

1 **Parameterizing the Impact of Unresolved Temperature**
2 **Variability on the Large-Scale Density Field: Part 2.**
3 **Modeling.**

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14 **Key Points:**

15 • Coarse-resolution ocean model density fields contain systematic errors due to subgrid-
16 scale variance

17 • Parameterizations of density correction cause circulation changes in the North and
18 South Atlantic Ocean

19 • Parameterizations reduce coarse-resolution model biases in the Gulf Stream rep-
20 resentation

21 **Abstract**

22 Ocean circulation models have systematic errors in large-scale horizontal density gradients due to estimating the grid-cell-mean density by applying the nonlinear seawater equation of state to the grid-cell-mean water properties. In frontal regions where unresolved subgrid-scale (SGS) fluctuations are significant, dynamically relevant errors in the representation of current systems can result. A previous study developed a novel and computationally efficient parameterization of the unresolved SGS temperature variance and resulting density correction. This parameterization was empirically validated but not tested in an ocean model. In this study, we implement deterministic and stochastic variants of this parameterization in the pressure-gradient force term of a coupled ocean-sea ice configuration of CESM-MOM6 and perform a suite of hindcast sensitivity experiments to investigate the ocean response. The parameterization leads to coherent changes in the large-scale ocean circulation and hydrography, particularly in the Nordic Seas and Labrador Sea, which are attributable in large part to changes in the seasonally varying upper-ocean exchange through Denmark Strait. In addition, the separated Gulf Stream strengthens and shifts equatorward, reducing a common bias in coarse-resolution ocean models. The ocean response to the deterministic and stochastic variants of the parameterization is qualitatively, albeit not quantitatively, similar, yet qualitative differences are found in various regions.

40 **Plain Language Summary**

41 In ocean models, the location and strength of current systems are related to horizontal gradients of the seawater density. The density of seawater is calculated using an equation of state which depends on the temperature and salinity. These water properties could vary considerably over the spatial scale of one model grid box, yet ocean models resolve only grid-cell-mean water properties. As the seawater equation of state is nonlinear, density gradients which are calculated by applying this equation to the grid-cell-mean water properties could contain errors which result in the misrepresentation of current systems. Therefore, parameterizations have been developed to represent the unknown subgrid-scale water property variance in terms of resolved variables, allowing for a correction to the resolved density field. In this study, we implement and test a parameterized density correction in a coupled ocean-sea ice configuration of CESM-MOM6. The ocean response to the density correction consists of large-scale circulation changes, particularly in the Atlantic Ocean. The representation of the Gulf Stream, a dynamically important current for the global ocean circulation, is found to improve.

55 **1 Introduction**

56 The horizontal resolution of ocean circulation models remains limited by computational constraints, particularly in global-scale climate simulations. Coarse-resolution ocean models, with a representative grid-cell size of $\sim 1^\circ$, generally do not resolve mesoscale and submesoscale variability. Yet such variability makes critical contributions to the ocean circulation and heat transport, and parameterizations are commonly used to represent unresolved subgrid-scale (SGS) processes in terms of resolved variables. Recently, it has been appreciated that unresolved SGS water property variability, associated both with eddies and the mean state, introduces uncertainties and errors into the large-scale density field. These errors, in turn, can result in the misrepresentation of the buoyancy force and thus the hydrostatic pressure gradient. Uncertainties in horizontal pressure gradients are a consequence of the nonlinearity of the seawater equation of state (EOS), which evaluates density as one of several empirically derived functions of temperature, salinity, and pressure. Such equations are valid for a water parcel in thermodynamic equilibrium. Yet coarse-resolution ocean general circulation models calculate the grid-cell-mean density by applying the EOS to the grid-cell-mean water properties, an approx-

71 imation which introduces dynamically relevant errors in frontal regions where unresolved
 72 SGS variability can be significant (Brankart, 2013).

73 To account for these errors, Brankart (2013) implemented a stochastic parameter-
 74 ization of the corrected density in the pressure gradient force (PGF) of the horizontal
 75 momentum equation in a coarse-resolution ($\sim 2^\circ$), stand-alone configuration of NEMO
 76 (ORCA2). Specifically, a corrected density was calculated as a mean of densities obtained
 77 by applying the seawater EOS to the grid-cell-mean temperature and salinity, perturbed
 78 via p local random walks, where p is a model parameter. In one sensitivity experiment,
 79 this parameterization reduced biases in the mean state which are widely observed in coarse-
 80 resolution ocean models, such as in the Gulf Stream separation and transport, air-sea
 81 heat fluxes in the North Atlantic Ocean, and the sea surface height (SSH) gradient be-
 82 tween the tropical Atlantic and Pacific Oceans. Furthermore, in an eddy-resolving ($\sim 0.25^\circ$)
 83 configuration of NEMO (NATL025), the stochastic density correction of Brankart (2013)
 84 improved the Gulf Stream representation (Zanna et al., 2019). Using a different approach,
 85 Williams et al. (2016) implemented a stochastic perturbation of the temperature ten-
 86 dency in a coarse-resolution ($\sim 2.5 \times 3.75$) configuration of the FAMOUS ocean-atmosphere
 87 general circulation model, which reduced biases in the sea surface temperature and salin-
 88 ity.

89 However, the relationship between the Brankart (2013) density parameterization
 90 and the true mean density has not been validated quantitatively. For this reason, Stanley
 91 et al. (2020) derived a mathematical relationship between the true mean density and the
 92 unresolved SGS temperature variability. (The contribution from the unresolved SGS salin-
 93 ity variability was found to be negligible; see also Williams et al. (2016)). Using this re-
 94 lationship, they developed two versions of a SGS temperature variance parameterization
 95 which takes into account the spatial and temporal structure as diagnosed from an eddy-
 96 resolving ($\sim 0.1^\circ$) configuration of the POP model. One parameterization is deter-
 97 ministic, expressing the SGS temperature variance in terms of horizontal gradients of the
 98 resolved temperature field. The stochastic variant conceptualizes individual realizations
 99 of SGS turbulence as fundamentally random, thus representing spread about the deter-
 100 ministic model; in particular, the stochastic parameterization replaces the fixed ampli-
 101 tude of the deterministic parameterization with a log-normally distributed random pro-
 102 cess. Both versions of the SGS temperature variance parameterization have been shown
 103 to demonstrate high goodness-of-fit to that diagnosed from the eddy-resolving simula-
 104 tion (Stanley et al., 2020). However, the parameterization has not previously been im-
 105 plemented in an ocean model, and the ocean response to the density corrections remains
 106 unknown. Furthermore, as the Brankart (2013) study tested a stochastic parameteriza-
 107 tion exclusively, the attribution of the mean ocean response to the mean of the param-
 108 eterization versus the rectified effects of the noise was not possible.

109 In this study, we perform a suite of sensitivity experiments to investigate the dy-
 110 namical effects of the Stanley et al. (2020) parameterizations in a coarse-resolution ($\sim 0.66^\circ$)
 111 configuration of the Modular Ocean Model version 6 (MOM6). The parallel use of de-
 112 terministic and stochastic versions of the SGS temperature variance parameterization
 113 permits the attribution of the mean ocean response to mean density correction versus
 114 noise. Section 2 describes the model configuration and provides a detailed summary of
 115 the experimental protocol. Section 3 examines the effects of the parameterizations, both
 116 globally and in selected dynamically relevant current systems. Finally, Section 4 sum-
 117 marizes this study, compares the results with prior stochastic modeling studies, and states
 118 the conclusions.

119 2 Methods

120 MOM6 solves the hydrostatic primitive equations on an Arawaka C-grid and fea-
 121 tures several advancements over its predecessors, including the use of vertical Lagrangian

122 discretization with remapping to permit arbitrary vertical coordinates (Adcroft et al.,
 123 2019). MOM6 was coupled to the Los Alamos Sea Ice Model (CICE) version 5 (Hunke
 124 et al., 2010) in the framework of the Community Earth System Model (CESM) version
 125 2.2 (Danabasoglu et al., 2020). The CESM-MOM6 model was configured as a coupled
 126 ocean-sea ice model forced at the surface with historical atmospheric state and flux fields
 127 provided by the Japanese 55-year atmospheric reanalysis product (JRA55-do; Tsujino
 128 et al. (2018)). The model has a nominal spatial resolution of $\sim 0.66^\circ$ and 65 vertical layers
 129 in z^* coordinates with depths ranging from $\sim 1.25\text{-}5876$ m; the layer spacing varies
 130 and is refined in the upper ocean (see the description of z^* coordinates in Adcroft and
 131 Campin (2004)). The GEOMETRIC parameterization of Marshall et al. (2012) was used
 132 to represent the extraction of available potential energy from the mean flow by mesoscale
 133 eddies within the Gent-McWilliams (GM) framework. Vertical mixing in the turbulent
 134 boundary layer is represented using the K -Profile Parameterization (KPP) of Large et
 135 al. (1994), implemented via the CVMix package. MOM6 was configured to use the Wright
 136 seawater EOS (Wright, 1997). At the time the simulations described herein were per-
 137 formed, the model parameters were chosen for consistency with one of several “best” con-
 138 figurations of MOM6 within the CESM 2.2 framework, although MOM6 configurations
 139 remain under development within this framework and model tuning is ongoing.

140 The density corrections resulting from the SGS temperature variance parameter-
 141 izations of Stanley et al. (2020) were implemented in the PGF. Specifically, Stanley et
 142 al. (2020) used a Taylor expansion to relate the mean density to the unresolved SGS tem-
 143 perature variance:

$$\bar{\rho} \approx \hat{\rho}(\bar{T}, \bar{S}) + \frac{\partial_T^2 \hat{\rho}(\bar{T}, \bar{S})}{2} \sigma_T^2. \quad (1)$$

144 Here $\bar{\rho}$ is the true grid-cell-mean density; $\hat{\rho}$ is the seawater EOS (dependence on pres-
 145 sure has been omitted to condense notation); \bar{T}, \bar{S} are the grid-cell-mean temperature
 146 and salinity; and σ_T^2 is the unresolved SGS temperature variance.

147 In the deterministic parameterization, the SGS temperature variance is expressed
 148 in terms of finite differences of the resolved temperature field. In particular,

$$\sigma_T^2 \approx c |\delta x \circ \nabla \bar{T}|^2, \quad (2)$$

149 where δx is the horizontal grid cell size, c is a parameter to be tuned, and \circ is the Hadamard
 150 product. In the stochastic parameterization, the amplitude of σ_T^2 is scaled by a lognor-
 151 mally distributed random variable e^χ ; here χ is a Gaussian random field with zero mean
 152 and constant variance σ_X^2 :

$$\sigma_T^2 \approx c e^\chi |\delta x \circ \nabla \bar{T}|^2. \quad (3)$$

153 (Therefore, the medians of the deterministic and stochastic parameterizations are equal,
 154 while the mean of the stochastic parameterization is 21.5% greater than the mean of the
 155 deterministic parameterization, as discussed in Stanley et al. (2020).) Here χ is uncor-
 156 related in space and has an AR(1) structure in time:

$$\chi(x, y, t) = \phi(x, y, t) \chi(x, y, t - \delta t) + \epsilon(x, y, t), \quad (4)$$

157 where ϵ is a Gaussian random field with zero mean and no correlations in horizontal space
 158 (x, y) or time (t). The variance of the noise ϵ varies with the AR(1) parameter ϕ so as
 159 to keep the process variance σ_X^2 constant; Stanley et al. (2020) diagnosed $\sigma_X^2 \approx 0.39$
 160 from a high-resolution POP simulation (see Equation 44 of that study and surrounding
 161 discussion). Here ϕ is related to a decorrelation timescale τ via

$$\phi(x, y, t) = e^{-\delta t / \tau(x, y, t)}, \quad (5)$$

162 where $\delta t = 1800$ s is the model baroclinic timestep, and

$$\tau = k \sqrt{\frac{u^2(x, y, t) + v^2(x, y, t)}{\delta x^2 + \delta y^2}}. \quad (6)$$

163 Here (u, v) represent the instantaneous velocity of the upper-most layer. Stanley et al.
 164 (2020) diagnosed the scaling factor $k = 3.7$ from the same high-resolution simulation
 165 described previously, and our study retains this choice. For the spatial variability of the
 166 diagnosed and modeled decorrelation timescale, the reader is referred to Figure 6 therein.

167 The amplitude of the density corrections is determined by the scaling factor c in
 168 Equation 2. While Stanley et al. (2020) developed heuristics for choosing the magnitude
 169 of this constant based on the horizontal grid-cell size, it is expected that the optimal value
 170 of c is a model- and physics- dependent quantity. In addition, Stanley et al. (2020) es-
 171 timated optimal choices of c as a function of grid-cell resolution by coarse-graining tem-
 172 perature data (referenced to an eddy-resolving, nominally 0.1° POP grid) offline. Some
 173 heuristics for the choice of c are provided in Figure A1 of that study. This analysis sug-
 174 gests a prior choice of $c \approx 0.2$ for a $\sim 1^\circ$ model. However, the temperature gradient in
 175 a coarse-resolution model is generally weaker than that obtained by coarse-graining the
 176 temperature field of a high-resolution model. In this study, we therefore expected a pri-
 177 or to require a larger value of c and performed only preliminary tuning, given that MOM6
 178 is under active development in CESM and tuning of parameters in the base case is cur-
 179 rently ongoing. In particular, a factor of $c = 0.5$ was selected for the Experimental Runs.
 180 The scheme was effectively disabled along boundaries, as the representation of layers in
 181 shallow regions in this configuration of MOM6 posed numerical difficulties, and Stanley
 182 et al. (2020) had not validated the parameterization there.

183 All runs were initialized from the ocean/sea ice state that resulted from a prior sim-
 184 ulation forced with 58 years of the JRA55-do historical atmospheric variables (Tsujino
 185 et al., 2018). A Control Run, in which the SGS temperature variance parameterization
 186 was disabled, was then integrated for 1 year (to complete the sequence of 59 years of avail-
 187 able JRA55-do v1.3, historically varying forcing fields), and then for an additional 59
 188 years, i.e., for the period of 1958-2016. Next, two Experimental Runs were performed
 189 in which either the deterministic or stochastic version of the parameterization was en-
 190 abled in the PGF (hereafter referred to as Deterministic PGF Run and Stochastic PGF
 191 Run, respectively). Other model parameters were chosen for consistency with the prior
 192 run.

193 MOM6 was configured to output monthly mean data which was remapped verti-
 194 cally from its native 65 layers in z^* -coordinates to a set of 34 standard levels from the
 195 WOA09 (White et al., 2009; Griffies et al., 2020). To represent the ocean long-term mean
 196 state, the initial nine years of the simulation were disregarded as a spinup and the final
 197 50 years were averaged in time; monthly and seasonal climatologies were formed for the
 198 same period. To represent the variability, variances on timescales ranging from twice the
 199 thermodynamic timestep ($dt = 3600$ s) to 50 years were calculated. Although the model
 200 was configured to output monthly mean data, it was possible to compute variance on
 201 shorter time scales than monthly by configuring the model to output monthly means of
 202 the square of the water properties. Variance was then subsequently computed as the mean
 203 of the square minus the square of the mean.

204 Hereafter, the ocean response to the density corrections will be assessed by con-
 205 sidering anomalies of the long-term mean state and variability in the Experimental Runs
 206 relative to the Control Run. Subsequent analysis will focus upon changes in the ocean
 207 circulation in the range of ~ 0 -400 m, as lateral temperature gradients and hence the SGS
 208 temperature variance parameterizations are expected to be greatest near the surface based
 209 on the results of Stanley et al. (2020). The spatial pattern of the SGS temperature vari-
 210 ance parameterization (Figure 1a) and that of the associated density corrections (Fig-

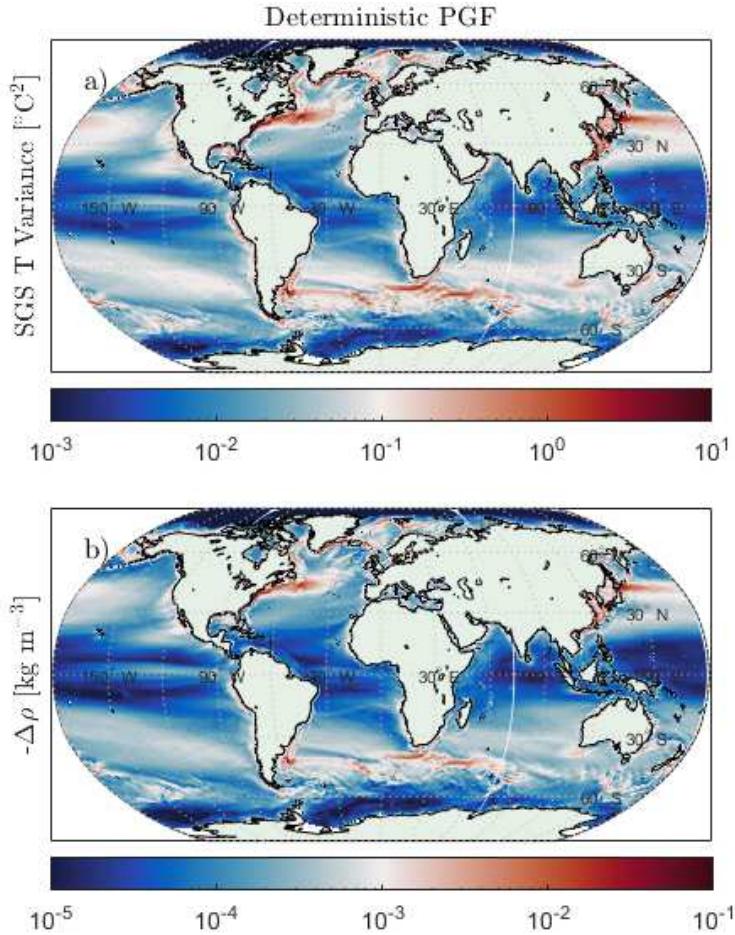


Figure 1. a) Parameterized SGS temperature variance in the Deterministic PGF Run. b) $-\Delta\rho$, the negative of the density correction from Equation 1 corresponding to the SGS temperature variance parameterization in panel a. Note the use of logarithmic scales. Data are 50-year means at a depth of 2.5 m.

ure 1b) in MOM6 are generally consistent with that of Stanley et al. (2020); the parameterization has the greatest amplitude in frontal regions with large lateral temperature gradients.

3 Ocean Dynamical Response to the Density Corrections

3.1 Summary of Ocean Response

The ocean mean state of temperature and salinity, at 200 m, and SSH as simulated in the Control Run are shown in Figures 2a-c (for completeness, we also show fields at depths 2.5 m, 100 m and 400m in S1-S3a-b). Here the SSH field represents dynamic sea level with a correction for the sea ice inverse barometer effect and zero global area mean (Griffies et al., 2016). In the range of \sim 0-400 m, the SSH and water property changes in the Experimental Runs relative to the Control Run are most pronounced near the Gulf Stream and North Atlantic Current, the Kuroshio Current, the Agulhas Current, and the Brazil-Malvinas Confluence region (Figures 2d-i, S1-S3c-f). Near the Gulf Stream, the circulation change is indicated by a dipole pattern in the SSH field (Figure 2f,i) the

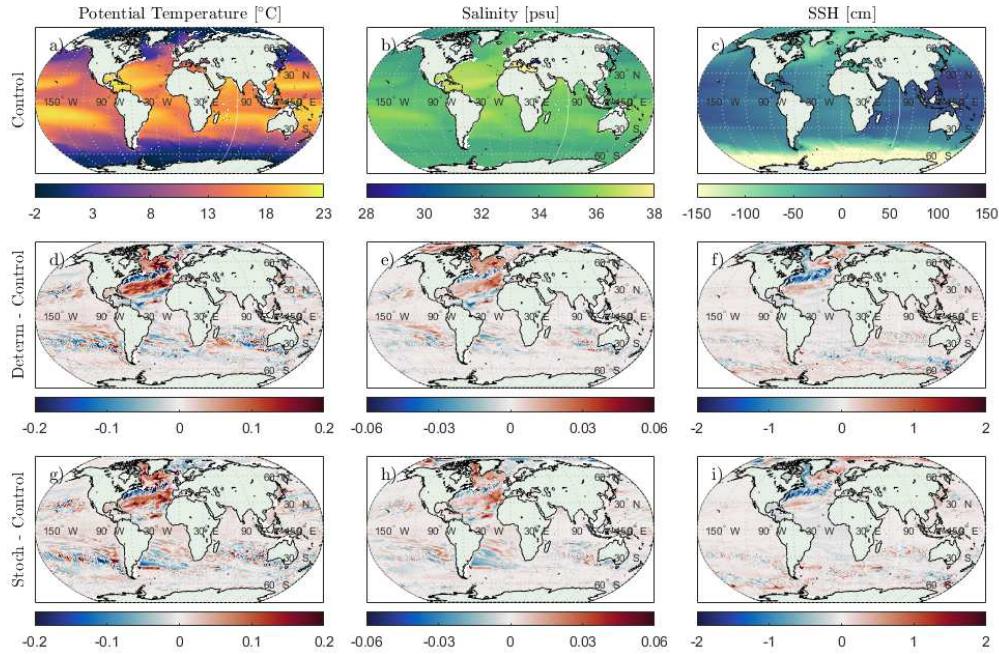


Figure 2. a-c) Control Run 50-year-mean state of a) potential temperature at 200 m, b) salinity at 200 m, c) SSH. d-f) Deterministic PGF Run minus Control Run of indicated variable. g-i) Same as d-f, but for Stochastic PGF Run minus Control Run.

cause of which is discussed in Section 3.3. The upper-ocean temperature and salinity along the path of the separated Gulf Stream and North Atlantic Current are observed to decrease in the Experimental Runs relative to the Control Run (Figures 2d,e,g,h, S1-S3c-f). On the other hand, along the southern flank of the Gulf Stream and in the subtropical gyre, the upper-ocean temperature and salinity increase. The Nordic Seas are cooler and fresher, while the Labrador Sea is warmer and saltier, in the Experimental Runs than in the Control Run (Figures 2d,e,g,h, S1-S3c-f). Associated with these changes, the SSH generally increases in the Nordic Seas and decreases in the Labrador Sea (Figure 2f,i). Section 3.2 investigates the physical mechanisms which account for the circulation and hydrographic changes in the subpolar North Atlantic Ocean. Finally, in the South Atlantic Ocean, circulation changes near the Brazil-Malvinas Confluence are evidenced by the water property anomalies (relative to the Control Run) and SSH dipole there; these signals are predominantly seen in the Stochastic PGF Run, and are weak or absent in the Deterministic PGF Run (Figures 2d-i, S1-S3c-f). The ocean dynamical response in the South Atlantic Ocean is the focus of Section 3.4.

In general, the spatial pattern of the upper-ocean response is qualitatively similar in the Deterministic PGF Run and Stochastic PGF Runs, although differences can be observed, such as near the Brazil-Malvinas Confluence (Figures 2d-i, S1-S3c-f). In many regions, the amplitude of the response is greater in the Stochastic PGF Run than in the Deterministic PGF Run, yet some exceptions exist; specific regional characteristics of the differences will be investigated in Sections 3.2, 3.3, and 3.4. These findings suggest that the majority of the mean ocean response is associated with the mean density correction, while the rectified effects of the noise play a lesser, albeit non-negligible, role.

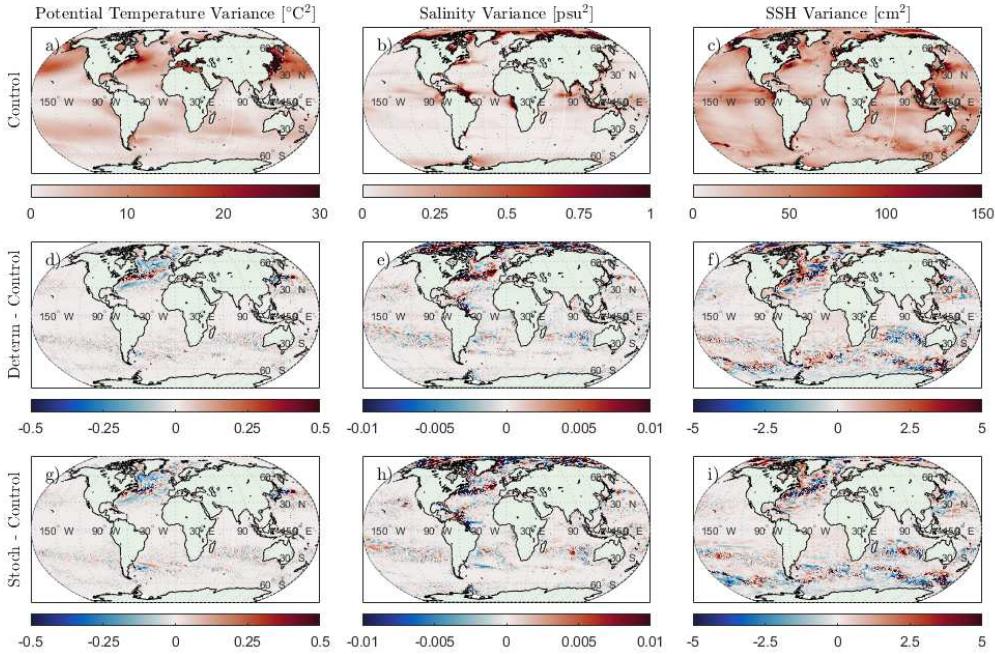


Figure 3. a-c) Control Run variance of a) potential temperature at 2.5 m, b) salinity at 2.5 m, c) SSH. d-f) Deterministic PGF Run minus Control Run of indicated variable. g-i) Same as d-f, but for Stochastic PGF Run. The timescales represented range from 2 hours to 50 years.

The Control Run variance of temperature, salinity (at 2.5 m), and SSH are shown in Figure 3a-c. Overall, the changes in water property and SSH variance in the Experimental Runs relative to the Control Run are modest (Figure 3d-i). Despite the introduction of random noise into the SGS temperature variance parameterization in the Stochastic PGF Run, the differences in variance relative to the Control Run are similar to those of the Deterministic PGF Run (Figure 3d-i). Notably, the parameterized density corrections reduce the SSH and/or water property variance in certain regions of the separated Gulf Stream, Labrador Sea, and Irminger Sea. In the subpolar gyre of the North Atlantic Ocean, the reduced variance in the 2.5-m temperature may be related to the deepening of wintertime mixed-layer depths (MLDs), as a homogenization of water properties over a greater depth of the water column would be expected to result in reduced variability due to surface forcing and entrainment; however, this remains uncertain. The Gulf Stream region will be explored in greater detail in Section 3.3.

The stochastic perturbations introduced to the density by the parameterization are uncorrelated in space. The lateral gradient of these perturbations appears in the momentum equations, which implies that the noise forcing is concentrated at the smallest scales that can be represented on the grid; in a spatially homogeneous setting the spectrum of the noise forcing in the momentum equation would be proportional to k^3 where k is the magnitude of the spatial wavenumber. This is also the spectrum associated with backscatter due to Reynolds stresses in quasigeostrophic turbulence (Grooms et al., 2015). Grooms et al. (2015) found that the combination of viscous closures and low-order discretizations combine to prevent noise forcing concentrated at the smallest scales of a parameterized model from cascading back up to large scales; the same mechanism is presumably at play here, damping the ability of the model to respond to the stochastic forcing with increased variability at large scales and long times. This is exacerbated by the fact that backscatter

274 ter associated with Reynolds stresses (as studied by Grooms et al. (2015)) occurs wher-
 275 ever the eddy kinetic energy is large, whereas the backscatter associated with density
 276 corrections is confined to a smaller subset of the domain where there are large resolved
 277 temperature gradients. Some combination of Reynolds-stress-driven backscatter (e.g. Bach-
 278 man, 2019; Jansen et al., 2019; Juricke et al., 2020), backscatter associated with density
 279 corrections, and other forms of stochastic noise (e.g. Grooms, 2016; Grooms & Kleiber,
 280 2019) may be necessary to achieve a large increase in low-resolution model variability.

281 3.2 Nordic Seas and Labrador Sea

282 The Nordic Seas are a region at the interface of the Arctic Ocean and North At-
 283 lantic Ocean, and the hydrographic conditions reflect the exchange of water masses with
 284 both sources. Along with the Labrador Sea, the Nordic Seas are a principal site of deep
 285 convective mixing in the North Atlantic Ocean, a process which takes place when strong
 286 air-sea heat fluxes cause upper-ocean buoyancy loss and destabilize the stratification. In
 287 the Control Run, deep convection takes place within the Nordic Seas, Labrador Sea, Irminger
 288 Sea, and in other localized regions (Figure 5a). Ocean modeling studies have linked win-
 289 tertime deep convection in these regions with the strength of the Atlantic Meridional Over-
 290 turning Circulation (Heuzé, 2017), which has wide-reaching interconnections with the
 291 global ocean circulation. Given the importance of these regions for the accurate repre-
 292 sentation of the ocean circulation, we summarize the ocean general circulation pathways
 293 and hydrographic conditions here.

294 Along the western boundary of the Nordic Seas, the East Greenland Current trans-
 295 ports relatively cold, fresh water south from the Arctic Ocean through the Nordic Seas
 296 via Fram Strait. The East Greenland Current flows southward along the coast of Green-
 297 land through Denmark Strait, continuing west of Cape Farewell as the West Greenland
 298 Current, and forming the northern flank of the subpolar gyre in the North Atlantic Ocean.
 299 A branch of the East Greenland Current, the East Icelandic Current, bifurcates from the
 300 East Greenland Current north of Denmark Strait and flows eastward to the north of Ice-
 301 land. At the same time, the Irminger Current, as well as its extension, the North Ice-
 302 lantic Irminger Current, transport relatively warm, saline Atlantic water northward via
 303 the eastern Denmark Strait and then eastward around Iceland into the Nordic Seas. Ad-
 304 ditional branches of the North Atlantic Current enter the Nordic Seas across the Greenland-
 305 Scotland Ridge, and warm, saline currents flow northward along the eastern boundary
 306 of the Nordic Seas before partially recirculating south of Fram Strait. The Nordic Seas
 307 circulation and hydrographic conditions are detailed by Latarius and Quadfasel (2016,
 308 see their Figure 1). The Polar Front represents the boundary between the Arctic-origin
 309 inflow in the East Greenland Current and the Nordic Seas interior and is present in the
 310 Control Run as a region of strong lateral temperature and salinity gradients (Figures 4a-
 311 b, other depths shown in S4-S6a-b). In the Control Run, sea ice is present in winter near
 312 the cold, fresh boundary currents of the Nordic Seas and subpolar gyre (Figure 5b).

313 The ocean response to the density corrections in the near-surface (i.e., 2.5 m) is
 314 characterized by cooling and freshening in the Nordic Seas and warming and salinification
 315 in the Labrador Sea (here we focus particularly on wintertime conditions; Figure
 316 4c-f). In the upper ocean, this signal generally attenuates with depth, and in certain re-
 317 gions reverses, by 400 m (Figures 4c-f, S4-S6c-f). Within the Nordic Seas, the Iceland
 318 Sea is the epicenter of the near-surface changes in the ocean mean state, which locally
 319 reach $\sim 0.5^{\circ}\text{C}$ and ~ 0.1 psu or more in the Stochastic PGF Run. Consistent with these
 320 hydrographic changes, a band of increased sea ice area (not shown) and thickness (Figure
 321 5d-f) is present in the western Nordic Seas; in contrast, the sea ice cover is reduced
 322 along the Labrador Current. The responses are similar in the Stochastic PGF Run and
 323 Deterministic PGF Run, although the amplitude of the changes is generally greater in
 324 the Stochastic PGF Run (Figures 4-5g-h, S4-S6g-h). However, exceptions exist, as can

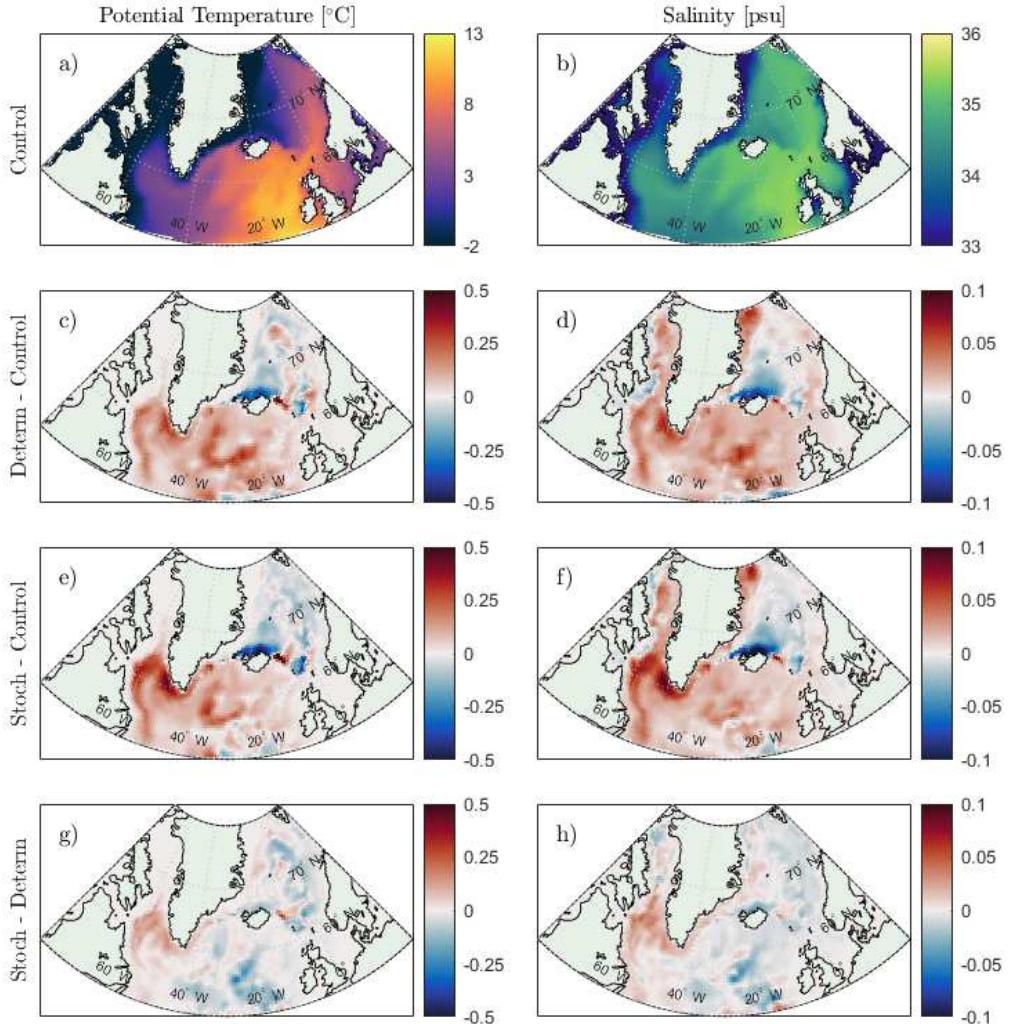


Figure 4. a-b) Control Run 50-year wintertime (JFM) mean state of a) potential temperature at 2.5 m, and b) salinity at 2.5 m. c-d) Deterministic PGF Run minus Control Run of indicated variable. e-f) Stochastic PGF Run minus Control Run of indicated variable. g-h) Stochastic PGF Run minus Deterministic PGF Run of indicated variable.

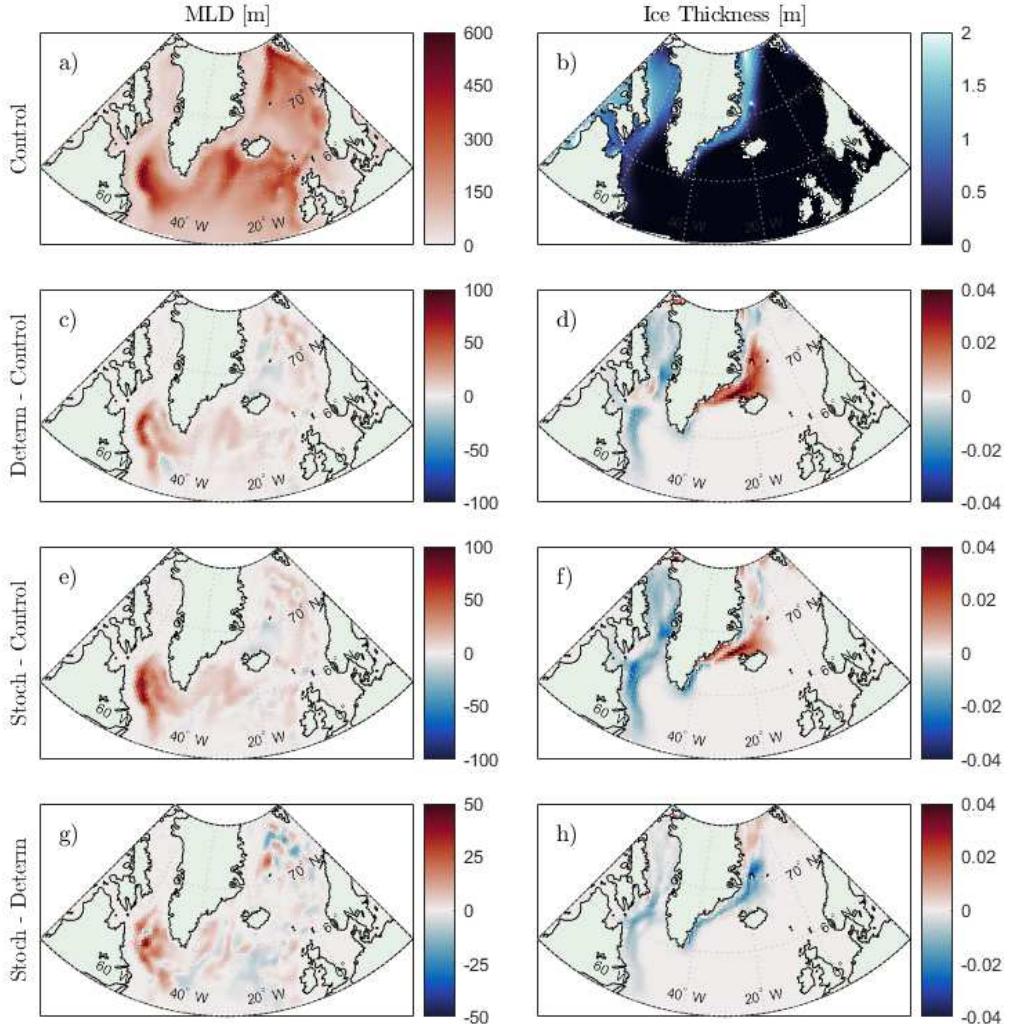


Figure 5. a-b) Control Run 50-year wintertime (JFM) mean state of a) MLD from KPP, and b) sea ice thickness. c-d) Deterministic PGF Run minus Control Run of indicated variable. e-f) Stochastic PGF Run minus Control Run of indicated variable. g-h) Stochastic PGF Run minus Deterministic PGF Run of indicated variable.

325 be seen, e.g., from the changes in the potential temperature and sea ice fields in the western
326 Nordic Seas at 2.5 m (Figures 4-5g-h).

327 Changes in the ocean circulation and exchange between the Nordic Seas and sub-
328 polar North Atlantic Ocean are implicated in the hydrographic changes in the Nordic
329 Seas and Labrador Sea in the Experimental Runs relative to the Control Run. Associated
330 with the lateral temperature gradients near the East Greenland Current, the SGS
331 temperature variance and density parameterizations are highly active (Figure 1a-b), and
332 near the surface, there is considerable seasonal variability of the position and strength
333 of maximum temperature gradients (Figure S7a-d). In the Control Run, the simulated
334 exchange exhibits seasonal variability, and the East Greenland Current outflow is weak-
335 est in summer (JAS; Figure 6a-d). The density corrections modify the PGF near the East

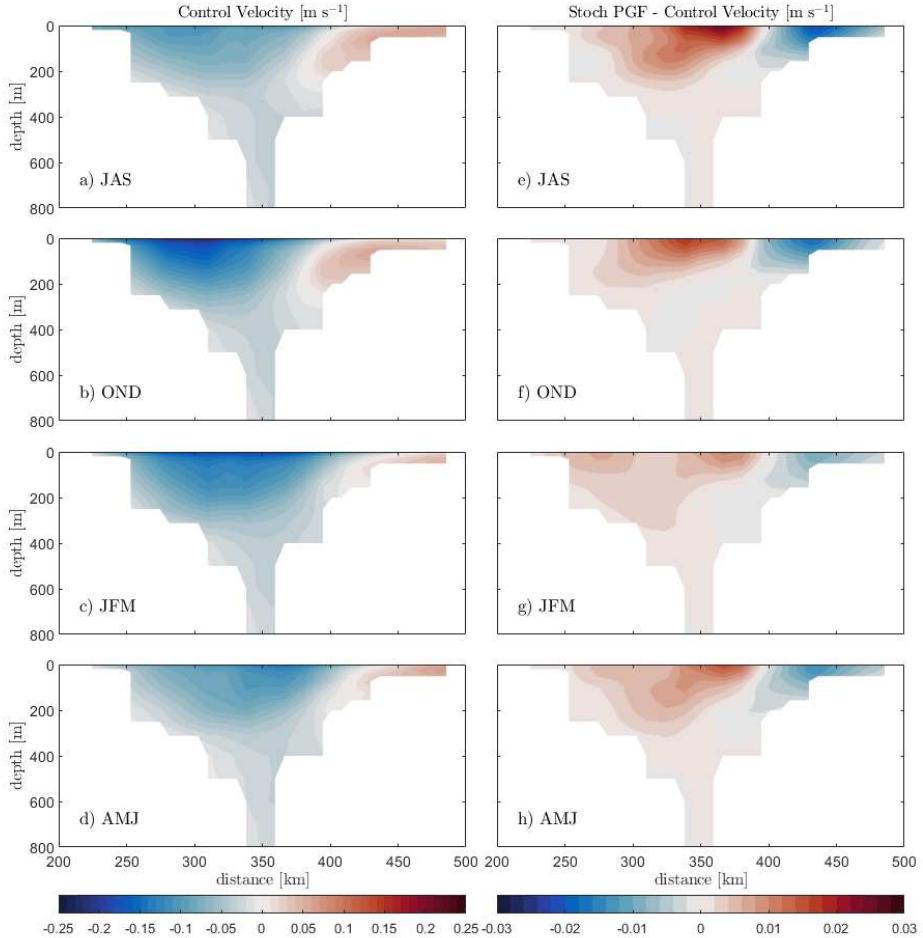


Figure 6. a-d) Control Run velocity through Denmark Strait (i.e., normal to the transect given in Figure 7a) by indicated season. Transect is oriented such that distance increases to the east of Greenland, and positive velocities indicate northeastward flow. e-h) Stochastic PGF Run minus Control Run of velocity.

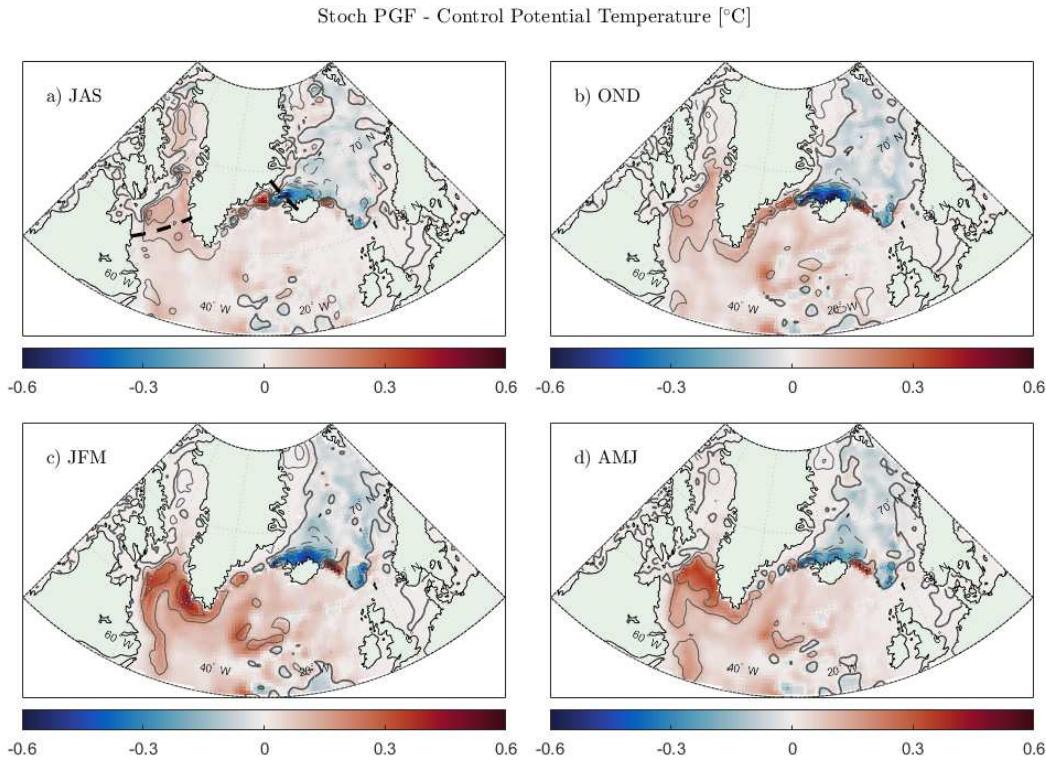


Figure 7. a-d) Stochastic PGF Run minus Control Run of indicated 50-year seasonal mean of potential temperature (colormap) and salinity (contour lines in increments of 0.03 psu) at 2.5 m. The dashed black line near Denmark Strait in panel a represents the location of the transect in Figure 6, oriented such that distance increases to the east of Greenland. The dashed black line in the Labrador Sea represents the location of the transect in Figure S9.

336 Greenland Current north of Denmark Strait, leading to a reduction in the upper-ocean
 337 exchange through Denmark Strait. In particular, there is a decrease in the southward
 338 transport of cold, fresh water and northward transport of warm, saline water in the core
 339 of the simulated East Greenland Current and North Icelandic Irminger Currents, i.e.,
 340 above ~ 200 m (Figure 6e-h). The use of the corrected density reduces the exchange through-
 341 out the year, although the absolute change is greatest during the summer (Figure 6e).
 342 The circulation changes are associated with a dipole in upper-ocean temperature and
 343 salinity anomalies (relative to the Control Run) which develops northwest of Iceland in
 344 summer (Figure 7a). The negative temperature and salinity anomaly circulates into the
 345 Nordic Seas north of Iceland and follows cyclonic circulation pathways through the Nordic
 346 Seas interior, resulting in cooling and freshening relative to the Control Run (Figure 7a-
 347 d). The positive temperature and salinity anomaly is exported in the East Greenland
 348 Current, as indicated by the band of relatively warm and saline water along the Green-
 349 land shelf between Iceland and Cape Farewell in autumn (Figure 7b). The warm and saline
 350 signal is present in the West Greenland Current and Labrador Current by winter and
 351 is amplified (Figure 7c). Qualitatively similar responses are found in the Deterministic
 352 PGF Run as in the Stochastic PGF Run, albeit with a reduced amplitude (Figures 4g-
 353 h, S4-S6g-h; also compare Figures 7a-d and S8a-d).

354 The amplification of the temperature and salinity anomalies in the Labrador Sea
 355 is associated with an enhancement of vertical mixing, as indicated by the deepening of

wintertime MLDs there (Figure 5c,e). In much of the Labrador Sea, the stratification associated with vertical temperature gradients is unstable, i.e., the temperature profile generally decreases with depth in the wintertime (Figure S9a, 0-1000 m) and in the annual mean below ~ 50 m (not shown). On the other hand, the profile of salinity tends to stabilize the stratification, i.e., salinity increases with depth (Figure S9b). In this regime, the stratification is strongly influenced by vertical salinity gradients, and the warm and saline anomaly tends to destabilize the stratification, leading to increased convection depths and the vertical redistribution of the relatively warm, salty water at depth. Associated with the warming of the Labrador Sea, wintertime sea ice cover is reduced near the Labrador Current (Figure 5d,f). Conversely, in the Nordic Seas, particularly in the Iceland Sea, cooler surface temperatures lead to a reduction in sea ice melting which manifests as a region of increased sea ice thickness. Consistent with the increase in wintertime convection in the Labrador Sea, the maximum of the overturning streamfunction increases by ~ 0.3 Sv in the Deterministic PGF Run and ~ 0.4 Sv in the Stochastic PGF Run, relative to a baseline of ~ 14.1 Sv in the Control Run (Figures S10-S12).

3.3 Gulf Stream Position and Transport

The Gulf Stream, the western boundary current of the subtropical gyre in the North Atlantic Ocean, and the North Atlantic Current, its northward extension, are upper-ocean components of the Atlantic Meridional Overturning Circulation and play a major role in the northward heat transport. In the process of North Atlantic Deep Water formation, the warm waters of the Gulf Stream undergo heat and buoyancy loss associated with strong air-sea heat fluxes in the North Atlantic Ocean. The representation of the Gulf Stream in ocean models thus has implications for the representation of dynamic and thermodynamic processes more broadly. Yet in coarse-resolution ocean models, the separation latitude of the Gulf Stream suffers from a persistent poleward bias which tends to abate as the horizontal grid-cell resolution is refined to that of the first baroclinic mode Rossby radius (e.g., Bryan et al. (2007); Chassignet and Xu (2017)). A wide variety of factors have been proposed to influence the separation latitude of the Gulf Stream in numerical ocean models, including boundary conditions, coastline geometry, topographic effects, subgrid-scale parameterizations, and others. However, a unifying theory has not yet emerged (Chassignet & Marshall, 2008).

The separated Gulf Stream is associated with the boundary of the North Atlantic (cyclonic) subpolar and (anticyclonic) subtropical gyres (Figure 8a); variability in the separation region is reflected by the strong variance in the SSH field (Figure 8a). A proxy for the Gulf Stream North Wall position (i.e., the 12°C isotherm at 400 m) in the Control Run is compared with that of the gridded EN4 analysis (Good et al., 2013), an observational data product, for the same 50-year period (Figure 9); this comparison suggests that the Control Run has a mean longitudinal bias of $\sim 0.6^{\circ}$ in the separation region of $\sim 73\text{--}65^{\circ}\text{W}$.

In the Gulf Stream region, the SGS temperature variance parameterization and density corrections are highly active (Figure 1a-b), resulting in large-scale changes in the regional ocean circulation as indicated by the SSH fields (Figure 8c-f). The deterministic and stochastic variants of the parameterization give rise to qualitatively similar changes in the circulation (8c,e,g). In general, the ocean response in this region is characterized by a strengthening and equatorward shift (Figure 9) of the Gulf Stream. According to geostrophy, the Gulf Stream transport is related to the cross-stream SSH gradient (Figure 8a). The change in the cross-stream SSH gradient is longitudinally varying and increases by $\sim 5\%$ in the Experimental Runs relative to the Control Run in the range of $\sim 65\text{--}55^{\circ}\text{W}$. Furthermore, the southward displacement of the Gulf Stream reduces the bias in the Control Run by $\sim 0.2^{\circ}$ in the region of $\sim 73\text{--}65^{\circ}\text{W}$ relative to the EN4 analysis product.

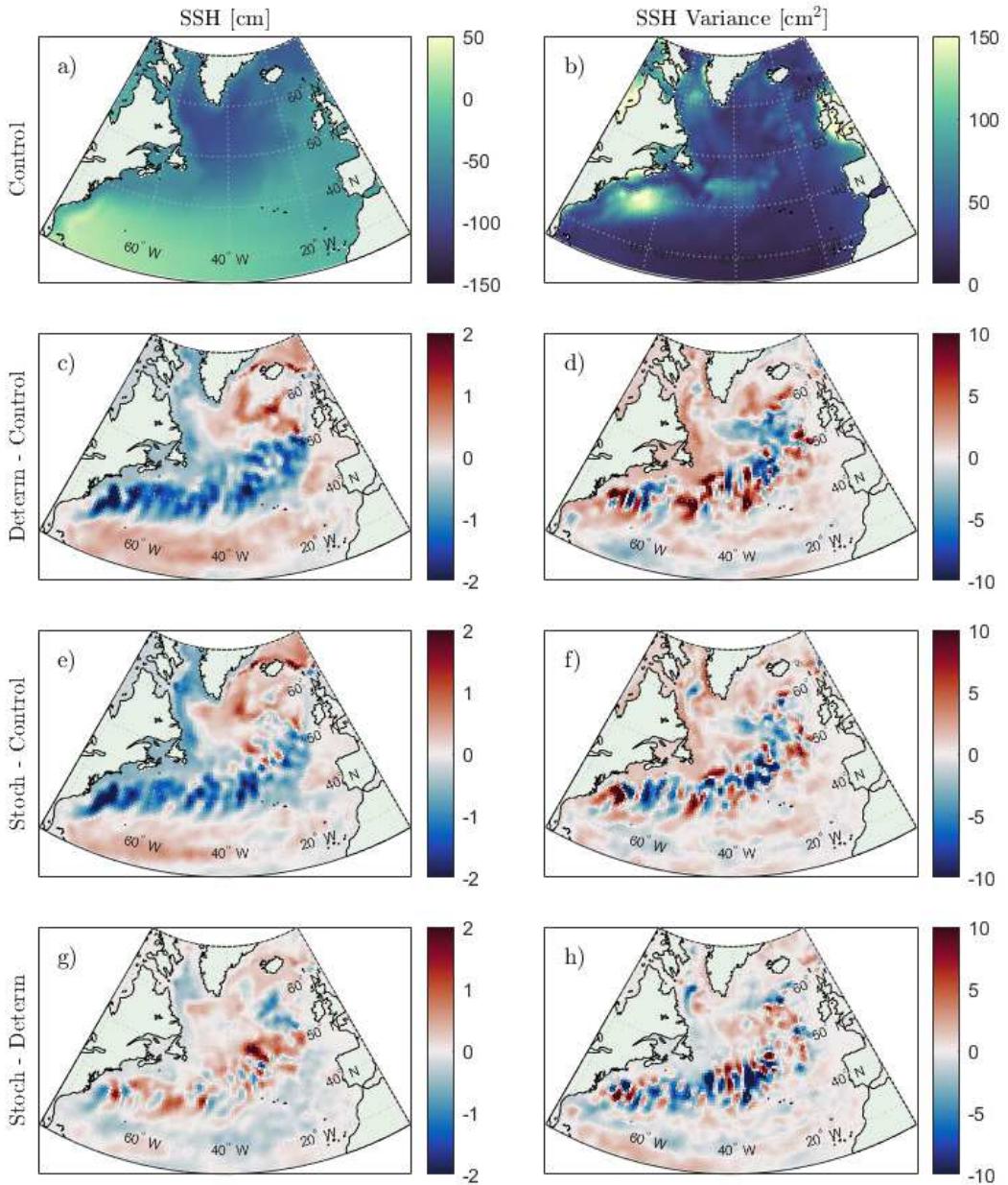


Figure 8. a) Control Run 50-year mean state of SSH. b) Control Run variance, including timescales from two hours to 50 years. c-d) Deterministic PGF Run minus Control Run of indicated variable. e-f) Stochastic PGF Run minus Control Run of indicated variable. g-h) Stochastic PGF Run minus Deterministic PGF Run of indicated variable.

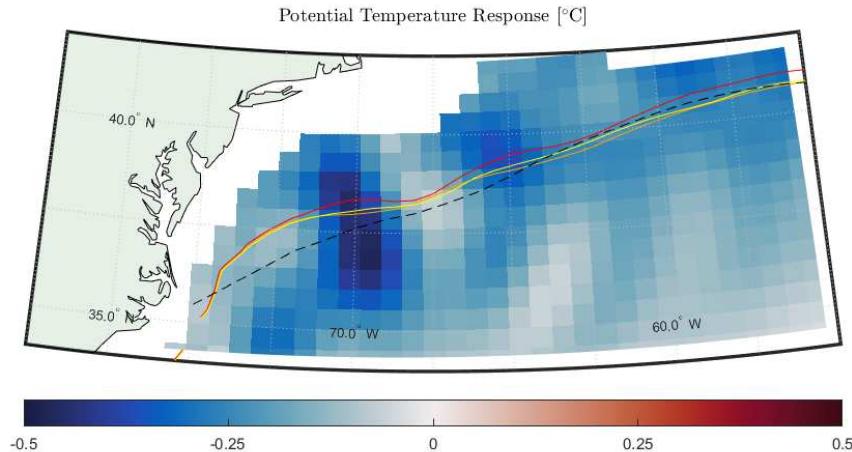


Figure 9. Deterministic PGF Run minus Control Run of potential temperature at 400 m (colormap). Red, orange, yellow, and black dashed contours indicate the 12°C isotherm at 400 m in the Control Run, Deterministic PGF Run, and Stochastic PGF Run, and EN4 analysis, respectively. Data are 50-year means.

407 However, a mechanistic description for the shift of the Gulf Stream remains unclear.
 408 Viewing the Gulf Stream as a geostrophically balanced flow, corrections to the merid-
 409 ional PGF along the mean path of the separated current do not directly account for the
 410 southward displacement. We explain this using a simplified model in section S1 but the
 411 direct effect should be to displace the current poleward. It remains uncertain why there
 412 is an equatorward shift of the Gulf Stream in the Experimental Runs relative to the Con-
 413 trol Run but the response is consistent with prior studies of ocean stochastic parame-
 414 terizations (Brankart, 2013; Zanna et al., 2019).

415 As described in Section 3.1, the SSH variance does not increase considerably in the
 416 Experimental Runs relative to the Control Run, globally or in the Gulf Stream region
 417 (Figure 8d-f). In certain regions, the variance is reduced in the Experimental Runs, in
 418 some cases more so with the stochastic version of the parameterization, although there
 419 is considerable uncertainty as the signal is very noisy (8d,f,h). These differences are likely
 420 related to the southward displacement of the mean current, which is associated with a
 421 reduction in variability along its prior mean path.

422 3.4 South Atlantic Ocean

423 The Brazil Current, the western boundary current of the subtropical gyre in the
 424 South Atlantic Ocean, transports relatively warm, saline subtropical water poleward, while
 425 the Malvinas Current, a branch of the Antarctic Circumpolar Current, transports rel-
 426 atively cold, fresh subantarctic water equatorward. The Brazil-Malvinas Confluence refers
 427 to the intersection of these currents off the eastern coast of South America, which is
 428 a region of strong water property gradients. The South Atlantic Current, representing

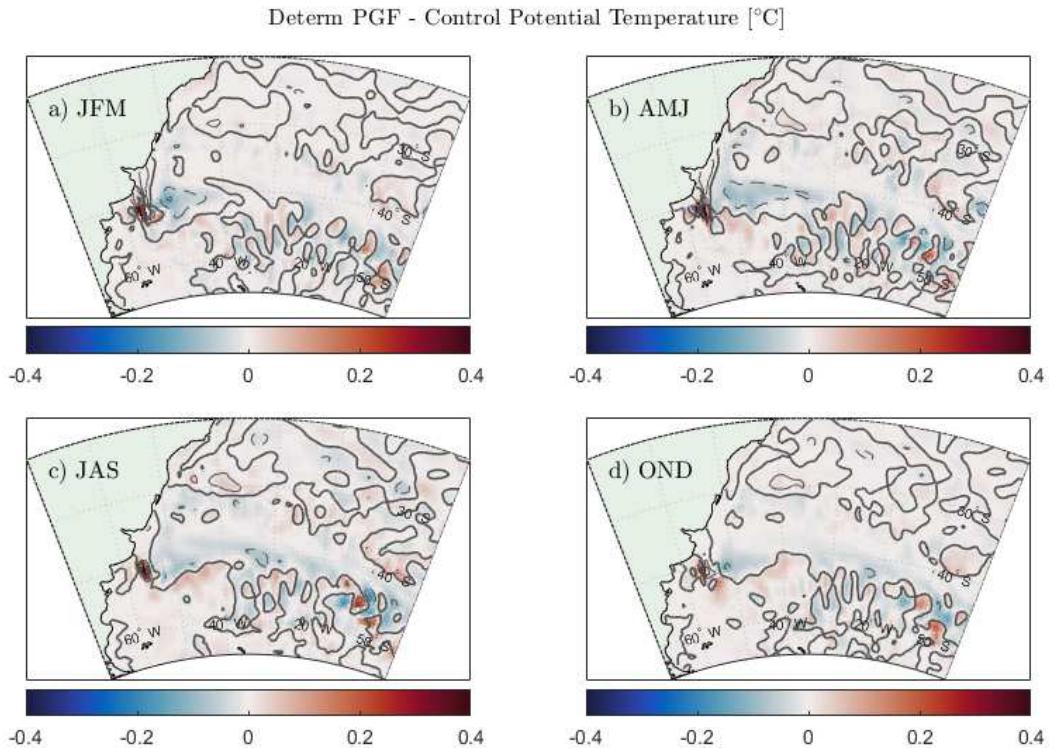


Figure 10. a-d) Deterministic PGF Run minus Control Run of indicated 50-year seasonal mean of potential temperature (colormap) and salinity (contour lines in increments of 0.02 psu) at 2.5 m.

the southern boundary of the subtropical gyre in the South Atlantic Ocean, is fed by the Brazil Current and flows eastward alongside the Antarctic Circumpolar Current to the south. In the Control Run, the South Atlantic Current is characterized by a tongue of relatively warm, salty water (compared with the hydrographic conditions to the north and south) extending eastward across the basin at $\sim 40^{\circ}\text{S}$ (Figures S17-S20a-d).

Owing to the strong lateral temperature gradients near the Brazil-Malvinas Confluence, the SGS temperature variance parameterization and density corrections are locally elevated (Figure 1a-b). In both the Experimental Runs, the Brazil Current extends further south before separating from the coast relative to the Control Run, as evidenced by the dipole pattern of temperature and salinity anomalies near the front (i.e., the positive anomaly of temperature and salinity at $\sim 41^{\circ}\text{S}$, $\sim 57^{\circ}\text{W}$ and the negative anomaly to the northeast at 2.5 m; Figure 10-11a-d). The dipole pattern of temperature and salinity anomalies develops along the coast of South America in summer (JFM), when the Brazil Current/South Atlantic Current is fed by particularly warm subtropical water (Figure 11a). The cold, fresh signal is observed to propagate westward along the prior path of the South Atlantic Current during subsequent seasons and to dissipate by winter (JAS; Figure 11a-d). (The depth-dependence of the seasonality is characterized in section S2.) Nevertheless, the ocean response to the deterministic and stochastic parameterizations differs both qualitatively and quantitatively. In the Stochastic PGF Run, the water property dipole is present, with varying amplitude, from the surface to 400 m and below (Figures 11a-d, S21-S23a-d), while it is considerably weaker in the Deterministic PGF Run (compare Figures 10a-d and 11a-d) and is not significantly present at 400 m (not shown).

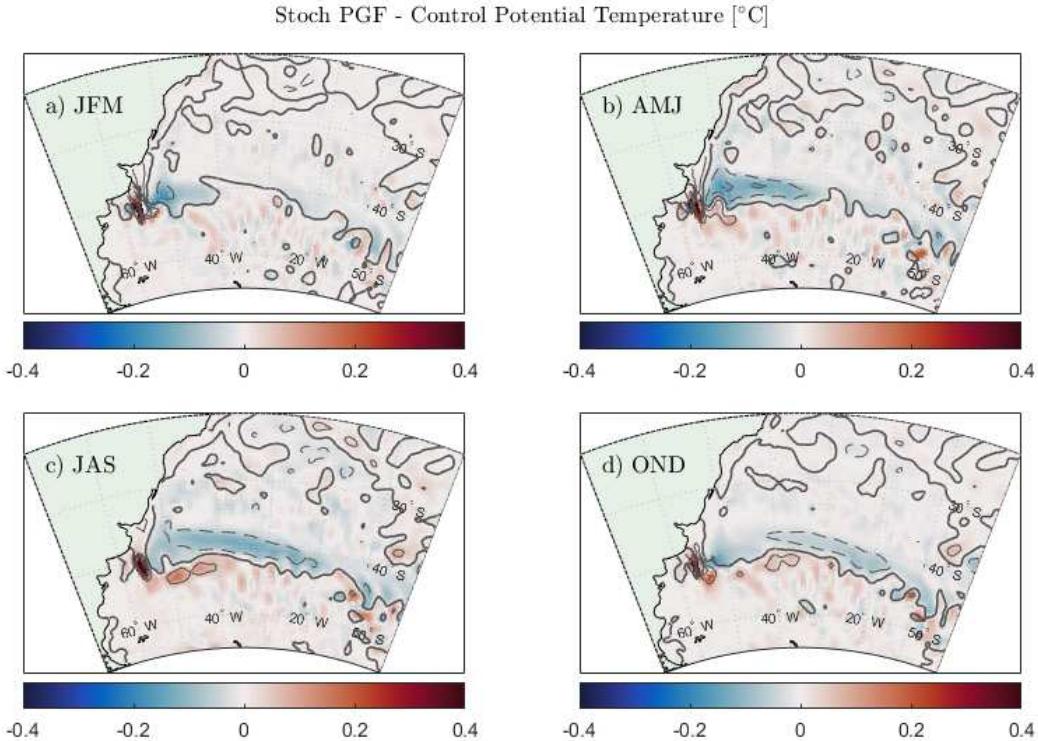


Figure 11. Same as Figure 10, but for the Stochastic PGF Run.

451 The greater ocean response in the Stochastic PGF Run relative to the Deterministic PGF
 452 Run is a rectified effect of the noise.

453 4 Discussion and Conclusions

454 Coarse-resolution ocean models contain dynamically significant errors in horizontal
 455 density gradients associated with estimating the mean density by applying the non-
 456 linear EOS to the grid-cell-mean temperature and salinity. The SGS temperature vari-
 457 ance parameterization of Stanley et al. (2020) was developed to provide an empirically
 458 validated and computationally efficient correction to the large-scale density field, but the
 459 ocean response to this correction has been hitherto unknown. In this study, we have im-
 460 plemented the deterministic and stochastic versions of this parameterization in the hor-
 461 izontal PGF of CESM-MOM6 and performed hindcast sensitivity experiments to quan-
 462 tify changes in the long-term mean and variability of the ocean state.

463 In general, the parameterization leads to multifaceted changes in the mean hydro-
 464 graphic conditions and circulation, particularly in the Atlantic Ocean (Figure 2d-i). In
 465 many regions, the changes associated with the deterministic and stochastic parameter-
 466 izations are qualitatively, if not quantitatively, similar. In the North Atlantic Ocean, cool-
 467 ing and freshening of the Nordic Seas and warming and salinification of the Labrador
 468 Sea result (Figure 4c-f). These changes are primarily attributed to a seasonally varying
 469 reduction in the upper-ocean exchange at Denmark Strait in the Experimental Runs rel-
 470 ative to the Control Run (Figure 6e-h), resulting in a dipole pattern of temperature and
 471 salinity anomalies which propagates with the mean circulation (Figure 7a-d). Thus, the
 472 upper ocean in the Nordic Seas cools and freshens, while the Labrador Sea warms and
 473 salinifies (Figure 4c-f). The warming of the Labrador Sea is amplified by the intensifi-

474 cation of wintertime deep convection which is triggered by the positive salinity anomaly
 475 (Figure 5c,e). The changes in upper-ocean temperature are associated with the enhanced
 476 growth of sea ice in the western Nordic Seas and reduced growth along the Labrador Cur-
 477 rent (Figure 5d,f).

478 Other changes in the position of dynamically important boundary currents result
 479 from the SGS temperature variance parameterization. The separated Gulf Stream shifts
 480 to the south and strengthens in the Experimental Runs relative to the Control Run, re-
 481 ducing a well-known bias in coarse-resolution ocean models (Figure 9). In the South At-
 482 lantic Ocean, the separation of the Brazil Current from the coast shifts to the south, lead-
 483 ing to the formation of seasonally propagating hydrographic anomalies near the Brazil-
 484 Malvinas Confluence (Figure 11a-d).

485 Overall, both the deterministic and stochastic density corrections elicit qualitatively
 486 similar ocean responses, although amplitudes are generally greater with the stochastic
 487 correction; however, regional exceptions have been noted. The majority of the response
 488 is associated with the mean correction, and the rectified effects of the noise play a lesser
 489 role, albeit a substantial one in certain regions such as the Brazil-Malvinas Confluence.
 490 Furthermore, the stochastic version has additional applications in the context of ensem-
 491 ble generation and data assimilation. Both parameterizations are highly efficient; the cost
 492 of computing the deterministic and stochastic parameterizations of SGS temperature vari-
 493 ance amounts to $\sim 1\%$ and $\sim 2\%$ of the total cost, respectively. In ocean modeling ap-
 494 plications, the deterministic version of the parameterization could offer a sufficient ap-
 495 proximation of the mean effect of the stochastic parameterization, depending on the tar-
 496 get application.

497 It is a priori unclear to what extent the dynamical response to the Brankart (2013)
 498 and Stanley et al. (2020) parameterizations should qualitatively differ, given the theo-
 499 retical and mathematical differences in their formulations. Nevertheless, certain changes
 500 in the ocean mean state, such as the equatorward shift and strengthening of the Gulf
 501 Stream, have been reported to result from the Brankart (2013) parameterization in both
 502 a coarse-resolution and an eddy-permitting configuration of NEMO (Zanna et al., 2019).
 503 Yet the SSH changes in the North Atlantic Ocean associated with the Stanley et al. (2020)
 504 parameterization in MOM6 differ somewhat from those associated with the Brankart (2013)
 505 parameterization in NEMO. For instance, both Brankart (2013) and Zanna et al. (2019)
 506 have found that the SSH in the Labrador Sea and along the northwestern North Atlantic
 507 shelf generally increases in response to their parameterizations, in contrast with our find-
 508 ings (compare Figure 8c,e with Figure 4 of Zanna et al. (2019)). However, as demonstrated
 509 in Section 3.2, the hydrographic changes in the Labrador Sea are largely attributable to
 510 remotely generated anomalies (i.e., changes in the circulation near Denmark Strait which
 511 result in the export of a warm and saline anomaly to the Labrador Sea). These mech-
 512 anisms may be relatively weak or absent in Brankart (2013) and Zanna et al. (2019), since
 513 the coarse horizontal resolution of ORCA2 may preclude the resolution of some features
 514 of the East Greenland Current and Denmark Strait circulation. Moreover, in the eddy-
 515 permitting NEMO experiments of Zanna et al. (2019), the density correction is scaled
 516 by a function of latitude which attenuates the parameterization at both low and high
 517 latitudes. Differences between the ocean responses to the Brankart (2013) parameter-
 518 ization in ORCA2 and the Stanley et al. (2020) parameterization in MOM6 in the Brazil-
 519 Malvinas Confluence region may be attributable in part to differences in resolution and
 520 hence representation of boundary currents. The eddy-permitting NEMO configuration
 521 was regional, precluding a comparison of our studies in that region.

522 While this study has assessed the ocean response to a parameterization which cor-
 523 rects errors in the large-scale horizontal PGF, the errors associated with the unresolved
 524 SGS temperature variability are not confined to this term. For instance, the GM frame-
 525 work is widely used to parameterize unresolved mesoscale eddy variability as an advect-
 526 tive flux of buoyancy which reduces the slope of isopycnals, thus simulating the effects

527 of baroclinicity in extracting available potential energy from the fluid (Gent & McWilliams,
 528 1990). In the GM parameterization, the eddy-induced streamfunction is proportional via
 529 an eddy buoyancy diffusivity to the isopycnal slope (i.e., a ratio of lateral and vertical
 530 buoyancy gradients), which may contain errors associated with applying the nonlinear
 531 EOS to grid-cell-mean quantities. Although stochastic perturbations of eddy buoyancy
 532 fluxes have recently been tested in an ocean model (Grooms, 2016; Grooms & Kleiber,
 533 2019), a correction to the buoyancy gradient for the interaction of the unresolved SGS
 534 variability with the nonlinear EOS has not. Other potential areas where the density cor-
 535 rection may be relevant include the mixed-layer restratification parameterization of Fox-
 536 Kemper et al. (2008). Ongoing and future work will investigate the ocean response to
 537 applying the Stanley et al. (2020) parameterization to correct the isopycnal slope in the
 538 GM and Fox-Kemper et al. (2008) frameworks.

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 556 10.5281/zenodo.5793964. All model data are stored on the NCAR/CGD storage sys-
 557 tem.

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