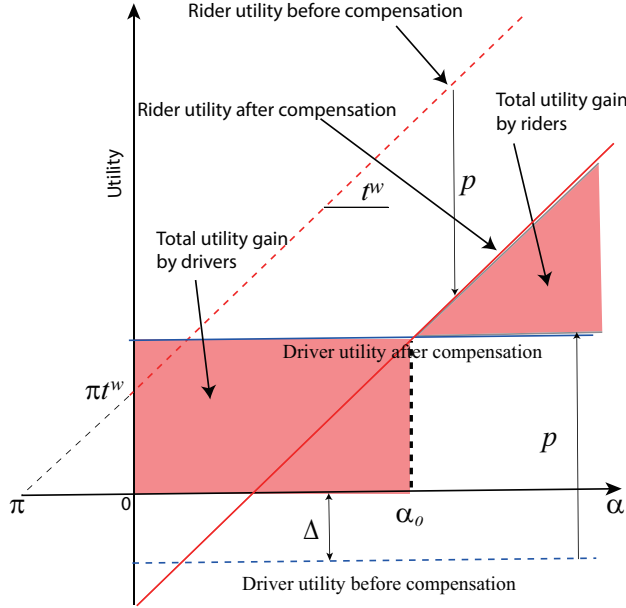


Figure 2: Utility of riders and drivers before and after compensation.

Problem (5) can be solved using Algorithm 1. The idea is straightforward: we check commuters following a descending order of the value of v_r^j , and if $v_r^j - \Delta \geq 0$, the commuter is matched with a driver with the lowest PGR among all commuters who have not been matched. This process continues until either there is no more commuter left for matching, or the benefit of riding is no longer enough to offset the inconvenience cost. Since the algorithm always picks the rider with the highest value of v_r^j from the commuters left to be matched, the allocative efficiency (5a) must be maximized.

Proposition 1. *With Assumptions 1 - 4, the solution to Problem (5) always leads to a perfect match.*

Proof: Assumption 4 implies that for at least half of the commuters, $v_r^j > \Delta$. Thus, when Algorithm 1 terminates, it should have matched exactly $0.5q^w$ riders. Given Assumption 1, everyone must have been matched either as a rider or as a driver. $\square$

Algorithm 1 Matching algorithm

- 1: **Inputs:** $\mathcal{I}^w = \{1, \dots, i, \dots, q^w\}$ ranked in descending order of α^i ; v_d^i, v_r^i .
 - 2: **Output:** Solution to Problem (5): $x_{i,j}$.
 - 3: Initialize $j = 1$, set $x_{i,j} = 0, \forall i, j$.
 - 4: **while** $j \leq 0.5q^w$ and $v_r^j > \Delta$ **do**
 - 5: Set $x_{(q^w+1-j),j}=1$, i.e., match rider j with driver $q^w + 1 - j$.
 - 6: Set $j = j + 1$.
 - 7: **end while**
 - 8: **return** $x_{i,j}$
-

Whether or not a perfect match can be achieved in practice depend on the platform's pricing strategy (p_r, p_d) . First and foremost, the pricing strategy must ensure no individual is worse off after ridesharing, commonly known as *individual rationality* (IR). Mathematically, this means the utility of any driver or rider must be non-negative, i.e.,

$$u_d^i \geq 0; u_r^i \geq 0, \forall i \in \mathcal{I}^w. \quad (6)$$

Second, the prices must be such set that no commuters have the incentive to misreport their PGR values, which is known as *incentive compatibility* (IC) in auction. Third, the platform may desire

to balance budget (BB), i.e.,

$$p_d f_d^w \leq p_r f_r^w. \quad (7)$$

We proceed to explore the pricing strategy that can both achieve a perfect match and satisfy the above three properties.

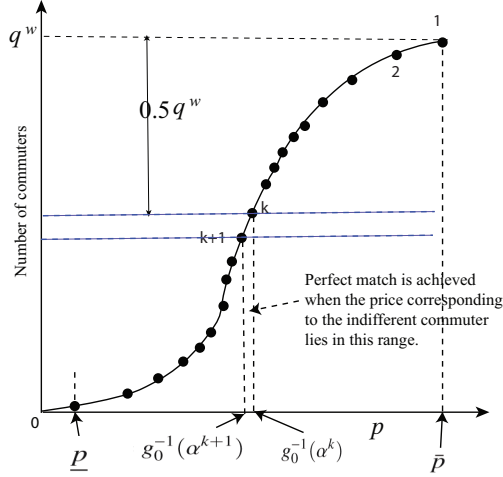


Figure 3: Illustration of a perfect match. Each solid dot corresponds to a commuter i , with the y value equal to i , and the x value equal to $g_0^{-1}(\alpha^i)$. that $\alpha^k > \hat{\alpha} > \alpha^{k+1}$. See Figure 3 for an illustration.

Let p be the base price used by the platform to adjust the utility of drivers and riders. We note that this price always corresponds to an *imaginary* commuter whose PGR value is such that s/he is indifferent to his/her role in ridesharing given p . That is,

$$u_d = u_r \rightarrow \alpha_0 = \frac{2p - \Delta}{t^w} - \pi, \quad (8)$$

where α_0 is referred to as the indifferent PGR value. Because α_0 can be viewed as a linear function of p , we formally define it as $g_0(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$. As $g_0(\cdot)$ is invertible, we define

$$\bar{p} = g_0^{-1}(\bar{\alpha}); \hat{p} = g_0^{-1}(\hat{\alpha}); \underline{p} = g_0^{-1}(\underline{\alpha}),$$

respectively as the maximum, median and minimum base price, corresponding to the maximum, median and minimum PGR value in $\mathfrak{A}^w$. We shall focus on the median base price $\hat{p}^w$ hereafter. Because of Assumptions 1 and 2, we can always find a commuter $k = 0.5q^w$ such

Proposition 2 (IR and BB). *Given Assumptions 1 - 4 and a perfect match obtained by Algorithm 1, the pricing policy*

$$p_r^* = p_d^* = g_0^{-1}(\alpha^k), k = 0.5q^w \quad (9)$$

is IR and BB.

Proof: (i) IR. According to Assumption 1 and Algorithm 1, any commuter $i \leq k$ will be matched as a rider and commuter $i \geq k + 1$ will be matched as a driver. For any commuter $i \geq k + 1$, the utility

$$p_d^* - \Delta = 0.5(t^w(\alpha^k + \pi) + \Delta) - \Delta = 0.5(t^w(\alpha^k + \pi) - \Delta) > 0,$$

because of Assumption 4. For the rider $i \leq k$, the utility is

$$\begin{aligned} t^w(\alpha^{i,w} + \pi) - p_r^* &= 0.5(t^w(\alpha^i + \pi) + \Delta) + 0.5(t^w(\alpha^i + \pi) - \Delta) - p_r^* \\ &\geq 0.5(t^w(\alpha^i + \pi) + \Delta) - p_r^* = g_0^{-1}(\alpha^i) - g_0^{-1}(\alpha^k) \geq 0. \end{aligned}$$

The first inequality is due to Assumption 4, the second equality is obtained from the definition of g_0 , and the second inequality is due to $\alpha^i \geq \alpha^k, \forall i \leq k$. This proves IR is satisfied at p^w , since

no commuter would have a negative utility.

(ii) BB. Because $p_d^* = p_r^*$ and a perfect match is achieved, we have $p_d^* f_d^w = p_r^* f_r^w$. Thus, the budget is balanced. This completes the proof. $\square$

The pricing policy given in Proposition 2 will not ensure IC, however. The next result provides an incentive compatible pricing policy.

Proposition 3 (IC). *With Assumptions 1 - 4, commuters have no incentive to misreport their PGR values if*

$$p_r^* = g_0^{-1}(\alpha^{k+1}); p_d^* = g_0^{-1}(\alpha^k); k = 0.5q^w.$$

Proof: First, we note that, since $\alpha^k > \alpha^{k+1}$, we have $p_d^* > p_r^*$. That is, each driver is paid by the platform more than each rider pays the platform.

For any matched driver $i \geq k + 1$ who intends to manipulate the matching result by lying about the PGR value, there are only two possible consequences: s/he remains a driver; or s/he is matched as a rider. In the first case, s/he can either report a higher or a lower PGR value. S/He can never improve his/her utility if s/he reports a lower PGR value, because s/he is already paid based on the highest PGR value of any drivers. If s/he tries to report a higher PGR value, s/he can only change the utility if s/he reports a PGR value higher than α^k (since s/he is paid by $g^{-1}(\alpha^k)$). If s/he does so, s/he would now become a rider and paid exactly at the same price as before (since now commuter k would have become the driver with highest PGR value). We note that

$$p_d^* > 0.5((\alpha^{k+1} + \pi)t^w + \Delta) \rightarrow (\alpha^{k+1} + \pi)t^w - p_d^* < p_d^* - \Delta.$$

The first equality is due to $\alpha^k > \alpha^{k+1}$, and note that $(\alpha^{k+1} + \pi)t^w - p_d^*$ is the new utility of the former driver who is now matched as a rider. Thus, switching the role will only reduce the utility of commuter i .

The case when i is a rider can be proved similarly, and omitted here for brevity. $\square$

It is straightforward to show the pricing strategy given by Proposition 3 can also ensure individual rationality. To see this, note that the driver is paid exactly the same as the price given in Proposition 2, and the price that a rider must pay is lower. Thus, neither the driver nor the rider would be worse off under the incentive compatible pricing policy of Proposition 3.

In order to achieve IC, the platform has to run a deficit equal to $0.5q^w(p_d^* - p_r^*)$. Thus, the requirement for budget balancing is violated. It is well known that in a double auction, it is impossible to achieve AE, IR, IC and BB in general (Myerson and Satterthwaite, 1983).

4.3 Imperfect match solutions

Achieving a perfect match is not always possible. Sometimes it may not even be desirable. We address these situations in this section.

When Assumption 1 is not satisfied, it is impossible to achieve a perfect match. Since in this case, q^w is an odd number, the median $\hat{\alpha} = \alpha^l, l = 0.5(q^w + 1)$. Intuitively, applying Algorithm 1 would yield a match result in which commuter l is the only s-driver. Because s-driver has a utility of zero, an incentive compatible policy must ensure commuter l has the same utility for other two roles (driver and rider). Thus,

$$u_d^l = p_d - \Delta = 0 \rightarrow p_d = \Delta,$$

and

$$u_r^l = (\alpha^l + \pi)t^w - p_r = 0 \rightarrow p_r = (\alpha^l + \pi)t^w.$$

Proposition 4. *With Assumptions 2 - 4 and assume q^w is an odd number, the solution to Problem (5) leaves only commuter $l = 0.5(q^w + 1)$ unmatched. Also the following pricing policy satisfies IR, IC and BB.*

$$p_d^* = \Delta, p_r^* = v_r^l = (\alpha^l + \pi)t^w, \quad (10)$$

where $l = 0.5(q^w + 1)$.

Proof: Using Algorithm 1, we will match everyone except l , because there are exactly $0.5(q^w - 1)$ commuters whose PGR value is larger than α^l and per Assumption 4, their valuation for the rider role is higher than Δ . We next prove the pricing policy given in Eq. (10) satisfies IR, IC and BB.

(i) IR. First, the policy has no effect on the utility of commuter l . For any driver $i \geq l + 1$, the utility

$$p_d^* - \Delta = 0.$$

For the rider $i \leq k - 1$, the utility is

$$t^w(\alpha^i + \pi) - p_r^* = t^w(\alpha^i - \alpha^l) \geq 0.$$

(ii) BB. Per Assumption 4, we have $p_r^* > p_d^*$. Hence $f_r^w p_r^* > f_d^w p_d^*$.

(iii) IC. Let's first consider commuter l . Clearly, s/he has no incentive to report a PGR lower than his/her true value, because if s/he does so, s/he would either remain a s-driver or become a driver. In both cases, his/her utility would still be zero. Suppose now that commuter l reports a PGR value so high that s/he becomes a rider. S/He would have to pay at $p_r^* = (\alpha^{l-1} + \pi)t^w$, which is higher than the original price, because $\alpha^{l-1} > \alpha^l$. Thus, commuter l has no incentive to misreport. For any driver $i \geq l + 1$, we only need to check the case when misreporting can change his/her role to a rider. If that does happen, s/he has to pay at $p_r^* = (\alpha^{l-1} + \pi)t^w$, which will give him/her a negative utility. Now consider rider $i \leq l - 1$. S/He has a positive utility being a rider. Thus, s/he would have no interest in switching to a s-driver and rider role by lowering the reported PGR value. S/He cannot improve utility by reporting PGR higher than the true value, because by doing so s/he would always remain a rider, and pay the same price.

This complete the proof. $\square$

Interestingly, when q^w is an odd number, the platform can actually make profit, even though one commuter will be left unmatched. For large q^w , the real impact of this type of imperfect match on traffic conditions is negligible. However, if the inconvenience cost is so large relative to the operating cost that Assumption 4 no longer holds, the portion of unmatched commuters could become significant.

To relax Assumption 4, let $k = \max\{k \in \mathcal{I} | v_r^k > \Delta\}$, and $k < 0.5q^w$. In this case, a perfect match cannot be achieved by Algorithm 1 because it will terminate at $j = k$, leaving $q^w - 2k$ commuters unmatched. Thus, any commuter $i \in \{k + 1, \dots, q^w - k\}$ will be s-driver. In order to ensure all s-drivers have no interest in riding or driving for ridesharing we need to set

$$p_d^* = \Delta, p_r^* = (\alpha^{k+1} + \pi)t^w. \quad (11)$$

Given the above pricing policy, an s-driver has no incentive to become a rider because everyone would have to pay a higher price than their valuation, except for commuter $k + 1$, who would break even. They have no incentive to be a driver either, because their utility would remain to be zero. By the same token, drivers and riders have no interest in alternative roles, which can be shown similarly as in the proof of Proposition 3. Because riders pay a higher price than the payment received by the driver, the system will generate a positive profit.

We close by noting that relaxing Assumption 4 implies

$$(\alpha^{k+1} + \pi)t^w < \Delta, k < 0.5q^w,$$

i.e., the inconvenience cost from ridesharing is significantly higher than the operating cost of the entire trip. This is certainly not likely to occur in reality. Nevertheless, if the inconvenience cost does rise to this level, ridesharing needs to be carefully planned to avoid hurt system efficiency.

4.4 VCG pricing policy

A standard approach to addressing incentive compatibility (also known as truthful reporting) in auction is to invoke the VCG policy (Vickrey, 1961; Clarke, 1971; Groves, 1973). Algorithm 2 describes how the VCG policy sets prices p_d and p_r after the optimal matching is obtained from solving Problem (5). Note that V^* and V_{-i}^* denote, respectively, the optimal objective function value of the original Problem (5), and that of Problem (5) without commuter i .

Algorithm 2 VCG pricing policy

```

1: Input: Solution to Problem (5),  $x_{i,j}, \forall i, j \in \tilde{\mathcal{I}}$ , i.e., the set of matched commuters.
2: Output: A pricing policy  $p_d^i$  and  $p_r^j$  for any  $i, j \in \tilde{\mathcal{I}}$ .
3: for every commuter  $i \in \tilde{\mathcal{I}}$  do
4:   Calculate  $V_{-i}^*$  by re-solving Problem (5) without commuter  $i$ .
5:   Set the bonus for commuter  $i$  as  $\rho_i = V^* - V_{-i}^*$ .
6:   if Commuter  $i$  is a driver then
7:     set  $p_d^i = v_d^i + \rho_i = \Delta + \rho_i$ .
8:   else if Commuter  $i$  is a rider then
9:     set  $p_r^i = v_r^i - \rho_i$ .
10:  end if
11: end for
12: Return  $p_d^i$  and  $p_r^j$ .

```

Proposition 5. *With Assumptions 2 - 4 and assume q^w is an odd number, the pricing policy given by Proposition 4 equals the VCG policy.*

Proof. The basic idea of the VCG pricing mechanism is to have the winner who offers the highest bid pay the price offered by the second highest bidder in a single round sealed auction. Under Assumptions 2 - 4 and given q^w as an odd number, the system achieves a perfect match except for commuter $l = 0.5(q^w + 1)$. Thus, when a driver i is removed, the only s-driver l will be assigned as a driver and the system will achieve a perfect match that provides the same commuters'

welfare. This is because the contribution of any driver to the commuters' welfare is always $-\Delta$. Since $V^* = V_{-i}^*$, the bonus for driver i is

$$\rho_i = V^* - V_{-i}^* = 0.$$

According to Algorithm 2, $p_d^i = \Delta$. When a rider i is removed, the only s-driver l will be assigned as a rider. Thus

$$\rho_i = V^* - V_{-i}^* = v_r^i - v_r^l.$$

Thus, the payment to the rider i is $p_r^i = v_r^i - (v_r^i - v_r^l) = v_r^l$. Clearly, any commuter will pay the same price as long as his/her role is fixed, i.e., $p_d^* = \Delta$, $p_r^* = v_r^l$. The proof is completed. $\square$

It is easy to prove that the pricing strategy given by Eq. (11) also equals the VCG policy for the case when Assumption 4 is relaxed. However, the pricing policy given in Proposition 3 is not a VCG policy. We leave it to the reader to verify in that case (q^w is even and Assumptions 1 - 4 hold), the VCG policy would prescribe

$$p_r^* = \Delta; p_d^* = v_r^{0.5q^w}, \quad (12)$$

which may lead to a larger deficit than allowed by the policy given in Proposition 3.

5 Two O-D Problem

When the system has two O-D pairs, let $\mathcal{I} = \{1, \dots, q^1 + q^2\}$ represent the set of all commuters. For each commuter $i \in \mathcal{I}$, we use $w(i)$ to map the commuter i to his/her O-D pair. We proceed to define the cost for each role. For a driver i who is matched with a rider from the same O-D pair, we have

$$c_d^i = (\beta_d^i + \pi)t^{w(i)} + \Delta. \quad (13)$$

If the driver i is matched with a rider from a different O-D pair, then

$$c_{dc}^i = \begin{cases} (\beta_d^i + \pi)t^{w(i)} + \Delta' & w(i) = 1, \\ \infty & w(i) = 2. \end{cases} \quad (14)$$

Here the subscript dc is used to differentiate from c_d^i , highlighting the fact that the driver performs a cross O-D pickup. Note that in our setting, drivers from O-D pair 2 are forbidden to take riders from O-D pair 1. That is why the cost for doing so is set to infinity.

If the commuter i is a rider or a solo driver, then his/her cost is, respectively

$$c_r^i = \beta_r^i t^{w(i)}, c_s^i = (\beta_d^i + \pi)t^{w(i)}. \quad (15)$$

If a driver i is matched with someone from the same O-D pair, his/her valuation of the trip becomes

$$v_d^i = c_s^i - c_d^i = -\Delta; \quad (16)$$

otherwise, it is

$$v_{d^c}^i = c_s^i - c_{d^c}^i = \begin{cases} -\Delta' & w(i) = 1, \\ -\infty & w(i) = 2. \end{cases} \quad (17)$$

The valuation of the commuter i for being a rider and solo driver is respectively

$$v_r^i = c_s^i - c_r^i = (\alpha^i + \pi)t^{w(i)}; v_s^i = 0. \quad (18)$$

341 With the above definitions, we can similarly define the bipartite matching problem for the
 342 two O-D problem, by introducing the same binary assignment variable x_{ij} . The new formulation
 343 reads:

$$\max \sum_{i \in \mathcal{I}} \left(\sum_{j \in \mathcal{I}, j \neq i, w(j)=w(i)} (v_r^j + v_d^i) x_{ij} \right) + \sum_{i \in \mathcal{I}} \left(\sum_{j \in \mathcal{I}, j \neq i, w(j) \neq w(i)} (v_r^j + v_{d^c}^i) x_{ij} \right) \quad (19a)$$

subject to:

$$\sum_{j \in \mathcal{I}, j \neq i} x_{ij} + \sum_{k \in \mathcal{I}, k \neq i} x_{ki} \leq 1, \forall i \in \mathcal{I}, \quad (19b)$$

$$x_{ij} \in \{0, 1\}, \forall i \in \mathcal{I}, \forall j \in \mathcal{I}, i \neq j. \quad (19c)$$

344 The formulation is identical to Problem (5) except its objective function is divided to account for
 345 the within O-D matching (the first term) and the cross O-D matching (the second term). Again,
 346 Problem (19) is a nonlinear integer program if $t^w, w = 1, 2$ depends on x_{ij} . Even if we assume t^w
 347 is a flow-independent constant, obtaining analytical solutions and pricing strategies for Problem
 348 (19) is much more complicated. Instead, we will derive the VCG policy using Algorithm 2, and
 349 explore the properties through numerical experiments.

350 6 Numerical experiments

351 We first conduct a couple of numerical experiments to demonstrate and validate the analytical
 352 results obtained for the one O-D case (Section 6.1). In Section 6.2, we explore how flow-dependent
 353 travel time might affect matching results. Finally, Section 6.3 tests a two O-D example which is
 354 not amenable to analysis. The goal is to determine if and when cross O-D matching is necessary
 355 for welfare maximization.

356 6.1 One O-D

357 Consider a simple case with only four commuters, whose PGR values are reported in the second
 358 row in Table 1. We set the inconvenience cost $\Delta = 4$ and the flow-independent travel time $t^w = 2$.
 359 The vehicle operating cost per unit time $\pi = 5$. Hence, we have $\Delta < t^w \pi$.

360 According to Eq. (3), the valuation of commuter i as a driver is $v_d^i = -4, \forall i \in I$, and if s/he is
 361 matched as rider, his/her valuation v_r^i is reported in the third row in Table 1.

Table 1: Valuation in the four-commuter matching problem

i	4	3	2	1
α^i	1	2	3	4
v_r^i	12	14	16	18
v_d^i	-4	-4	-4	-4

Using Algorithm 1, it is easy to see that the solution to Problem (5) is to assign commuters 1 and 2 as riders and commuters 3 and 4 as drives. The outcome obviously achieves perfect match and a vehicular flow of 2. According to Proposition 3, a rider should pay $p_r^* = 0.5(v_r^2 + \Delta) = 9$ and a driver will receive $p_d^* = 0.5(v_r^3 + \Delta) = 10$. The system ends up with a deficit of 2.

When commuter 3 truthfully reports the PGR value as 2, his/her utility is $p_d^* - v_r^2 = 12 - 4 = 8$. When s/he reports the valuation $0 \leq \alpha_2 < 3$, s/he remains a driver and the payment is still 10. When s/he reports the valuation $\alpha > 3$, s/he will be assigned as a rider. Suppose s/he reports α as 3.5. In this case, the rider's payment will become 10 (based on the valuation of commuter 2). The utility then becomes $14 - 10 = 4$, a loss compared to the utility under truthful reporting. Thus, under no circumstance, commuter 3 could improve the utility by misreporting the valuation.

Let us now add a fifth commuter ($i = 0$) whose PGR value = 5 to the matching problem. The valuation for the driver role remains 4, and that for the rider role is 20. All the other parameters remain the same.

In this case, Algorithm 1 will assign commuters 0 and 1 as riders and commuters 4 and 3 as drivers. Commuter 2 will have to drive alone since s/he cannot find a match. Proposition 4 dictates that each driver receives a payment of 4, and each rider pays the valuation of the solo driver, i.e., 16. Thus, the system generates a net profit of 24. The solo driver, whose utility is zero, would have no incentive to become a carpool driver, since that would not change the utility. If s/he is to report the PGR value as 4.5, commuter 4 would now become the solo driver, changing the rider payment from 16 to 18. This means by misreporting the PGR value as 4.5, the original solo driver would become a rider, and reduce the utility from 0 to -2.

Finally, let us consider a case with an unusually high inconvenience cost, say $\Delta = 18.5$. Note that in this case $v_r^1 < \Delta < v_r^0$. According to Algorithm 1, only commuter 0 will be assigned as a carpool driver in order to maximize the welfare. Attempting to match any other commuters would lead to a direct loss. Eq. (11) suggests that a driver receives a payment of $p_d^* = 18.5$ and a rider pays at the highest rider valuation of any solo driver, in this case, that of commuter 1, i.e., $p_r^* = 18$. In total, the system leads to a total vehicular flow of 4, a welfare of 1.5, and a profit of -0.5.

6.2 Congestion effect

In the example presented in Section 6.1, t^w is assumed to be flow-independent. The matching problem always ensures, for any matched pair, the inconvenience cost of the driver not exceed the valuation of the rider. This means certain matching would be forbidden in favor of welfare maximization. However, if t^w depends on flow, a lower matching rate means more vehicles

on road, hence a larger t^w . There is a clearly a trade-off between the welfare defined in the matching problem and the cost of congestion. In this section, we explore the congestion effect by constructing a slightly more realistic single O-D example. Let the distance between the origin

Algorithm 3 Matching algorithm with congestion effect

```

1: Inputs:  $\mathcal{I}^w = \{1, \dots, i, \dots, q^w\}$  and  $\alpha^i, \forall i$ ;
2: Output: Solution to Problem (5):  $\mathbf{x}^*$  and maximal commuters' welfare  $V^*$ .
3: Initialize:  $V^* = 0, f^* = -1, \mathbf{x}^* = 0$ .
4: for  $\frac{q^w}{2} \leq f \leq q^w$  do
5:   Update  $t^w$  using Eq. (20). Calculate  $v_r^i$  for each commuter  $i \in \mathcal{I}^w$  according to  $t^w$ .
6:   Rank  $v_r^i$  in the descending order.
7:   Set  $V = 0, l = 1, \mathbf{x} = 0$ .
8:   while  $l < q^w - f$  do
9:     Set  $l' = q^w - l + 1, x_{ll'} = 1$ , and  $V = V + v_r^l + v_r^{l'}$ .
10:    Update  $l = l + 1$ .
11:   end while
12:   if  $V^* < V$  then
13:     Set  $V^* = V, \mathbf{x}^* = \mathbf{x}, f^* = f$ .
14:   end if
15: end for

```

and the destination be $d = 11.5$ miles and the free flow speed $v_0 = 40$ mph, which leads to a free flow travel time $t_0 = d/v_0 = 0.2875$ hour. Assume the travel time t^w be a function of vehicular flow f , which takes the following form

$$t^w = t_0(1 + 0.15(f/C)^4), \quad (20)$$

where $C = 2000$ vph (vehicle per hour) is road capacity. The total number of commuters $q^w = 3000$. Thus, a perfect match case yields $t^w = 0.3$ hours and non-sharing case yields $t^w = 0.51$ hours.

We estimate π as \$8/hour³. The value of time (VOT) (Lam and Small, 2001) is estimated based on the median wage rate (taken as \$21/hour), and the mean PGR value is taken as 1/3 of the VOT, i.e., \$7/hour. We assume $\tilde{\alpha}$ follows a log-normal distribution with a mean of 7\$ and a variance of 16 (this corresponds to $\text{lognormal}(\mu, \sigma), \mu = 1.8, \sigma = 0.53$). Δ can be estimated based on the detour time valued at the median VOT. For a 5 minute detour (on top of a 17.25 minute trip without congestion), $\Delta = \$1.75 < \pi \times t_0 = \2.3 . For a 10 minute detour, $\Delta = \$3.5 > \2.3 . For a 15 minute detour, $\Delta = \$5.25 > (\tilde{\alpha} + \pi)t_0 = \4.3 .

To solve Problem (5) with congestion effect, we implement a simple enumeration method detailed in Algorithm 3. The idea is to examine commuters' welfare corresponding to each possible matching result. The algorithm searches between a perfect match scenario (when the vehicular flow $f = 0.5q^w$) and a no-match scenario (when the vehicular flow $f = q^w$). For each matching scenario, the algorithm first updates the travel time and then compares the welfare associated with the scenario against the maximum welfare recorded so far.

³It consists of both the fuel cost and vehicle depreciation for driving 11.5 miles using a car purchased at \$ 20,000 with a lifetime millage of 200,000 miles. The fuel cost is estimated at \$3 per gallon and the fuel efficiency of the car is 30 mile per gallon. Travel speed is assumed to be constant in this estimation.

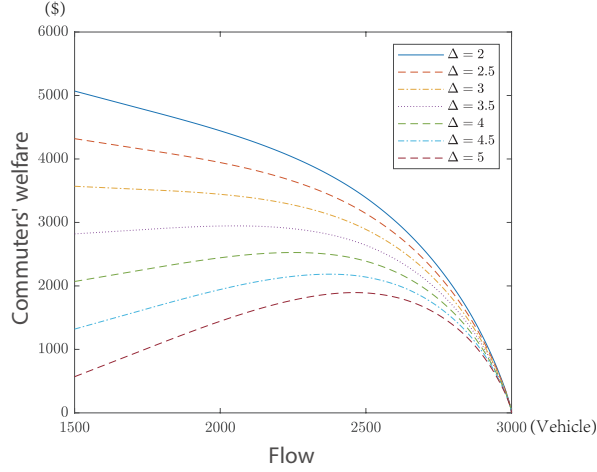


Figure 4: Objective function value of Problem (5) corresponding to different matching results (determined by vehicular flow f) and different inconvenience cost Δ ($q^w = 3000$). The high point on each curve corresponds to the optimal solution.

(each time a 3000-traveler sample is drawn from the Log-normal distribution). In each experiment, we find the optimal vehicular flow, commuters' welfare and profit based on the VCG pricing scheme. Figure 5 (a) shows the average optimal vehicular flow begins to rise above 1500 (i.e., the value corresponding to the perfect match) after $\Delta > \$3$. This trend is similar to that established in Figure 4. Thus, the actual PGR values of a sample does not seem to have much impact on the optimal traffic pattern in this case. The dominating factor is Δ .

Figures 5 (b) and (c) show, as ridesharing becomes more inconvenient, the system's commuters' welfare and profits both plunge. Importantly, when $\Delta < \$3$, the welfare gain of the users is more than enough to cover the subsidy required to sustain the operation. When $\Delta \geq 3$, the large deficit of the VCG scheme begins to exceed the welfare gain. Clearly, to run a sustainable ridesharing program in those cases is challenging and would require pricing mechanisms more sophisticated than what are considered herein.

6.3 Two O-D pair

Consider the network shown in Figure 1. Let $q^1 = 2000, q^2 = 1000$. For each link a , we denote its length, travel time, and capacity by d_a , T_a and C_a , respectively. We set $d_1 = 2$ mile, $d_2 = 8$ mile, and $d_3 = 1.5$ mile, and $C_1 = 1500$ vph, $C_2 = 3000$ vph, and $C_3 = 1500$ vph. Similar to the example reported in Section 6.2, the free flow speed $v_0 = 40$ mph and the vehicle operating cost $\pi = \$8/\text{hour}$. The link travel time

$$T_a = \frac{d_a}{v_0} \left(1 + 0.15 \left(\frac{y_a}{C_a} \right)^4 \right), \quad (21)$$

⁴An inconvenience cost of \$3 roughly equals 8.6 minute detour time, about half of the free flow travel time.

Figure 4 illustrates how the welfare varies with f with different inconvenience cost Δ . When Δ is within the "reasonable range" (e.g., at or below \$3⁴), the perfect match is always optimal and the platform can achieve it through the pricing scheme described in Proposition 3 or the VCG scheme. Thus, for modest Δ , the system can be analyzed as if there was no congestion effect, since the outcome is always a perfect match. When Δ reaches \$3.5 (a ten minute detour and about 50% more than the operating cost), the perfect match is no longer the best choice for the system. Instead, the optimal number of matched pairs is reduced by a third to about 1000 (for a vehicular flow of about 2000). As Δ continues to rise, matching becomes less and less attractive. For $\Delta = \$5$, the number of matched pairs is just a little more than 500 at optimum.

We repeat the above experiment 100 times

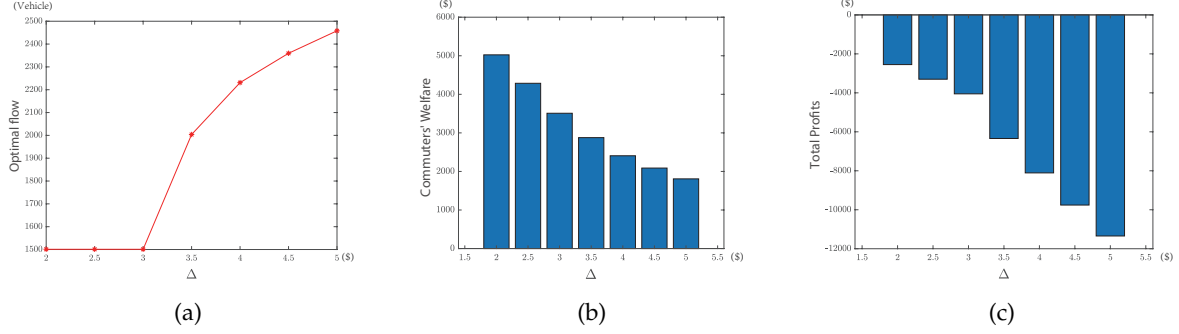


Figure 5: Relationship between inconvenience cost Δ and average vehicular flow (a), commuters' welfare (b), and profits (c) in 100 simulations.

Table 2: Test scenarios for the two O-D problem.

Scenario	Description	Δ	Δ'
1	Cross-OD matching is more expensive	Δ_0	$2\Delta_0$
2	Within O-D matching is more expensive	$2\Delta_0$	Δ_0
3	Identical	Δ_0	Δ_0

where y_i is given in Eq. (1). The optimal solution to the two O-D matching problem (19) can also be obtained through a simple enumeration procedure, detailed in Algorithm 4.

We set a base inconvenience cost $\Delta_0 = \$2$, and consider three combinations of Δ (within O-D inconvenience cost) and Δ' (cross O-D inconvenience cost), as shown in Table 2. Table 3 reports the number of cross and within-OD matched pairs in the three scenarios. In the first scenario, in which $\Delta' = 2\Delta$, cross-OD matching never occurs, regardless of the value of Δ_0 . For $\Delta_0 \leq 2.5$, both O-D pairs can achieve the perfect match on its own. As Δ_0 continues to rise, the matching ratio at both O-D pairs begin to decline, as expected. Moreover, matching for O-D 1 is evidently less sensitive to the value of Δ_0 than that for O-D 2. For example, when Δ_0 increases from 2 to 5, the number of matched pair for O-D 1 is reduced to a little more than 1/3, whereas for O-D pair 2, less than 10 percent of the matched pair remains. This discrepancy is clearly driven by the fact that it is twice as expensive to match cross O-D as within O-D.

Matching within O-D 2 becomes completely unattractive in Scenario 2. Although there are still many matches within O-D 1, the number of cross O-D matches is always greater at every level of Δ_0 . Moreover, when Δ_0 is larger, twice as many pairs are matched for cross O-D than within O-D 1. Again, this is expected since within O-D matching is more expensive than cross O-D matching.

Compared to Scenario 1, the cross O-D inconvenience cost is reduced to the same level as that within O-D in Scenario 3. This change has surprisingly little impact on the results, however. For Δ_0 up to 3.5, the matching results are identical in both scenarios. Only for $\Delta_0 \geq 4$ do we observe the emergence of a small number of cross O-D matches. The travelers demonstrate a strong preference for matching within O-D pair 1, likely because O-D pair 1 has a longer travel

Algorithm 4 Matching algorithm for the two-OD problem with congestion effect

```

1: Inputs:  $\mathcal{I} = \{1, \dots, i, \dots, q^1 + q^2\}$  and  $\alpha^i, \forall i$ ;
2: Output: Solution to Problem (19):  $\mathbf{x}^*$  and maximal commuters' welfare  $V^*$ .
3: Initialize:  $V^* = 0, \mathbf{x}^* = 0$ .
4: Define:  $l_w$  as the number of matched pairs within O-D pair  $w = 1, 2$ ,
5: and  $l_{12}$  as the number of cross-OD matched pairs.
6: for  $l_1 \in [0, 0.5q^1]$  do
7:   for  $l_2 \in [0, 0.5q^2]$  do
8:     Calculate the maximum number of possible cross O-D matches as  $L_{12}$  given  $l_1$  and  $l_2$ .
9:     for  $l_{12} \in [0, L_{12}]$  do
10:      Update  $y_a$  based on  $l_1, l_2$  and  $l_{12}$ ,
11:      Calculate link travel time  $T_a$  using Eq. (21) and update O-D travel time  $t^1 = \sum_{a=1}^3 T_a$  and
       $t^2 = T_2$ .
12:      Compute  $v_r^i, \forall i \in \mathcal{I}$  and rank  $v_r^i$  in descending order.
13:      Set  $\mathbf{x}$  and  $V$  based on  $l_1, l_2$  and  $l_{12}$  using a similar procedure included in Algorithm 3 (line
      7 -11).
14:      if  $V^* < V$  then
15:        Set  $V^* = V, \mathbf{x}^* = \mathbf{x}$ .
16:      end if
17:    end for
18:  end for
19: end for

```

Table 3: Matching outcome with different base inconvenience cost Δ_0

Δ_0 (\$)	Matched pair for O-D 1			Matched pair for O-D 2			Cross O-D matched pair		
	Scenario			Scenario			Scenario		
	1	2	3	1	2	3	1	2	3
2	1000	564	1000	500	0	500	0	872	0
2.5	1000	364	1000	500	0	500	0	626	0
3	1000	197	1000	352	0	352	0	375	0
3.5	1000	111	1000	188	0	188	0	238	0
4	706	75	706	111	0	105	0	155	6
4.5	506	48	506	65	0	48	0	103	17
5	384	27	384	42	0	32	0	62	10

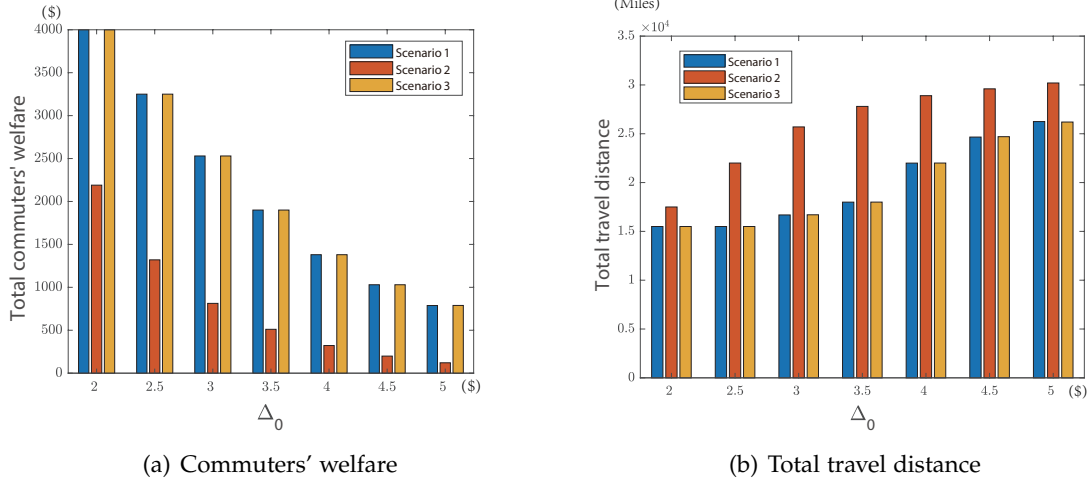


Figure 6: System performance of the two O-D example under different value of Δ_0 .

time, which gives riders a greater saving to offset the inconvenience cost.

Figure 6 shows a rising inconvenience cost reduces system welfare and increases travel distance because it discourages carpool. Also, Scenarios 1 and 3 lead to very similar commuters' welfare and travel distance, whereas the performance of Scenario 2 is much less desirable. A larger share of cross O-D matching (as in Scenario 2) implies that a greater number of drivers from O-D pair 1 would find it cheaper to share a ride with someone from O-D pair 2. Since these drivers must travel alone on links 1 and 3, the total travel distance is bound to increase, as well as traffic congestion, see Figure 6 (a). Evidently, this increase in the travel distance coincides with a decrease in the commuters' welfare.

In reality, it is more than likely that inconvenience cost incurred within the same O-D would be comparable to or lower than that incurred cross O-D. Thus, the amount of cross O-D matching should be relatively small at social optimum.

7 Conclusions

This study aims to examine the impact of carpool on network traffic in a highly idealized futuristic world, where all travelers are willing to participate in carpool arranged by a Transportation Network Company. The parsimonious model built for this purpose centers on the trade-off between the inconvenience cost imposed by carpool and the travel cost savings from (i) less vehicle miles travelled, (ii) driving relief and (iii) reduced congestion. Underlying the model is a nonlinear bipartite matching problem that seeks to maximize the commuters' welfare of the system.

By assuming the congestion effect is negligible, we obtained several useful analytical results for the single O-D problem. When the inconvenience cost is less than the median trip valuation of a rider, the platform could always achieve an *almost* perfect match while maximizing commuters' welfare. In this case, if there are an even number of travelers, we propose a pricing policy that possesses all desired properties of the VCG policy but runs a lower deficit; otherwise, we show

the VCG policy always generates a profit. If the inconvenience cost is too high, the perfect match is no longer socially optimal, but the VCG policy still yields a positive profit.

Results from numerical experiments generally agree with the analytical solutions obtained without considering the congestion effect. Specifically, when the inconvenience cost is relatively low, the system can reach a perfect match among travelers from the same O-D pair, which corresponds to a 50% reduction in vehicular traffic flow. Moreover, even when the system is forced to run a deficit, the welfare gain generated from carpool is large enough to offset it. The results also suggest that matching travelers from different O-D pairs is rarely desired by the system. It occurs only when matching across O-D has a significantly lower inconvenience cost than matching within, an unlikely event in reality. Also, when cross O-D matching does become prevalent, it leads to higher vehicle miles travelled, hence worse congestion. Thus, from the point of view of traffic management, cross O-D carpool should not be encouraged.

The present analysis has several limitations that a future study could try to address. First and foremost, the current model assumes the inconvenience cost as a publicly known constant cost applied only to drivers. In reality, this cost not only is heterogeneous and private, but could also be shared by driver and riders. Second, the network considered in our model has a very special topology, which might affect the generalization of the results, especially those related to cross O-D carpool. Developing a nonlinear bipartite matching model over a general network is an intriguing challenge, both analytically and computationally. Finally, the study can be enhanced by considering other real world features such as traffic dynamics, multiple carpooling platforms, and one-to-many matching.

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