

Probabilistic Grid Strength Assessment of Power Systems with Uncertain Renewable Generation based on Probabilistic Collocation Method

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Abstract—The increasing integration of renewable energy resources (RERs) such as wind and solar onto the electric power grid through power electronic interface is challenging safe and reliable grid operation. Particularly, the high penetration of the inverter-based RERs (IB-RERs) may drive the grid towards weak grid conditions, which may cause grid stability issues. Grid strength assessment is helpful to identify these weak grid issues. However, it is challenging to assess grid strength while considering the impact of uncertain renewable generation. This paper presents an approach for quantifying the probabilistic characteristics of grid strength under uncertain renewable generation based on the probabilistic collocation method, which is a computationally efficient technique to reduce the computational burden without compromising the result accuracy compared with traditional Monte Carlo simulation. The efficacy of the proposed approach is demonstrated on the modified IEEE 9-bus system.

Keywords—renewable energy resources, grid strength, probabilistic collocation method

I. INTRODUCTION

Renewable energy resources (RERs) are being increasingly integrated into the electric power grid to reduce greenhouse gas emissions. Most renewable energy resources are interfaced with the power grid through power electronic inverters. While the inverter-based RERs (IB-RERs) supply clean energy to electricity customers, they are also challenging grid planning and operation. The IB-RERs provide expected real and reactive power based on their electronic controls, which link renewable energy resources to the grid. These controls may in turn depend on a stable voltage reference from the grid. As the grid is weakened due to the increasing displacement of synchronous generators by IB-RERs, the voltage reference becomes less stable, and control dynamics and tuning become increasingly influential on overall system behavior [1]. The weak grid issues may become prominent due to the variability of IB-RER generation under uncertain weather conditions [2][3].

Potential weak grid issues are usually analyzed and identified based on grid strength assessment. Short-circuit ratio (SCR) is an index recommended by North American Electric Reliability Corporation (NERC) to quantify the grid strength [1][4]. Commonly used SCR calculation methods ignore the impact of interactions among IB-RERs on grid strength and thus may cause an inaccurate estimation of grid strength at points of interconnection (POI) for IB-RERs [1][5]. To consider the effect of IB-RERs interactions on the grid strength, several new methods have been developed, such as the weighted short-circuit ratio (WSCR) method developed by the Electric Reliability Council of Texas [5] and the composite short-circuit ratio (CSCR) method

developed by GE Energy Consulting [6]. Both the CSCR and the WSCR methods do not consider the real electrical network connections among IB-RERs, which may not reflect the actual strength of the grid at the POIs. Also, both the CSCR and WSCR methods mainly provide the aggregated strength of a power grid in the area where the IB-RERs are interconnected electrically close, but they do not calculate the strength of the grid at each individual POIs in the specific area. To overcome these shortcomings, the site-dependent short-circuit ratio (SDSCR) method is proposed in [7].

While various methods have been proposed to improve the accuracy of grid strength assessment, it is still challenging to assess grid strength under uncertain IB-RER generation. Generally, the impact of uncertain IB-RERs on grid strength can be evaluated by integrating the methods for grid strength assessment with Monte Carlo simulation (MCS). Thus, MCS can be used to obtain a large number of IB-RER generation samples [8]-[10], and grid strength assessment is repeated with these samples to render the uncertainty characteristics of the results. For large power systems with large-scale integration of IB-RERs, such uncertainty evaluation for grid strength needs tens of thousands of simulations to achieve accurate results.

To improve the computational efficiency, the paper proposes a probabilistic approach for assessing the impact of uncertain IB-RER generation on grid strength by integrating the probabilistic collocation method (PCM) with the SDSCR-based method. The PCM has been studied for uncertainty analysis in numerous power system studies [11]-[15]. Compared to other probabilistic methods used in power system probabilistic studies, the PCM can characterize the uncertainty of renewable generation by more types of distributions obtained from the historical or predicted data. Thus, the proposed method can use the probability distributions of renewable generation to quantify the probabilistic characteristics of grid strength through a set of orthogonal polynomials to establish the probabilistic approximation functions for grid strength analysis. Since a small number of grid strength assessment are required to determine the parameters in the approximation functions, the proposed method can significantly reduce computational burden and outperforms MCS in terms of simulation efficiency.

The rest of this paper is organized as follows. In Section II, grid strength assessment is discussed. In Section III, the principle of PCM is introduced. Section IV presents our proposed method for grid strength analysis of power systems under uncertain renewable generation. The efficacy of the proposed method is demonstrated in Section V. In Section VI, the conclusions are drawn.

II. GRID STRENGTH ASSESSMENT

Grid strength assessment can help grid engineers identify and understand weak grid issues for reliably planning and operating the power grid. Grid strength is a characteristic of an electrical power system that relates to the size of the change in voltage following a fault or disturbance on the power system [16]. The stronger a grid is, the less risks the grid will have for weak grid issues. The strength of a power grid at (POI) is commonly quantified by SCR, which is the ratio of the short circuit capacity at the POI to the rated capacity or injected power from the IB-RER [17]. That is,

$$\text{SCR}_i = \frac{|S_{ac,i}|}{P_{R,i}} = \frac{|V_{R,i}|^2}{P_{R,i}} \cdot \frac{1}{|Z_{R,i}|} \quad (1)$$

where symbol $|\cdot|$ indicates the magnitude of a complex quantity; $S_{ac,i} = |V_{R,i}|^2 / |Z_{R,i}|$ is the short-circuit capacity of the grid at POI i ; $V_{R,i}$ is the voltage at POI i ; $Z_{R,i}$ is the Thevenin equivalent impedance seen at POI i ; and $P_{R,i}$ is the rated capacity or injected power from the IB-RER to be integrated at POI i .

Since the SCR defined in (1) does not account for the interactions among multiple IB-RERs, its evaluation results may be inaccurate, especially for grid strength assessment at POIs, where IB-RERs are electrically close. To improve the accuracy of grid strength assessment, the SDSCR was proposed in [7] by analyzing the relationship between the SCR and voltage stability in a power grid with a single IB-RER and then extending this relationship to a power grid with multiple IB-RERs. The SDSCR is defined as [7],

$$\text{SDSCR}_i = \frac{|V_{R,i}|^2}{(|P_{R,i} + \sum_{j \in R, j \neq i} w_{ij} P_{R,j}|) |Z_{RR,ii}|} \quad (2)$$

$$w_{ij} = \frac{Z_{RR,ij}}{Z_{RR,ii}} \times \left(\frac{V_{R,i}}{V_{R,j}} \right)^* \quad (3)$$

where $\mathbf{R}$ is the set of all POIs for IB-RER integration; $Z_{RR,ij}$ is the $(i^{\text{th}}, j^{\text{th}})$ element in submatrix of bus impedance matrix that is only related to buses connected to IB-RERs; and symbol $*$ indicates the conjugate of a complex quantity.

Compared with the SCR in (1), the SDSCR in (2) improves the accuracy of grid strength assessment by considering the impact of the interactions among IB-RER generation on the assessment results at POI i via the interaction factor w_{ij} . The SCR in (1) can be considered as a special case of the SDSCR when only one IB-RER is connected to the power grid. Thus, the SCR ranges for grid strength evaluation can also be applied to SDSCR. That is, the grid is strong at a POI if its SDSCR value is larger than 3; the grid is weak at a POI if its SDSCR value is between 2 and 3; and the grid is very weak at a bus if its SDSCR value is smaller than 2 [9]. To accurately assess grid strength at POI i using the SDSCR, it needs to consider not only the renewable generation $P_{R,i}$ to be integrated at POI i but also the renewable generation $P_{R,j}$ to be integrated at the other POIs. Thus, increasing renewable generation at any of these POIs may reduce grid strength.

The uncertainty and intermittency characteristics of renewable energy could affect grid strength and thus grid stability, and this impact could be aggravated with the increase in the penetration level of renewable energy. To evaluate the impact of uncertain renewable generation on grid strength, MCS can typically be used to generate a large number of IB-RER generation samples; then, grid strength assessment is repeated with these samples to render the

uncertainty characteristics of the results. Such uncertainty evaluation needs to consider various combinations of IB-RER generations at different POIs. In large power systems with large-scale integration of IB-RERs, the uncertainty evaluation is computational expensive. To improve the computational efficiency, the paper proposes a probabilistic approach by integrating the PCM with the SDSCR-based method to quantify the probabilistic characteristics of grid strength under uncertain renewable generation.

III. PRINCIPLE OF PROBABILISTIC COLLOCATION METHOD

The PCM is an approach using Gaussian quadrature to map the relationship between the uncertain input parameters and the output [11]. It simplifies the relationship between the uncertain parameters and the desired output by identifying a good set of simulations for correctly and robustly determining the mapping. The coefficients of this polynomial mapping equation are determined by methodically selecting the collocation points. The basic principles of the PCM are derived from the concepts of orthogonal polynomials and Gaussian quadrature integration [12].

A. Orthogonal polynomials and Gaussian quadrature

For a single uncertain parameter x , two polynomial functions $g(x)$ and $h(x)$ are orthogonal only if their inner product is zero [18]. The inner product of $g(x)$ and $h(x)$ is defined as

$$\langle g(x), h(x) \rangle = \int_A f(x) g(x) h(x) dx \quad (4)$$

where $f(x)$ is any non-negative weighting function defined in a space A , and it can be represented with the probability density function (pdf) in the PCM. A set of orthogonal polynomial functions $\{H_1(x), H_1(x), \dots, H_n(x)\}$ can be defined,

$$\langle H_i, H_j \rangle = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad (5)$$

where H_i is a polynomial of order i . For each order i , H_i has exactly i roots within the space of A . These roots are the collocation points to evaluate the coefficients of $g(x)$ used in the PCM. The $(-1)^{\text{th}}$ and 0^{th} order polynomials are defined to be 0 and 1, respectively.

$$H_{-1}(x) = 0$$

$$H_0(x) = 1 \quad (6)$$

Gaussian quadrature integration in (4) approximates the numeric value for the integral by selecting appropriate x values to evaluate $g(x)$ and calculate the integral,

$$\int_A f(x) g(x) h(x) dx \approx \sum_{i=1}^n f_i g(x_i) \quad (7)$$

where f_i is the coefficient determined by the weighting function $f(x)$, and $g(x_i)$ is computed based on x_i , which is the roots of the higher orthogonal polynomials selected based on the order of the PCM model.

B. PCM with a single input parameter

For a single uncertain parameter x with its pdf $f(x)$, the output Y is a function of the input uncertain parameter x . The function $g(x)$ maps the relationship between x and Y . The estimated output $\hat{Y}$ in PCM can be represented

$$\hat{Y} = \hat{g}(x) = k_0 H_0(x) + k_1 H_1(x) + \dots + k_{n-1} H_{n-1}(x) \quad (8)$$

where k_i are constant coefficients, H_i are orthogonal polynomials of uncertain input x , and n is the order of the PCM model. The coefficients k_i are solved by replacing the

estimated output and orthogonal polynomials for the collocation points.

The collocation points are selected as the roots of the next higher order orthogonal polynomial H_{n+1} of the uncertain parameter x for the n^{th} order PCM approximation model. This approach allows the collocation points to traverse the high probability regions of their distribution and to capture the behavior of the estimated output to the fullest extent [13]. The coefficients k_i can be solved by

$$\begin{bmatrix} k_{n-1} \\ \vdots \\ k_0 \end{bmatrix} = \begin{bmatrix} H_{n-1}(x_1) & \cdots & H_0(x_1) \\ \vdots & \ddots & \vdots \\ H_{n-1}(x_n) & \cdots & H_0(x_n) \end{bmatrix}^{-1} \begin{bmatrix} \hat{g}(x_1) \\ \vdots \\ \hat{g}(x_n) \end{bmatrix} \quad (9)$$

where $x_1, \dots, x_n$ are the collocation points, $\hat{g}(x_1), \dots, \hat{g}(x_n)$ are the responses of the output at the collocation points, and $H_0(x), \dots, H_{n-1}(x)$ are the orthogonal polynomials calculated at the collocation points. These coefficients are replaced in (8) to obtain the PCM approximation model. The statistics of the output response for a given range of the uncertain input parameter can be calculated simply using these coefficients. The expected value of output is $E[\hat{g}(x)] = k_0$ and the variance of the output value is $\sigma^2[\hat{g}(x)] = \sum_{i=1}^{n-1} k_i^2$.

C. PCM with multiple input parameters

When multiple uncertain parameters $x_1, x_2, \dots, x_n$ with independent pdfs $f(x_1), f(x_2), \dots, f(x_n)$ are considered in a system, the approximation for the output can be obtained by,

$$\hat{Y} = k_0 + \sum_{i=1}^n [k_{i1}H_{i1}(x_i) + \dots + k_{im}H_{im}(x_i)] + \sum_{i=1}^n \sum_{j=1}^n [k_{ij}H_{ij}(x_i)H_{j1}(x_j)] \quad (10)$$

where $k_0, k_{i1}, \dots, k_{im}$ are the coefficients, and $H_{i1}(x_i), \dots, H_{im}(x_i)$ are orthogonal polynomials for uncertain parameter x_i [13].

Equation (10) shows that as the number of input parameters and the order of orthogonal polynomials grow, the size and complexity of the approximation polynomial functions increases. Thus, the number of simulation samples required for determining the coefficients also increases dramatically. Therefore, to retain the advantages of PCM, the number of input variables and the order of polynomials need to be relatively small [14].

IV. PROBABILISTIC APPROXIMATION METHOD FOR GRID STRENGTH ASSESSMENT

To assess the impact of uncertain renewable generation on grid strength, the SDSCR-based method is integrated with the PCM, which models the impact of uncertain renewable generation probabilistically based on their historical data and evaluates the probabilistic results.

A. Probabilistic model of renewable generation

The actual historical data can be used to characterize the uncertainty of renewable generation. For example, uncertain wind generation is related to variable wind speed. Thus, the forecasted or the historical data for wind speed can be used for probabilistically modeling the uncertain feature of wind generation. The mechanical power output P of wind turbine can be calculated using the equation below [19],

$$P(w) = \begin{cases} 0, & \text{if } w \leq w_c^{\text{in}} \\ P_R \frac{w - w_c^{\text{in}}}{w_c^{\text{out}} - w_c^{\text{in}}}, & \text{if } w_c^{\text{in}} < w \leq w_c^{\text{out}} \\ P_R, & \text{otherwise} \end{cases} \quad (11)$$

where w is the wind speed, P_R is the rated power of wind generator, w_c^{in} and w_c^{out} are the cut-in and cut-out wind speed, and the uncertainty of the wind speed can be represented by the Weibull distribution below [13],

$$f(w) = \left(\frac{k}{c}\right) \left(\frac{w}{c}\right)^{k-1} e^{-\left(\frac{w}{c}\right)^k} \quad (12)$$

where $f(w)$ is the pdf of the wind speed, k is the shape factor and c is the scale factor of the distribution of wind speed w . Fig. 1 shows the pdf curve of wind speed modeled as Weibull distribution to represent the uncertainty in wind generation.

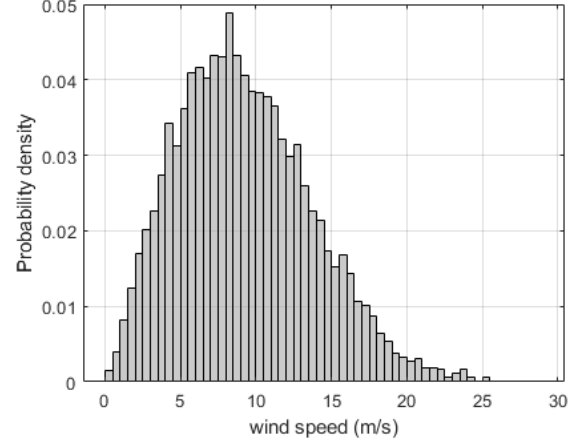


Fig. 1. Probability density of wind speed with Weibull distribution.

TABLE I. ORTHOGONAL POLYNOMIALS FOR WIND GENERATORS

Wind Generator 1	
$G_1(y_1) = y_1 - 1.4872$	
$G_2(y_1) = y_1^2 - 4.9743y_1 + 3.6988$	
$G_3(y_1) = y_1^3 - 10.4615y_1^2 + 26.0197y_1 - 12.8986$	
$G_4(y_1) = y_1^4 - 17.9487y_1^3 + 93.8856y_1^2 - 155.6735y_1 + 57.8786$	
...	
Wind Generator 2	
$H_1(y_2) = y_2 - 1.5454$	
$H_2(y_2) = y_2^2 - 5.0909y_2 + 3.9338$	
$H_3(y_2) = y_2^3 - 10.6363y_2^2 + 27.0743y_2 - 13.9474$	
$H_4(y_2) = y_2^4 - 18.1818y_2^3 + 96.6942y_2^2 - 164.0871y_2 + 63.3973$	
...	

To derive the orthogonal polynomials of wind speed for the application of the PCM, we consider the associated Laguerre polynomial, which is orthogonal over $[0, \infty)$ with respect to the weighting function $v^z e^{-v}$ with an arbitrary real z .

$$\int_0^\infty v^z e^{-v} L_i^{(z)}(v) L_j^{(z)}(v) dv = 0, \quad i \neq j \quad (13)$$

The weighting function for the Weibull distribution can be rearranged to be expressed in the form of the Laguerre polynomial [13]. Thus, the orthogonal polynomials for the representative distribution can be derived by the method described in [20]. Equation (10) can be rearranged by comparing it with the Weibull distribution and is written as in terms of y as a transitional variable.

$$\int_0^\infty y^{\frac{k-1}{k}} e^{-y} H_i(y) H_j(y) dy = 0, \quad i \neq j \quad (14)$$

$$\text{where} \quad y = \left(\frac{v}{c}\right)^k \quad (15)$$

Based on (14) and the Gram-Schmidt process [18], we can derive the orthogonal polynomials for the wind speed in wind generators, such as those listed in Table I, where $G_i(y_1)$ represents the orthogonal polynomials for wind generator 1 and $H_i(y_2)$ for wind generator 2. The roots from $G_i(y_1)$ and

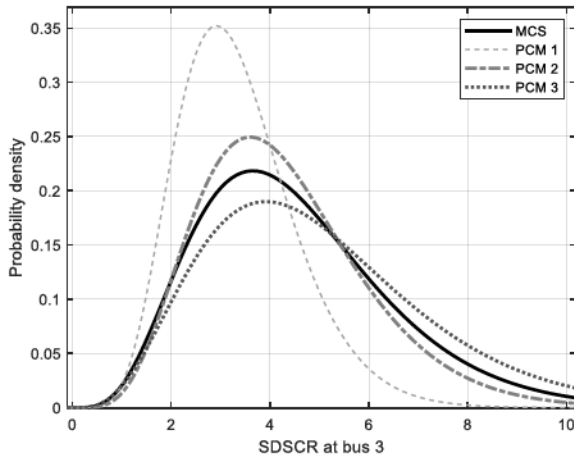


Fig. 3. pdf curves for grid strength evaluated based on SDSCR at bus 3 for different polynomial orders.

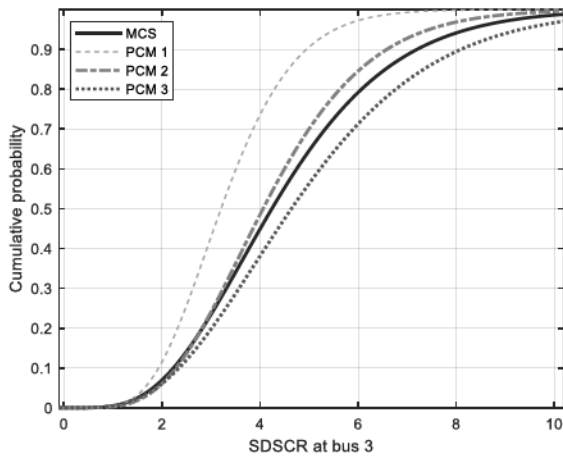


Fig. 4. cdf curves for grid strength evaluated based on SDSCR at bus 3 for different polynomial orders.

It can be observed from Fig. 3 and Fig. 4 that among all three approximation models with different orders of orthogonal polynomials, the approximation model with the first two polynomial orders provides more accuracy fitting results. In Fig. 3 and Fig. 4, the approximation model with the first two polynomial orders provides the estimated pdf and cdf curves closer to those obtained from MCS than the approximation models with the other polynomial orders. Moreover, Table III shows that the means and variances from the approximation model with the first two polynomial orders are 4.28 and 2.95, which are closer to 4.53 and 3.97 from MCS results. In addition, the approximation model with the first two polynomial orders has the smallest error (i.e., 0.27) among the three approximation models with different orders of orthogonal polynomials. Thus, the approximation model with the first three polynomial orders provides more accuracy approximation results than the other two approximation models with different orders of orthogonal polynomials.

It can be seen from Table III that the proposed method can save a huge amount of simulation time, since it reduces simulation burden significantly. To obtain the results of Figs. 2 and 3, the approximation model with the first two polynomial orders requires only 8 simulation runs, but the MCS requires 5000 simulation runs. The time spent by MCS method is also almost 20 times larger than the proposed method.

TABLE III. COMPARISON OF SDSCR AT BUS 3 FOR MCS AND APPROXIMATION MODELS WITH DIFFERENT POLYNOMIALS ORDERS

	MCS	PCM 1st	PCM 2nd	PCM 3rd
mean	4.53	3.35	4.28	4.99
variance	3.97	1.45	2.95	3.08
e_{SSR}	-	0.59	0.27	0.29
runs	5000	3	8	15
Time (s)	102.71	4.32	4.91	5.11

B. Estimating SDSCR for Different Wind Penetration

To analyze the impact of uncertain wind generation on grid strength, the wind power penetration is increased from 20% of total load power to 40% in the system, and the power outputs of synchronous generators are also adjusted to balance the power in the system. Fig. 5 shows the cdf curves of the grid strength evaluated based on the SDSCR at bus 3 for the two different wind penetrations, respectively. The approximation model with the first two polynomial orders is used for the evaluation. Table IV compares the mean value and variance value for the different penetration cases.

It can be observed from Fig. 5 and Table III that the increasing penetration of uncertain wind generation reduces the grid strength, thus increasing the risk of weak grid issues. As shown in Fig. 6, when the wind penetration is 20% of total load power, the SDSCR values changes between 0.93 and 15.88; when the wind generation is increased to 40% of total load power, the SDSCR values are reduced to the range between 0.41 to 8.04. Also, Table IV shows that the means and variances of SDSCR at bus 3 are reduced after the wind power penetration is increased from 20% of total load power to 40% in the system. Thus, the increasing wind generation reduces the changing range of the SDSCR and thus the grid strength. Particularly, as discussed in Section II, the weak grid issues may become significant when SDSCR is smaller than 3. It can be observed from Fig. 5 that when the wind penetration is 20% of total load power, the probability of SDSCR smaller than 3 is 0.027; on the other hand, when the wind generation is increased to 40% of total load power, the probability is increased to 0.2. This indicates that the risk of weak grid issues increases with the wind generation penetration. Thus, the proposed method can quantify the probabilistic impact on grid strength due to uncertain wind generation.

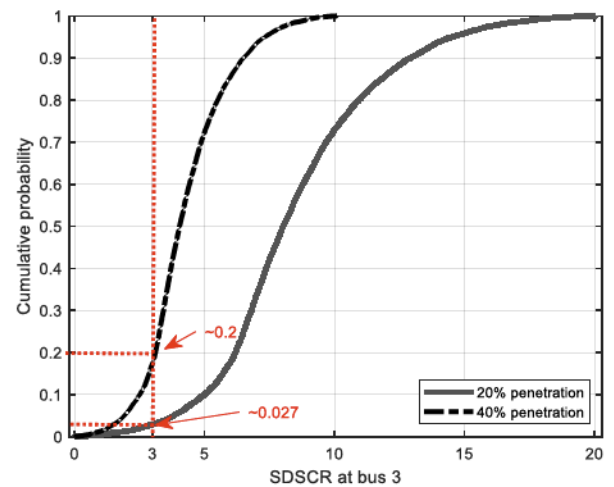


Fig. 5. cdf curves of SDSCR at bus 3 for different wind power penetration based on the approximation model with the first two polynomial orders.

TABLE IV. COMPARISON OF SDSCR AT BUS 3 FOR DIFFERENT WIND PENETRATION USING THE APPROXIMATION MODEL WITH THE FIRST TWO POLYNOMIAL ORDERS

Wind Generation	40% of total load (150 MW)	20% of total load (75 MW)
mean	4.28	8.52
variance	2.56	10.14

VI. CONCLUSION

This paper proposed an approach for assessing the impact of uncertain renewable generation on grid strength by integrating the PCM with the SDSCR-based method. In the proposed approach, the PCM was used to establish the approximation polynomial functions with multiple input variables for modeling the probabilistic impact of uncertain renewable generation on grid strength evaluated based on the SDSCR-based method. The PCM is a computationally efficient technique, which can reduce the computation burden without compromising the result accuracy compared to traditional Monte Carlo simulation. The efficacy of the proposed method is demonstrated on the modified IEEE 9-bus system. The proposed approach is promising to guide grid planning and operation for identifying potential weak grid issues in power systems under uncertain renewable generation. In our further research, we will further improve the proposed method for large-scale power system application. We will design algorithms to select the approximation samples of input variables to improve the approximation accuracy. Also, the actual historical data of renewable generation will be used to capture the distribution of renewable generation.

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REFERENCES

- [1] NERC Integrating Inverter-Based Resources into Low Short Circuit Strength Systems. December 2017. Available: https://www.nerc.com/comm/PC_Reliability_Guidelines_DL/Item_4a_Integrating%20InverterBased_Resources_into_Low_Short_Circuit_Strength_Systems_-_2017-11-08-FINAL.pdf.
- [2] Y. Ding, P. Wang, L. Goel, P. C. Loh, and Q. Wei, "Long-term reserve expansion of power systems with high wind power penetration using universal generating function methods," *IEEE Trans. Power Syst.*, vol. 26, no. 2, pp. 766-774, May 2011.
- [3] Y. Ding, C. Singh, L. Goel, J. Østergaard, P. Wang, "Short-term and medium-term reliability evaluation for power systems with high penetration of wind power," *IEEE Trans. Sust. Energy*, vol. 5, no. 3, pp. 896-906, July 2014.
- [4] NERC essential reliability services task force: measures framework report. November 2015. Available: <http://www.nerc.com/comm/Other/essntlrbltysrvcskfrDL/ERSTF%20Framework%20Report%20-%20Final.pdf>.
- [5] Zhang, S. H. Huang, J. Schmall, J. Conto, J. Billo and E. Rehman, "Evaluating system strength for large-scale wind plant integration", *IEEE Power and Energy Society General Meeting*, July 2014, pp. 1-5.
- [6] Minnesota renewable energy integration and transmission study final report. GE Energy Consulting. October 2014. Available: <http://www.minnelectrans.com/documents/mrits-report.pdf>.
- [7] D. Wu, G. Li, M. Javadi, A. Malyscheff, Member, M. Hong, and J. Jiang, "Assessing impact of renewable energy integration on system strength using site-dependent short circuit ratio," *IEEE Trans. on Sust. Energy*, vol. 9, No. 3, pp.1072-1080, 2018.
- [8] R. Billinton and W. Li, *Reliability Assessment of Electrical Power Systems Using Monte Carlo Methods*. New York: Springer, 1994.

- [9] IEEE guide for planning dc links terminating at ac locations having low short-circuit capacities, IEEE Standard 1204-1997, 1997.
- [10] J. F. Zhang, C. Y. Chung, C. T. Tse, and K. W. Wang, "Voltage Stability Analysis considering the uncertainties of dynamic load parameters," *IET Gen., Trans.&Dist.*, vol. 3, no.10, pp.941-948, 2009.
- [11] J. R. Hockenberry and B. C. Lesieutre, "Evaluation of uncertainty in dynamic simulations of power system models: The probabilistic collocation method," *IEEE Trans. on Power Syst.*, vol. 19, no. 3, pp. 1483-1491, 2004.
- [12] M. Webster, M. A. Tatang, G. J. McRae, "Application of the probabilistic collocation method for an uncertainty analysis of a simple ocean model", Tech. Rep. 4, Joint Program on the Science and Policy of Global Change, MIT, Cambridge, MA, Jan. 1996.
- [13] C. Zheng and M. Kezunovic, "Impact of wind generation uncertainty on power system small disturbance voltage stability: A PCM-based approach," *Elec. Power Syst. Research*, vol. 84, no. 1, pp. 10-19, 2012.
- [14] R. Preece, N. C. Woolley, and J. V. Milanovic, "The probabilistic collocation method for power-system damping and voltage collapse studies in the presence of uncertainties," *IEEE Trans. on Power Syst.*, vol. 28, no. 3, pp. 2253-2262, 2012.
- [15] M. Fan, Z. Li, T. Ding, L. Huang, F. Dong, Z. Ren, and C. Liu, "Uncertainty Evaluation Algorithm in Power System Dynamic Analysis with Correlated Renewable Energy Sources," *IEEE Trans. on Power Syst.*, vol. 36, no. 6, pp. 5602-5611, 2021.
- [16] Australian Energy Market Operator, "System strength in the NEM explained," AEMO Information & Support Hub, March 2020.
- [17] Y. Zhu and D. Brown, "Prepare to meet the challenges in regional transmission planning and development", IEEE Power and Energy Society General Meeting, pp. 1-5, July 2015.
- [18] P. Davis and P. Rabinowitz, "Numerical Methods of Integration," ed: Academic Press, New York, 1975.
- [19] "Type 3 Wind Turbine Generators (WTG)." <https://www.pscad.com/knowledge-base/article/496>.
- [20] W. Gautschi, "OPQ: A Matlab suite of programs for generating orthogonal polynomials and related quadrature rules," <http://www.cs.purdue.edu/archives/2002/wxg/codes/OPQ.html>, 2002.
- [21] M. C. Shekar and N. Aarthi, "Contingency Analysis of IEEE 9 Bus System," 2018 3rd IEEE International Conference on Recent Trends in Electronics, Information & Communication Technology (RTEICT), 2018, pp. 2225-2229, doi:10.1109/RTEICT42901.2018.9012467.
- [22] K.N. Hasan, R. Preece, and J. V. Milanović, "Existing approaches and trends in uncertainty modelling and probabilistic stability analysis of power systems with renewable generation," *Renewable and Sustainable Energy Reviews*, vol. 101, pp. 168-180, 2019.