Modeling Resilient Modulus of Unsaturated Subgrade Soils under Concurrent Changes in Water Content and Temperature

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ABSTRACT

Seasonal variations and climatic events cause fluctuations of water content and temperature in shallow unsaturated soils. Such fluctuations can alter the resilient modulus (M_R) of subgrade, which is an important parameter in the design and evaluation of pavements. This paper presents a new model to determine M_R of unsaturated subgrade soils under concurrent changes in water content and temperature. The proposed analytical model offers the following two new features distinguishing it from alternative models: (1) the model separately accounts for two different soil water retention mechanisms, namely capillary and adsorption, which enables it to predict M_R over a wide range of suctions, and (2) it explicitly incorporates the effect of temperature in the calculation of M_R through employing temperature-dependent expressions for matric suction and the soil water retention curve (SWRC). The proposed model showed high accuracy when validated against experimentally measured M_R values for several different soils reported in the literature. The presented model is simple and can readily be employed in practice to determine M_R of subgrade soils under concurrent variations of water content and temperature.

INTRODUCTION

Resilient modulus of subgrade soil, M_R, is defined as the ratio of the applied deviatoric stress (σ_d) to the recoverable strain (ε_r) (Seed et al. 1962). It serves as an indication of the subgrade soil stiffness and its stress-strain behavior under traffic loads. M_R has been extensively used in the evaluation and design of pavements (e.g., AASHTO 2003). M_R of subgrade soil is not a constant material property but rather a function of both the soil type and characteristics and state variables such as degree of saturation, stress state, and temperature (e.g., Ng and Zhou 2014; Han and Vanapalli 2016). Seasonal variations and weather events (e.g., drought, rainfall) induce concurrent changes in water content and temperature within surface and near surface unsaturated soils. Such fluctuations alter M_R of unsaturated subgrade soils, which in turn can affect the performance of pavements.

Previous studies have used repeated load triaxial (RLT) testing to investigate the effect of stress level and water content on M_R (e.g., Sawangsuriya et al. 2009; Cary and Zapata 2011; Han and Vanapalli 2016). In general, M_R increases as the water content decreases (or suction increases), attributed to suction hardening effect. The majority of the existing experiments are performed over a low range of matric suction (high degrees of saturation) where capillarity is the only mechanism of soil water retention. However, more recent studies (e.g., Banerjee et al. 2020) showed that the trend of variation of M_R is completely different at low and high values of suction, and that the rate of hardening of M_R is significantly higher at higher suctions. Limited experimental studies have been conducted to examine the effect of temperature on M_R. Ng and Zhou (2014) performed a set of laboratory tests and showed that, at a constant water content, M_R has a higher magnitude at lower temperatures. Temperature-induced changes in M_R can be attributed to the temperature effects on matric suction and the soil water retention curve (SWRC) through changes in the surface tension of the pore water, soil–water contact angle, soil fabric, water absorption potential, and pore size distribution (Abdollahi and Vahedifard 2022).

Several predictive models have been proposed in the literature to estimate M_R as a function of the stress level and water content (or soil suction). The existing models include semiempirical models such as the Mechanistic-Empirical Pavement Design Guide (MEPDG) model (NCHRP 2004), models incorporating suction through the concept of effective stress like the Liang et al. (2008) model, or models that treat soil suction as an independent stress state variable like the Cary and Zapata (2011) model. Experimentally measured data (e.g., Banerjee et al. 2020, Han and Vanapalli 2016) have shown that the existing models fail to reasonably capture the variation of M_R with water content especially at high values of suction. This deficiency can be attributed to the failure of existing models to properly distinguish between M_R variation and hardening rate due to different water retention mechanisms (Abdollahi and Vahedifard 2022). In addition, the existing models do not explicitly consider the effect of temperature on soil suction and M_R.

To address the aforementioned limitations, this study aims to present a generalized model to calculate M_R of unsaturated subgrade soils while accounting for concurrent changes in water content and temperature. The model was recently proposed by Abdollahi and Vahedifard (2022) and separately accounts for two different soil water retention mechanisms, namely capillary and adsorption, which enables it to predict M_R over a wide range of suctions including very high suctions. Further, the model explicitly considers the effect of temperature by incorporating temperature-dependent expressions for matric suction and SWRC in the calculation of M_R. In this paper, the model is employed to demonstrate the effect of changes in water content and temperature on M_R. The proposed model is validated against experimentally measured M_R values for different soils reported in the literature and compared against two alternative models.

MODEL FORMULATION

Base Model. The base model is built upon the hypothesis that the variation of M_R with water content can be attributed to two different water mechanisms, i.e., capillary and adsorption. Considering a two-part SWRC model proposed by Lu (2016), it is known that the water content (θ) at any suction can be defined as the summation of capillary water content (θ_c) and adsorption water content (θ_a). The adsorption water content can be defined as (Lu 2016):

$$\theta_a = \theta_{a,max} \left\{ 1 - \left[\exp\left(-\frac{\psi_{max} + \psi}{\psi} \right) \right]^m \right\} \tag{1}$$

where ψ is the soil suction, $\theta_{a,max}$ is the maximum adsorption water content, and ψ_{max} is the maximum suction, and m is a parameter reflecting adsorption strength. Lu (2016) adopted the van Genuchten (1980) model to define capillary water content as:

$$\theta_c = \frac{1}{2}\left[1 - \operatorname{erf}\left(\sqrt{2}\frac{\psi - \psi_c}{\psi_c} \right) \right] \times [\theta_s - \theta_a(\psi)] \times [1 + (\alpha\psi)^n]^{\left(\frac{1}{n}-1\right)} \tag{2}$$

where erf () is an error function, θ_s is the saturated volumetric water content, α is a fitting parameter related to air entry suction, n is related to pore size distribution where large values of n reflect a relatively narrow pore size distribution, and ψ_c is the mean cavitation pressure. The first term on the right-hand side of the equation represents the cumulative distribution function corresponding to the standard normal distribution of ψ_c. The error function is used to minimize the error in the equation due to uncertainty in ψ_c.

Following the same logic and due to the close relation between M_R and θ, the overall resilient modulus at any suction is defined as follow (Abdollahi and Vahedifard 2022):

$$M_R = M_{R,c}(\theta) + M_{R,a}(\theta) \tag{3}$$

where $M_{R,C}(\theta)$ and $M_{R,a}(\theta)$ are the capillary and adsorptive parts, respectively, of the total M_R.

Fig. 1 schematically illustrates different components of the proposed model. In the model, $M_{R,c}(\theta)$ is defined as a power law function of θ as:

$$M_{R,c}(\theta) = \left(M_{R,SAT} - M_{Mc}\right)\left(\frac{\theta}{\theta_s}\right)^{m_M} + M_{Mc} \tag{4}$$

where $M_{R,SAT}$ is resilient modulus of subgrade soil when it is saturated, M_{Mc} is the maximum M_R due to capillarity at the dry state, and m_M is a fitting parameter controlling the rate of M_R hardening in the capillary zone. $M_{R,a}(\theta)$ is defined as:

$$M_{R,a}(\theta) = -r\theta + (M_{Md} - M_{Mc}) \tag{5}$$

where M_{Md} is the maximum M_R at dry state and r is the rate of adsorptive hardening defined as:

$$r = \frac{(M_{Md} - M_{Mc})\theta}{2\theta_{a,max}^{m}} \tag{6}$$

where $\theta_{a,max}^{m}$ is the water content after which and up to complete dry state, adsorption becomes the dominant mechanism controlling M_R. A detailed derivation of the base model is presented by Abdollahi and Vahedifard (2022).

Considering Effect of Temperature. In the proposed model, the effect of temperature on M_R is considered through its impact on matric suction and the SWRC. At a constant water content, increasing temperature will decrease the suction, inducing a downward shift in the SWRC (McCartney et al. 2019; Vahedifard et al. 2018, 2019). Changes in temperature affect the soil suction by altering the soil fabric and surface tension in low and high-water contents, respectively. Low suction within the soil matrix at higher temperatures contributes less to soil stiffness and therefore reduces M_R values.

The Freundlich model (Ponec et al. 1974; Jeppu and Clement 2012) is used to define the amount of adsorbate (liquid) on a flat adsorbent (solid) in thermodynamic energy equilibrium with the ambient adsorbate (in vapor phase) as follows:

$$\theta_a = \theta_{a,max}(RH)^{1/m} \tag{7}$$

where RH is the relative humidity. As defined previously, m represents the adsorption strength. Revil and Lu (2013) recast Eq. 7 by imposing the Kelvin-Laplace equation as:

$$\theta_a = \theta_{a,max}\left[\exp\left(-\frac{M_w\psi}{RT}\right)\right]^{1/m} \tag{8}$$

where M_w is the molar volume of water and equal to 1.8×10^{-5} m^{3}mol^{-1}, R is the universal gas constant and equal to 8.314 Jmol^{-1}K^{-1}, and T is temperature in Kelvin. Eq. 8 degenerates to Eq. 1 if $\frac{RT}{M_w} = \psi_{max}$. A temperature-dependent equation for matric suction can be expressed by (Grant and Salehzadeh 1996):

$$\psi = \psi_{T_r}\left(\frac{\beta + T}{\beta_{T_r} + T_r}\right) \tag{9}$$

where T_r is a reference temperature, ψ_{T_r} is suction at the reference temperature, β and β_{T_r} are regression parameters defined in terms of surface tension, contact angle, and enthalpy of immersion. Following the procedure outlined in Vahedifard et al. (2018, 2019), Eqs. 7, 8, and 9 can be used to obtain θ_a and θ_c. The calculated water content can then be used to determine M_R at different temperatures.

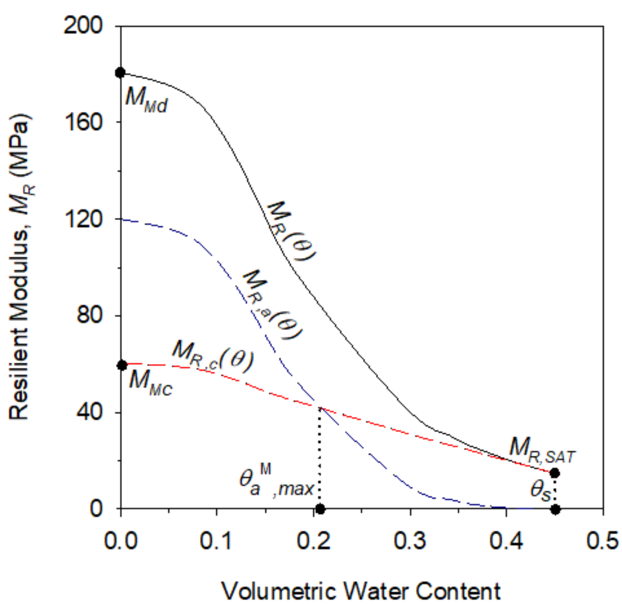

Figure 1. Schematic representation of the proposed generalized model for resilient modulus, M_R

VALIDATION OF THE PROPOSED MODEL

Experimentally measured results using RLT tests on 5 different compacted fine-grained subgrade soils (a total of 56 measured data points) are collected from the literature and used to evaluate the performance of the proposed model. Further, the performance of the model is compared with the MEPDG model and the Cary and Zapata (2011) model. The MEPDG model is used by several departments of transportation (DOTs) across the United States. The Cary and Zapata (2011) model is a result of a comprehensive set of experiments on granular materials and clayey sand over the suction range of 0-250 kPa. The MPEDG model is defined as:

$$\log\left(\frac{M_R}{M_{R,OPT}}\right) = a + \frac{b-a}{1+\exp[\ln\left(-\frac{b}{a}\right) + k_m(S - S_{OPT})]} \quad (10)$$

where $M_{R,OPT}$ is the resilience modulus at the optimum moisture content (OMC), S is the degree of saturation, S_{OPT} is the degree of saturation at OMC, $a = \min\log\left(M_R/M_{R,OPT}\right)$, $b = \max\log\left(M_R/M_{R,OPT}\right)$, and k_m is a fitting parameter. The Cary and Zapata (2011) model is presented in the following form:

$$M_R = n_1 P_a \left(\frac{\theta_{net} - 3\Delta u_{w-sat}}{P_a}\right)^{n_2} \left(\frac{\tau_{oct}}{P_a} + 1\right)^{n_3} \left(\frac{\psi_0 - \Delta\psi}{P_a} + 1\right)^{n_4} \quad (11)$$

where P_a is atmospheric pressure, $\theta_{net} = \theta_b - 3u_a$, Δu_{w-sat} is the buildup of pore-water pressure under saturated condition, ψ_0 is initial soil suction, $\Delta\psi$ is relative change of soil suction with respect to ψ_0 as a result of buildup of pore-water pressure under unsaturated conditions, in this case $\Delta u_{w-sat} = 0$, n_1 to n_4 are fitting parameters.

Table 1 summarizes the 5 data sets used in this section along with the proposed model, the MEPDG model, and the Cary and Zapata (2011) model fitting parameters. Considering that the MEPDG model and the Cary and Zapata (2011) model are both incapable of predicting M_R under varying temperature, they are not used to predict M_R values for the last test series. The root mean square error (*RMSE*) is used to statistically examine the predictive accuracy of the proposed model and compare its performance to the alternative models. *RMSE* is defined as:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(\widehat{M}_{R_i} - M_{R_i}\right)^2}{N}} \quad (12)$$

where M_{R_i} and $\widehat{M}_{R_i}$ are the measured and predicted resilient modulus at each water content, respectively, and N is the total number of measurements.

Validation of Base Model. The measured data from RLT experiments on the first 4 soils presented in Table 1 were used to validate the proposed model and to capture the variation of M_R over a wide range of soil water content (suction). The SWRC of these soils are presented in Fig. 2. The results were further used to compare the predictive accuracy of the proposed model with the widely-used MEPDG model. Data sets presented in this section include experiments conducted by Banerjee et al. (2020) and Sawangsuriya et al. (2009).

Banerjee et al. (2020) conducted suction-controlled RLT tests on silty and clayey samples over a wide range of suction using the axis translation technique (for low to moderate suction) and the vapor pressure technique (for high suction up to 600 MPa). They measured the variation of M_R for non-plastic sandy soil obtained from Denison, Texas and a mixture of locally available sandy-clayey soil and bentonite clay. Test results that were used in this study for the two aforementioned soils were obtained under confining pressure of 27.6 kPa and a maximum deviatoric stress of 41.4 kPa after 8 loading sequences. Banerjee et al. (2020) reported the saturated resilient modulus ($M_{R,SAT}$) for the silt and clay samples were 46.6 and 14.5 MPa, respectively. Figs. 3a and 3b show the measured values of $M_R$of the two soil samples corresponding to different soil water contents along with predicted values using two alternative models and the proposed model in this study.

Table 1. Parameters for the proposed model and two alternative models

	Proposed Model					MEPDG			Cary and Zapata (2011)			
Soil	θ^M_{ama}	M_{Mc} (MPa)	M_{Md} (MPa)	$M_{R,SAT}$ (MPa)	m_M	k_m	$M_{R,OPT}$	S_{OPT}	n_1	n_2	n_3	n_4
Silt[1]	0.12	204	785	46.6	0.60	6.12	87.1	0.65	1.67	0.03	-1.96	0.28
Clay[1]	0.21	70	223	14.5	0.84	7.12	26.2	0.83	0.17	0.03	-1.73	0.31
Duluth Slopes[2]	0.27	180	265	12	3.30	7.30	60.0	12	0.35	0.01	-2.14	0.48
MnRoad[2]	0.22	150	186	13	2.71	6.54	57.0	0.77	0.52	0.01	-1.67	0.28
Hong Kong Silt[3]	0.18	96	100	27	7.35	-	-	-	-	-	-	-

[1]measured data from Banerjee et al. (2020), [2] measured data from Sawangsuriya et al. (2009), [3] measured data from Ng and Zhou (2014)

Sawangsuriya et al. (2009) measured the M_R of subgrade soils from Minnesota. Their measured points for two clayey soils, namely MnRoad (CL) and Duluth Slopes (CH), are used in this study for validation purposes. All specimens were prepared at OMC and at a dry density corresponding to 98%-103% relative compaction. Based on NCHRP 1-28A (2004), a deviatoric stress of 41 kPa and a confining pressure of 14 kPa were applied to the specimens in order to measure their M_R values while soil suction was controlled using the axis-translation technique. Figs. 3c and 3d shows the measured values by Sawangsuriya et al. (2009) along with the proposed model in this study and two alternative models.

As Fig. 3 shows, the proposed model can capture the variation of M_R with water content (suction) and has an excellent prediction accuracy of above 99%. While the two alternative models can predict M_R with reasonable accuracy at low suction values (high water content), they considerably lose their accuracy at lower water contents. Both alternative models only consider the capillary water content and ignore the role of adsorption in water retention. Thus, when they are used to fit the data at low water contents (high suctions), using the method of least squares, they lose their accuracy (Fig. 3). The proposed model, on the other hand, considers two different mechanisms of capillary and adsorption for soil hardening and can capture the increased rate of soil hardening when it is approaching dryer states. In Fig. 3, the proposed model is decomposed to its consisting component $M_{R,c}$ and $M_{R,a}$. It is noted that at lower water contents, especially for the first two soils (Figs. 3a and 3b) where suction gained extremely high values, $M_{R,a}$ has considerably high values compared to $M_{R,c}$. Table 2 provides a comparison of the predictive accuracy (in terms *RMSE*) of the proposed model versus the two alternative models at ambient temperature.

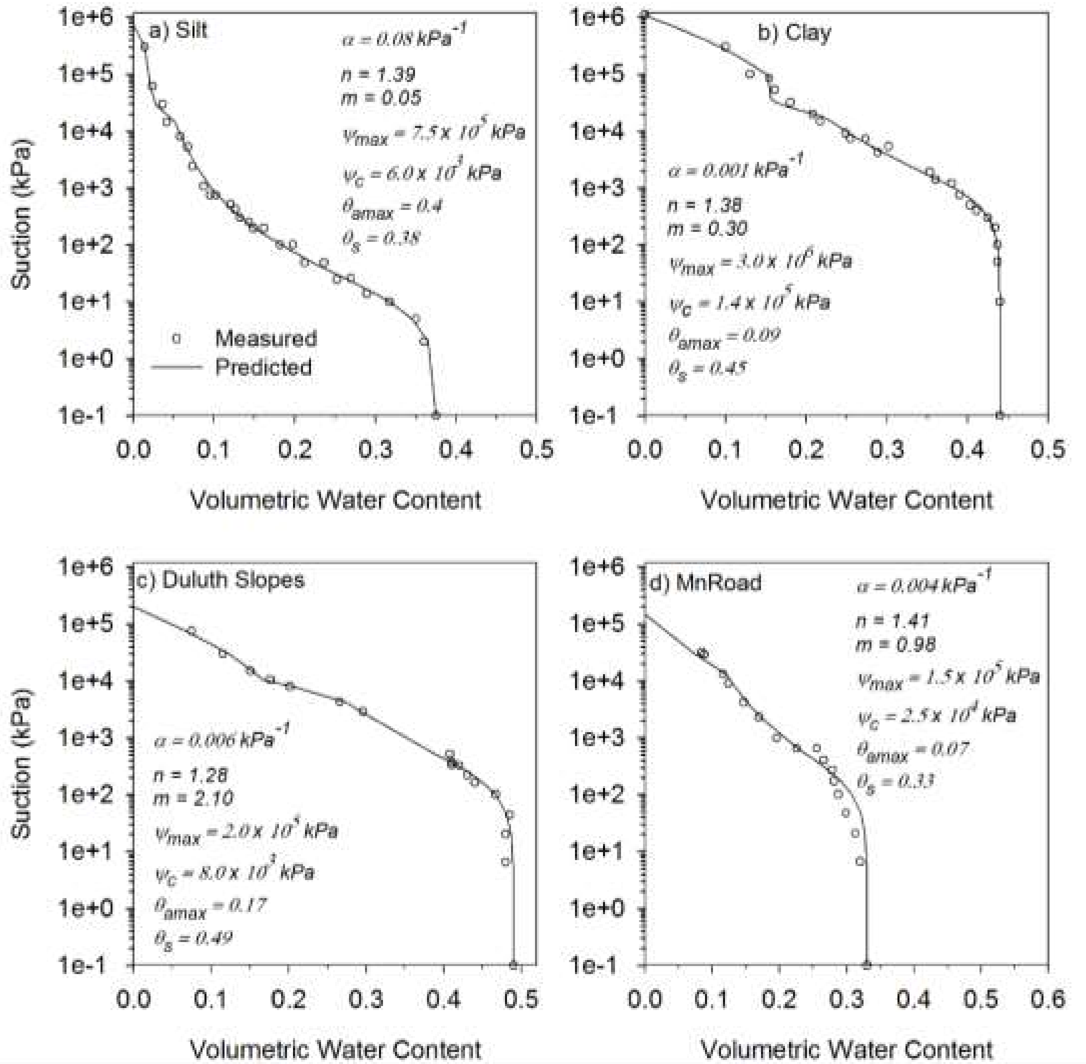

Figure 2. Measured and predicted SWRC for different soils: a) Silt, b) Clay, c) Duluth Slopes, and d) MnRoad

Validation at Elevated Temperatures. Ng and Zhou (2014) conducted seven suction and temperature controlled RLT tests on Hong Kong silt samples at an ambient temperature of 20°C and an elevated temperature of 40°C. The soil specimens were compacted at an initial water content of 16.3% in order to reach the maximum dry unit weight of 17.3 kN/m^3. Specimens had an initial suction of 95 kPa and then were gradually wetted to reach suctions of 60, 30, and 0 kPa. Fig. 4 shows the SWRC of Hong Kong silt along the wetting path. While no measured SWRC data at 40°C was presented in Ng and Zhou (2014), the SWRC at 40°C is predicted using Eqs. 1, 2, and 9. The specimens were tested under a confining pressure of 30 kPa and deviatoric stresses of 30, 40, 55, and 70 kPa. Suction was controlled using the axis-translation technique, while a heating system consisted of a thermostat, heater and thermocouple that was used to heat the specimen. Fig. 5 shows a comparison between the measured and predicted M_R values for four different σ_ds. As shown, M_R decreases with an increase in temperature. For each set, Fig. 5

includes *RMSE* of the proposed model considering the effect of temperature. Results show the capability of the proposed model in capturing the effect of temperature on M_R.

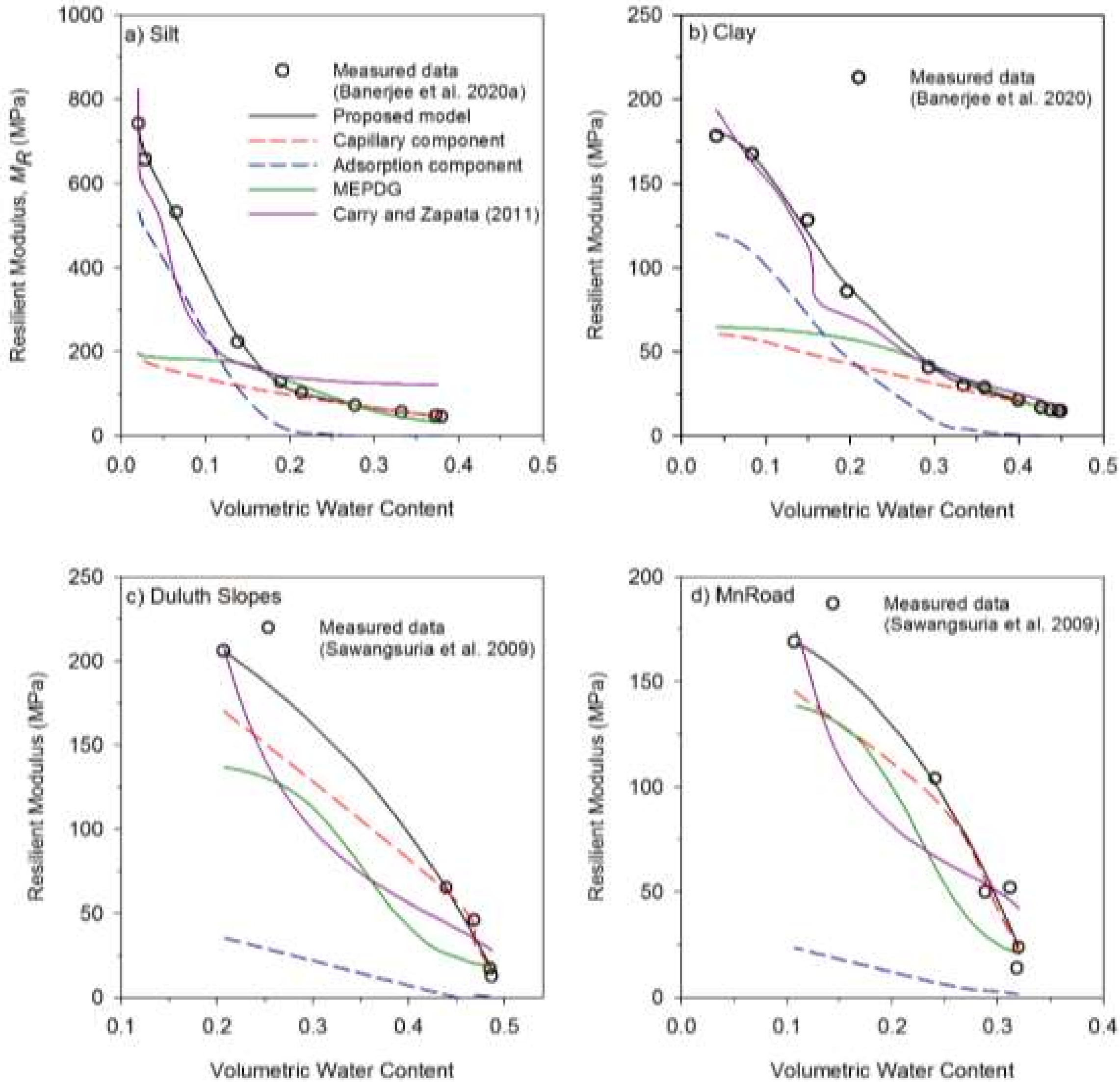

Figure 3. Measured and predicted resilient modulus for different soils: a) Silt, b) Clay, c) Duluth Slopes, and d) MnRoad

Table 2. Comparison of prediction accuracy of the proposed model with alternative models

	MEPDG	Cary and Zapata (2011)	Proposed Model
Soil ID	*RMSE* (MPa)	*RMSE* (MPa)	*RMSE* (MPa)
Silt	243.0	79.0	6.4
Clay	21.1	8.6	2.6
Duluth Slopes	34.3	12.4	5.2
MnRoad	26.8	20.0	9.7

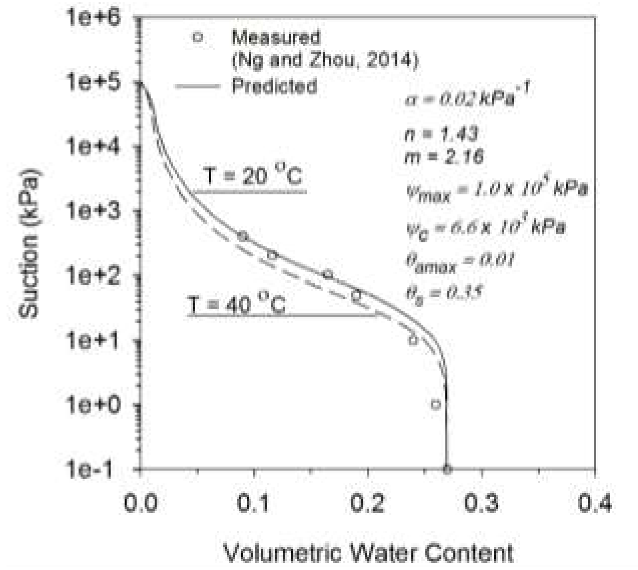

Figure 4. Measured and predicted SWRC of Hong Kong silt

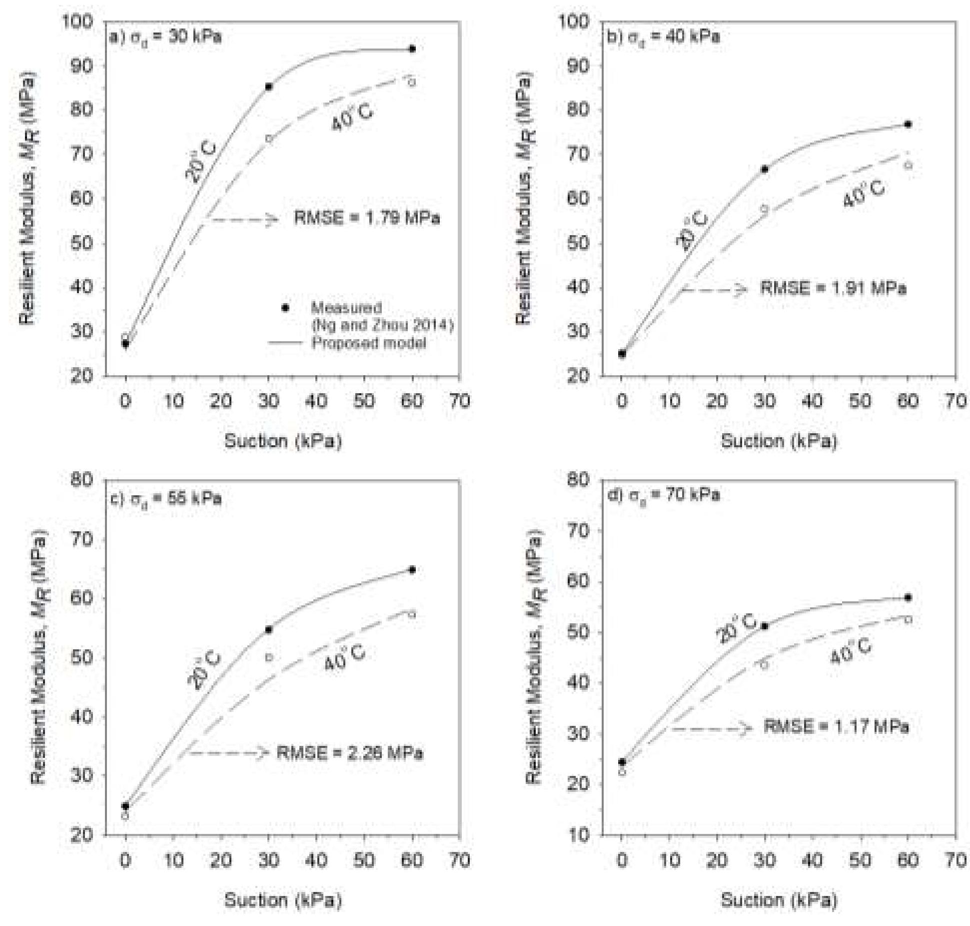

Figure 5. Measured and predicted resilient modulus of Hong Kong Silt at different deviatoric stress (σ_d) of a) 30 kPa, b) 40 kPa, c) 55 kPa, and d) 70 kPa

CONCLUSION

Resilient modulus of subgrade soils, M_R, is an important parameter in the design and evaluation of pavements. In unsaturated subgrade soils, M_R can vary with changes in the water content and temperature of the subgrade soil. In this study, a general model is presented to predict the variation of M_R with changes in subgrade soil water content while considering the effect of temperature. The proposed model explicitly accounts for the impact of different water retention mechanisms, capillary and adsorption, on subgrade soil hardening rate. Thus, it is capable of predicting M_R over a wide range of water content (suction) with excellent accuracy. The proposed model was validated against measured data for 5 different subgrade soils and set of experiments under ambient and elevated temperatures reported in the literature. The model showed great performance with high prediction accuracy, with errors significantly lower than those from two alternative models. The model uses the volumetric water content as a direct input, which, unlike suction, is commonly measured in field conditions. The model is straightforward and can be implemented along with the mechanistic-empirical pavement design method in practice to design and evaluate pavements under unsaturated conditions.

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