Transportation Research Part E 145 (2021) 102165

Contents lists available at ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

Sustainable multi-commodity capacitated facility location problem with complementarity demand functions

Weiwei Liu [a], Nan Kong [b], Mingzheng Wang [c,*], Lingling Zhang [a]

[a] School of Economics and Management, Dalian University of Technology, Dalian 116024, China
[b] Weldon School of Biomedical Engineering, Purdue University, West Lafayette, USA
[c] School of Management, Zhejiang University, Hangzhou 310058, China

ARTICLE INFO

Keywords:
Facility location
Multi-commodity
Carbon emission
Complementarity demand function
Mixed-integer quadratic program

ABSTRACT

We investigate a multi-commodity capacitated facility location problem involving sustainability concerns (e.g., restrained carbon emission). In addition, this problem incorporates a complementarity demand function. We show that this incorporation can lead to superior decisions both analytically and computationally. The resultant formulation is a 0–1 mixed-integer non-concave quadratic program with equilibrium constraints. We adopt the piecewise-linear envelope method to transform the formulation into a 0–1 mixed-integer concave program. We then propose an efficient branch-and-refine algorithm with global convergence. Numerical examples demonstrate the effect of carbon emission limit and carbon trading on company decisions.

1. Introduction

In many industries, companies produce various commodities to satisfy diverse customer demands (Geoffrion and Graves, 1974). It is critical that these companies intelligently identify production facilities and set commodity prices based on production capacity and demand forecast to maximize profit. In this paper, we investigate a type of sustainable multi-commodity capacitated facility location problem (MCCFLP) with a focus on carbon emission and a refined characterization of the demand-price relationship.

Rapid global industrialization has led to environmental issues and climate change, which have captured worldwide attentions (Saberi, 2018). Sustainability is currently a key issue in supply chain management (Choi et al., 2019). Achieving sustainability implies that companies are concerned with economic objectives as well as relevant environmental influences (Phillis et al., 2010; Azevedo et al., 2012). Companies therefore have to adjust their production and operations in response to external pressure due to government regulations and growing consumer concerns about environmental issues (e.g., carbon emission (Drake et al., 2016) and pollution (Choi and Cai, 2020)). In particular, the government and consumers have identified carbon emission as one of the most important metrics related to environmental issues (Nouira et al., 2016). Wu and Dunn (1995) pointed out that transportation is the largest source of environmental hazards in logistic systems. Tsao and Thanh (2019) reported that transportation provides a quarter of the total amount of carbon emission and the proportion continues to grow. Elhedhli and Merrick (2012) indicated that reducing vehicle total travel distance via strategically placed facilities could reduce transportation-related carbon emission, thus playing a vital role in reducing carbon footprint. In this paper, we investigate the MCCFLP with consideration of carbon emission generated by transportation.

* Corresponding author.
E-mail addresses: liuweiwei887145@mail.dlut.edu.cn (W. Liu), nkong@purdue.edu (N. Kong), wangmzh@zju.edu.cn (M. Wang), zhanglldlut@163.com (L. Zhang).

https://doi.org/10.1016/j.tre.2020.102165
Received 17 February 2020; Received in revised form 8 November 2020; Accepted 8 November 2020
Available online 5 December 2020
1366-5545/

Increasing globalization and diverse preferences of customers have led companies to consider offering substitutable products (i.e., products that consumers perceive as similar or comparable, such that one product renders others less desirable) and/or complementary products (i.e., products that consumers typically purchase together). Substitutable products can elicit positive cross-elasticity of demands, whereas complementary products contribute to negative cross-elasticity. Consumer theory generally emphasizes the need to incorporate potentially complex demand patterns into multi-product location decisions (Kalcsics et al., 2000).

In much of the existing literature, MCCFLPs only involve cases in which the demand-price relationship is defined on a restricted price domain. This restriction is applied to simplify the characterization of the demand-price relationship by only considering local areas where product demands are price-sensitive. We refer to the demand-price relationship as demand function in this paper. In the literature, for mathematical convenience, demand functions of simplistic form, such as linear functions, are frequently used, and subsequently, nonnegativity restriction is imposed in location optimization. We investigate the MCCFLP with complementarity demand function. The motivation behind this work is that a company could set the price of one product extremely high, even presented with the possibility of having no demand. Nevertheless, this product could still affect consumer demands for other products. The following question naturally arises, namely whether limiting the prices to some restricted domain (e.g., the nonnegative domain) would necessarily lead to maximum revenue. This situation further implies that location decisions based on a demand function defined in such a domain will be suboptimal. As pointed out by Soon et al. (2009), the complementarity demand function offers a way to relax the aforementioned domain restriction and is applicable to a set of interrelated products.

The following numerical example of pricing in a tight market, adopted from Soon et al. (2009), motivates the need of defining the price domain less restrictively. A single seller provides two mutually substitutable products (e.g., business-class and economy-class airfares). Demand functions for these two products are given by $d^1(p) = 20 - 3p^1 + 2p^2$ and $d^2(p) = 100 + p^1 - 4p^2$, where $p = (p^1, p^2)$. In a tight market, we have $0 \leqslant y^k \leqslant d^k(p), k = 1, 2$, where y^k represents the quantity of product k to be sold. Assuming that the inventory level/production capacity is 30, then we would have $y^1 + y^2 \leqslant 30$. Without loss of generality, we further assume that the price of Product 1 is higher than that of Product 2; that is, $p^1 \geqslant p^2 \geqslant 0$. The objective is to determine prices of both products and quantities to be sold, such that the seller's revenue $p^1y^1 + p^2y^2$ is maximized.

To solve this two-product pricing problem, let us define $\Omega := \{p \in R_+^2 | d^k(p) \geqslant 0, k = 1, 2\}$, which is the price domain (see Fig. 1). From the constraints $p^1 \geqslant p^2$ and $d^1(p) \geqslant 0$, one has that $p^1 \leqslant 3p^1 - 2p^2 \leqslant 20$ is derived. Hence, for any $p \in \Omega$, the maximum revenue is bounded from above by $p^1y^1 + p^2y^2 \leqslant p^1y^1 + p^1y^2 \leqslant 30p^1 \leqslant 30 \times 20 = 600$. Nevertheless, we can set the prices outside Ω to yield higher revenue. For example, when setting the price as $p' = (p'^1, p'^2) = (24, 23)$, customers would only buy Product 2 in this case. It would therefore be optimal to sell 30 units of Product 2 and none of Product 1, that is, $y^1 = 0, y^2 = 30$. It is easy to verify that each constraint above is satisfied and the revenue yielded is $0 + 30 \times 23 = 690 > 600$. However, note that $d^1(p') = 20 - 3 \times 24 + 2 \times 23 = -6 < 0$, which implies that $p' \notin \Omega$. This example clearly shows that the company can achieve higher revenue by setting prices outside Ω. Soon et al. (2014) confirmed this result based on simulated customer behavior data. Overall, demand functions defined on a restricted price domain may lead to inferior pricing decisions.

With this conclusion and given the impact of pricing on location decisions, we investigate the sustainable MCCFLP with complementarity demand function to answer the following two research questions:

(1) Whether the demand function defined on the entire nonnegative price domain necessarily leads to an improved solution to the MCCFLP over some restricted domain (e.g., Ω in the above example); and

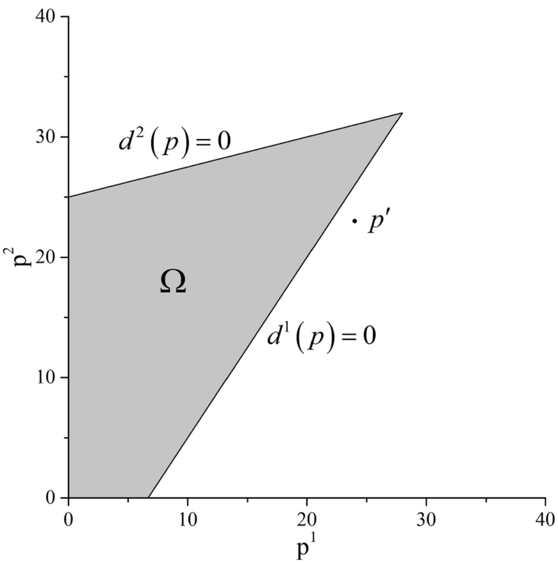

Fig. 1. Relationship between price and demand.

(2) Whether the improved MCCFLP solution impacts the logistic decisions subject to sustainability concerns.

The resultant MCCFLP model is novel and practical but challenging to solve. We thus develop an efficient global optimization algorithm via the piecewise-linear envelope method. Specifically, the MCCFLP is a 0–1 mixed-integer non-concave quadratic program with equilibrium constraints. Under mild conditions, we first transform this problem into a 0–1 mixed-integer non-concave program with linear constraints. Nevertheless, a computational challenge persists in dealing with bilinear terms. To efficiently address the nonlinear and non-concave functions, we further transform them into linear functions by relaxing bilinear terms using piecewise-linear envelopes. We also design a branch-and-refine algorithm with global convergence, which iteratively refines the linear envelopes along the branch-and-bound procedure.

The main contributions of our paper are twofold:

(1) We are the first that study the MCCFLP with incorporation of complementarity demand functions to capture the substitutable and/or complementary relationship among various products.
(2) We demonstrate the benefit of considering complementarity demand functions for the MCCFLP in the context of sustainable low-carbon economy.

The remainder of this paper is organized as follows. Section 2 reviews literature on four topics relevant to our study. Section 3 presents a sustainable MCCFLP formulation. Section 4 outlines an in-depth analysis of our model with input of a less restrained complementarity demand function. Section 5 details the branch-and-refine algorithm and proves its global convergence. Section 6 reports numerical experiments and presents a case study on carbon emission limit and carbon trading. Section 7 concludes the paper and outlines future research.

2. Literature review

We review four relevant areas in the literature, namely multi-source facility location decision, demand-price relationship modeling, mixed-integer programming with equilibrium constraints, and sustainable supply chain design.

2.1. Multi-source capacitated facility location problem

Extended and in-depth research on facility location problems has been conducted over the last decades (Zhang et al., 2016). Each customer's demand may be split to different facilities. There is extended research on the multi-source capacitated facility location problem with one product; see Fernández and Landete (2015) for a literature survey. Many efficient algorithms have been proposed, including cutting plane method (Avella and Boccia, 2009), a Lagrangian-based branch-and-bound method (Görtz and Klose, 2012), and kernel search heuristic method (Guastaroba and Speranza, 2012). Nevertheless, the multi-source capacitated facility location problem with multiple products has received far less attention. Askin et al. (2014) designed the distribution network of a logistic provider with multiple products. Akyüz et al. (2012), Akyüz et al. (2012), Akyüz et al. (2019) considered multiple commodities and capacitated facilities for the Weber problem. Among existing studies, Nouira et al. (2016) is most relevant to our work. The authors investigated a sustainable supply chain design problem considering multiple products and carbon emission. However, they did not deal with substitutable nor complementary relationship among the products.

2.2. Demand functions for interrelated products

We only review the demand functions relevant to our study, i.e., demand functions defined on the entire nonnegative price domain. Generally, there are two such types of demand functions. The first type is to use one differentiable function to approximate the demands, e.g., Cobb-Douglas function (Bernstein and Federgruen, 2004) and logit function (Milgrom and Roberts, 1990). While these functions make the ensuing analysis more tractable, they may not necessarily reflect the interrelation between customer demands on substitutable and/or complementary products. For example, the demand of one product using a logit model tends to converge to infinity as the prices of other products converge to infinity. The other type is to use piecewise-smooth functions to approximate the demands, e.g., Boyer and Moreaux (1987) and Kübler and Müller (2002). Nevertheless, these demand functions have similar undesirable properties as those of the first type.

The so-called complementarity demand functions are also a class of piecewise-smooth functions. This class of consumer demand functions has several advantages, which are highlighted in Soon et al. (2009) and Federgruen and Hu (2016, 2019). First, these functions possess desirable properties such as monotonicity (i.e., the law of demand in economics). Second, these functions, particularly asymmetric ones, are widely applicable to combinations of direct and cross-price elasticity. Third, these functions can specify a product portfolio together with the products' prices and demand quantities. This specification is unlike many widely used demand models. For instance, with many multinomial logit models, each product attains certain market share, irrespective of its absolute and relative price level. Such a property is, in practice, usually violated in the real market. On the other hand, complementarity demand functions approximate the demand-price relationship in a real market more accurately. Federgruen and Hu (2015, 2016, 2019) used complementarity demand functions in supply chain decisions. Nevertheless, the authors focused on product pricing and assortment decisions. In conclusion, it is important to use complementarity demand functions in many decision problems, including the facility location problem in this paper.

2.3. Mixed-integer programming with equilibrium constraints

The optimization model proposed in this paper is a mixed-integer nonlinear program (MINLP) with bilinear objective and equilibrium constraints, which is a type of mathematical program with equilibrium constraints (MPEC). MPEC is difficult to solve, especially for a global optimality. The common approach is to transform the equilibrium constraints (Luo et al., 1996). For equilibrium constraints in our model, we equivalently transform them to linear constraints with the big-M method. The resultant program becomes an MINLP with bilinear objective and linear constraints.

MINLPs with bilinear objective are a special type of MINLPs, which can be divided into concave MINLPs and non-concave MINLPs. Although both types are NP-hard (Sherali and Adams, 2013), concave MINLPs are typically easier to solve than their counterparts as their continuous relaxations remain concave. Significant progress has been made in solving concave MINLPs over the past decades with such methods as generalized benders decomposition (Geoffrion, 1972), branch-and-price (Sharkey et al., 2011), outer approximation (Hijazi et al., 2014), supporting hyperplane (Kronqvist et al., 2016) and LP/NLP-based branch-and-bound (Vielma et al., 2008). On the other hand, algorithm development for solving non-concave MINLPs lags far behind.

It is common to convert a non-concave MINLP to a concave MINLP via semidefinite programming (see Helmberg et al., 2002; Sun et al., 2012; Dong, 2016). Another common approach is to adopt piecewise-linear functions to approximate nonlinear functions. Then the original MINLP is approximated by an MILP, which can be solved with using existing algorithms; see, e.g., Vielma et al. (2010) and Nowak et al. (2018). Unfortunately, there is usually a duality gap between the original formulation and the relaxed formulation. For the MINLP with bilinear objective and equilibrium constraints proposed in this paper, we, under mild conditions, can relax it to a 0–1 mixed-integer concave program through piecewise-linear envelopes. To eliminate the duality gap, we propose a branch-and-refine algorithm with global convergence.

2.4. Supply chain design with consideration of carbon emission

Traditionally, the supply chain management literature focuses on economic performance measures considering a single product. Under the pressure of the government and consumers, recent studies have taken carbon emission into account (e.g., Cachon, 2014; Chan et al., 2018; Sun et al., 2018; Chen and Bidanda, 2019; Jabbarzadeh et al., 2019). Studies addressing environmental issues in the forward supply chain design are not yet abundant, as highlighted in Tang and Zhou (2012).

Compared with studies considering a single product, there are a few studies focused on the MCCFLP with consideration of carbon emission reduction. Closely related to our study are the following two articles. Diabat and Simchi-Levi (2009) developed a two-stage capacitated facility location model with multiple products under carbon emission cap. Abdallah et al. (2010) studied a similar problem with multiple products and the focus on carbon offset. The objectives of these two papers are to minimize the operational costs while satisfying customer demand for a variety of products. However, it is usually impossible to meet all the demands in reality. Different from previous work, we investigate the MCCFLP combined with emission limit and carbon trading, where joint decisions of product pricing and multi-source are also made.

From the above literature review, we conclude that almost all prior studies focus on some specific aspects of capacitated facility location, including carbon emission, product pricing, and multiple sourcing. Our study, by addressing these aspects more holistically, is thus more reflective of the current industry practice. Moreover, for the proposed MPEC model, we transform it into a 0–1 mixed-integer concave program and propose a branch-and-refine algorithm with global convergence.

3. A sustainable MCCFLP formulation

In this section, we present the sustainable MCCFLP formulation. We consider a one-tier supply chain system for a monopoly company, which intends to locate a set of capacitated production facilities to produce multiple products while satisfying customers' demands (see Fig. 2). Some of the products are substitutable and some others are complementary.

Further, we consider the carbon trading scheme, which is one of the two popular carbon regulatory policy mechanisms, together

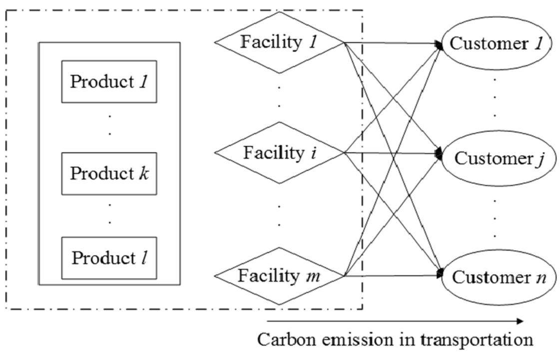

Fig. 2. Illustration of the MCCFLP.

with carbon pricing (taxation) scheme (Zakeri et al., 2015; Drake et al., 2016). Zakeri et al. (2015) indicated that the carbon trading scheme has better supply chain performance compared with the carbon pricing scheme. In accordance with the carbon trading scheme, the amount of carbon emission generated by shipping products to customers must be no more than a predetermined limit the government sets, denoted by E. If the company's carbon emission amount is above (below) E, it has a chance to buy additional (sell remaining) carbon capacity from a carbon trading market. Carbon price is assumed to be constant for its medium to long-term stability. Our notation is introduced in Table 1. In addition, we assume that the production cost of product k $(\in K)$ at facility i $(\in I)$ is depicted by a quadratic function: $G_i^k(s_i^k) = \gamma_i^k s_i^k + \frac{\delta_i^k}{2}(s_i^k)^2$. This convex cost arises possibly because more staff work overtime, costly materials are used, or equipment maintenance schedules are neglected or postponed (Harkness and ReVelle, 2003). Such function has been used in the supply chain design literature, see, e.g., Wolf and Smeers (1997),DeMiguel and Xu (2009), and Ghaddar and Naoum-Sawaya (2012).

We next describe the structure of complementarity demand function, denoted by $D_j(p) := (D_j^1(p), \cdots, D_j^k(p), \cdots, D_j^l(p))^T$ for all $j \in J$. Let $d_j(p) : R_+^l \to R^l$ be a given demand function, i.e.,

$$d_j(p) := (d_j^1(p), \cdots, d_j^k(p), \cdots, d_j^l(p))^T,$$

where $p = (p^1, \cdots, p^k, \cdots, p^l)^T, R_+^l = \{p \in R^l | p \geqslant 0\}$. With $d_j(p)$, specify $\Omega := \{p \in R_+^l | d_j(p) \geqslant 0, j \in J\}$. For $p \in \Omega$, let $D_j(p) = d_j(p)$. For $p \notin \Omega$, let $D_j(p) = d_j(B(p))$, where $B(p) = (\bar{p}^1, \cdots, \bar{p}^k, \cdots, \bar{p}^l)^T$ on Ω is obtained from a nonlinear complementarity problem given by p, i.e., $NCP(p) : 0 \leqslant d_j(B(p)) \perp (p - B(p)) \geqslant 0$, for $j \in J$, where $\perp$ stands for orthogonality and $d_j(B(p)) \perp (p - B(p))$ is equivalent to $d_j(B(p))^T (p - B(p)) = 0$. $NCP(p)$ is assumed to have a unique solution $B(p)$ and $B(p) \in \Omega$ for all $p \in R_+^l$ throughout this paper. For an overview on solution uniqueness about complementarity problems, see Facchinei and Pang (2007).

With the above introduction, the sustainable MCCFLP is formulated as:

Table 1
Nomenclature.

Parameters	
$I = \{1, \cdots, i, \cdots, m\}$	Set of candidate facility locations;
$J = \{1, \cdots, j, \cdots, n\}$	Set of customers;
$K = \{1, \cdots, k, \cdots, l\}$	Set of products;
c_i	Fixed cost of opening facility i;
m_i^k	Production capacity of facility i for product k;
t_{ij}^k	Per unit cost for shipping product k from facility i to customer j;
g_{ij}^k	Per unit emission for shipping product k from facility i to customer j;
N	Maximum number of facilities to open;
E	Carbon emission limit;
p_c	Carbon price;
η_k	Price cap of product k.
Miscellaneous	
$G_i^k(s_i^k)$	Production cost of product k at facility i;
s_i^k	Total production quantity of product k of facility i;
γ_i^k	Coefficient for linear term in production cost;
δ_i^k	Coefficient for quadratic term in production cost;
$D_j^k(p)$	Complementarity demand function of customer j for product k;
$D_j(p)$	Complementarity demand function of customer j;
$d_j^k(p)$	A demand function of customer j for product k;
$d_j(p)$	A demand function of customer j;
p	Price vector;
$B(p)$	Projected price vector;
Ω	Price domain.
Decision variables	
y_{ij}^k	Quantity of product k supplied by facility i to customer j;
p^k	Price of product k;
z_i	1, If location i is selected; 0, otherwise.

$$(a) \quad \max \quad f(p^k, y_{ij}^k, z_i, i \in I, j \in J, k \in K)$$

$$= \sum_{k\in K}(\sum_{i\in I}\sum_{j\in J} y_{ij}^k)p^k - \sum_{k\in K}\sum_{i\in I}[\gamma_i^k \sum_{j\in J} y_{ij}^k + \frac{\delta_i^k}{2}(\sum_{j\in J} y_{ij}^k)^2] \tag{1}$$

$$-\sum_{i\in I} c_i z_i - \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} t_{ij}^k y_{ij}^k - p_c[\sum_{k\in K}\sum_{i\in I}\sum_{j\in J} g_{ij}^k y_{ij}^k - E]$$

$$\text{s.t.} \sum_{i\in I} y_{ij}^k \leqslant D_j^k(p), \ \forall j \in J, k \in K; \tag{2}$$

$$\sum_{j\in J} y_{ij}^k \leqslant m_i^k z_i, \ \forall i \in I, k \in K; \tag{3}$$

$$\sum_{i\in I} z_i \leqslant N; \tag{4}$$

$$y_{ij}^k \geqslant 0, \ \forall i \in I, j \in J, k \in K; \tag{5}$$

$$0 \leqslant p^k \leqslant \eta_k, \ k \in K; \tag{6}$$

$$z_i \in \{0, 1\}, \ \forall i \in I. \tag{7}$$

The objective function (1) reflects the company's profit, that is calculated as the sales revenue minus the production cost, the facility setup cost, the transportation cost, and the price of carbon amount (i.e., either revenue of selling redundant carbon if the total carbon emission amount is less than the carbon limit, or cost of purchasing external carbon if the total amount extends the carbon limit). Constraints (2) ensure that the supply does not exceed the demand for each customer j and each product k. Constraints (3) enforce that production only takes place at an open facility and must not exceed its capacity. Constraint (4) limits the number of opened facilities by some upper bound N. Constraints (5) and (6) ensure that the distribution quantities and prices to be nonnegative.

As defined earlier, $D_j^k(p) = d_j^k(B(p))$ for each $j \in J$ and $k \in K$, constraints (2) can be rewritten as:

$$\sum_{i\in I} y_{ij}^k \leqslant d_j^k(B(p)), \forall j \in J, k \in K; \tag{8}$$

$$0 \leqslant \sum_{j\in J} d_j^k(B(p)) \perp (p^k - \overline{p}^k) \geqslant 0, \forall k \in K. \tag{9}$$

By replacing constraints (2) in model (a) with (8) and (9), we obtain an equivalent model as:

$$(b) \quad \max \quad f(p^k, y_{ij}^k, z_i, i \in I, j \in J, k \in K)$$
$$\text{s.t.} \quad \text{Constraints } (3) - (9).$$

Model (b) is a 0–1 mixed integer nonlinear program with equilibrium constraints, which is difficult to solve directly. Hence we conduct in-depth model analysis for developing an efficient solution approach.

4. Model analysis

In this section, we further analyze model (b) for solution consideration. With mild assumptions, we can equivalently transform model (b) into a 0–1 mixed-integer non-concave program with linear constraints, by introducing auxiliary binary variables (Section 4.1). We then use piecewise-linear envelopes to linearize inherent bilinear terms when the given function $d(p)$ is the frequently used linear demand function (Section 4.2). The resultant reformulation includes a concave objective function and a bounded convex constraint set.

4.1. Equivalent transformation of equilibrium constraints

Generally speaking, optimization models with equilibrium constraints are difficult to deal with because their feasible regions are not necessarily convex or even connected. For each equilibrium constraint in model (b), we can use the big-M method, a standard technique used to reformulate equilibrium terms in Ghaddar and Naoum-Sawaya (2012), to equivalently transform it into two linear inequalities. This ensures that the feasible region corresponding to the equilibrium constraints is transformed into a convex set, thus reducing the complexity of solving model (b).

In practice, each customer location has limited demand for each product, such that there must exist an upper bound for $\sum_{j\in J} d_j^k(B(p)$) ($\forall k \in K$). Equilibrium constraints (9) are linearized by using binary variables $\nu^k \in \{0, 1\}$, for all $k \in K$, and identical big-M

coefficients, denoted by M. Then we obtain a model as follows:

$$(c)\quad \max \quad f(p^k, y_{ij}^k, z_i, i \in I, j \in J, k \in K)$$
$$\text{s.t.}\quad \text{Constraints } (3)-(8); \qquad (10)$$
$$0 \leqslant \sum_{j\in J} d_j^k(B(p)) \leqslant M v^k,\ \forall k \in K;$$

$$0 \leqslant p^k - \overline{p}^k \leqslant M(1-v^k),\ \forall k \in K; \qquad (11)$$

$$v^k \in \{0,1\},\ \forall k \in K. \qquad (12)$$

By introducing auxiliary binary variables, we transform model (b) into model (c), a 0–1 MINLP, pending equivalence of the two models. Due to the equivalent transformation of equilibrium constraints, models (c) and (b) are equivalent. We then solve model (c) instead of model (b). Model (c) is a 0–1 mixed-integer quadratic program. We present the property of model (c) as follows.

Proposition 1. *The objective function of model* (c) *is not a concave function.*

Please see the proof in Appendix A.

Note that all decision variables in model (c) are nonnegative and bounded; thus, the constraint set of model (c) is a bounded set. In the next section, for linear demand function $d(p)$, we further reformulate the non-concave objective function, which contributes to development of an efficient optimization algorithm.

4.2. Further reformulation for linear demand function $d(p)$

Demand for each product is related to the prices of all products available in the market. As the linear relationship prevails in theoretical models and empirical research, we put all our emphasis on linear structure in the following analysis. This gives rise to the demand function of customer j, i.e.,

$$d_j(p) := (d_j^1(p), \cdots, d_j^k(p), \cdots, d_j^l(p))^T = b_j - A_j(p)$$

$$= \begin{pmatrix} b_1^j \\ \vdots \\ b_k^j \\ \vdots \\ b_l^j \end{pmatrix} - \begin{pmatrix} a_{11}^j & \cdots & a_{1k}^j & \cdots & a_{1l}^j \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{k1}^j & \cdots & a_{kk}^j & \cdots & a_{kl}^j \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{l1}^j & \cdots & a_{lk}^j & \cdots & a_{ll}^j \end{pmatrix} \begin{pmatrix} p^1 \\ \vdots \\ p^k \\ \vdots \\ p^l \end{pmatrix}$$

$$= \begin{pmatrix} b_1^j - (a_{11}^j p^1 + \cdots + a_{1k}^j p^k + \cdots + a_{1l}^j p^l) \\ \vdots \\ b_k^j - (a_{k1}^j p^1 + \cdots + a_{kk}^j p^k + \cdots + a_{kl}^j p^l) \\ \vdots \\ b_l^j - (a_{l1}^j p^1 + \cdots + a_{lk}^j p^k + \cdots + a_{ll}^j p^l) \end{pmatrix}.$$

If $d_j(p)$ is linear, then A_j is a P-matrix (i.e., all principal minors of A_j are positive) if and only if there exists a unique solution to the linear complementarity problem; that is, equilibrium constraints (9) yield a unique solution $B(p)$. We thus assume that $\sum_{j\in J} A_j$ is a P-matrix in the remainder of this paper. Note that we have made the assumption: $\forall p \in R_+^l,\ B(p) \in \Omega$.

Remark 1. Under the condition that $d_j(p)$ is linear and $\sum_{j\in J} A_j$ is a P-matrix, to ensure this assumption, Soon et al. (2009) provided a necessary and sufficient condition: $\forall p \in R_+^l, B(p) \in \Omega$ if and only if $(\sum_{j\in J} A_j)_{K'K'}^{-1} b_{K'} \geqslant 0, \forall K' \subseteq K$, where $(\sum_{j\in J} A_j)_{K'K'}$ and $b_{K'}$ are the principal submatrix and corresponding vector for any index set $K' \subseteq K$, respectively.

Remark 2. Consider a collection of substitutable commodities; in this case, the non-negativity of all off-diagonal entries of $\sum_{j\in J} A_j$ is guaranteed. Furthermore, if $\sum_{j\in J} A_j$ is a P-matrix, the non-negativity of its inverse and all its principal submatrices is guaranteed (see Berman and Plemmons, 1994). That is to say, if $b_j \geqslant 0$, then $(\sum_{j\in J} A_j)_{K'K'}^{-1} b_{K'} \geqslant 0$ will always be satisfied for all $K' \subseteq K$ (Soon et al., 2009).

Note that in addition to the P-matrix specification in the model, we may consider cases where $\sum_{j\in J} A_j$ are other forms of matrices to ensure the uniqueness of solution of equilibrium constraints (9). Furthermore, we may not rely on the equivalence to conduct our investigation. Instead, we may attempt to solve model (b) directly, which is an MPEC. We leave these two potential research items to future.

Next, by further specifying constraints (8) and (10) with the linear form of the demand function $d(p)$, we obtain the following equivalent model:

$$(d)\max \quad f(p^k, y_{ij}^k, z_i, i \in I, j \in J, k \in K)$$
$$\text{s.t.} \quad \text{Constraints } (3)-(7); (11)-(12); \tag{13}$$
$$\sum_{i\in I} y_{ij}^k + (a_{k1}^j \overline{p}^1 + \cdots + a_{kk}^j \overline{p}^k + \cdots + a_{kl}^j \overline{p}^l - b_k^j) \le 0, \forall j \in J, k \in K;$$

$$\sum_{j\in J}(a_{k1}^j \overline{p}^1 + \cdots + a_{kk}^j \overline{p}^k + \cdots + a_{kl}^j \overline{p}^l - b_k^j) \le 0, \ \forall k \in K; \tag{14}$$

$$\sum_{j\in J}(-a_{k1}^j \overline{p}^1 - \cdots - a_{kk}^j \overline{p}^k - \cdots - a_{kl}^j \overline{p}^l + b_k^j) - Mv^k \le 0, \forall k \in K. \tag{15}$$

Note that the nonlinear term in the objective function of model (d) is $\sum_{k\in K}[\sum_{i\in I}\sum_{j\in J} y_{ij}^k p^k - \sum_{i\in I}\frac{\delta_i^k}{2}(\sum_{j\in J} y_{ij}^k)^2]$. The bilinear terms $y_{ij}^k p^k$ $(\forall i \in I, j \in J, k \in K)$, lead to non-concavity of the objective function. Leyffer et al. (2008) originated the idea of piecewise-linear envelopes to approximate nonlinear functions, based on the special ordered set approximation method (Tomlin, 1981; Martin et al., 2006). We adopt piecewise-linear envelopes of bilinear functions to transform the objective function into a concave quadratic function.

Lemma 1. *For each* (x, y, xy) *that satisfies* $L_x \leqslant x \leqslant U_x$ $(L_x < U_x)$ *and* $L_y \leqslant y \leqslant U_y$ $(L_y < U_y)$*, there exists a unique* $\lambda_i \geqslant 0$ $(i = 1,2,3,4)$*, such that*

$$\begin{pmatrix} x \\ y \\ xy \end{pmatrix} = \lambda_1 \begin{pmatrix} L_x \\ L_y \\ L_x L_y \end{pmatrix} + \lambda_2 \begin{pmatrix} L_x \\ U_y \\ L_x U_y \end{pmatrix} + \lambda_3 \begin{pmatrix} U_x \\ L_y \\ U_x L_y \end{pmatrix} + \lambda_4 \begin{pmatrix} U_x \\ U_y \\ U_x U_y \end{pmatrix}$$

and

$$\sum_{i=1}^{4} \lambda_i = 1.$$

By Lemma 1, the linear envelope of bilinear function xy is

$$w = \sum_{s=1}^{\tau_x} \sum_{t=1}^{\tau_y} \lambda^{st} x^s y^t,$$

where x^s, y^t are breakpoints along each dimension; and τ_x, τ_y are the numbers of breakpoints in each dimension. For each bilinear term $y_{ij}^k p^k$ $(\forall i \in I, j \in J, k \in K)$ in model (d), we obtain the linear envelopes as

$$w_{ij}^k = \sum_{s=1}^{\tau_{(i,j,k)}} \sum_{t=1}^{\tau_k} \lambda^{kst} y_{ij}^{ks} p^{kt}, y_{ij}^k = \sum_{s=1}^{\tau_{(i,j,k)}} \sum_{t=1}^{\tau_k} \lambda^{kst} y_{ij}^{ks}, p^k = \sum_{s=1}^{\tau_{(i,j,k)}} \sum_{t=1}^{\tau_k} \lambda^{kst} p^{kt},$$

$$\sum_{s=1}^{\tau_{(i,j,k)}} \sum_{t=1}^{\tau_k} \lambda^{kst} = 1, \lambda^{kst} \geqslant 0,$$

where y_{ij}^{ks} (p^{kt}) are the breakpoints in dimension y_{ij}^k (p^k), and $\tau_{(i,j,k)}$ (τ_k) is the corresponding number of breakpoints. Following the above results, we can further transform model (d) to the following model:

$$(e) \quad \max \quad g(p^k, y_{ij}^k, z_i, i \in I, j \in J, k \in K)$$

$$= \sum_{k\in K}(\sum_{i\in I}\sum_{j\in J} w_{ij}^k) - \sum_{k\in K}\sum_{i\in I}[\gamma_i^k \sum_{j\in J} y_{ij}^k + \frac{\delta_i^k}{2}(\sum_{j\in J} y_{ij}^k)^2] \tag{16}$$
$$- \sum_{i\in I} c_i z_i - \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} t_{ij}^k y_{ij}^k - p_c[\sum_{k\in K}\sum_{i\in I}\sum_{j\in J} g_{ij}^k y_{ij}^k - E]$$

$$\text{s.t.} \quad \text{Constraints } (3)-(7); (11)-(15);$$
$$w_{ij}^k = \sum_{s=1}^{\tau_{(i,j,k)}} \sum_{t=1}^{\tau_k} \lambda^{kst} y_{ij}^{ks} p^{kt}; \tag{17}$$

$$y_{ij}^{k} = \sum_{s=1}^{\tau_{(i,j,k)}} \sum_{t=1}^{\tau_k} \lambda^{kst} y_{ij}^{ks}; \tag{18}$$

$$p^{k} = \sum_{s=1}^{\tau_{(i,j,k)}} \sum_{t=1}^{\tau_k} \lambda^{kst} p^{kt}; \tag{19}$$

$$\sum_{s=1}^{\tau_{(i,j,k)}} \sum_{t=1}^{\tau_k} \lambda^{kst} = 1; \tag{20}$$

$$\lambda^{kst} \geqslant 0. \tag{21}$$

Note that, compared with model (d), $\sum_{k\in K}\sum_{i\in I}[\frac{\delta_i^k}{2}(\sum_{j\in J} y_{ij}^k)^2]$ is the only nonlinear term in model (e). Next, we show that with this further reformulation with linear envelopes, a concave objective function is obtained.

Theorem 1. *The objective function* $g(p^k, y_{ij}^k, z_i, i \in I, j \in J, k \in K)$ *of model* (e) *is concave.*

Please see the proof in Appendix B.

Remark 3. Although $y_{ij}^k p^k$ and $(\sum_{j\in J} y_{ij}^k)^2$ both lead to non-concavity of the objective function in model (d), the objective function of model (e) becomes a concave function once the bilinear terms $y_{ij}^k p^k$ are replaced by their linear envelopes. Thus, we simply need to linearize $y_{ij}^k p^k$, not $(\sum_{j\in J} y_{ij}^k)^2$.

All constraints in model (e) are linear; thus, the constraint set is bounded and convex. From the above analysis, we deduce that to solve the MCCFLP with complementarity demand functions, we can instead solve a 0–1 mixed-integer concave program with linear constraints. But compared with model (a), the scale of model (e) becomes larger due to a rising number of constraints. In the next section, we propose an efficient optimization algorithm to address larger-scale 0–1 mixed-integer concave program problems.

5. A branch-and-refine algorithm

In this section, we present a branch-and-refine algorithm to solve model (e). Leyffer et al. (2008) dealt with mixed-integer nonlinear non-convex optimization problems by relaxing the non-convex problem to obtain a convex approximation. For the relaxation, they introduced intermediate variables to decompose problem functions into unary and binary functions, which lead to a large convex approximation problem. The bounds of intermediate variables also needed to be derived, which is computationally expensive. Unlike Leyffer et al. (2008), we avoid the decomposition of each function and simply relax the bilinear terms by using their linear envelopes. Accordingly, a 0–1 mixed-integer concave program with linear constraints is obtained. To solve model (e) effectively, we present a branch-and-refine algorithm, which adds more breakpoints than Leyffer et al. (2008) as we explore the branch-and-bound tree. We thus gradually generate tighter outer approximations.

For convenience, this paper considers the minimization model with negation of the objective function in model (e), which is termed as model (e'). Denote $x = (y_{ij}^k, p^k, i \in I, j \in J, k \in K)$ and $X = \{x : L_x \leqslant x \leqslant U_x\}$, where L_x and U_x are column vectors consisting of lower and upper bounds of all components of x, respectively. Denote $q = (z_i, i \in I) \in R^m$ and $Q = [0,1]^m$, as all components of y are 0–1 binary variables.

To obtain subproblems along the branch-and-bound tree, we begin by solving the continuous relaxation program of model (e'). If this program is infeasible, then the original problem is infeasible. Otherwise, we choose a variable to branch, generate two subproblems accordingly, and place them in a stack. The algorithm repeatedly removes subproblems from the stack, solves them, and adds new subproblems after branching. The algorithm terminates when the stack is empty.

Let $LPR(X_\kappa, Q_\kappa)$ denote the subproblem, where κ labels the current node, and X_κ and Q_κ form the current feasible solution subspaces. $(\overline{x}^\kappa, \overline{q}^\kappa)$ and z_{LPR_κ} denote the optimal solution and the associated optimal objective function value, respectively.

We implement the branching process as follows. If q_ς is a fractional value, we branch on it as

$$Q_{2\kappa} := \{q \in Q_\kappa : q_\varsigma = 0\}, X_{2\kappa} := X_\kappa; \tag{22}$$

$$Q_{2\kappa+1} := \{q \in Q_\kappa : q_\varsigma = 1\}, X_{2\kappa+1} := X_\kappa. \tag{23}$$

Similarly, we branch on continuous variable x_ς at a value x_ς':

$$X_{2\kappa} := \{x \in X_\kappa : x_\varsigma \leqslant x_\varsigma'\}, Q_{2\kappa} := Q_\kappa; \tag{24}$$

$$X_{2\kappa+1} := \{x \in X_\kappa : x_\varsigma \geqslant x_\varsigma'\}, Q_{2\kappa+1} := Q_\kappa. \tag{25}$$

While diving down through the tree, we alternately select an integer variable and a continuous variable to branch; the location decisions (modeled with integer variables) interact with pricing and supply quantity decisions (modeled with continuous variables).

Once the variable type is determined for branching, we randomly select one such variable. To create tighter outer approximations, after each branching, we need to refine the linear envelopes (17)–(21) in the resultant two subproblems by selecting more breakpoints from the feasible solution subspace of each subproblem. For detailed information on breakpoint selection, please see the appendix on proving Theorem 2.

We define $NLP(X_\kappa, \overline{q}^\kappa)$ with fixed integer variables $q = \overline{q}^\kappa$ and scope X_κ in model (d'), which is the equivalent convex model by negating the objective function of model (d). $(\widehat{x}^\kappa, \overline{q}^\kappa)$ and z_{NLP_κ} denote the optimal solution and the associated optimal objective function value, respectively. We denote εas the accuracy level. After solving $LPR(X_\kappa, Q_\kappa)$, we can fathom nodes in the branch-and-bound tree as long as one case occurs:

(1) If $LPR(X_\kappa, Q_\kappa)$ is infeasible, there is no feasible point in $X_\kappa \times Q_\kappa$. Then the node is fathomed.
(2) If $LPR(X_\kappa, Q_\kappa)$ has an integer solution $\overline{q}^\kappa$, and if the solution to $NLP(X_\kappa, \overline{q}^\kappa)$ satisfies $|z_{LPR_\kappa} - z_{NLP_\kappa}| \leqslant \varepsilon$, the node is fathomed. Then, either a new incumbent is obtained, or a better solution dominates the node.
(3) If $|U - z_{LPR_\kappa}| \leqslant \varepsilon$, where U is the optimal value of the incumbent, it is impossible to find a better solution in $X_\kappa \times Q_\kappa$. Then this node is fathomed.

Summarizing the above, we formally state the branch-and-refine algorithm.

The branch-and-refine algorithm.

```
Initialize U = ∞, choose an accuracy ε > 0, integer σ > 0, κ = 1, and set L = {X_κ × Q_κ}.
 while L ≠ ∅:
  Solve LPR(X_κ, Q_κ), (x̄^κ, q̄^κ) is obtained.
    If LPR(X_κ, Q_κ) is infeasible or z_{LPR_κ} ≥ U − ε, fathom this node.
    If LPR(X_κ, Q_κ) cannot be pruned:
      If q̄^κ is integer feasible, solve NLP(X_κ, q̄^κ):
        If z_{NLP_κ} < U − ε, update U = z_{NLP_κ}, new incumbent (x*, q*) = (x̂^κ, q̄^κ).
          If |z_{LPR_κ} − z_{NLP_κ}| ≤ ε, fathom this node.
      If q̄^κ is non-integer, select the branching variable, choose σ + κ − 2 equidistantly distributed
       breakpoints from the dimension of y_{ij}^k, p^k (∀i ∈ I, j ∈ J, k ∈ K), and refine envelopes (17)-(21):
          If the selected variable is fractional, branch according to (22)-(23).
          If the selected variable is continuous, branch according to (24)-(25).
   Remove X_κ × Q_κ from L.
 Return optimal solution (x*, q*).
```

Theorem 2. *The branch-and-refine algorithm converges to a global optimum with ε-accuracy in a finite number of iterations.*

Please see the proof in Appendix C.

6. Numerical study

We implement the proposed branch-and-refine algorithm in Matlab2014a and run the algorithm on a Windows 7 workstation with 3.0 GHz Intel CPU and 16 GB RAM. We conduct five sets of experiments. First, we consider a simple scenario with two substitutable products, one demand location, and two candidate facility locations, to assess the impact of considering the complementarity demand function for facility location decisions (Section 6.1). Second, we consider a scenario with a list of complementary and substitutable products (Section 6.2). We then assess the effect of carbon emission limit on carbon offset and total profit (Section 6.3) and the impact of carbon pricing on the amount of carbon traded (Section 6.4). Finally, we test the performance of the proposed branch-and-refine algorithm on larger instances by comparing the computational time against the branch-and-bound algorithm in He et al. (2014) (Section 6.5). We recode the branch-and-bound algorithm in the same Matlab environment and run it on the same machine as the proposed algorithm.

We assume that the company's candidate locations and demand points are evenly distributed in a field of $150 \times 150km^2$. We use Euclidean distance and assume $\varepsilon = 10^{-4}$,$M = 10^{10}$, and $\sigma = 6$. We extract the transportation cost of Product 1, the emission associated

Table 2
Instance parameters.

Transportation cost of Product 1	\$5/km/unit	Emission associated with shipping Product 1	0.425 kg/km/unit
First-order coefficient of production function for Product 1	0.42	Second-order coefficient of production function for Product 1	0.2
Transportation cost of Product 2	\$6/km/unit	Emission associated with shipping Product 2	0.225 kg/km/unit
First-order coefficient of production function for Product 2	0.45	Second-order coefficient of production function for Product 2	0.225
Capacity for Product 1	3000 units	Capacity for Product 2	2500 units
Cost of building facility	\$7500	Carbon emission limit	100 kg
Carbon price	\$150/ton	Price cap of Product 1(2)	\$800(\$1000)

with shipping Product 2, and the cost of facility construction and carbon limit from Ghaddar and Naoum-Sawaya (2012). We specify the other parameters used in the numerical experiments, as reported in Table 2.

6.1. A scenario with two candidate locations, a demand domain, and two substitutable products

For this scenario, we consider identical production cost functions for two candidate locations. We further assume they take the forms $G_i^1(s) = 0.42s + 0.2s^2$ and $G_i^2(s) = 0.45s + 0.225s^2$, $i = 1, 2$. The linear demand function takes the form $d(p^1, p^2) = \begin{pmatrix} 3600 \\ 2400 \end{pmatrix} - \begin{pmatrix} 40 & -10 \\ -30 & 20 \end{pmatrix} \begin{pmatrix} p^1 \\ p^2 \end{pmatrix}$. Clearly, the coefficient matrix of the linear demand function is a P-matrix. We summarize the results in Fig. 3 and Table 3.

As shown in Fig. 3, by considering the linear demand function, the company would install a facility at both Locations 1 and 2. At each location, the facility would produce Products 1 and 2 to service the demand region. The facility at Location 1 would offer 1 unit of Product 1 and 2 units of Product 2; the facility at Location 2 would offer 1 unit of Product 1 and 354 units of Product 2. In this case, the total expected profit is $14090. By contrast, with consideration of the complementarity demand function, the company would only install a facility at Location 2 and only produce Product 2 at this location. In this case, the company produces 56 units of Product 2 at a total expected profit of $24359. This profit is 72.9% higher than that obtained when considering the linear demand function.

As listed in Table 3, the market price of Product 2 is $800 in both cases; the market price of Product 1 is $489 based on the linear demand function whereas it is $614 based on the complementarity demand function. These results implies that in the latter case, the company increases the sales of Product 2 by raising the market price of Product 1. The profit obtained from the sales of Product 2 in this case is higher than that obtained by selling both products at Locations 1 and 2 in the linear demand function case. Further, we find that $d_1(p^1, p^2) = 3600 - 40p^1 + 10p^2 < 0$ if $p^1 = 614, p^2 = 800$; that is, the optimal decision is attained only at points outside the non-negative price domain of the linear demand function case.

In conclusion, these results show that restriction of admissible prices may lead to inferior decisions, which implies some profit loss. This suggests the necessity of considering a complementarity demand function when making location decisions.

6.2. A three-product scenario with two substitutable products and one complementary product

For this scenario, we again consider one demand domain and two candidate locations. Different from the previous scenario, we consider two substitutable products here (i.e., Products $1'$ and $2'$) and one complementary product (i.e., Product $3'$). For each product, we assume the two candidate locations have identical production cost functions: $G_i^k(s) = 0.42s + 0.2s^2 (k = 1', 2')$, and $G_i^{3'}(s) = 0.45s + 0.225s^2, i = 1,2$. The linear demand function takes the form $d(p^1, p^2, p^3) = \begin{pmatrix} 1200 \\ 2400 \\ 2400 \end{pmatrix} - \begin{pmatrix} 40 & -10 & 20 \\ -10 & 80 & 30 \\ 20 & 30 & 60 \end{pmatrix} \begin{pmatrix} p^1 \\ p^2 \\ p^3 \end{pmatrix}$. It is easy to know that the coefficient matrix of the linear demand function is a P-matrix and satisfies the assumption. Thus, there exists a unique projected price $B(p)$ in Ω.

Some parameters used here can be found in Table 2. The parameters of substitutable Products $1'$ and $2'$ are the same as that of Product 1; the parameters of complementary Product $3'$ are the same as that of Product 2. We summarize the results in Table 4. With this experiment, we conclude that the proposed model is viable to the MCCFLP with a group of products containing substitutable or complementary products or products of both types.

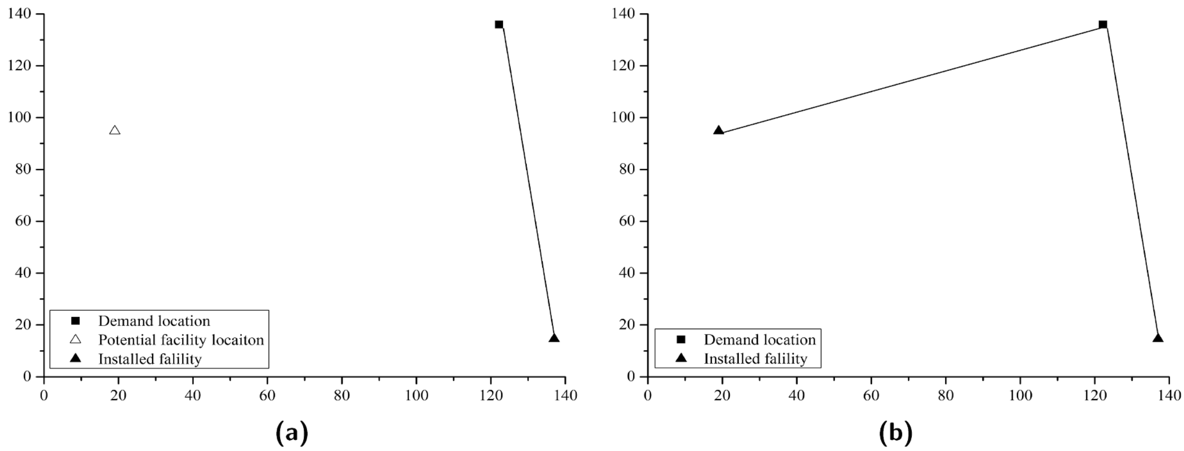

Fig. 3. Solution when considering the (a) complementarity demand function; (b) linear demand function.

Table 3
Optimal location and pricing results.

	MCCFLP with complementarity demand function	MCCFLP with linear demand function
Demand for Product 1	0	2
Market price for Product 1	614	489
Demand for Product 2	356	355
Market price for Product 2	800	800
Supply from Facility 1 for Product 1	–	1
Supply from Facility 1 for Product 2	–	2
Supply from Facility 2 for Product 1	0	1
Supply from Facility 2 for Product 2	356	354
Supply from Facility 2 for Product 2	356	354
Profit ($)	24359	14090

Note. "–" implies that the facility is not installed.

Table 4
Optimal location and pricing results.

	Price	Supply from Facility 1	Supply from Facility 2
Product1′	115	171	–
Product2′	112	167	–
Product3′	55	361	–
			Profit $885920

Note. "–" implies that the facility is not installed.

6.3. Evaluating the effect of carbon limit on carbon emission and profit

In this experiment, we consider a more complex scenario with two demand regions. Our focus is to assess the impact of carbon emission limit on carbon offset and subsequent profit. We set the baseline carbon emission limit to be 100 kg. The optimal solution for the corresponding baseline MCCFLP instance implies a total emission amount of 73.3 kg of CO_2 and a profit of $61735. To assess the effect of carbon emission limit, we decrease the limit from 100 kg at a decrement of 10 kg in each experiment. We consider two cases: carbon trading allowed and prohibited, with results reported in Figs. 4 and 5.

If carbon trading is allowed, the amount of carbon offset purchased would decline significantly as the emission limit increases over a wide range (i.e., from 0 to more than 70). By contrast, the profit would be insensitive to the limit change (Fig. 4). When carbon trading is prohibited, decreasing the limit on carbon emission would lead to a lower profit (Fig. 5). We report more detailed experimental results in Table 5.

If the government prohibits carbon trading, as if with a reduced carbon emission limit, then the company's profit and emission would essentially have the same magnitude of reduction. Thus, with a reduced carbon emission limit, carbon trading would not hinder the company's development. When the government allows carbon trading, under tightened limit, the company would consistently purchase enough carbon offset to maintain its carbon emission capacity at a constant level. Interestingly, carbon emission can decrease significantly even with a small penalty on the profit: the additional carbon offset pays a price on the profit, such that the profit would fall by 1.3% if the carbon limit drops to 0 kg. Therefore, the company would purchase additional carbon offset from the market when the allowed emission limit is insufficient to maintain production and logistic activities. This pattern confirms the role of carbon trading in reducing carbon footprint.

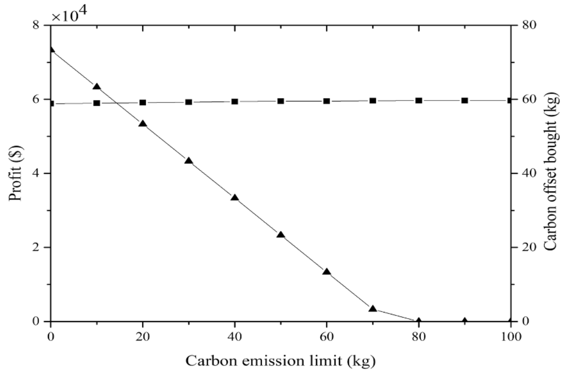

Fig. 4. Effect of carbon emission limit (carbon trading allowed).

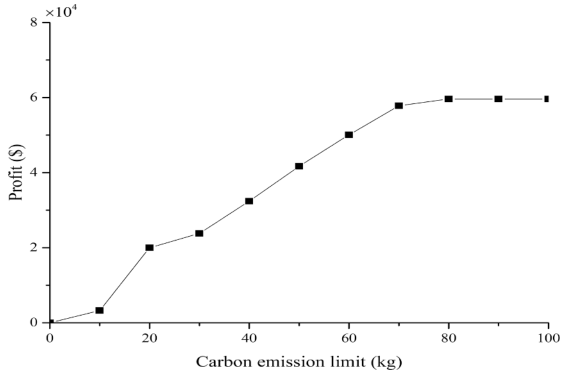

Fig. 5. Effect of carbon emission limit (carbon trading prohibited).

Table 5
Detailed sensitivity analysis results for two cases.

Carbonemissionlimit (kg)	Carbon trading prohibited			Carbon trading allowed		
	Profit ($)	Decrease in carbon emission(%)	Decrease in profit(%)	Profit ($)	Carbon offset bought(kg)	Decrease in profit(%)
70	57845	4.5	3.0	59591	3.3	0.08
60	50063	18.2	16.1	59505	13.3	0.23
50	41697	31.8	30.1	59497	23.3	0.24
40	32388	45.4	45.7	59386	33.3	0.43
30	23804	59.1	60.1	59243	43.3	0.67
20	20000	72.7	66.5	59105	53.3	0.90
10	3253	86.4	94.6	58960	63.3	1.14
0	0	100	100	58840	73.3	1.34

6.4. Evaluating the effect of carbon price on the amount of carbon traded

In this section, we use the same scenario as in Section 6.3 with an emission limit of 50 kg to illustrate the effect of carbon price on the amount of carbon traded in the market. The baseline carbon price is set to be $150/ton. To assess the impact of carbon price, we vary the price from $0/ton to 2500$/ton. We display our findings in Fig. 6.

When the carbon price is $0/ton, the amount of carbon offset purchased is 23.3 kg. As the price increases (i.e., from $0/ton to $1860/ton), the amount of carbon offset purchased decreases. When the carbon price reaches $1860/ton, the amount of carbon offset purchased would reach 0. As the carbon price continues to climb, the company would shift from a carbon buyer to a carbon seller. Carbon price thus has an impact on emission reduction. However, excessive price would cause more companies to become carbon sellers, even with the possibility of producing few products. This trend implies that companies would likely exit the market. In conclusion, although the carbon price is affected by supply and demand in the market, government intervention may still be needed for company's long-term development when the carbon price is set too high.

6.5. Computational results for larger instances

This section reports computational results on solving larger MCCFLP instances with the branch-and-refine algorithm. We compare the computational time with that with the branch-and-bound algorithm in He et al. (2014). We consider the scenario of two substitutable products (similar to the scenario in Section 6.1). All test instances are generated randomly. Different from previous experiments, we consider more candidate facility locations and demand domains with these instances. We run the algorithm five times for each instance and report the averaged CPU time in Table 6. From left to right, Table 6 reports the instance number, the number of candidate locations, the number of demand domains, and the instance size, i.e., the number of variables and constraints in the corresponding model (e'), as well as the computational time of the branch-and-refine algorithm and the branch-and-bound algorithm (labeled "b&r" and "b&b").

For Instances 1–4, we consider scenarios with six candidate facility locations and increase the number of demand domains from 6 to 9. Compared to Instance 1, extra CPU time required for Instances 2–4 are 65s, 70s, and 183s, respectively. For Instances 5–7, we consider scenarios with six demand domains and increase the number of candidate locations from 8 to 10. Compared to Instance 5, extra CPU time required for Instances 6–7 are 94s and 166s, respectively. The computational time increases somewhat noticeably. For Instances 8–13, we increase the number of candidate locations from 11 to 16 and the number of demand domains from 8 to 13. Compared to Instance 8, extra CPU time required for Instances 9–13 are 140s, 321s, 537s, 791s, and 1018s, respectively. Small to medium instances can be solved within a reasonable amount of time. Nevertheless, solving large instances, especially those with a

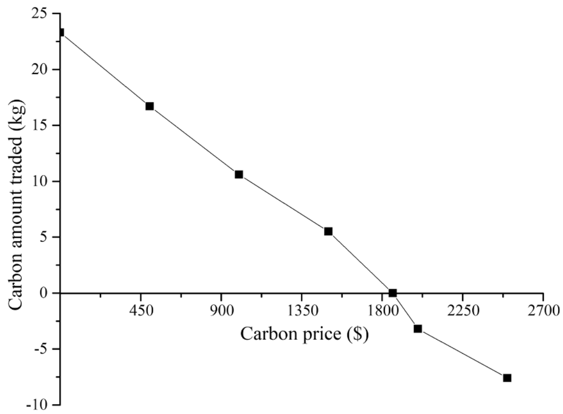

Fig. 6. Effect of carbon price on the amount of carbon traded.

Table 6
Computational results.

InstanceNo.	Candidatefac. loc.	Dem.dom.	Instance size		Time (sec)	
			var.	con.	b&r	b&b
1	6	6	1954	454	253	729
2	6	7	2278	518	328	924
3	6	8	2602	582	398	1138
4	6	9	2926	576	581	1502
5	8	6	2604	637	482	1392
6	9	6	2929	646	576	1526
7	10	6	3254	698	742	2113
8	11	8	4767	987	987	2841
9	12	9	5848	1192	1127	–
10	13	10	7037	1417	1448	–
11	14	11	8332	1662	1985	–
12	15	12	9737	1927	2776	–
13	16	13	10418	2212	3794	–

Note. "–" implies that the branch-and-bound algorithm does not terminate within 1 h.

large number of products, may be computationally expensive. However, compared to the algorithm in He et al. (2014), the proposed branch-and-refine algorithm is still much faster. The larger an instance is, the more obvious the advantage is. However, when the instance increases to 16 candidate facility locations and 13 demand domains, the branch-and-refine algorithm does not terminate within 1 h, thus motivating the development of a more efficient algorithm, e.g., decomposition-based algorithms.

7. Conclusions

In this paper, we investigate the effect of incorporating the complementary demand function in multi-commodity capacitated facility location optimization. To conform to the recent trend of requiring carbon emission reduction, we formulate the MCCFLP in the context of sustainable supply chain design; the corresponding model thus includes carbon trading scheme. The resultant problem, a joint optimization of location and pricing decisions, is a 0–1 mixed-integer non-concave quadratic program with equilibrium constraints. Acknowledging that the optimization model is difficult to solve, we transform it into a 0–1 mixed-integer concave program with linear constraints and design a branch-and-refine algorithm with global convergence.

With the experiments, we conclude that the proposed algorithm works more efficiently than a state-of-the-art branch-and-bound algorithm. Our model analysis and numerical example analysis provide the following insight to practitioners:

(1) Incorporating the complementarity demand function into facility location problems can lead to more profitable decisions for monopoly companies than commonly assumed demand-price relationships in the literature.
(2) A small penalty on profit may help companies maintain a necessary level of production and logistic activities. This measure could in turn alleviate a company's concerns about implementing carbon emission reduction mechanisms.
(3) A substantial fluctuation in carbon price could shift a company from a carbon buyer to a carbon seller. Government intervention may therefore still be needed.

Our findings point out new research questions and directions for the future. First, we plan to develop a more efficient algorithm for large MCCFLP instances. In particular, the decomposable structure of the problem warrants further exploration. Second, we intend to characterize a demand function that reflects the reality for a wide range of price. Third, we will consider possible model extensions, such as considering carbon emission on the production side and customer diverse interests on low-carbon products.

CRediT authorship contribution statement

Weiwei Liu: Conceptualization, Methodology, Formal analysis, Writing - original draft. **Nan Kong:** Supervision, Validation, Writing - review & editing. **Mingzheng Wang:** Conceptualization, Supervision, Validation. **Lingling Zhang:** Validation, Writing - review & editing.

Acknowledgment

This research is partly supported by the National Natural Science Foundation of China (71 671 023), the Key Program of National Natural Science Foundation of China (71 931 009), and the Science Fund for Creative Research Groups of the National Natural Science Foundation of China (71421001). This research is also partly supported by the U.S. National Science Foundation grant (1761022). The authors sincerely thank the Editors and four anonymous Reviewers for their constructive and insightful comments, which helped to improve the quality of the paper considerably.

Appendix A. Proof of Proposition 1

The objective function of model (c)

$$f(p^k, y_{ij}^k, z_i, i \in I, j \in J, k \in K)$$

$$= \sum_{k\in K}(\sum_{i\in I}\sum_{j\in J} y_{ij}^k)p^k - \sum_{k\in K}\sum_{i\in I}[\gamma_i^k \sum_{j\in J} y_{ij}^k + \frac{\delta_i^k}{2}(\sum_{j\in J} y_{ij}^k)^2]$$

$$-\sum_{i\in I} c_i z_i - \sum_{k\in K}\sum_{i\in I}\sum_{j\in J} t_{ij}^k y_{ij}^k - p_c[\sum_{k\in K}\sum_{i\in I}\sum_{j\in J} g_{ij}^k y_{ij}^k - E],$$

is a quadratic function, which is second-order differentiable, and the quadratic term is

$$\sum_{k\in K}[\sum_{i=1}^{m}\sum_{j=1}^{n} y_{ij}^k p^k - \sum_{i=1}^{m}\frac{\delta_i^k}{2}(\sum_{j=1}^{n} y_{ij}^k)^2].$$

Next, we transform the quadratic term into the corresponding quadratic form. Let E be a matrix whose elements are all 1 and e be a column vector whose elements are all 1.

For $\forall k \in K$, let

$$Y_{ik} = (y_{i1}^k, y_{i2}^k, \cdots, y_{in}^k)^T, i = 1, 2, \cdots, m;$$

$$Y_k = (y_{11}^k, y_{12}^k, \cdots, y_{1n}^k; y_{21}^k, y_{22}^k, \cdots, y_{2n}^k; \cdots; y_{m1}^k, y_{m2}^k, \cdots, y_{mn}^k)^T = (Y_{1k}^T, Y_{2k}^T, \cdots, Y_{mk}^T);$$

$$\begin{aligned} X = \; & (y_{11}^1, y_{12}^1, \cdots, y_{1n}^1; \cdots; y_{m1}^1, y_{m2}^1, \cdots, y_{mn}^1; y_{11}^2, y_{12}^2, \cdots, y_{1n}^2; \cdots; y_{m1}^2, y_{m2}^2, \cdots, y_{mn}^2; \\ & \cdots; y_{11}^l, y_{12}^l, \cdots, y_{1n}^l; \cdots; y_{m1}^l, y_{m2}^l, \cdots, y_{mn}^l; p^1; p^2; \cdots; p^l). \end{aligned}$$

Since

$$\begin{aligned} (\sum_{i=1}^{n} x_i)^2 = \; & [(x_1, x_2, \cdots, x_n)(1, 1, \cdots, 1)^T]^2 \\ = \; & (x_1, x_2, \cdots, x_n)(1, 1, \cdots, 1)^T(1, 1, \cdots, 1)(x_1, x_2, \cdots, x_n)^T \\ = \; & (x_1, x_2, \cdots, x_n)E_{n\times n}(x_1, x_2, \cdots, x_n)^T, \end{aligned}$$

then $\frac{\delta_i^k}{2}(\sum_{i=1}^{n} x_i)^2 = (Y_{ik})^T(\frac{\delta_i^k}{2}E_{n\times n})Y_{ik}$, thus

$$\sum_{i=1}^{m}\frac{\delta_i^k}{2}(\sum_{i=1}^{n}x_i)^2 = [Y_{1k}^T, Y_{2k}^T, \cdots, Y_{mk}^T]diag\{\frac{\delta_1^k}{2}E_{n\times n}, \frac{\delta_2^k}{2}E_{n\times n}, \cdots, \frac{\delta_m^k}{2}E_{n\times n}\}\begin{bmatrix} Y_{1k} \\ Y_{2k} \\ \cdots \\ Y_{mk} \end{bmatrix}$$

$$= [Y_{1k}^T, Y_{2k}^T, \cdots, Y_{mk}^T]E^k\begin{bmatrix} Y_{1k} \\ Y_{2k} \\ \cdots \\ Y_{mk} \end{bmatrix},$$

where

$$E^k \triangleq diag\{\frac{\delta_1^k}{2}E_{n\times n}, \frac{\delta_2^k}{2}E_{n\times n}, \cdots, \frac{\delta_m^k}{2}E_{n\times n}\} = \begin{bmatrix} \frac{\delta_1^k}{2}E_{n\times n} & 0_{n\times n} & \cdots & 0_{n\times n} \\ 0_{n\times n} & \frac{\delta_2^k}{2}E_{n\times n} & \cdots & 0_{n\times n} \\ \cdots & \cdots & \cdots & \cdots \\ 0_{n\times n} & 0_{n\times n} & \cdots & \frac{\delta_m^k}{2}E_{n\times n} \end{bmatrix}.$$

Moreover,

$$(\sum_{i=1}^{m}\sum_{j=1}^{n}y_{ij}^k)p^k = \sum_{i=1}^{m}\sum_{j=1}^{n}(\frac{1}{2}y_{ij}^k)p^k + (\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}y_{ij}^k)p^k + (Y_k^T, p^k)(\frac{1}{2}p^k, \cdots, \frac{1}{2}p^k, \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n}y_{ij}^k)$$

$$= [Y_k^T, p^k]\begin{bmatrix} 0_{mn\times mn} & \frac{1}{2}e_{mn} \\ \frac{1}{2}e_{mn}^T & 0 \end{bmatrix}\begin{bmatrix} Y_k^T \\ p^k \end{bmatrix},$$

then

$$\sum_{k\in K}\left[\sum_{i=1}^{m}\frac{\delta_i^k}{2}(\sum_{j=1}^{n}y_{ij}^k)^2 - \sum_{i=1}^{m}\sum_{j=1}^{n}y_{ij}^k p^k\right]$$

$$= X^T\begin{bmatrix} diag\{E^1, \cdots, E^l\} & diag\{(-\frac{1}{2})e_{mn}, \cdots, (-\frac{1}{2})e_{mn}\} \\ diag\{(-\frac{1}{2})e_{mn}^T, \cdots, (-\frac{1}{2})e_{mn}^T\} & diag\{0, \cdots, 0\} \end{bmatrix}X.$$

Based on the above analysis, we obtain the negative Hessian matrix of $f(p^k, y_{ij}^k, z_i, i\in I, j\in J, k\in K)$ as

$$\begin{bmatrix} diag\{E^1, \cdots, E^l\} & diag\{(-\frac{1}{2})e_{mn}, \cdots, (-\frac{1}{2})e_{mn}\} \\ diag\{(-\frac{1}{2})e_{mn}^T, \cdots, (-\frac{1}{2})e_{mn}^T\} & diag\{0, \cdots, 0\} \end{bmatrix}.$$

A matrix is positive semidefinite if all its principal minors are non-negative. Obviously, this matrix is not positive semidefinite because there are negative principal minors. For instance, for the principal minor consisting of row 1, row $2mn+1$, column 1, column $2mn+1$, we have $\begin{vmatrix} \frac{\delta_1^1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{vmatrix} = -\frac{1}{4}$. Then the objective function of (c) is not concave.

Appendix B. Proof of Theorem 1

It suffices to prove that $\sum_{k\in K}\sum_{i\in I}\left[\frac{\delta_i^k}{2}(\sum_{j\in J}y_{ij}^k)^2\right]$ is convex, which is the collection of quadratic terms in objective function $g(p^k,w_{ij}^k,y_{ij}^k,z_i,i\in I,j\in J,k\in K)$. For any $x,y\in R$ and $\alpha\in[0,1]$, we have.

$(\alpha x+(1-\alpha)y)^2-(\alpha x^2+(1-\alpha)y^2)=\alpha^2x^2+2\alpha(1-\alpha)xy+(1-\alpha)^2y^2-\alpha x^2-(1-\alpha)y^2$

$=(\alpha)(\alpha-1)x^2+2\alpha(1-\alpha)xy+(1-\alpha)(-\alpha)y^2=(\alpha)(\alpha-1)(x^2-2xy+y^2)\leqslant 0$, which implies that $(\alpha x+(1-\alpha)y)^2\leqslant\alpha x^2+(1-\alpha)y^2$.

Let $h(y_{ij}^k,i\in I,j\in J,k\in K):=\sum_{i\in I}\frac{\delta_i^k}{2}(\sum_{j\in J}y_{ij}^k)^2$. Next, we analyze two arbitrary points in the value range of y_{ij}^k, denoted by $y_{ij}^{k(1)}$ and $y_{ij}^{k(2)}$. Based on the above derivation, we have

$$\begin{aligned}
&h(\alpha y_{ij}^{k(1)}+(1-\alpha)y_{ij}^{k(2)},i\in I,j\in J,k\in K)\\
&=\sum_{i\in I}\frac{\delta_i^k}{2}\left[\sum_{j\in J}(\alpha y_{ij}^{k(1)}+(1-\alpha)y_{ij}^{k(2)})\right]^2=\sum_{i\in I}\frac{\delta_i^k}{2}\left[\alpha\sum_{j\in J}y_{ij}^{k(1)}+(1-\alpha)\sum_{j\in J}y_{ij}^{k(2)}\right]^2\\
&\leqslant\sum_{i\in I}\frac{\delta_i^k}{2}\left[\alpha(\sum_{j\in J}y_{ij}^{k(1)})^2+(1-\alpha)(\sum_{j\in J}y_{ij}^{k(2)})^2\right]\\
&=\alpha\sum_{i\in I}\frac{\delta_i^k}{2}(\sum_{j\in J}y_{ij}^{k(1)})^2+(1-\alpha)\sum_{i\in I}\frac{\delta_i^k}{2}(\sum_{j\in J}y_{ij}^{k(2)})^2\\
&=\alpha h(y_{ij}^{k(1)},i\in I,j\in J,k\in K)+(1-\alpha)h(y_{ij}^{k(2)},i\in I,j\in J,k\in K).
\end{aligned}$$

Hence, the function $h(y_{ij}^k,i\in I,j\in J,k\in K)$ is convex. Note that the sum of finite convex functions is still convex. Then $\sum_{k\in K}\sum_{i\in I}\left[\frac{\delta_i^k}{2}(\sum_{j\in J}y_{ij}^k)^2\right]=\sum_{k\in K}h(y_{ij}^k,i\in I,j\in J,k\in K)$ is convex. This implies that the function $-g(p^k,y_{ij}^k,z_i,i\in I,j\in J,k\in K)$ is also convex, which completes the proof.

Appendix C. Proof of Theorem 2

Because the objective function of model (d') is continuous, the gap between the objective function of model (e') and that of model (d') must decrease with additional breakpoints and continuously refined envelopes. Furthermore, the difference in the two continuous objective functions is still continuous; there thus must exist a maximum and minimum in the closed constraint set.

At each node associated with $X_\kappa\times Q_\kappa$, because the approximated part in the objective function of model (e') involves bilinear functions $y_{ij}^kp^k(i\in I,j\in J,k\in K)$, we simply need to analyze these bilinear functions. Per Lemma 1, for each $(y_{ij}^k,p^k,y_{ij}^kp^k)$ such that $y_{ij}^{k(1)}\leqslant y_{ij}^k\leqslant y_{ij}^{k(2)}$ $(0\leqslant y_{ij}^{k(1)}\leqslant y_{ij}^{k(2)}),p_{(1)}^k\leqslant p^k\leqslant p_{(2)}^k$ $(0\leqslant p_{(1)}^k\leqslant p_{(2)}^k)$, there exists a unique convex combination of the four points $(y_{ij}^{k(1)},p_{(1)}^k,y_{ij}^{k(1)}p_{(1)}^k)$, $(y_{ij}^{k(1)},p_{(2)}^k,y_{ij}^{k(1)}p_{(2)}^k)$, $(y_{ij}^{k(2)},p_{(1)}^k,y_{ij}^{k(2)}p_{(1)}^k)$ and $(y_{ij}^{k(2)},p_{(2)}^k,y_{ij}^{k(2)}p_{(2)}^k)$. Therefore, the gap between the objective function of model (e') and that of model (d') satisfies the following relationship:

$$\begin{aligned}
&|y_{ij}^kp^k-(\lambda_1y_{ij}^{k(1)}p_{(1)}^k+\lambda_2y_{ij}^{k(1)}p_{(2)}^k+\lambda_3y_{ij}^{k(2)}p_{(1)}^k+\lambda_4y_{ij}^{k(2)}p_{(2)}^k)|(\sum_{i=1}^4\lambda^i=1,\lambda^i\geqslant 0,i=1,2,3,4)\\
=\;&|(\lambda_1+\lambda_2+\lambda_3+\lambda_4)y_{ij}^kp^k-(\lambda_1y_{ij}^{k(1)}p_{(1)}^k+\lambda_2y_{ij}^{k(1)}p_{(2)}^k+\lambda_3y_{ij}^{k(2)}p_{(1)}^k+\lambda_4y_{ij}^{k(2)}p_{(2)}^k)|\\
=\;&|\lambda_1(y_{ij}^kp^k-y_{ij}^{k(1)}p_{(1)}^k)+\lambda_2(y_{ij}^kp^k-y_{ij}^{k(1)}p_{(2)}^k)+\lambda_3(y_{ij}^kp^k-y_{ij}^{k(2)}p_{(1)}^k)+\lambda_4(y_{ij}^kp^k-y_{ij}^{k(2)}p_{(2)}^k)|\\
\leqslant\;&\lambda_1|y_{ij}^kp^k-y_{ij}^{k(1)}p_{(1)}^k|+\lambda_2|y_{ij}^kp^k-y_{ij}^{k(1)}p_{(2)}^k|+\lambda_3|y_{ij}^kp^k-y_{ij}^{k(2)}p_{(1)}^k|+\lambda_4|y_{ij}^kp^k-y_{ij}^{k(2)}p_{(2)}^k|\\
\leqslant\;&\lambda_1(y_{ij}^{k(2)}p_{(2)}^k-y_{ij}^{k(1)}p_{(1)}^k)+\lambda_2\max\{y_{ij}^{k(2)}p_{(2)}^k-y_{ij}^{k(1)}p_{(2)}^k,y_{ij}^{k(1)}p_{(2)}^k-y_{ij}^{k(1)}p_{(1)}^k\}\\
&+\lambda_3\max\{y_{ij}^{k(2)}p_{(2)}^k-y_{ij}^{k(2)}p_{(1)}^k,y_{ij}^{k(2)}p_{(1)}^k-y_{ij}^{k(1)}p_{(1)}^k\}+\lambda_4(y_{ij}^{k(2)}p_{(2)}^k-y_{ij}^{k(1)}p_{(1)}^k)\\
\leqslant\;&\lambda_1(y_{ij}^{k(2)}p_{(2)}^k-y_{ij}^{k(1)}p_{(1)}^k)+\lambda_2(y_{ij}^{k(2)}p_{(2)}^k-y_{ij}^{k(1)}p_{(1)}^k)+\lambda_3(y_{ij}^{k(2)}p_{(2)}^k-y_{ij}^{k(1)}p_{(1)}^k)\\
&+\lambda_4(y_{ij}^{k(2)}p_{(2)}^k-y_{ij}^{k(1)}p_{(1)}^k)\\
\leqslant\;&y_{ij}^{k(2)}p_{(2)}^k-y_{ij}^{k(1)}p_{(1)}^k.
\end{aligned}$$

To estimate the last right-hand side item in the above inequality, we select equidistant $\sigma+\kappa-2$ breakpoints in the corresponding dimension of $L_{y_{ij}^k}\leqslant y_{ij}^k\leqslant U_{y_{ij}^k}$ $(0\leqslant L_{y_{ij}^k}\leqslant U_{y_{ij}^k})$ and $L_{p^k}\leqslant p^k\leqslant U_{p^k}$ $(0\leqslant L_{p^k}\leqslant U_{p^k})$. That is, the breakpoints in dimension y_{ij}^k are chosen as $L_{y_{ij}^k}+i'\frac{U_{y_{ij}^k}-L_{y_{ij}^k}}{\sigma+\kappa-1}$, $i'=1,2,\cdots,\sigma+\kappa-2$, and the breakpoints in dimension p^k are chosen as $L_{p^k}+j'\frac{U_{p^k}-L_{p^k}}{\sigma+\kappa-1},j'=1,2,\cdots,\sigma+\kappa-2$.

Based on the above analysis, the maximal gap between the objective function of model (e') and that of model (d') is estimated as

$$(L_{y_{ij}^k} + (i' + 1)\frac{U_{y_{ij}^k} - L_{y_{ij}^k}}{\sigma + \kappa - 1})(L_{p^k} + (j' + 1)\frac{U_{p^k} - L_{p^k}}{\sigma + \kappa - 1}) - (L_{y_{ij}^k} + i'\frac{U_{y_{ij}^k} - L_{y_{ij}^k}}{\sigma + \kappa - 1})(L_{p^k} + j'\frac{U_{p^k} - L_{p^k}}{\sigma + \kappa - 1})$$
$$= L_{y_{ij}^k}\frac{U_{y_{ij}^k} - L_{y_{ij}^k}}{\sigma + \kappa - 1} + L_{p^k}\frac{U_{p^k} - L_{p^k}}{\sigma + \kappa - 1} + \frac{(U_{y_{ij}^k} - L_{y_{ij}^k})(U_{p^k} - L_{p^k})}{(\sigma + \kappa - 1)^2}(i' + j' + 1)$$

in each domain

$$[L_{y_{ij}^k} + i'\frac{U_{y_{ij}^k} - L_{y_{ij}^k}}{\sigma + \kappa - 1}, L_{y_{ij}^k} + (i' + 1)\frac{U_{y_{ij}^k} - L_{y_{ij}^k}}{\sigma + \kappa - 1}] \times [p^k + j'\frac{U_{p^k} - L_{p^k}}{\sigma + \kappa - 1}, p^k + (j' + 1)\frac{U_{p^k} - L_{p^k}}{\sigma + \kappa - 1}]$$

where $i', j' = 0, 1, 2, \cdots, \sigma + \kappa - 2$.

Given that $i', j' = 0, 1, 2, \cdots, \sigma + \kappa - 2$, one has $i' + j' + 1 \leqslant 2\sigma + 2\kappa - 3$. Thus, the maximal gap between the objective function of model (e') and that of model (d') is further estimated as

$$L_{y_{ij}^k}\frac{U_{y_{ij}^k} - L_{y_{ij}^k}}{\sigma + \kappa - 1} + L_{p^k}\frac{U_{p^k} - L_{p^k}}{\sigma + \kappa - 1} + \frac{(U_{y_{ij}^k} - L_{y_{ij}^k})(U_{p^k} - L_{p^k})}{(\sigma + \kappa - 1)^2}(2\sigma + 2\kappa - 3)$$

in each domain $L_{y_{ij}^k} \leqslant y_{ij}^k \leqslant U_{y_{ij}^k}, L_{p^k} \leqslant p^k \leqslant U_{p^k}$.

Because $L_{y_{ij}^k}, U_{y_{ij}^k}, L_{p^k}, U_{p^k}$ are constant values, we have

$$\lim_{\kappa \to \infty} L_{y_{ij}^k}\frac{U_{y_{ij}^k} - L_{y_{ij}^k}}{\sigma + \kappa - 1} + L_{p^k}\frac{U_{p^k} - L_{p^k}}{\sigma + \kappa - 1} + \frac{(U_{y_{ij}^k} - L_{y_{ij}^k})(U_{p^k} - L_{p^k})}{(\sigma + \kappa - 1)^2}(2\sigma + 2\kappa - 3) = 0.$$

We know from the above limit that for some given tolerance $\varepsilon > 0$, there exists a positive integer T such that

$$|-g(p^k, w_{ij}^k, y_{ij}^k, z_i, i \in I, j \in J, k \in K) - (-f(p^k, y_{ij}^k, z_i, i \in I, j \in J, k \in K))| < \frac{\varepsilon}{2}$$

holds for any $\kappa > T$.

Based on the construction method of Q_κ, the number of integer components in Q_κ increases as κ increases. Furthermore, the number of all integer variables is finite. Thus, after $\kappa(> T)$ steps, the algorithm will stop; that is, $|z_{LPR_\kappa} - z_{NLP_\kappa}| \leqslant \varepsilon$ must be satisfied. As z_{LPR_κ} and z_{NLP_κ} are the lower and upper bounds of model (d') in $X_\kappa \times Q_\kappa$, respectively, then the difference between the optimal value of model (e') and that of model (d') in $X_\kappa \times Q_\kappa$ is less than $\frac{\varepsilon}{2}$. As model (d') and the original problem are equivalent, the gap between the optimal value of model (e') and that of the original problem is less than ε.

References

Abdallah, T., Diabat, A., Simchi-Levi, D., 2010. A carbon sensitive supply chain network problem with green procurement. In: The 40th International Conference on Computers & Indutrial Engineering. IEEE, pp. 1–6.
Akyüz, M., Öncan, T., Altinel, Í., 2012. Efficient approximate solution methods for the multi-commodity capacitated multi-facility Weber problem. Comput. Oper. Res. 39 (2), 225–237.
Akyüz, M., Öncan, T., Altinel, Í., 2012. Solving the multi-commodity capacitated multi-facility Weber problem using Lagrangean relaxation and a subgradient-like algorithm. J. Oper. Res. Soc. 63 (6), 771–789.
Akyüz, M., Öncan, T., Altinel, Í., 2019. Branch and bound algorithms for solving the multi-commodity capacitated multi-facility Weber problem. Ann. Oper. Res. 279 (1–2), 1–42.
Askin, R., Baffo, I., Xia, M., 2014. Multi-commodity warehouse location and distribution planning with inventory consideration. Int. J. Prod. Res. 52 (7), 1897–1910.
Avella, P., Boccia, M., 2009. A cutting plane algorithm for the capacitated facility location problem. Comput. Optim. Appl. 43 (1), 39–65.
Azevedo, S.G., Carvalho, H., Duarte, S., Cruz-Machado, V., 2012. Influence of green and lean upstream supply chain management practices on business sustainability. IEEE Trans. Eng. Manage. 59 (4), 753–765.
Berman, A., Plemmons, R., 1994. Nonnegative Matrices in the Mathematical Sciences. Society for Industrial and Applied Mathematics.
Bernstein, F., Federgruen, A., 2004. Dynamic inventory and pricing models for competing retailers. Nav. Res. Log. 51 (2), 258–274.
Boyer, M., Moreaux, M., 1987. On Stackelberg equilibria with differentiated products: the critical role of the strategy space. J. Ind. Econ. 36 (2), 217–230.
Cachon, G., 2014. Retail store density and the cost of greenhouse gas emissions. Manage. Sci. 60 (8), 1907–1925.
Chan, H., Choi, T., Cai, Y., Shen, B., 2018. Environmental taxes in newsvendor supply chains: a mean-downside-risk analysis. IEEE Trans. Syst., Man, Cybern., Syst., 1–14.
Chen, Z., Bidanda, B., 2019. Sustainable manufacturing production-inventory decision of multiple factories with jit logistics, component recovery and emission control. Transp. Res. Part E: Logist. Transp. Rev. 128, 356–383.
Choi, T.M., Cai, Y.J., 2020. Impacts of lead time reduction on fabric sourcing in apparel production with yield and environmental considerations. Ann. Oper. Res. 290, 521–542.
Choi, T.M., Cai, Y.J., Shen, B., 2019. Sustainable fashion supply chain management: a system of systems analysis. IEEE Trans. Eng. Manage. 66 (4), 730–745.
DeMiguel, V., Xu, H., 2009. A stochastic multiple-leader Stackelberg model: analysis, computation, and application. Oper. Res. 57 (5), 1220–1235.
Diabat, A., Simchi-Levi, D., 2009. A carbon-capped supply chain network problem. In: 2009 IEEE International Conference on Industrial Engineering and Engineering Management. IEEE, pp. 523–527.

Dong, H., 2016. Relaxing nonconvex quadratic functions by multiple adaptive diagonal perturbations. SIAM J. Optimiz. 26 (3), 1962–1985.
Drake, D.F., Kleindorfer, P.R., Wassenhove, L.N.V., 2016. Technology choice and capacity portfolios under emissions regulation. Prod. Oper. Manag. 25 (6), 1006–1025.
Elhedhli, S., Merrick, R., 2012. Green supply chain network design to reduce carbon emissions. Transp. Res. Part D 17, 370–379.
Facchinei, F., Pang, J., 2007. Finite-dimensional Variational Inequalities and Complementarity Problems. Springer Science & Business Media.
Federgruen, A., Hu, M., 2015. Multi-product price and assortment competition. Oper. Res. 63 (3), 572–584.
Federgruen, A., Hu, M., 2016. Technical note-sequential multiproduct price competition in supply chain networks. Oper. Res. 64 (1), 135–149.
Federgruen, A., Hu, M., 2019. Stability in a general oligopoly model. Nav. Res. Log. 66, 90–102.
Fernández, E., Landete, M., 2015. Fixed-charge facility location problems. In: Location Science. Springer, pp. 44–77.
Geoffrion, A., 1972. Generalized Benders decomposition. J. Optimiz. Theory. App. 10 (4), 237–260.
Geoffrion, A., Graves, G., 1974. Multicommodity distribution system sesign by Benders decomposition. Manage. Sci. 20, 822–844.
Ghaddar, B., Naoum-Sawaya, J., 2012. Environmentally friendly facility location with market competition. J. Oper. Res. Soc. 63 (7), 899–908.
Görtz, S., Klose, A., 2012. A simple but usually fast branch-and-bound algorithm for the capacitated facility location problem. Informs J. Comput. 24 (4), 597–610.
Guastaroba, G., Speranza, M., 2012. Kernel search for the capacitated facility location problem. J. Huristics 18 (6), 877–917.
Harkness, J., ReVelle, C., 2003. Facility location with increasing production costs. Eur. J. Oper. Res. 145 (1), 1–13.
He, H., Daume, H., Eisner, J.M., 2014. Learning to search in branch and bound algorithms. In: Advances in Neural Information Processing Systems, pp. 3293–3301.
Helmberg, C., Rendl, F., Weismantel, R., 2002. A semi-definite programming approach to the quadratic knapsack problem. J. Comb. Optim. 4 (2), 197–215.
Hijazi, H., Bonami, P., Ouorou, A., 2014. An outer-inner approximation for separable mixed-integer nonlinear programs. Informs J. Comput. 26 (1), 31–44.
Jabbarzadeh, A., Fahimnia, B., Rastegar, S., 2019. Green and resilient design of electricity supply chain networks: a multiobjective robust optimization approach. IEEE Trans. Eng. Manage. 66 (1), 52–72.
Kalcsics, J., Melo, T., Nickel, S., Schmid-Lutz, V., 2000. Facility location decisions in supply chain management. Oper. Res. Proc. 1999, 467–472.
Kronqvist, J., Lundell, A., Westerlund, T., 2016. The extended supporting hyperplane algorithm for convex mixed-integer nonlinear programming. J. Global. Optim. 62 (2), 249–272.
Kübler, D., Müller, W., 2002. Simultaneous and sequential price competition in heterogeneous duopoly markets: experimental evidence. Int. J. Ind. Rrgan. 20 (10), 1437–1460.
Leyffer, S., Sartenaer, A., Wanufellez, E., 2008. Branch-and-refine for mixed-integer nonconvex global optimization. Preprint ANL/MCS-P1547-0908.
Luo, Z., Pang, J., Ralph, D., 1996. Mathematical Programs with Equilibrium Constraints. Cambridge University Press, Cambridge.
Martin, A., Möller, M., Moritz, S., 2006. Mixed integer models for the stationary case of gas network optimization. Math. Program. 105 (2–3), 563–582.
Milgrom, P., Roberts, J., 1990. Rationalizability, learning and equilibrium in games with strategic complementarities. Econometrica 58 (6), 1255–1277.
Nouira, I., Hammami, R., Frein, Y., Temponi, C., 2016. Design of forward supply chains: Impact of a carbon emissions-sensitive demand. Int. J. Prod. Econ. 173, 80–98.
Nowak, I., Breitfeld, N., Hendrix, E., Njacheun-Njanzoua, G., 2018. Decomposition-based inner-and outer-refinement algorithms for global optimization. J. Global. Optim. 72 (2), 305–321.
Phillis, Y.A., Kouikoglou, V.S., Manousiouthakis, V., 2010. Technology choice and capacity portfolios under emissions regulation. IEEE Syst. J. 4 (1), 15–25.
Saberi, S., 2018. Sustainable, multiperiod supply chain network model with freight carrier through reduction in pollution stock. Transp. Res. Part E: Logist. Transp. Rev. 118, 421–444.
Sharkey, T., Geunes, J., Romeijn, H., Shen, Z., 2011. Exact algorithms for integrated facility location and production planning problems. Nav. Res. Log. 58, 419–436.
Sherali, H., Adams, W., 2013. A Reformulation-Linearization Technique for Solving Discrete and Continuous Nonconvex Problems. Springer Science & Business Media.
Soon, W., Ang, K., Teo, K., 2014. Comparison of pricing models with simulated demand data. Electron. J. Math. Technol. 8 (3), 195–210.
Soon, W., Zhao, G., Zhang, J., 2009. Complementarity demand function and pricing models for multi-product markets. Eur. J. Appl. Math. 20 (5), 399–430.
Sun, X., Liu, C., Li, D., Gao, J., 2012. On duality gap in binary quadratic programming. J. Global. Optim. 53 (2), 255–269.
Sun, Z., Dababneh, F., Li, L., 2018. Joint energy, maintenance, and throughput modeling for sustainable manufacturing systems. IEEE Trans. Syst., Man, Cybern., Syst. 50 (6), 1–12.
Tang, C., Zhou, S., 2012. Research advances in environmentally and socially sustainable operations. Eur. J. Oper. Res. 223 (3), 585–594.
Tomlin, J., 1981. A suggested extension of special ordered sets to non-separable non-convex programming problems. North. Holl. Math. Stud. 59, 359–370.
Tsao, Y., Thanh, V., 2019. A multi-objective mixed robust possibilistic flexible programming approach for sustainable seaport-dry port network design under an uncertain environment. Transp. Res. Part E: Logist. Transp. Rev. 124, 13–39.
Vielma, J., Ahmed, S., Nemhauser, G., 2008. A lifted linear programming branch-and-bound algorithm for mixed-integer conic quadratic programs. Informs J. Comput. 20 (3), 438–450.
Vielma, J., Ahmed, S., Nemhauser, G., 2010. Mixed-integer models for nonseparable piecewise-linear optimization: unifying framework and extensions. Oper. Res. 58 (2), 303–315.
Wolf, D., Smeers, Y., 1997. A stochastic versions of a Stacklberg-Nash-Cournot equilibrium model. Manage. Sci. 43 (2), 190–197.
Wu, H., Dunn, S., 1995. Environmentally responsible logistics systems. Intern. J. Phys. Dis. Log. Manage. 25 (2), 20–38.
Zakeri, A., Dehghanian, F., Fahimnia, B., Sarkis, J., 2015. Carbon pricing versus emissions trading: a supply chain planning perspective. Int. J. Prod. Econ. 164, 197–205.
Zhang, Y., Snyder, L., Ralphs, T., Xue, Z., 2016. The competitive facility location problem under disruption risks. Transp. Res. Part E: Logist. Transp. Rev. 93, 453–473.