Understanding Queries by Conditional Instances

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ABSTRACT

A powerful way to understand a complex query is by observing how it operates on data instances. However, *specific* database instances are not ideal for such observations: they often include large amounts of superfluous details that are not only irrelevant to understanding the query but also cause cognitive overload; and one specific database may not be enough. Given a relational query, is it possible to provide a simple and *generic* "representative" instance that (1) illustrates how the query can be satisfied, (2) summarizes all specific instances that would satisfy the query in the same way by abstracting away unnecessary details? Furthermore, is it possible to find a collection of such representative instances that together completely characterize all possible ways in which the query can be satisfied? This paper takes initial steps towards answering these questions. We design what these representative instances look like, define what they stand for, and formalize what it means for them to satisfy a query in "all possible ways." We argue that this problem is undecidable for general domain relational calculus queries, and develop practical algorithms for computing a minimum collection of such instances subject to other constraints. We evaluate the efficiency of our approach experimentally, and show its effectiveness in helping users debug relational queries through a user study.

CCS CONCEPTS

• **Theory of computation → Data modeling**; *Incomplete, inconsistent, and uncertain databases.*

KEYWORDS

conditional tables, domain relational calculus, satisfying instances

ACM Reference Format:
Amir Gilad, Zhengjie Miao, Sudeepa Roy, and Jun Yang. 2022. Understanding Queries by Conditional Instances. In *Proceedings of the 2022 International Conference on Management of Data (SIGMOD '22), June 12–17, 2022, Philadelphia, PA, USA.* ACM, New York, NY, USA, Article 111, 14 pages. https://doi.org/10.1145/3514221.3517898

1 INTRODUCTION

A powerful way to understand a complex query is by observing how it operates on data instances. A further in-depth approach may also consider the provenance of the query [10, 17, 29] as an indicator how different combinations of tuples in the database satisfy the query and generate each result. Another approach [41] finds a minimal satisfying instance of the query. However, these characterizations are highly dependent on the given database instance, even when provenance is employed. In addition, such database instances can contain many details that divert attention from the query features themselves. For example, some queries are satisfied by an empty instance, but there may be other satisfying instances that are not trivial. Thus, some parts of the query may be ignored since they are not satisfied by the instance. Furthermore, the evaluation on an instance leads to a satisfaction of specific combination of query atoms, but a different combination of atoms that is not satisfied by the specific database instance may reveal new insights. While there are approaches to finding a query solution without a given database instance [15], they provide only one way to satisfy a query, thereby again possibly missing different paths toward satisfying the query.

On the other hand, a single query can have infinitely many satisfying instances so showing all of them will not just be confusing, it may be impossible. Therefore, to understand all possible solutions to a query, we study the question of whether it is possible to provide a simple and *generic* "representative" instance that (1) illustrates how the query can be satisfied, and (2) summarizes all specific instances that would satisfy the query in the same way by abstracting away unnecessary details. Further, we ask how we can find a collection of such representative instances that together completely characterize all possible ways to satisfy the query.

To answer these questions, we propose a novel approach to understanding queries based on *conditional instances* or *c-instances*, by adapting the notion of *c-tables* [33] from the literature on incomplete databases, which are abstract database instances comprising variables (labeled nulls) along with a condition on those variables. Thus, each c-instance can be considered a representative of all grounded instances that replace its variables with constants that satisfy the conditions they are involved in. However, it may be impossible to capture all satisfying instances with a single c-instance. Therefore, we use the idea of *coverage*, borrowed from the field of software validation [5, 42, 43], where it has been well-studied in the context of software testing. For example, a test suite is said to *cover* a function if the function is invoked during the test. This idea can be abstracted to program flows, where an edge/branch in the control-flow graph (see [4] for details) is said to be covered if the edge/branch has been executed. For our use, when given a query Q and a satisfying c-instance $\mathcal{I}$, the atoms and conditions of Q that are satisfied by all ground instances that $\mathcal{I}$ represents are said to be covered by $\mathcal{I}$. We intend to find a set of c-instances such that for every grounded instance that satisfies Q with some coverage C, we have a c-instance that satisfies Q with the same coverage C.

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SIGMOD '22, June 12–17, 2022, Philadelphia, PA, USA

ACM ISBN 978-1-4503-9249-5/22/06…$15.00
https://doi.org/10.1145/3514221.3517898

name	addr
Eve Edwards	32767 Magic Way

(a) Drinker relation

name	brewer
American Pale Ale	Sierra Nevada

(b) Beer relation

name	addr
Restaurant Memory	1276 Evans Estate
Tadim	082 Julia Underpass
Restaurante Raffaele	7357 Dalton Walks

(c) Bar relation

drinker	beer
Eve Edwards	American Pale Ale

(d) Likes relation

bar	beer	price
Restaurant Memory	American Pale Ale	2.25
Restaurante Raffaele	American Pale Ale	2.75
Tadim	American Pale Ale	3.5

(e) Serves relation

Figure 1: Database instance K_0 of the Beers dataset. We assume natural foreign key constraints from Serves and Likes to Drinker, Bar, Beer.

The idea of providing a compact representation of all instances that satisfy a query is appealing not just from a theoretical perspective, but also for multiple practical reasons.

- First, this approach can be used for explaining why a given (wrong) query is different than another (correct) one, as studied in works on counterexamples [15, 41]. In this scenario, we are given a query Q_1 and another query Q_2, the output is an instance K such that $Q_1(K) \neq Q_2(K)$ (i.e., K is a satisfying instance for $Q_1 - Q_2$ or $Q_2 - Q_1$). In an educational setting, such instances would help instructors and students understand why a query is wrong and debug it, without revealing the correct query to students.
- Second, developers and data scientists who work with complex queries can this approach to explore how various parts of the queries can be triggered by different data, or to help them debug or refine these queries. For example, if there are no instances that can trigger some part of a query, it may be possible to simplify the query to remove "dead code" that logically contradicts other necessary conditions in the query.
- Third, this approach offers a method for generating a suite of test instances for a complex query such that together they "exercise" all parts of the query. In the field of synthetic data generation, previous works have proposed different approaches to generating data for testing workload queries [9, 37, 47]. Using our approach, given a workload query Q, we can generate a set of instances to provide coverage testing for all parts of Q; furthermore, given a set of workload queries, we can generate test instances where a given subset of queries are satisfied but others are not. These generated instances can be used for automated, comprehensive testing of queries.

We illustrate the first use case above with an example below.

Example 1. *Consider the database K_0 shown in Figure 1 containing information about drinkers (Drinker table), beers (Beer table), bars (Bar table), which beer does a drinker like (Likes table), and which bar serves which beer (Serves table). Also consider the queries Q_A and Q_B written in Domain Relational Calculus (DRC) in Figures 2a and 2b, respectively. The correct query Q_A returns a list of bars that serve the most expensive beer liked by any drinker whose first name is 'Eve', whereas Q_B is a very similar query that chooses bars serving beers not at the lowest price and only requires first names to have a prefix of 'Eve'. Figure 3 shows the formula for $Q_B - Q_A$ but is not easily understandable and does not clearly show the difference between the*

$$Q_A = \{(x_1, b_1) \mid \exists d_1, p_1 (\text{Serves}(x_1, b_1, p_1) \wedge \text{Likes}(d_1, b_1) \wedge d_1 \text{ LIKE 'Eve}\_\%\text{'} \wedge \forall x_2, p_2 (\neg \text{Serves}(x_2, b_1, p_2) \vee p_1 \geq p_2))\}$$

(a) *Query Q_A: for each beer liked by any drinker whose first name is Eve, find the bars that serve this beer at the* highest *price*

$$Q_B = \{(x_1, b_1) \mid \exists d_1, p_1 (\exists x_2, p_2 (\text{Serves}(x_1, b_1, p_1) \wedge \text{Likes}(d_1, b_1) \wedge d_1 \text{ LIKE } 'Eve\%' \wedge \text{Serves}(x_2, b_1, p_2) \wedge p_1 > p_2))\}$$

(b) *Query Q_B which is similar to Q_A but does not use the difference operator and instead, find beers served at a* non-lowest *price*

Figure 2: Correct query Q_A and incorrect query Q_B. Note that the formula in Q_A has a space after 'Eve' whereas Q_B does not. Here and later, ␣ denotes the space symbol.

$$Q_B - Q_A = \{(x_1, b_1) \mid \exists d_1, p_1 (\exists x_2, p_2 (\text{Serves}(x_1, b_1, p_1) \wedge \text{Likes}(d_1, b_1) \wedge d_1 \text{ LIKE } 'Eve\%' \wedge \text{Serves}(x_2, b_1, p_2) \wedge p_1 > p_2) \wedge \forall d_2, p_3 (\neg \text{Likes}(d_2, b_1) \vee \neg(d_2 \text{ LIKE 'Eve}\_\%\text{'}) \vee \neg \text{Serves}(x_1, b_1, p_3) \vee (\exists x_3, p_4 (\text{Serves}(x_3, b_1, p_4) \wedge p_3 < p_4))))\}$$

Figure 3: The difference query $Q_B - Q_A$ from Figure 2.

name	addr
d_1	*

(a) Drinker relation

name	addr
x_1	*
x_2	*
x_3	*

(b) Bar relation

bar	beer	price
x_1	b_1	p_1
x_2	b_1	p_2
x_3	b_1	p_3

(c) Serves relation

name	brewer
b_1	*

(d) Beer relation

drinker	beer
d_1	b_1

(e) Likes relation

d_1 LIKE 'Eve%' $\wedge p_1 > p_2 \wedge p_2 > p_3$

(f) Global condition

Figure 4: C-instance $\mathcal{I}_0$ that satisfies $Q_B - Q_A$ and generalizes the counterexample K_0 in Figure 1.

queries. In this case, Figure 1 gives the minimum counterexample K_0 for the difference between Q_A and Q_B [41]. In particular, Q_B returns the tuples (Restaurante Raffaele, American Pale Ale) and (Tadim, American Pale Ale) while Q_A only returns the latter tuple.

Now consider the more general counterexample as a c-instance *(defined in the next section) showing the differences between the queries $Q_B - Q_A$ in Figure 4. This c-instance, $\mathcal{I}_0$, shows abstract tuples with variables instead of constants ($*$ are 'don't care' variables) and a condition that the variables must satisfy (there should be a drinker whose name is 'Eve' with a space after and the order of the prices in Serves table should be $p_1 > p_2 > p_3$). Thus, $\mathcal{I}_0$ not only generalizes the counterexample in Figure 1 (i.e., there exists an assignment to the variables that results in the instance in Figure 1 and satisfies the global condition), but, it also specifies the 'minimal' condition for which Q_B differs from Q_A (the global condition). The ground instance in Figure 1 contains specific values that may confuse the user and divert attention from the core differences. This is one of the c-instances in our* universal solution *that includes three c-instances. Each of the c-instances captures a facet of the difference between Q_B and Q_A.*

Our contributions. Our contributions are summarized below.

- We propose a framework for characterizing all query answers using c-instances using the notion of *coverage* and a *universal solution* that captures different ways a given DRC query can be satisfied.
- We argue that deciding whether a universal solution or even any satisfying c-instance exists is undecidable for general DRC queries, by giving a reduction from the finite satisfiability problem for First Order Logic formulas [50]. However, for the class of conjunctive queries with negation $CQ^{\neg}$, the universal solution can be found in poly-time in the query size.

- Since the problem in general is undecidable, we give two practical algorithms for finding a minimal set of satisfying c-instances. The first algorithm runs an exhaustive search subject to a size limit on the c-instances inspired by the chase procedure from Data Exchange [21, 22]. Then, we provide a more efficient chase algorithm that may return a smaller set of satisfying c-instances.
- We experimentally show scalability and quality of the c-instances returned by our algorithms varying different parameters. Our experiments use a real collection of wrong queries submitted by students of an undergraduate database course as well as queries over the TPC-H schema. Finally, we provide a comprehensive user study and a case study that show the usefulness of our approach.

2 RELATED WORK

Test data generation. QAGen [9] was among the first systems that focused on data generation in a query-aware fashion. The system aimed at testing the performance of a database management system given a database schema, one parametric Conjunctive Query, and a collection of constraints on each operator. MyBenchmark [37] extends [9] by generating a *set* of database instances that approximately satisfies the cardinality constraints from a set of query results. HYDRA [47] uses a *declarative approach* that allows for the generation of a database summary that can be used for dynamically generating data for query execution. Cosette [15], which targets checking SQL equivalence without any test instances, encodes SQL queries to constraints using symbolic execution, and uses a constraint solver to find one counterexample that differentiates two input queries. RATest [41] proposes an instance-based counterexample for distinguishing two queries, where the emphasis is on the cardinality of the generated counterexample. The main differences between our work and [41] are that (1) we provide abstract instances with variables and conditions to pinpoint the source of error while comparing a wrong query against a correct query, (2) we only use the query and the schema to generate the counterexample and *do not need a database instance* to provides a counterexample, whereas [41] requires a database instance to output one sub-instance as the counterexample, (3) [41] provides one grounded instance where the correct and wrong queries differ, while we generate *a set of c-instances* aims to show all possible ways two queries can differ. XData [11] generates test data by covering different types of query errors that can commonly occur. Qex [52] is a tool for generating input relations and parameter values for a parameterized SQL query and aims at unit testing of SQL queries. It also generates one instance that satisfies the query and does not support nested queries and set operations, and thus does not support the full class of DRC queries. Olston et. al. [44] studied the problem of generating small example data for dataflow programs to help users understand the behavior of their programs.

Explanations for query results using provenance Data provenance has been studied from many aspects [6, 10, 13, 24, 25, 27, 29, 48]. A multitude of approaches have used provenance for *query answers and non-answers explanations* [12, 18, 19, 30–32, 35, 36, 40, 46, 51] using various approaches including ideas inspired by causality [45] and Shapley value [49]. Our approach suggests a query characterization that is independent of a specific database instance and thus also its provenance.

Coverage in software testing Test coverage is used to measure the percentage of a given software that is executed during the tests. Intuitively, the higher the test coverage, the lower the likelihood of the software containing bugs and unforeseen errors [39, 42, 53]. Different criteria for coverage have been proposed [43, 53], e.g., function coverage (checking the percentage of functions in the program that are executed during testing) and branch coverage (checking the percentage of branches, i.e., decisions that have true and false outcomes, in the program that are executed during testing). These ideas have certainly influenced our model.

Chase in schema mappings The chase procedure was originally suggested in the context of database dependencies [3, 38] and later used for generating schema mappings [14, 21]. The latter application aims to map one database schema to another, by using tuple-generating and equality-generating dependencies. These dependencies are then used in a chase procedure to generate the mapping between the schemas. Previous work has explored the complexity of the chase procedure and the types of solutions it is able to generate [20, 23, 28]. Our notion of a universal solution is therefore inspired by the notion of universal solution in the schema mapping problem [21]. Recent work has proposed an abstraction of schema mappings [7], which allows reusing meta schema mappings. In this paper, the c-instances can be thought of as "meta-instances" that can be mapped to concrete ones (e.g., [41]) as needed.

3 MODEL FOR QUERY CHARACTERIZATION

In this section, we describe some basic concepts and our framework.

3.1 Databases and Relational Calculus

First, we review and define domains and Domain Relational Calculus which will be used for express queries in this paper. A database schema $\mathbf{R}$ is a collection $(R_1, ..., R_r)$ of relation schemas. Each R_i is defined over a set of attributes denoted by $\texttt{Attr}(R_i)$. For each attribute $\texttt{A} \in \texttt{Attr}(R_i)$, its **domain** is a set of (possibly infinite) **constants** and is denoted as $\textsc{Dom}(\texttt{A})$, and $\textsc{Dom} = \cup_{\texttt{A}}\textsc{Dom}(\texttt{A})$; for simplicity, we will frequently use $\textsc{Dom}$ instead of $\textsc{Dom}(\texttt{A})$ with the implicit assumption that the constants are from the right domain $\textsc{Dom}(\texttt{A})$. Two relations can share the same attribute A; we use $R_i.\texttt{A}$ to explicitly denote an attribute $\texttt{A} \in \texttt{Attr}(R_i)$. Further, two attributes may share the same domain (e.g., when they share the same name or are related by foreign key constraints). A **ground instance** (or simply an **instance** when it is clear from the context) is a (possibly empty) finite set of tuples with constant attribute values that conform to the schema and corresponding domains. In addition, we allow standard constraints like key constraints, foreign key constraints, and functional dependencies in our framework.

DRC queries and tree representation. We next review the definition of Domain Relational Calculus (DRC) [34] and use it to define queries and syntax trees. It has been shown that DRC is equivalent to Relational Algebra [16], which provides the theoretical foundation to query languages such as SQL.

Definition 1 (DRC Queries). *Given a schema* $\mathbf{R}$*, a* **DRC query** Q *has the form* $Q = \{(x_1, x_2, ..., x_p) \mid P_Q(x_1, ...x_p)\}$ *where* P_Q *is a standard first order logic (FOL) formula [1] involving relation names* $R_1, \cdots, R_r$*, constants from* $\textsc{Dom}$*, a set of* **query variables** $\mathcal{V}_Q$ *for attribute values, quantifiers* $\exists, \forall$*, operators* $\neg, =, >, \geq, <, \leq, \neq$*, LIKE*

etc., and connectives $\wedge, \vee$. *Here,* $\mathcal{V}_Q^{out} = \{x_1, \cdots, x_p\} \subseteq \mathcal{V}_Q$ *denote* **output variables** *of the query* Q*, which can be an empty set for* a Boolean query. *The output variables are* free variables *in* P_Q*; the remaining variables in* P *are* quantified *under* $\forall$ *or* $\exists$.

The formula P_Q *is built up from* **DRC atoms** *of the following forms: (1)* $R(y_1..., y_k)$ *or* $\neg R(y_1..., y_k)$*, where* $R \in \mathbf{R}$ *is a relation, and each* $y_i \in \mathcal{V}_Q \cup \textsc{Dom}$ *is a query variable or a constant, and (2) conditions* x_1 *op* x_2 *or* x_1 *op* c*, where* $x_1, x_2 \in \mathcal{V}_Q$*,* $c \in \textsc{Dom}$*, and op is a binary operator.*

A ground instance D *is said to* **satisfy** *a DRC query* Q *(denoted by* $D \models Q$*) if* $Q(D) \neq \emptyset$ *(for a Boolean query,* $Q(D) = \{\{\}\}$ *or* true*), i.e., there is a* **satisfying assignment** $\alpha : \mathcal{V}_Q^{out} \rightarrow \textsc{Dom}$ *of the output variables of* Q *to the constants in* D *such that* P_Q *evaluates to true.*

We make a few assumptions without loss of generality: (1) in the FOL formula P_Q, all negations appear in DRC atoms (which can be achieved by repeated applications of standard equivalences like $\neg(\forall x P(x)) = \exists x(\neg P(x))$ and De Morgan's laws like $\neg(a \vee b) = \neg a \wedge \neg b$, etc.); (2) the DRC queries are **safe** or domain independent [1], i.e., any variable y_i that appears in a negated relation $\neg R(y_1..., y_k)$ also appear under a positive relation, e.g., queries like $\{x : \neg R(x)\}$ are not allowed; and (3) each quantified variable is unique in P_Q (which can be achieved by renaming).

Example 2. *The queries* Q_A *and* Q_B *are shown in Figure 2, whereas Figure 3 gives the difference query* $Q_B - Q_A$ *that identifies beers at a non-lowest price but also not at the highest price. In Figure 3, the output variables are* x_1, b_1 *and the FOL formula is specified on the right hand side by renaming the variables in* Q_A, Q_B *in Figure 2. The ground instance* K *in Figure 1 satisfies* $Q_B - Q_A$ *in Figure 3, since there is a satisfying assignment* α *from the output variables* x_1, b_1 *in the query* $Q_B - Q_A$*, i.e.,* $\alpha(x_1)$ = "Restaurant Raffaele", $\alpha(b_1)$ = "American Pale Ale", *such that the formula in the query is satisfied when* d_1 = "Eve_Edwards", $p_1 = 2.75$, x_2 = "Restaurant Memory", $p_2 = 2.25$*, therefore the first part of the FOL formula from* Q_B *is true. The second part of the FOL formula with* $\wedge$ *is also true for all* d_2, p_3*: while the first two disjuncts under* $\neg$ *evaluates to false,* $\neg Serves(x_1, b_1, p_3) = true$ *for* $p_3 = 2.25, 3.5$*, and for* $p_3 = 2.75$ *the fourth disjunct is satisfied with* x_3 = "Tadim" *and* $p_4 = 3.5$.

Definition 2 (Syntax Tree of Query). *A syntax tree of a query* Q *is tree for the FOL formula* P_Q *satisfying the following rules:*

(1) Each leaf node is a DRC atom.
(2) Each internal node is either a quantifier with a single variable (e.g., $\forall x$ *and* $\exists x$*) with a single child, or a connective (*$\wedge$ *and* $\vee$*) with two children.*

Further, since in the formula P_Q *all negations appear in the DRC atoms, all negations in the syntax tree appear in the leaves; we do not use separate nodes for negation.*

Given a DRC query Q, we can have a unique syntax tree following the order of quantifiers in Q (e.g., for $\exists xy$, $\exists y$ appears as the child of $\exists x$, and a fixed order of associative connectives with appropriate parentheses, e.g., $(p_1 \vee p_2 \vee p_3)$ is always assumed to be $((p_1 \vee p_2) \vee p_3)$. However, two equivalent DRC queries may have different syntax trees, e.g., $\{x \mid R(x)\}$ and $\{x \mid (R(x) \wedge T(x)) \vee (R(x) \wedge \neg T(x))\}$. The syntax tree for the query $Q_B - Q_A$ from Figure 3 is shown in Figure 5 (to save space, we put multiple quantifiers with variables in the same node). The special treatment of the negation operator $\neg$ is for the sake of convenience in our algorithms.

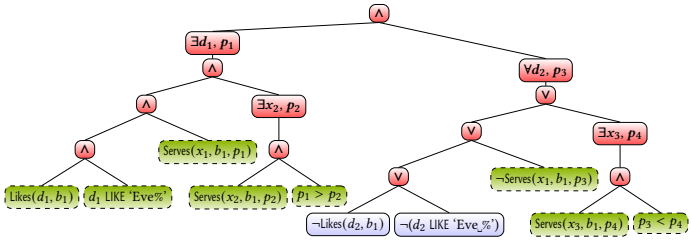


Figure 5: Syntax tree of $Q_B - Q_A$ in Example 2. Atoms covered by the c-instance in Figure 4 are in green dashed boxes.

name	addr
d_1	*

(a) Drinker relation

name	addr
x_1	*
x_2	*

(b) Bar relation

bar	beer	price
x_1	b_1	p_1
x_2	b_1	p_2

(c) Serves relation

name	brewer
b_1	*

(d) Beer relation

drinker	beer
d_1	b_1

(e) Likes relation

d_1 LIKE 'Eve%' $\wedge \neg(d_1$ LIKE 'Eve_%') $\wedge p_1 > p_2$

(f) Global condition

Figure 6: C-instance $\mathcal{I}_1$ that satisfies $Q_B - Q_A$.

3.2 Conditional Instances or C-Instances

We next give the definition of a *c-instance* adapting the concepts of *v-tables* and *c-tables* from the literature [33]. We distinguish between query variables whose domain is denoted by $\mathcal{V}$ (Definition 1), and variables in the c-instances whose domain is denoted by $\mathcal{L}$; we refer to the latter as **labeled nulls** (called *marked nulls* in [33]) for clarity. The c-instances involve conditions using **atomic conditions**, which are either (1) an atom of the form $[x\ op\ c]$ ($\neg[x\ op\ c]$) or $[x\ op\ y]$($\neg[x\ op\ y]$) where x and y are labeled nulls in $\mathcal{L}$, c is a constant in $\textsc{Dom}(x)$, and $op \in \{<, >, \leq, \geq, =, \neq, LIKE, ...\}$ is a binary operator, or (2) a condition of the form $\neg R(x_1, \ldots, x_k)$ where R is a relation on k attributes.

Definition 3 (Conditional Instance or c-instance). *A v-table with a relational schema* $R_i \in \mathbf{R}$ *is a table* T_i *in which for each tuple* $t \in T_i$ *and each attribute* $\mathtt{A} \in \mathtt{Attr}(R_i)$*,* $t[\mathtt{A}]$ *is either a constant from* $\textsc{Dom}(\mathtt{A})$ *or is a labeled null from* $\mathcal{L}$.

A **c-instance** $\mathcal{I}$ *of* $\mathbf{R}$ *is a tuple of the form* $(\{T_1, \ldots, T_r\}, \phi)$*, where for each* $i \in [1, r]$*,* T_i *is a v-table with schema* R_i*, and* ϕ *is a conjunction of atomic conditions, which is associated with the c-instance, denoted as the* **global condition**.

Our definition of c-instances is slightly different from those found in previous literature [33], as we only associate the instance with a global condition, while there are no local conditions associated with a single tuple or even a single table in the instance. Table-level conditions might still appear in the global condition as a conjunct that contains labeled nulls from a single table, e.g. in Figure 4, the condition $p_1 > p_2$ is only relevant to the *Serves* table.

A conditional table (c-table) is a special case of a c-instance when there is only one relation in the instance, hence we only discuss c-instances in the rest of this paper. Note that we also allow for labeled nulls that do not affect the evaluation of ϕ, and are not needed for joins between tables. These are called "don't care" labeled nulls and are denoted by $*$ for simplicity instead of having unique names.

Example 3. *Consider the c-instance $\mathcal{I}_0$ shown in Figure 4. The single tuple in the drinker table says that the name of the drinker is d_1, its address is a "don't care" token, and the condition says that the name d_1 must start with "Eve". The condition also enforces an order on the prices. b_1 cannot be replaced with $*$ because all three beers in Serves must be the same.*

C-instances define a set of possible worlds, each defined by a *mapping* (see below) to the labeled nulls in the c-instance.

Definition 4 (Mapping for C-instances). *Given a c-instance $\mathcal{I} = (T_1, \ldots, T_r, \phi)$ over a schema* **R**, *a* **mapping** $\mu : \mathcal{L} \rightarrow$ Dom *maps the labeled nulls $\mathcal{L}$ in $\mathcal{I}$ to their respective domains.*

Extending μ to the c-instance $\mathcal{I}$, we denote $\mu(\mathcal{I})$ as the ground instance $\mu(\mathcal{I}) = \{\mu(x) : x \text{ is a labeled null in } \mathsf{T}_i, \text{ for } i \in [1, r]\}$. Let $\mathcal{L}' \subseteq \mathcal{L}$ be the labeled nulls appearing in ϕ of $\mathcal{I}$. Then, $\phi_\mu = \phi(\mu(\mathcal{L}'))$ yields the evaluation of ϕ (True or False) with the mappings given by μ.

Definition 5 (Possible Worlds and Consistency for C-instances). *Given a c-instance $\mathcal{I} = (T_1, \ldots, T_r, \phi)$ of schema* **R**, *the* **set of possible worlds** *for $\mathcal{I}$ is $PWD(\mathcal{I}) = \{\mu(\mathcal{I}) : \mu$ is a mapping $\wedge$ $\phi_\mu = True\}$. A c-instance $\mathcal{I}$ is said to be* **consistent**, *denoted by IsConsistent($\mathcal{I}$) if $PWD(\mathcal{I}) \neq \emptyset$.*

Note that ground instances in $PWD(\mathcal{I})$ cannot contain extra tuples that are not in the c-instance $\mathcal{I}$. They may, however, have fewer tuples than the c-instance mapped to them because we consider set semantics, where a mapping may map two tuples with labeled nulls to the same tuple with constant values.

Example 4. *One possible world of the c-instance in Figure 4 is the ground instance in Figure 1. In this ground instance, all labeled nulls have a mapping such that the global condition is satisfied.*

3.3 Query Characterization by C-Instances

We next give definitions for our framework of query characterization through c-instances. The intuition is that the c-instances should be *sound*, i.e., they should correctly capture a query, as well as *"complete"* in terms of *different ways* a query can be satisfied by ground instances, which requires more careful considerations using *"coverage"* of ground and c-instances as we discuss below.

Definition 6. *[Satisfying c-instances] Given a query Q, a c-instance $\mathcal{I}$ is said to* **satisfy** *Q (denoted $\mathcal{I} \models Q$) if $\mathcal{I}$ is consistent (Definition 5) and for every ground instance D in $PWD(\mathcal{I})$, $D \models Q$.*

Example 5. *Consider the query $Q_B - Q_A$ in Figure 3, and the c-instance $\mathcal{I}_0$ shown in Figure 4. $\mathcal{I}_0 \models Q_B - Q_A$ since every mapping of constants from the domain of its labeled nulls that satisfies the global condition will generate a ground instance D such that $D \models Q_B - Q_A$; $D = K_0$ in Figure 1 is an example.*

Coverage. Given a DRC query, the *Coverage* of a (ground or c-) instance captures different parts of a DRC query, that are satisfied by the instance, and helps us define the *completeness* of a set of c-instances. An inductive definition of coverage is given below.

Definition 7. *[Coverage of ground instances] Given a DRC query syntax tree Q, a ground instance K such that $K \models Q$, and a satisfying assignment $\alpha : \mathcal{V}_Q^{out} \rightarrow$ Dom of the output variables of Q, the* **coverage cov**(Q, K, α) **of** Q **by** K **under** α *identifies a subset of the DRC atoms (leaves) of the query syntax tree recursively top-down by extending α to all free variables in a subtree as follows:*

(1) If Q consists of a single DRC atom (here all variables in Q are free), then cov(Q, K, α) contains the DRC atom of Q if:
  (a) $Q = R(x_1, ..., x_k)$ and $R(\alpha(x_1), ..., \alpha(x_k))$ appears in K, or
  (b) $Q = \neg R(x_1, ..., x_k)$ and $R(\alpha(x_1), ..., \alpha(x_k))$ is not in K, or
  (c) $Q = x$ op y and $\alpha(x)$ op $\alpha(y)$ evaluates to True; otherwise cov$(Q, K, \alpha) = \emptyset$.

(2) If $Q = Q_1 \wedge Q_2$ or $Q = Q_1 \vee Q_2$ (here the sets of free variables in Q, Q_1, Q_2 are the same), then cov$(Q, K, \alpha) =$ cov$(Q_1, K, \alpha) \cup$ cov(Q_2, K, α).

(3) If $Q = \exists x Q'(x)$ or $Q = \forall x Q'(x)$ (here x is a new free variable in Q'), then cov$(Q, K, \alpha) = \cup_{c \in Dom_K}$ cov$(Q', K, \alpha \cup \{x \rightarrow c\})$, where Dom_K denotes the constants appearing in K.

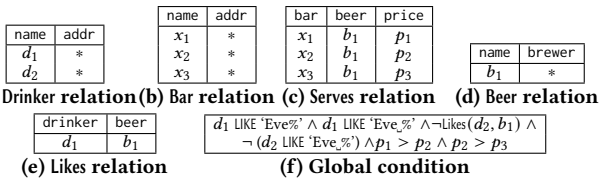
The **coverage of** K **for** Q *is defined as cov$(Q, K) = \bigcup_\alpha$ cov(Q, K, α).*

Intuitively, the coverage cov(Q, K) is the set of atoms and conditions of Q that can be covered by a ground instance K, eventually leading to a satisfying assignment of the output variables of Q. Therefore, we use union to combine the coverages in Definition 7 for all cases, since we are interested in all possible ways to satisfy a query. Since α is a satisfying assignment of the output variables, the coverages of Q' in case (3) in Definition 7 for both $\exists, \forall$, and for both Q_1, Q_2 for a $\wedge$ node and for at least one of them for a $\vee$ node in case (2) above is non-empty. For universal quantifiers, it is worth noting that when the quantified variable takes different constants, different branches of the inner query (Q') may be satisfied, and thus provide different coverages, and to take all of them into account, we again employ union.

Example 6. *The only satisfying assignment to the difference query $Q_B - Q_A$ depicted in Figure 3 w.r.t. the ground instance shown in Figure 1 is given by the assignment $\alpha(x_1) =$ "Restaurant Raffaele", $\alpha(b_1) =$ "American Pale Ale" described in Example 2. By applying the recursive top-down process implied by Definition 7, the DRC atoms of the query covered by this assignment are the leaves colored in green in Figure 5. Note that different assignments of a $\forall$ variable can cover different leaves, e.g., for $\forall p_3$ node in the right subtree, $p_3 = 2.25, 3.5$ covers the node $\neg Serves(x_1, b_1, p_3)$ atom whereas $p_3 = 2.75$ covers $Serves(x_3, b_1, p_4)$ and $p_3 < p_4$ atoms as discussed in Example 2.*

Definition 8. *[Coverage of satisfying c-instances] Given a DRC query Q and a c-instance $\mathcal{I}$ such that $\mathcal{I} \models Q$, the* **coverage of** $\mathcal{I}$ **for** Q *is defined as cov$(Q, \mathcal{I}) = \bigcap_{K \in PWD(\mathcal{I})}$ cov(Q, K).*

Since $PWD(\mathcal{I})$ can contain ground instances with different coverages, the coverage of $\mathcal{I}$ is defined as the common coverage of all possible worlds. Therefore, the coverage of a c-instance $\mathcal{I}$ is always a (not necessarily strict) subset of any ground instance in $PWD(\mathcal{I})$.



(a) Drinker relation (b) Bar relation (c) Serves relation (d) Beer relation

(e) Likes relation (f) Global condition

Figure 7: C-instance $\mathcal{I}_2$ that satisfies $Q_B - Q_A$. It is not minimal since one of the Serves tuples can be removed without changing the coverage.

Example 7. *Reconsider the syntax tree of the query $Q_B - Q_A$ shown in Figure 5. Every ground instance in $PWD(\mathcal{I}_0)$ ($\mathcal{I}_0$ is depicted in Figure 4) has to have exactly three Serves tuples due to the global conditions (recall that the mapping from a c-instance to each ground instance in PWD has to be onto). Thus, the coverage of the c-instance $\mathcal{I}_0$ depicted in Figure 4 is exactly the coverage of the ground instance in Figure 1/Example 6 and highlighted in green with dashed frames in the tree shown in Figure 5. The c-instance $\mathcal{I}_2$ in Figure 7 covers all DRC atoms in the query $Q_B - Q_A$. To see this, note that there are two drinker variables d_1, d_2 in $\mathcal{I}_2$ such that $\neg$Likes(d_2, b_1) as well as $\neg(d_2$ LIKE 'Eve_%') hold in all ground instances in $PWD(\mathcal{I}_2)$. Therefore, for the above satisfying assignments of output variables, when the $\forall d_2$ in the right subtree iterates over the constants corresponding to d_2, the two remaining uncovered leaves are covered as well.*

Minimality of a c-instance. We now define minimality w.r.t. the coverage of a c-instance. In the next definition, we denote by $|\mathcal{I}|$ the **size of a c-instance** $|\mathcal{I}|$, defined as the total number of tuples and atomic conditions of $\mathcal{I}$. For instance, $|\mathcal{I}_0| = 12$ in Figure 4 (9 tuples and 3 atomic conditions).

Definition 9 (Minimal satisfying c-instance). *Given a DRC query Q, a c-instance $\mathcal{I}$ with coverage C satisfying Q is* **minimal** *if for every other satisfying c-instance $\mathcal{I}'$ with coverage C, $|\mathcal{I}| \leq |\mathcal{I}'|$.*

Example 8. *Following Example 7, the c-instance $\mathcal{I}_0$ in Figure 4 is a minimal satisfying c-instance assuming natural foreign key constraints from Serves, Likes to Drinker, Beer, Bar, since any other c-instance with the same coverage has a larger size. In particular, $\mathcal{I}_1$ of smaller size (=10) in Figure 6 does not have the same coverage, since in the $\forall$ nodes of right subtree when d_2 (in query) = d_1 (in $\mathcal{I}_1$) and p_3 (in query) = p_1 (in $\mathcal{I}_1$), the two rightmost leaves Serves(x_3, b_1, p_4) and $p_3 < p_4$ are not covered by any ground instance in $PWD(\mathcal{I}_1)$.*

Minimality ensures that we do not include redundant tuples or conditions in our c-instances. While we adopt the above simple notion of minimality, it is ensured by a post-processing step in our algorithms, so any other reasonable form of minimality can also be used in this framework.

Universal solution. Our framework for finding solutions for a query Q is given in terms of a set of minimal c-instances $\mathcal{I} \models Q$, which immediately ensures soundness of our solutions, since any output c-instance is guaranteed to satisfy the query. Conversely, the notion of completeness is more challenging since there can be satisfying ground instances of unbounded size with redundant tuples that do not affect the query answer. The notion of coverage helps us define a notion of completeness using a *universal solution*, which ensures that for all satisfying ground instances of a certain coverage, a c-instance with the same coverage is included.

Definition 10 (Minimal c-Solution and Universal Solution). *Let Q be a DRC query over a schema* **R** *and domain Dom. A* **minimal c-solution** *of Q is a set of minimal c-instances of Q, $\mathcal{S}_\mathcal{I} = \{\mathcal{I}_1, \ldots, \mathcal{I}_k\}$, such that for all $\mathcal{I}_i$, $\mathcal{I}_i \models Q$, and for any two $\mathcal{I}_i, \mathcal{I}_j$ where $i \neq j$, $cov(Q, \mathcal{I}_i) \neq cov(Q, \mathcal{I}_j)$.*

A **universal solution** *of Q is a minimal c-solution $\mathcal{S}_\mathcal{I} = \{\mathcal{I}_1, \ldots, \mathcal{I}_k\}$ such that (1) if there exists a ground instance K, where $K \models Q$, with coverage C, there is a c-instance $\mathcal{I}_i \in \mathcal{S}_\mathcal{I}$ with coverage $C = C_i$, (2) if we remove any c-instance from $\mathcal{S}_\mathcal{I}$, condition (1) does not hold.*

Note that for the universal solution, we do not require that $K \in PWD(\mathcal{I}_i)$ since K may contain more tuples than $\mathcal{I}_i$ and thus may not be part of the set $PWD(\mathcal{I}_i)$.

Example 9. *Reconsider the $Q_B - Q_A$ shown in Figure 3. The set $\{\mathcal{I}_0, \mathcal{I}_1\}$ is a minimal c-solution for $Q_B - Q_A$ since they have different coverages and both satisfy $Q_B - Q_A$. Two of the three c-instances in the universal solution are $\mathcal{I}_0$, $\mathcal{I}_1$ shown in Figures 4, 6, 7, respectively (the rest are shown in Section 5.2 in the case study).*

3.4 Computability of the Universal Solution

Proposition 3.1. *The computability and complexity of finding a universal solution is as follows:*

1. *Finding a universal solution is poly-time in the size of the query for $CQ^\neg$ (and therefore also for CQs), where $CQ^\neg$ is the class of conjunctive queries with negation that includes operators $\exists$, $\wedge$, $\neg$ for individual atoms and conditions.*
2. *Checking whether a universal solution exists (or even any minimal c-solution exist) is undecidable for general DRC queries that may include the operators $\forall$, $\exists$, $\vee$, $\wedge$, and $\neg$.*

Proof. (1) The universal solution of $q \in CQ^\neg$ is a single c-instance comprising all relational atoms $R(x_1, \cdots, x_k)$ of the query, and a global condition that is the conjunction of all comparisons (x *op* y, x *op* c with or without negation) and negated relational atoms $\neg R(x_1, \cdots, x_k)$ in the query. This implies a poly-time complexity in the size of the query.

(2) Finding whether a universal solution exists is an undecidable problem for general DRC queries due to a reduction from the *finite satisfiability problem in first order logic* (FOL) that is known to be undecidable by the Trakhtenbrot's Theorem [50]. An FOL sentence ϕ is finitely satisfiable if there exists a finite ground instance K such that ϕ is true over K, which is true if and only if there is a satisfying c-instance $\mathcal{I}$. Hence the universal solution for Q is non-empty if and only if the global condition ϕ_Q is finitely satisfiable, which is undecidable when Q is a general DRC query. □

Note that the inclusion of $\forall$ operators and arbitrary positioning of $\neg$ make general DRCs harder than $CQ^\neg$. In $CQ^\neg$, negations are only allowed in front of relational atoms and conditions subject to standard 'safety' constraints [2]. The query that returns all beers that are not liked by some drinker: $\{(b) \mid \exists x, d, a\ (\text{Beer}(b, x) \wedge \text{Drinker}(d, a) \wedge \neg\text{Likes}(d, b))\}$ in the DRC form, and in the equivalent (safe) Datalog with negation form: $Q(b) :- \text{Beer}(b, x), \neg\text{Likes}(d, b), \text{Drinker}(d, a)$, is an example of $CQ^\neg$.

Since the problem of finding a universal solution for general DRC queries is undecidable, in Section 4 we give an algorithm that builds an exhaustive minimal c-solution up to a certain limit on the size of the c-instances to ensure halting. We also give a more efficient algorithm in Section 4.3 that relaxes the requirement of generating all possible c-instances by providing a subset of satisfying c-instances.

4 ALGORITHM FOR MINIMAL C-SOLUTION

In this section we show how to compute an exhaustive set of satisfying c-instances up to a size limit for a DRC query Q by adapting ideas from the *chase* procedure [21, 22].

4.1 Basic Notions and Overview

Given a query syntax tree Q (Definition 2), our algorithm constructs an exhaustive set of c-instances in the minimal c-solution by recursively extending each c-instance in multiple ways. To explore the different options of extending each c-instance, our algorithm takes a similar approach to that of the chase procedure, that was originally proposed for database dependencies [3, 38]. It constructs the different options for c-instances using a breadth first search (BFS) procedure, thereby implicitly generating a chase tree [8]. However, unlike the classic chase algorithm that directly adds or modifies tuples, our procedure converts the syntax tree into a conjunction of atoms and then maps the atoms in the conjunction to tuples and conditions which are added to the c-instance. Each quantifier and connector in the tree triggers a tailored recursive call. While creating the c-instances, the algorithm keeps track of the mappings between the query variables and the labeled nulls in the c-instances. We next formally define this homomorphism between query and c-instance; since we build the homomorphism in steps, we define it as a partial function.

Definition 11 (Homomorphism between query and c-instance). *Given a query Q and a c-instance $\mathcal{I}$ on the same schema and domain Dom, with query variables $\mathcal{V}_Q$ and labeled nulls $\mathcal{L}_\mathcal{I}$, and constants $C_Q, C_\mathcal{I} \subseteq$ Dom respectively, a* **homomorphism** *h* **from** *Q* **to** *$\mathcal{I}$ is a partial function $h : \mathcal{V}_Q \cup C_Q \rightarrow \mathcal{L}_\mathcal{I} \cup C_\mathcal{I}$ such that, (1) for each constant $c \in C_Q$, $h(c) = c$, and (2) for an atom $a = R(x_1, ..., x_k)$ in Q, if all of $h(x_1), \cdots, h(x_k)$ are defined, $h(a) = R(h(x_1), ..., h(x_k))$ is in relation R in $\mathcal{I}$.*

As opposed to assignments from queries to ground instances, homomorphisms to c-instances are not restricted to output variables but can also map quantified variables to labeled nulls. This allows the algorithms to consider multiple different homomorphisms of variables to different labeled nulls. For universally quantified variables $\forall x$, the algorithm keeps track of multiple homomorphisms even within the same c-instance.

We also abuse terminology as follows. Given a query Q and a c-instance $\mathcal{I}$ such that there is a homomorphism h from Q to $\mathcal{I}$, we refer to the **domain of a query variable in** Q as the set of labeled nulls with the same domain in $\mathcal{I}$ according to h, i.e., the labeled nulls in the same attribute or in corresponding attributes in different tables that share the same domain (e.g., by foreign key dependencies).

The procedure starts by mapping the free variables of the query to fresh labeled nulls in the c-instance. Then, it performs a BFS where for each c-instance in the queue, it expands the homomorphism and the c-instance using a recursive procedure. For a syntax tree with no quantifiers, the recursive procedure adds its atoms as tuples to the c-instance, ensuring that the variables in the atoms are converted to their labeled null counterparts according to the homomorphism. Otherwise, it handles the syntax tree based on its root: each quantifier ($\forall$, $\exists$) or connective ($\wedge$, $\vee$) is handled separately.

To ensure the minimality of each c-instance in the obtained set and the minimality of the set itself (Definitions 9 and 10), we use a post-processing procedure that checks the coverage of each c-instance, and for any coverage, it keeps a single c-instance with minimum size (breaking ties arbitrarily).

Algorithm 1 Tree-Chase-BFS

```
Tree-Chase-BFS(R, Q, h_0, I_0, limit)
    Input: R: the database schema; Q: a DRC query;
    I_0: a c-instance of schema of R; h_0: a mapping;
    limit: the maximum number of tuples and conditions in the c-instance;
    Output: A list of satisfying c-instances for Q.
1   res = [], queue = an empty queue
2   for x ∈ FreeVar(Q)
3       Create a fresh variable x' in the domain of x
4       I_0.domain(x) = I_0.domain(x) ∪ {x'}
5       h_0 = h_0 ∪ {x → x'}
6   queue.push(I_0)
7   visited = ∅
8   while ¬ queue.isEmpty()
9       I = queue.pop()
10      if I ∈ visited or |I| > limit
11          continue
12      visited = visited ∪ {I}
13      if Tree-SAT(Q, I, ∅) and IsConsistent(I)
14          res.append(I)
15          continue
16      Ilist = Tree-Chase(R, Q, I, h_0, limit)
17      for J ∈ Ilist
18          if IsConsistent(J) and |J| ≤ limit and J ∉ visited
19              queue.push(J)
20  return res
```

4.2 Exhaustive Chase for C-Instances

Algorithm 1 and Algorithm 2 form the main body of our 'chase' procedure. The procedure starts by calling Algorithm 1 (Tree-Chase-BFS) on the schema $\mathbf{R}$, the entire query Q, an empty instance $\mathcal{I}_0$, and an empty mapping h_0. In addition, the size bound *limit* sets the maximum number of tuples and atomic conditions in the global condition allowed in the c-instance, and is meant to ensure halting of the algorithm since finding a satisfying c-instance for a general DRC query is undecidable (Proposition 3.1).

Breadth-first search. First, to initialize the instance $\mathcal{I}_0$ and the mapping h_0 from free variables in the query to labeled nulls and constants in $\mathcal{I}_0$, for each free variable x in Q, Algorithm 1 will create a new labeled null and add it to the domain of x in $\mathcal{I}_0$ and update h_0 (Line 2-5). Then, the algorithm runs in a Breadth-first search manner: $\mathcal{I}_0$ is initially added to the empty queue; every time the algorithm takes the c-instance from the head of the queue, checks whether the instance has already been generated and its size does not exceeds *limit* (Line 10). The procedure for checking $I \in visited$ takes into account renaming of variables; it first compares certain properties of the c-instances (e.g., number of tuples, size of conditions etc.) and filters out candidates that cannot be equivalent to I, and then it checks all possible mappings to previously generated c-instances. It also checks (Line 13) (1) whether $I \models Q$ by the Tree-SAT procedure, and (2) whether it is consistent, i.e., $PWD(I) \neq \emptyset$, (we use an SMT solver in our implementation). It then runs the recursive procedure on the current c-instance (Line 16) and adds each one of the resulting c-instances to the queue (Lines 17–19).

Recursive generation of c-instances. The recursive procedure Algorithm 2 (Tree-Chase) handles the query according to its root operator. It gets as input the schema of the relational database $\mathbf{R}$, the syntax tree of a DRC query Q, the current c-instance I, and the current homomorphism from Q to I. For the case where the query has no quantifiers (Line 2-7), the algorithm converts the syntax tree into a list of conjunction of atoms/atomic conditions, and then an instance is created for each conjunction under the homomorphism

Algorithm 2 Tree-Chase

```
Tree-Chase(R, Q, I, h, limit)
    Input: R: the database schema; Q: the query as its syntax tree;
    I: current c-instance; h: current homomorphism from Q to I;
    limit.
    Output: a list of c-instances
 1  res = []
 2  if there are no quantifiers in Q
 3      L = tree-to-conj(Q)
 4      for ψ ∈ L
 5          J = Add-to-Ins(R, I, h(ψ))
 6          if IsConsistent(J)
 7              res.append(J)
 8  elseif Q.root.operator ∈ {∧}
 9      res = Handle-Conjunction(R, Q, I, h, limit)
10  elseif Q.root.operator ∈ {∨}
11      res = Handle-Disjunction(R, Q, I, h, limit)
12  elseif Q.root.operator ∈ {∃}
13      res = Handle-Existential(R, Q, I, h, limit)
14  elseif Q.root.operator ∈ {∀}
15      res = Handle-Universal(R, Q, I, h, limit)
16  return res
```

Algorithm 3 Handle-Conjunction

```
Handle-Conjunction(R, Q, I, h, limit)
 1  res = []
 2  lres = Tree-Chase-BFS(R, Q.root.lchild, I, h, limit)
 3  for J ∈ lres
 4      if IsConsistent(J) = false
 5          Continue
 6      rres = Tree-Chase-BFS(R, Q.root.rchild, J, h, limit)
 7      for K ∈ rres
 8          if IsConsistent(K)
 9              res.append(K)
10  return res
```

h by Add-to-Ins. The algorithm proceeds according to the root of the syntax tree ($\wedge, \vee, \exists, \forall$) by calling procedures that get the same input as Algorithm 2, call Algorithm 1 recursively, and output a list of c-instances that are sent back to Algorithm 1.

Handling conjunction ($\wedge$). The Handle-Conjunction procedure recursively calls Algorithm 1 on both children of the root, and every pair of solutions to each child is merged into one instance by taking a union of every solution to the left subtree with every solution to the right subtree, and adding the consistent instances to the list of c-instances.

Handling disjunction ($\vee$). The Handle-Disjunction procedure[1] reduces the disjunctive tree into three conjunctive trees by replacing $Q = Q_1 \vee Q_2$ with $Q_1 \wedge Q_2$, $Q_1 \wedge \neg Q_2$, and $\neg Q_1 \wedge Q_2$, since one of the three is True iff $Q_1 \vee Q_2$ is True. This conversion introduces negation to some subtrees, such a negated subtree is translated into a syntax tree with negation only on the leaves. Then, Algorithm 1 is called with each of the modified trees, and each set of c-instances obtained from the three cases is added to the result set.

Handling existential ($\exists$) and universal ($\forall$) quantifiers. If the root node has $\exists x$, Algorithm 2 calls Handle-Existential (see footnote 1) that iterates over all labeled nulls or constants in the domain of x, also adds a fresh labeled null, updates the homomorphism (as the quantified variable becomes free in the subtree), and recursively calls Algorithm 1. Handle-Universal (see footnote 1) handles the case when the root has $\forall x$. The difference with $\exists$ is that, like the $\wedge$ case, the solutions to all labeled nulls and constants that x is mapped to are merged into one instance. The algorithm first checks whether there is no root and returns the inputted c-instance in that case (Lines 2–3) adds each mapping from x to a labeled null or constant to the homomorphism, runs the recursive procedure to find all c-instances with this mapping and merges it with other c-instances generated with other homomorphism that map x to other labeled nulls or constants (Lines 5–14). It further generates c-instances by mapping x to a fresh labeled null (Lines 19–24).

Ensuring minimality in post-processing. After Algorithm 1 returns a set of c-instances, we remove the c-instances that are not minimal in the following manner. For each c-instance in the set, we compute a hash string for its coverage (we keep track of the coverage of each c-instance as it is created). Then, for each c-instance in the set, we get all other c-instances in the set with the same string representing its coverage and remove all but the minimal one according to their size. Note that the hash function is applied to the *coverage of the c-instance rather than the c-instances themselves*, i.e., the function hashes the covered atoms of the query. Thus, it allows us to efficiently detect c-instances with the same coverage and remove those that are not minimal in terms of their number of tuples and atomic conditions (ref. Section 3.3).

Soundness, termination, and complexity. Given a syntax tree Q of a DRC query, Algorithm 1 when given $(\mathbf{R}, Q, \emptyset, \emptyset)$ will output a list of c-instances that are consistent, minimal, and satisfy the query denoted by Q (validated in Line 13), i.e., our procedure is sound and generates a valid minimal c-solution.

Although the problem of verifying if a satisfying c-instance exists is undecidable (Proposition 3.1), Algorithm 1 is guaranteed to terminate given the *limit* parameter. There are finitely many distinct c-instances (that are not isomorphic in terms of renaming of variables) up to size *limit* given a query. If the size of a c-instance increases over *limit*, the algorithm will ignore this c-instance and not push it into *queue* (Lines 10-11). The algorithm will also not get into an infinite loop because of the *visited* set. Every generated c-instance is placed into this set and c-instance already found is the set are ignored (including renaming of variables) and are not pushed into *queue* (Lines 18–19). Since the size of the schema is constant, given a limit on size of the c-instances, the domains of all labeled nulls in the c-instances are also finite since the domain is derived from existing labeled nulls in the c-instance.

The running time of the algorithm is exponential in the number of operators and size of the c-instances (bounded by *limit* $\times$ no. of relations $\times$ max no. of attributes) since the algorithm performs an exhaustive search on c-instances subject to the size limit, resulting in a high complexity. This motivates us to design a more efficient algorithm by generating a possibly smaller minimal c-solution that we describe in the next subsection.

4.3 Optimization by Conjunctive Tree Chase

The optimized approach converts the original syntax tree into a set of syntax trees where each tree does not contain disjunctions ($\vee$) and then performs the chase procedure described in Algorithm 1 on each one of the trees. This speeds up the solution dramatically since there is no need to expand every disjunctive operator in the tree into a set of trees that do not contain disjunction.

[1]The pseudo code can be found in the full version [26]

Conversion of a tree with $\vee$ into conjunctive trees. For sets, $Q_1 \vee Q_2$ is equivalent to three sets $\{Q_1 \wedge Q_2, \neg Q_1 \wedge Q_2, Q_1 \wedge \neg Q_2\}$. However, this is not always true for FOL formulae, since $f_1 = \forall x(P(x) \vee Q(x))$ is not equivalent to $f_2 = (\forall x P(x) \wedge Q(x)) \vee (\forall x \neg P(x) \wedge Q(x)) \vee (\forall x P(x) \wedge \neg Q(x))$ and only $f_2 \Rightarrow f_1$ holds; we demonstrate this below, first with a toy example and then using our running example.

Example 10 (Toy example). *Consider $f = \forall x\ (even(x) \vee odd(x))$ where the predicate $even(x)$ ($odd(x)$) is true if x is even (odd). Suppose the domain of x is $\{3, 4\}$. f is clearly satisfied with this domain. Now consider the conversion of f into three conjunctions: $f_1 = \forall x\ (even(x) \wedge odd(x))$, $f_2 = \forall x\ (\neg even(x) \wedge odd(x))$, and $f_3 = \forall x\ (even(x) \wedge \neg odd(x))$. f_1 is not satisfied since neither 3 nor 4 are both even and odd, f_2 is not satisfied since 4 is even, and f_2 is not satisfied since 3 is odd, thereby losing completeness in the conversion.*

Example 11. *Consider the syntax tree of $Q_B - Q_A$ in Figure 5, and a sub-formula of its right subtree:*

$$\forall p_3\big(\neg Serves(x_1, b_1, p_3) \vee \exists x_3, p_4(Serves(x_3, b_1, p_4) \wedge p_3 < p_4)\big)$$

We can convert this into the following three formulae:

$$\{\forall p_3\big(\neg Serves(x_1, b_1, p_3) \wedge \exists x_3, p_4(Serves(x_3, b_1, p_4) \wedge p_3 < p_4)\big),$$
$$\forall p_3\big(\neg Serves(x_1, b_1, p_3) \wedge \forall x_3, p_4(\neg Serves(x_3, b_1, p_4) \vee p_3 \geq p_4)\big),$$
$$\forall p_3\big(Serves(x_1, b_1, p_3) \wedge \exists x_3, p_4(Serves(x_3, b_1, p_4) \wedge p_3 < p_4)\big)\}.$$

These three formulae are equivalent to the original one if there is only one variable that p_3 can be mapped to in the price domain (in this case the original formula would be unsatisfiable). However, when there are more than two variables or constants in the domain, this conversion is not equivalence-preserving and will miss satisfying c-instances. For example, the c-instance $\mathcal{I}_0$ in Figure 4 satisfies the original formula but does not satisfy any of the three conjunctive formulae - assigning p_1 from $\mathcal{I}_0$ to the universally quantified p_3 will not satisfy any of the formulae.

Bearing this in mind, we describe an algorithm that performs this conversion. Given a syntax tree Q, the algorithm converts it into a set of conjunctive syntax trees based on the above principal.

We start with a syntax tree Q that may contain the $\vee$ operator in different nodes. The algorithm recurses on the tree where the base case is a Q that contains a single atom, then the algorithm simply creates a conjunctive tree from this atom, or its negation if it is negated. If the root of Q is an $\wedge$ node, the algorithm continues to recurse over the two children and joins each pair of obtained subtrees. If the root of Q is an $\vee$ node, the algorithm considers three cases, as mentioned above: (1) converting the root into $\wedge$ and recursing over both of its children, (2) converting the root into $\wedge$, negating the right child, and recursing over both of its children, and, finally, (3) converting the root into $\wedge$, negating the left child, and recursing over both of its children, All solutions to the three cases are added to the list of c-instances *res*. If the root of Q is a $\forall$ or $\exists$ quantifier, the algorithm recurses over the child of the root and adds the resulting trees to *res*.

Chase for conjunctive trees. The main chase procedure utilizes Algorithm 1 and applies it on the conjunctive tree obtained from the conversion algorithm. It gets a schema, a syntax tree, and a limit as input. It first converts the input tree into a set of conjunctive trees using the conversion algorithm. It then calls Algorithm 1 with each one of the conjunctive trees and adds the resulting c-instances to the list of results which is then outputted.

Soundness and complexity. The soundness of the algorithm, i.e., that it returns a set of minimal satisfying c-instances, follows from the fact that the final set is returned by Algorithm 1. Although there is an exponential dependency in the number of operators, the algorithm gives a better running time by avoiding recursive calls to split a query tree into three query trees recursively for the disjunction operator, at the cost of possibly not generating some satisfying c-instances that are generated by Algorithm 1.

Other optimizations. The time complexity is also largely affected by the number of labeled nulls in each domain, especially when handling universal quantifiers. Hence, we disallow universal quantifiers from adding new labeled nulls, though this might lose completeness. Moreover, notice that our Tree-Chase-BFS is always initially called with $\mathcal{I}_0$ =an empty c-instance, we could manipulate it to achieve different coverage by calling Tree-Chase-BFS on a c-instance that is properly initialized with the tuples or atomic conditions we target to cover. We further evaluate these optimizations in Section 5.

Setting the limit parameter in Algorithm 1. The *limit* parameter can be set in several different manners in practice. One approach is setting a default *limit* according to query complexity. *limit* determines the maximal size of the c-instance. In our experimental results (Section 5) we have seen that it can be set to some multiple of the number of query atoms to safely allow for a c-instance to covers all the atoms of the query (we have used a multiple of 2 in our evaluation). Another alternative, aimed at an interactive experience, is to set a timeout parameter instead of the *limit* (as done in Section 5), thus allowing Algorithm 1 to explore higher limits as needed up to the allotted time.

5 EXPERIMENTS

We investigate the performance of our approach and compare it to different variations of our approach in the following aspects: (1) runtime for varying query complexity (2) varying the limit parameter in Algorithm 1, (3) properties of the output c-instances, and (4) case studies showing the actual obtained c-instances for a sample of the experimental queries.

Setup. We implemented our methods in Python 3.7. We ran all experiments locally on a 64-bit Ubuntu 18.04 LTS server with 3.20GHz Intel Core i7-8700 CPU and 32GB 2666MHz DDR4 RAM. We compare the following variants of our algorithms and optimizations.

- **Disj-Naive**: this method implements the exhaustive chase procedure described in Section 4.2.
- **Conj-Naive**: this method implements the optimized conjunctive tree chase procedure described in Section 4.3 that converts the original syntax tree into a set of syntax trees without disjunction.
- **Disj-EO/Conj-EO**: this method adapts Disj-Naive/Conj-Naive by only allowing the algorithm to add labeled nulls to the c-instance when handling an existential quantifier node.
- **Disj-Add/Conj-Add**: this method first runs Disj-EO(or Conj-EO) on the empty c-instance, then gets the minimal c-solution. If there are still leaf atoms not covered by any of the c-instance in the c-solution, then it iterates over every

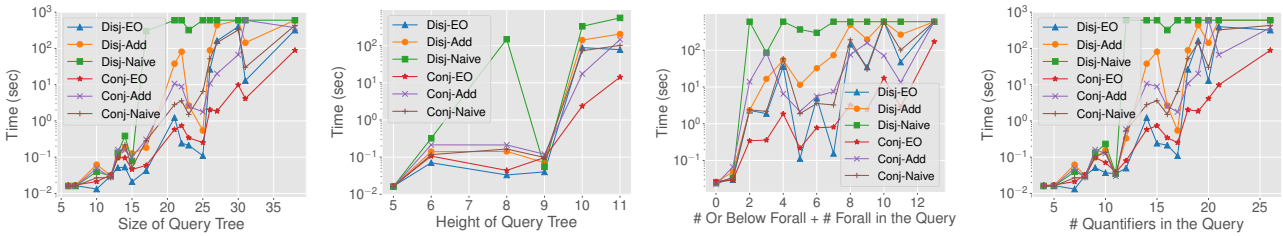


Figure 8: Running time vs. various measures of query complexity. $limit = 10, timeout = 600sec$.

Dataset	# Queries	Mean # Atoms	Mean # Quantifiers	Mean # Or	Mean Height
Beers	35	6.40	13.94	2.17	9.54
TPC-H	28	11.96	23.07	4.18	12.07

Table 1: Dataset statistics.

remaining uncovered leaf atom, creating corresponding labeled nulls and adding the atom to the initial c-instance, and runs Disj-EO (Conj-EO resp.) on the initialized c-instance.

We evaluate both the efficiency/scalability of our algorithms in terms of runtime and the quality of results with respect to different measures, and compare them against systems from related work.

Datasets. We used two datasets in our experiments, **Beers** and **TPC-H**. For the Beers dataset, queries in these experiments come from submissions by students for an assignment in an undergrad database course. We picked 5 questions (skipped those with only simple selection and join) and sampled a few students' queries, then manually rewrote them into domain relational calculus. There were 5 (correct) standard queries and 10 students' wrong queries; we also considered the difference between the standard queries and the wrong queries (and also the opposite direction), resulting in additional 20 queries. Some queries are very complex as they use the difference operator multiple times, resulting in nested universal quantification in the DRC query. Similarly, for TPC-H, we picked 4 queries (Q4, Q16, Q19, and Q21) and dropped their aggregate functions, then made two wrong queries each, resulting in 28 test queries in total [26]. The statistics of the datasets are in Table 1.

5.1 Performance Evaluation

Scalability. To evaluate the scalability of our approach, we study how query complexity affects the running time. We consider four measures of query complexity: (1) number of nodes in the query tree, (2) the height of the query tree, (3) number of universal quantifiers plus number of disjunction that is below a universal quantifier, and (4) the number of both universal and existential quantifiers. Although most of these parameters are specific to our algorithms that operate on DRC queries, the number of universal quantifiers also has a corresponding complexity notion in SQL form since each ∀ in DRC leads to at least one negated sub-query in SQL.

Example 12. *Recall the query in Figure 3 and its syntax tree in Figure 5. The number of nodes in the tree is* 27 *(measure (1)), the height of the tree is* 8 *(measure (2)), the query contains* 2 *universal quantifier,* 3 *disjunctions below it, and* 6 *existential quantifiers, so measure (3) is* 5 *and (4) is* 8. *For reference, the queries Q_A and Q_B from our running example are shown in SQL in Figure 9.*

We set the limit threshold to be 10 for the Beers dataset and 15 for TPC-H, and stops the algorithm if it does not finish in 10 minutes (20 minutes for TPC-H). The results are shown in Figure 8

```
SELECT l.beer, s.bar
FROM Likes l, Serves s
WHERE l.drinker LIKE 'Eve_%' AND
l.beer = s.beer
AND NOT EXISTS(
    SELECT * FROM Serves
    WHERE beer = s.beer AND price >
          s.price);
```

(a) Correct query Q_A

```
SELECT S1.beer, S1.bar
FROM Likes L, Serves S1, Serves S2
WHERE L.drinker LIKE 'Eve%' AND
L.beer = S1.beer AND L.beer = S2.
      beer
    AND S1.price > S2.price;
```

(b) Incorrect query Q_B

Figure 9: Queries from our running example in SQL.

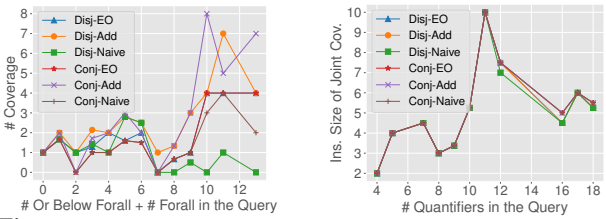


Figure 10: Result quality by query complexity. $limit = 10, timeout = 600sec$.

and Figure 11, respectively. We report the average running time for different queries with the same value of the complexity measure.

As shown in in Figure 8, the running time increases with query complexity. Disj-Naive has the worst time complexity (more than exponential), which did not finish for most of the complex queries with more than 10 quantifiers, followed by Disj-EO and Disj-Add, whereas Conj-Naive, Conj-EO, Conj-Add perform better. The running time of Conj-Naive, Conj-EO, Conj-Add increases exponentially as expected since their complexity largely depends on the number of conjunctive syntax trees generated from the original query. Compared to the total number of nodes and the height, the number of universal quantifiers and the number of disjunction nodes are more crucial to the growth in the running time. Similar trends are illustrated in Figure 11, whereas Conj-EO still performs better than Disj-EO, while the running time of Conj-Add is very close to Disj-Add. We conjecture that this is because the overall complexity of queries in the TPC-H dataset is much higher than the Beers dataset, as shown in Table 1, leading to more generated conjunctive syntax trees. Our results indicate that our solution scales well for complex schemas and queries (except for very complex and long queries: for only 4 extremely complex cases out of the 28 cases in the TPC-H dataset, our algorithm failed to return any results) .

Result quality. Our optimized approaches (Conj-EO, Disj-EO, Conj-Add and Disj-Add) run much faster than Disj-Naive by compromise on the completeness of the minimal c-solution to different extents. To evaluate the result quality of these approaches in terms of both completeness and minimality, we show in Figure 10 the number of distinct coverage from the returned c-solutions and the average size of the c-solutions. Notice that the number of returned

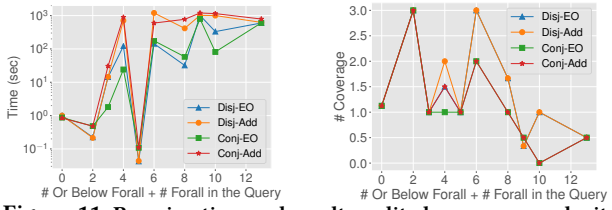


Figure 11: Running time and result quality by query complexity on TPC-H dataset $limit = 15, timeout = 1200sec$.

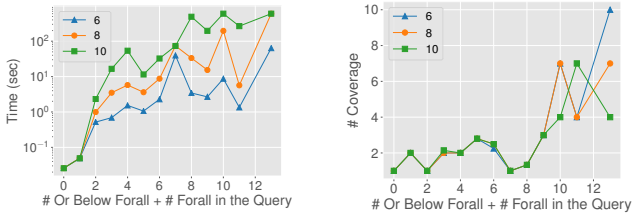


Figure 12: Parameter sensitivity varying limit. Disj-Add.

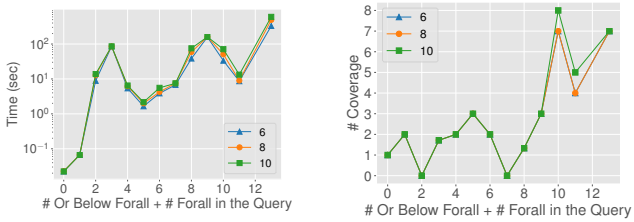


Figure 13: Parameter sensitivity varying limit. Conj-Add.

minimal c-solution of each variant can be different (either they are unable to find some results, e.g. Disj-EO returns a subset of Disj-Add; or some variants finish before the timeout but the others do not), to guarantee a fair comparison, for each query we only consider c-solutions with a coverage set returned by all of the variants. For example, for a query Q if Conj-Naive returns two c-solutions with coverage C_1 and C_2, and Disj-Add returns three c-solutions with coverage C_1, C_2, C_3, we will only report the average c-solution size of the two with coverage C_1 and C_2 for Q.

Figure 10 shows that Disj-Add returns more distinct coverage sets in most cases, while Conj-Add, Conj-Naive,Conj-EO, and Disj-EO might fail to return any satisfying instances. There are a few exceptions for some very complex queries where Disj-Add did not finish before timeout and thus Conj-Add returns more distinct coverage sets. Although Disj-Naive did not finish running in most cases, the c-solutions it returns can be smaller than other variants when there are more than 10 quantifiers in the query. There are few cases where the Disj-Add and Disj-EO return smaller c-solutions compared to the variants using conjunctive trees.

Parameter sensitivity. To ensure that the algorithm terminates, we used a *limit* parameter to restrict the size of the c-instances. Figure 12 and Figure 13 show how this limit affects the running time and completeness for Disj-Add and Conj-Add. Although the running time grows exponentially with the query complexity, Disj-Add runs one order of magnitude faster when limit=6 than limit=10, losing completeness only when the query tree is very complex. For Conj-Add, the difference in running time when varying the limit is negligible in most cases, and the number of distinct returned coverage sets only changes for the most complex queries.

Interactivity. To improve interactivity, our algorithms can output the instances one at a time as soon as they are generated, so users can start exploring immediately and have a more interactive experience. The time to produce the first instance for our algorithms on the Beers dataset is only 4.78 seconds on average (DisjAdd) or 0.77 seconds (ConjAdd), and the average delay between two consecutive output instances with different coverage is 18.34 seconds (DisjAdd) or 5.22 seconds (ConjAdd). While on the TPC-H dataset, the time to produce the first instance is longer but still tolerable: 101.25 seconds on average (DisjAdd) or 88.02 seconds (ConjAdd), and the average delay between two consecutive output instances with different coverage is 19.12 seconds (DisjAdd) or 54.16 seconds (ConjAdd). Note that doing so may risk returning non-minimal instances, as minimality is verified in postprocessing (Section 4.2). Another option is to start with the optimized version (Section 4.3) and if further insights are needed, run the exhaustive search (Section 4.2). We also note that slightly longer wait times might be acceptable in some scenarios, e.g., providing offline feedback to student solutions, or to help students/instructors when manual debugging would take significantly more effort for complex queries or subtly wrong solutions.

5.2 Case Study

By providing the "basis" to a query Q, our work yields a set of abstract instances that can help users understand and debug their query in practice. To evaluate the usefulness of the set of abstract instances (the minimal c-solution returned by the algorithms, providing a proxy for the universal solution), we report one case study on the same real-world dataset as the performance evaluation from an undergrad database course. We pick two most complex standard solution queries from an assignment each with one wrong query from student submissions. Table 2 shows the solution queries, wrong queries, and the universal solution for the difference query of the standard and wrong queries.

The universal solution captures different errors in the wrong query. To compare, we use the ground instances that serve as "counterexamples" for the wrong queries by a previous system [41] based on a randomly generated testing database instance.

For Q_1 (the same as our running example), the first and the second c-instances pinpoint that if the drinker's first name is not 'Eve' but has 'Eve' as its prefix. While the first c-instance does not contain the first name condition, it shows that if all three bars serve the same beer at different prices, the query would go wrong. Note that if we add to the last instance the condition $\neg(d_1 \texttt{LIKE 'Eve\_\%'})$, it is still a satisfying c-instance, but it is not minimal because its coverage is the same as the second c-instance. In comparison, while the ground instance by [41] (as in Figure 1) is in the represented world of the first c-instance, it does not highlight that the reason behind the wrong query result is that the prices are ordered in a particular way, but the actual values are unimportant.

For Q_2, the c-instances in the universal solution indicate that the query would go wrong if there is a drinker frequents to a bar that does not serve any beer, no matter the drinker likes a beer or not (the 1st and 3rd instances). This may pinpoint the error that the Likes table does not interact with the Serves table. Furthermore, the 2nd, 5th, and 6th c-instances imply that if there is a beer served at a bar, to make the query return a wrong result, the drinker should not

Query description	Queries (DRC)	c-instances
Q_1: for each beer liked by any drinker whose first name is Eve, find the bars that serve this beer at the highest price	Correct query: Q_A in Figure 2a. Wrong query: Q_B in Figure 2b.	I_0 in Figure 4 I_1 in Figure 6 $\text{Drinker}(d_1, *), \text{Drinker}(d_2, *), \text{Beer}(b_1, *), \text{Bar}(x_1, *), \text{Bar}(x_2, *), \text{Likes}(d_1, b_1), \text{Serves}(x_1, b_1, p_1),$ $\text{Serves}(x_2, b_1, p_2), d_1 \text{ LIKE 'Eve\%'} \wedge \neg(d_1 \text{ LIKE 'Eve\_\%'}) \wedge \neg(\text{Likes}(d_2, b_1)) \wedge p_1 < p_2$
Q_2: Find names of all drinkers who frequent only bars that serve some beer they like	Correct Query: $Q_{2A} = \{(d_1) \mid \exists a_1(\text{Drinker}(d_1, a_1) \wedge \forall x_1 \forall t_1(\neg\text{Frequents}(d_1, x_1, t_1) \vee \exists b_1, p_1 (\text{Serves}(x_1, b_1, p_1) \wedge \text{Likes}(d_1, b_1))))\}$ Wrong Query: $Q_{2B} = \{(d_1) \mid \exists a_1(\text{Drinker}(d_1, a_1) \wedge \forall b_1(\forall t_1, t_1, p_1(\neg\text{Frequents}(d_1, x_1, t_1) \vee \neg\text{Serves}(x_1, b_1, p_1)) \vee \text{Likes}(d_1, b_1)))\}$ Showing universal solution for $Q_{2B} - Q_{2A}$	$\text{Drinker}(d_1, *), \text{Beer}(b_1, *), \text{Bar}(x_1, *), \text{Likes}(d_1, b_1), \text{Frequents}(d_1, x_1, t_1)$ $\text{Drinker}(d_1, *), \text{Beer}(b_1, *), \text{Bar}(x_1, *), \text{Bar}(x_2, *), \text{Likes}(d_1, b_1), \text{Serves}(x_1, b_1, p_1),$ $\text{Frequents}(d_1, x_2, t_1), \neg(\text{Frequents}(d_1, x_1, t_1))$ $\text{Drinker}(d_1, *), \text{Beer}(b_1, *), \text{Bar}(x_1, *), \text{Frequents}(d_1, x_1, *),$ $\text{Drinker}(d_1, *), \text{Beer}(b_1, *), \text{Bar}(x_1, *), \text{Bar}(x_2, *), \text{Likes}(d_1, b_1),$ $\text{Serves}(x_1, b_1, p_1), \text{Frequents}(d_1, x_1, t_1), \text{Frequents}(d_1, x_2, t_1)$ $\text{Drinker}(d_1, *), \text{Beer}(b_1, *), \text{Bar}(x_1, *), \text{Bar}(x_2, *), \text{Serves}(x_1, b_1, p_1), \text{Frequents}(d_1, x_2, t_1),$ $\neg(\text{Likes}(d_1, b_1)) \wedge \neg(\text{Frequents}(d_1, x_1, t_1))$ $\text{Drinker}(d_1, *), \text{Drinker}(d_2, *), \text{Beer}(b_1, *), \text{Bar}(x_1, *), \text{Bar}(x_2, *), \text{Likes}(d_2, b_1),$ $\text{Serves}(x_1, b_1, p_1), \text{Frequents}(d_2, x_2, t_1), \neg(\text{Frequents}(d_2, x_1, t_1)) \wedge \neg(\text{Likes}(d_1, b_1))$ $\text{Drinker}(d_1, *), \text{Beer}(b_1, *), \text{Beer}(b_2, *), \text{Bar}(x_1, *), \text{Bar}(x_2, *), \text{Likes}(d_1, b_1),$ $\text{Serves}(x_1, b_1, p_1), \text{Frequents}(d_1, x_1, t_1), \text{Frequents}(d_1, x_2, t_1), \neg(\text{Likes}(d_1, b_2))$

Table 2: Queries used in the case study; universal solutions generated using Disj-Add, $limit = 10$. Note that the symbols in queries (representing the query variables) and the symbols in c-instances (representing labeled nulls) are not the same.

frequent this bar, which could be interpreted as the correct solution uses the frequents table together with negation in a different way. Actually, the wrong query joins Frequents with Serves, while the correct solution joins Likes with Serves. Therefore, the universal solution provides different perspectives in understanding the query and formulates a hint on how to modify the wrong query. The ground instance by [41] only consists of four tuples: Drinker("Bryan", "39934 Main St."), Beer("Amstel", "A. Brewer"), Bar("The Edge", "802 Morris St."), and Frequents("Bryan", "The Edge", 3), which is in the represented world of the third c-instance from our universal solution in Table 2. Such a simple counterexample might be less helpful for users to understand why the query goes wrong, as one would benefit from the explicit conditions with negation.

5.3 User Study

We conducted a user study for the Beers dataset to evaluate: (R1) how effective our approach is for explaining and understanding bugs in queries, and (R2) whether completeness as a quality metric is helpful. For (R1), we specifically compare our approach (c-instances) with **concrete instances [41]** having constant values. Note that our approach takes only the queries and the schema as input, whereas [41] also takes a database instance as input and outputs a sub-instance as a concrete counterexample[2].

Participants. We recruited 64 participants, including 22 graduate students from CS departments and 42 undergraduate students from an undergraduate database course. Participation was voluntary and anonymous, though the undergraduates were offered small souvenirs as a reward for their participation (we did not get enough responses from the undergraduates in our initial pilot surveys). The undergraduate students were already familiar with the schema of the Beers dataset from an earlier homework; while for the graduate students, we explained the schema details and also asked about their familiarity with SQL. We note that the undergraduate students have also been exposed to the tool using concrete instances developed by [41] (but only for relational algebra queries); because of this familiarity, concrete instances might hold a slight advantage over c-instances for these students. Half of the graduate students (11 out of 22) graduate students declared high familiarity with SQL queries and the rest reported moderate or low familiarity; our observations for both groups were similar in this study, therefore we report the overall statistics for graduate students.

Tasks. We asked all participants to spot errors in two SQL queries (each has two major errors, see Table 3) querying the Beers database, with the help of either our c-instances or concrete instances from [41]. We provided each participant with one query followed by c-instances and other query followed by concrete instances as counterexamples, randomly dividing them into two groups: one group saw (Q_1, c-instances) + (Q_2, concrete instances) and the other saw (Q_1, concrete instances) + (Q_2, c-instances). The order of showing these two questions for both groups was chosen at random to avoid any familiarity bias against either c-instances or concrete instances. Instead of showing completely abstract c-instances, we added an example concrete value to each variable in the c-instance, showing one way that it can be grounded. This is a trivial extension done to help alleviate novices' potential discomfort with seeing symbols and conditions alone. This approach somewhat blurs the line between c-instances and concrete instances, but faithfully represents how in practice c-instances would be deployed in an educational setting. Then, for each query we have the *treatment group* (with c-instances as explanation) and the *control group* (with concrete-value-only instances as explanation). To study (R2), following the task involving the first c-instance above, we presented a second c-instance for the same query but with a different coverage (which would illustrate a different error), and asked the participant what errors they found upon seeing both c-instances, and whether they felt the second c-instance provided additional help. (Note that [41] and other related work, there is no option for generating additional concrete instances that illustrate different errors in the same query.) At the end of the study, back to (R1), we also asked the participants about their preferences between c-instances and concrete instances for spotting errors in queries.

Results and analysis. Objectively, we evaluate the user performance by the number of errors they spotted. Figures 14 show the percentage of users who failed to spot any error, spotted one error, and spotted both errors in each group.For example, consider the last three bars in the left sub-figure in Figure 14, which show the

[2]In a pilot study we also compared these approaches against a baseline of not providing any instances (c-instance or concrete). We found that study questions involving this baseline significantly increased the length and difficulty of the survey demanding higher participant efforts to the point of discouraging participation, and that from the preliminary results we collected, participants did much better with the help of instances. Hence, in the final user study we excluded the baseline, but asked the question: *"When you learn SQL queries in the future, would you like to see the example instances shown in this survey to help you understand incorrect queries?"* All 64 participants answered yes.

overall statistics (combining both queries) for all undergraduate participants. Showing the concrete instance alone is already quite helpful: only 31% of the users failed to find an error ("total-conc"). Going from "total-conc" to "total-CI1," we see a clear performance improvement among users who were shown c-instances: percentage of the users failing to find an error goes down to 19%. Going further from "total-CI1" to "total-CI2," we see that as soon as users are show a second c-instance, practically all of them were able to spot at least one error, and the majority (64.3%) of them indeed spot both errors in the query; in contrast, no users were able to spot both errors with only a concrete instance ("total-conc"). Similar conclusions can be drawn from per-query statistics (shown by the first two batches of three bars in the left part of Figure 14) as well as from statistics for graduate students (shown in the right part of Figure 14). Overall, these results convincingly show that for (R1), c-instances hold a clear advantage over concrete instances in objectively improving participants' performance in spotting errors in queries; and for (R2), showing multiple c-instances with different coverage dramatically improves participants' ability in spotting remaining errors in queries.

A number of other observations from these results are worth noting but largely confirms intuition. First, Q_2 was easier to debug than Q_1. Second, graduate students overall perform better than undergraduates. The coupling of these factors explains why we did not see any difference from "Q2-conc" to "Q2-CI1" in the right part of Figure 14; apparently most graduate students got enough help from the concrete instance in order to spot at least one error in the simpler Q_2. Nonetheless, they still needed the help with an additional c-instance to uncover the second error.

Figures 15 and 16 summarize the subjective responses from participants regarding their preference for c-instances vs. concrete instances, and their opinion on the usefulness of additional c-instances. A clear majority of the participants found the additional c-instance useful per Figure 16. However, from Figure 15, it is apparent that many participants prefer viewing concrete instances, despite the fact that they perform objectively better with the help of c-instances. This preference is stronger among undergraduates—only a third preferred c-instances, compared with more than a half for concrete instances. A relatively lower fraction of graduate students—but still a half of them—preferred concrete instances. It would be interesting to conduct additional study to pinpoint their reluctance to embrace c-instances despite their objective advantages, but there are several possible explanations. First, the abstraction provided by variables and conditions the c-instances may be seen as more intimidating, especially for undergraduates. This conjecture is corroborated by some of the free-form feedback comments we received. Second, as mentioned earlier in this section, the undergraduates already had some familiarity working with concrete instances before this user study. Overall, the fact that still about a third of the participants preferred c-instances shows that there is a sizable and compelling demand for this approach. We also believe we can mitigate some of the reluctance in this user base with improved interfaces and familiarity.

6 CONCLUSIONS AND FUTURE WORK

We have defined and studied the problem of compact query characterization using the coverage of abstract c-instances. We have devised algorithms and optimizations for computing such characterizations building on the concept of chase and utilizing the structure of the syntax tree. We experimentally showed that our approach is effective at finding c-instances that characterize the query and examined the effect of query complexity and parameter changes on the scalability of our approach. In future work, we plan to study the development of further optimizations for finding such solution for more query classes different properties of c-instances. In this paper we showed that the problem of finding a universal solution is poly-time for $CQ^{\neg}$ queries, while the decision version is undecidable for general DRC queries: understanding the computability and complexity for universal solutions for other query classes in between is another interesting research direction. Finally, while our model can support queries with the same final aggregate and different bodies by removing the aggregate, extending our model to support arbitrary aggregate queries is another intriguing direction of future work.

Query description	Wrong Queries
Q_1: for each beer liked by any drinker whose first name is "Eve", find the bars that serve this beer at the highest price	Q_B in Figure 9 (our running example)
Q_2: Among the drinkers who frequent "The Edge", find the names of those who do not like "Erdinger".	**SELECT DISTINCT** S.beer **FROM** Serves S, Likes L **WHERE** S.bar = 'Edge' **AND** S.beer = L.beer **AND** L.drinker <> 'Richard';

Table 3: Queries used in the user study.

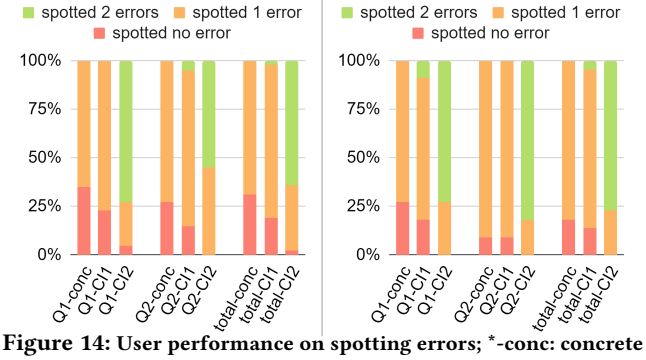


Figure 14: User performance on spotting errors; *-conc: concrete instance only, *-CI1: the first C-Instance, *CI2: the second C-Instance (left: undergrad, right: graduate).

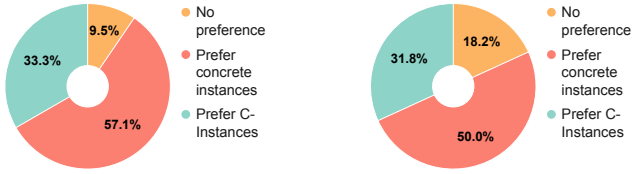


Figure 15: Preference on explanation types (left: undergraduate, right: graduate).

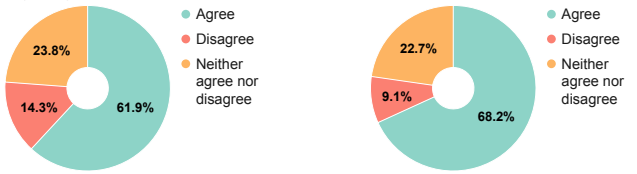


Figure 16: User feedback on "The second C-Instance provided additional help" (left: undergraduate, right: graduate).

ACKNOWLEDGMENTS

This work is supported by the NSF awards IIS-1552538, IIS-1703431, IIS-1814493, IIS-2008107, and by the NIH award R01EB025021.

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