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Brief paper

Minimum structural sensor placement for switched linear time-invariant systems and unknown inputs☆

Emily A. Reed [a,*], Guilherme Ramos [b], Paul Bogdan [a], Sérgio Pequito [c]

[a] Ming Hsieh Electrical and Computer Engineering Department, University of Southern California, USA
[b] Instituto Superior Técnico, Universidade de Lisboa, 1049-001, Lisbon, Portugal
[c] Department of Information Technology, Uppsala University, Uppsala, Sweden

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ABSTRACT

In this paper, we study the structural state and input observability of continuous-time switched linear time-invariant systems and unknown inputs. First, we provide necessary and sufficient conditions for their structural state and input observability that can be efficiently verified in $O((m(n+p))^2)$, where n is the number of state variables, p is the number of unknown inputs, and m is the number of modes. Moreover, we address the minimum sensor placement problem for these systems by adopting a feed-forward analysis and by providing an algorithm with a computational complexity of $O((m(n+p)+\alpha)^{2.373})$, where α is the number of target strongly connected components of the system's digraph representation. Lastly, we apply our algorithm to a real-world example in power systems to illustrate our results.

1. Introduction

Scientists and engineers model systems by describing the nature of their dynamics and the environment in which they interact. One powerful tool to model complex switching dynamics is to adopt a switched linear time-invariant framework. This model assumes that the system under scrutiny transitions between different (yet known) linear time-invariant dynamics, where such transitions are discrete in nature and are captured by a switching signal for which the sequence of the switches may not be known *a priori*. Examples of such systems include the power electric grid (Du, Jiang, & Shi, 2015), where the change in dynamics may be dictated by a faulty transmission line (Ramos, Pequito, Aguiar, & Kar, 2015; Ramos, Pequito, Aguiar, Ramos, & Kar, 2013), or a multi-agent system (Ramos, Silvestre and Silvestre, 2020; Sun, Tian, & Xie, 2017), where the dynamics may change due to a loss in communication among agents.

However, when modeling a process, it is common to neglect the fact that the interaction of a dynamical system with its environment introduces errors. We can describe these external environmental errors as unknown inputs entering into the dynamical system. For instance, in the power grid, the generated power and/or the customer demand behave as unknown inputs. Similarly, in multi-agent robotic systems, particularly in surface vehicles, friction behaves as an unknown input, whereas in the context of unmanned vehicles, airflow or ocean currents act as unknown inputs. An alternate scenario is in networked systems where the unknown input is due to the interconnections with the remaining hidden network (Alur, 2015; Corradini & Cristofaro, 2017; Farivar, Haghighi, Jolfaei, & Alazab, 2019; Gupta, Pequito, & Bogdan, 2018; Hutchison et al., 2013; Xie & Yang, 2018). As is evident in the previously mentioned examples, a reoccurring practice in control engineering is to model the unknown inputs in a latent space that can capture the main features of the incoming signal but does not model the system from which the unknown input originates.

To monitor such switched linear time-invariant systems under unknown inputs requires us to assess both the state and the inputs by guaranteeing that the system is state and input observable (Sundaram & Hadjicostis, 2012). Often, however, we cannot accurately know the parameters of the system. Moreover, if the parameters are known, the study of controllability and/or observability properties leads to NP-hard problems (Ramos, Pequito, & Caleiro, 2018). Hence, we assume that only the structure of the system is known meaning that a system parameter is either zero or could take on any real scalar value (Ramos, Aguiar, & Pequito, 2022). In this context, we can rely on the notion of structural state and input observability that yields state and input observability for almost all system parameterizations.

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* Corresponding author.
E-mail addresses: emilyree@usc.edu (E.A. Reed), guilherme.ramos@tecnico.ulisboa.pt (G. Ramos), pbogdan@usc.edu (P. Bogdan), sergio.pequito@it.uu.se (S. Pequito).

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Previous work has provided the necessary and sufficient conditions to ensure *structural* state and input observability for discrete-time systems under unknown inputs (Sundaram & Hadjicostis, 2006). Nonetheless, the counterpart for continuous-time switched linear-time invariant systems under unknown inputs were only studied in Boukhobza (2012), Boukhobza and Hamelin (2011) and Boukhobza, Hamelin, Kabadi, and Aberkane (2011). In particular, Boukhobza (2012) and Boukhobza and Hamelin (2011) consider the graph-theoretic necessary and sufficient conditions for generic discrete mode observability of a continuous-time switched linear system with unknown inputs and proposed a computational method to verify such conditions with a complexity of $O(n^6)$, where n is the number of states. The works of Boukhobza (2012) and Boukhobza et al. (2011) present sufficient conditions for the generic observability of the discrete mode of continuous-time switched linear systems with unknown inputs and find an exhaustive location set to place sensors when these conditions are not satisfied with a computational complexity of $O(n^4)$. However, none of these works considered the minimum number of required sensors and their placement to guarantee structural state and input observability as we consider in this work. This problem is important in designing control schemes for large scale systems and is often referred to as the *minimum sensor placement*. While this problem has been studied for a variety of systems (Pequito, Kar, & Aguiar, 2015), to the best of the authors' knowledge, it has not been studied in the context of continuous-time switched linear time-invariant systems under unknown inputs.

The main contributions of this manuscript are as follows. We first provide necessary and sufficient conditions for structural state and input observability of continuous-time switched linear-time invariant systems under unknown inputs. Moreover, we can verify these conditions in $O((m(n+p))^2)$, where n is the number of state variables, p is the number of unknown inputs, and m is the number of modes. Furthermore, we address the minimum sensor placement for these systems using a feed-forward analysis and an algorithm with a computational complexity of $O((m(n+p)+\alpha)^{2.373})$, where the $n\times n$ matrix multiplication algorithm with best asymptotic complexity runs in $O(n^{\varsigma})$, with $\varsigma \approx 2.3728596$ (Alman & Williams, 2021), and where α is the number of target strongly connected components of the system's digraph representation. Finally, we provide a real-world example from power systems to illustrate our results.

We structure the remainder of our paper as follows. Section 2 provides the addressed problem formulation. Section 3 presents the main results including two graph-theoretic conditions for structural state and input observability for switched linear time-invariant systems with unknown inputs as well as an algorithm that determines the minimum set of state and input variables for ensuring structural state and input observability. Section 4 presents a real-world example from power systems to illustrate our results. Finally, Section 5 concludes the paper.

2. Problem statement

In this paper, we consider a continuous-time switched linear time-invariant (LTI) system with (unknown) inputs that can be described as follows:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + F_{\sigma(t)}d(t), \tag{1a}$$

$$\dot{d}(t) = Q_{\sigma(t)}d(t), \tag{1b}$$

$$y(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}d(t), \tag{1c}$$

where $x(t)\in\mathbb{R}^n$ is the state, $d(t)\in\mathbb{R}^p$ represents the unknown inputs, $y(t)\in\mathbb{R}^n$ is the output, and $\sigma(t):[0,\infty)\to\mathbb{M}\equiv\{1,\ldots,m\}$ is the unknown switching signal. System (1) contains m possible known subsystems also known as *modes*, which we denote by the tuple $(A_k, F_k, Q_k, C_k, D_k)$, where $\sigma(t)=k\in\mathbb{M}$. Lastly, we implicitly assume that the dwell time of each mode is greater than zero.

In what follows, we seek to assess and determine the minimum sensor placement that ensures state and input observability for the continuous-time switched LTI system with unknown inputs in (1).

Definition 1 (*State and Input Observability Molinari, 1976*)**.** The switched LTI system described by $(A_{\sigma(t)}, F_{\sigma(t)}, Q_{\sigma(t)}, C_{\sigma(t)}, D_{\sigma(t)}, \sigma(t); T_f)$ is said to be state and input observable for a time horizon T_f *if and only if* the initial state $x(t_0)$ and the unknown inputs $d(t)$ where $t\in[t_0,T_f]$ can be uniquely determined, given $((A_{\sigma(t)}, F_{\sigma(t)}, Q_{\sigma(t)}, C_{\sigma(t)}, D_{\sigma(t)}, \sigma(t); T_f)$ and measurements $y(t)$ $(t_0\le t\le T_f)$. ∘

In this paper, we focus on the sensor placement problem. For the sake of simplicity, we assume that the measurements take the following form

$$y(t) = Cx(t) + Dd(t). \tag{1c$'$}$$

Simply speaking, we assume that the output and feed-forward matrices are the same across all modes. Notice that this assumption can be waived as we discuss in the following Remark 2.

Remark 2. We can consider a fixed set of measurements represented by C and D without loss of generality since taking the union of the measurements made in different modes, represented by $C_{\sigma(t)}$ and $D_{\sigma(t)}$, will result in the total set represented by C and D. ◇

We assume that each sensor is dedicated, meaning that each sensor can measure only one state or only one input. Considering an arbitrary set of sensors would lead to an NP-hard problem as this is the case for the linear time-invariant systems (Pequito et al., 2015). We state this formally in the following assumption.

A1 The output matrix and feed-forward matrix are written as $C=\mathbb{I}_n^{\mathcal{J}_x}$ and $D=\mathbb{I}_p^{\mathcal{J}_d}$, where $\mathbb{I}_n^{\mathcal{J}_x}$ is a matrix where its rows are composed of canonical identity matrix rows that are each multiplied with any arbitrary value. These canonical rows are indexed by $\mathcal{J}_x=\{1,\ldots,n\}$. Similarly, $\mathbb{I}_p^{\mathcal{J}_d}$ is a matrix where its rows are composed of canonical identity matrix rows that are each multiplied with any arbitrary value. These canonical rows are indexed by $\mathcal{J}_d=\{1,\ldots,p\}$.

Due to uncertainty in the system's parameters, we consider a structural systems framework (Ramos et al., 2022). We introduce the following definition for a structural matrix.

Definition 3 (*Structural Matrix*)**.** A matrix $\bar{M}\in\{0,\star\}^{m_1\times m_2}$ is referred to as a structural matrix if $\bar{M}_{ij}=0$, then $M_{ij}=0$, and if $\bar{M}_{ij}=\star$, then $M_{ij}\in\mathbb{R}$, so M_{ij} is any arbitrary real number and M_{ij} is assumed to be independent of $M_{i'j'}$ for all i,j,i',j' such that $i\neq i'$ and $j\neq j'$.

With this notion in mind, we next define *structural* state and input observability for the switched LTI system with unknown inputs in (1).

Definition 4 (*Structural State and Input Observability*)**.** The switched LTI system with unknown inputs described by the structural matrices $(\bar{A}_{\sigma(t)}, \bar{F}_{\sigma(t)}, \bar{Q}_{\sigma(t)}, \bar{C}_{\sigma(t)}, \bar{D}_{\sigma(t)}, \sigma(t); T_f)$ is said to be structurally state and input observable for a time horizon T_f *if and only if* there exists a system described by $(A_{\sigma(t)}, F_{\sigma(t)}, Q_{\sigma(t)}, C_{\sigma(t)}, D_{\sigma(t)}, \sigma(t); T_f)$ that is state and input observable and satisfies the structural pattern imposed by the structural matrices $(\bar{A}_{\sigma(t)}, \bar{F}_{\sigma(t)}, \bar{Q}_{\sigma(t)}, \bar{C}_{\sigma(t)}, \bar{D}_{\sigma(t)})$. ∘

Subsequently, the problem statement we seek to address in this paper is as follows: given $\bar{A}_{\sigma(t)}$, $\bar{F}_{\sigma(t)}$, $\bar{Q}_{\sigma(t)}$, which are the known structural matrices of system in (1a) and (1b), and time horizon T_f, we aim to find the minimum set of states $\mathcal{J}_x$ and inputs $\mathcal{J}_d$ that need to be measured to ensure structural state and input observability. We present this formally as

$$\begin{aligned} \min_{\substack{\mathcal{J}_x \subseteq \{1,\dots,n\} \\ \mathcal{J}_d \subseteq \{1,\dots,p\}}} \quad & |\mathcal{J}_x| + |\mathcal{J}_d| \\ \text{s.t. } & (\bar{A}_{\sigma(t)}, \bar{F}_{\sigma(t)}, \bar{Q}_{\sigma(t)}, \bar{\mathbb{I}}_n^{\mathcal{J}_x}, \bar{\mathbb{I}}_p^{\mathcal{J}_d}, \sigma(t); T_f) \\ & \text{is struct. state and input observable.} \end{aligned} \qquad (\mathcal{P}_1)$$

For the sake of clarity, we assume that the matrix $\bar{F}_{\sigma(t)}$ does not have zero columns as this would correspond to having disturbances that do not affect the dynamics of the system.

3. Minimum structural sensor placement for switched LTI systems with unknown inputs

In this section, we proceed as follows. First, we provide necessary and sufficient conditions for the feasibility of the optimization problem $\mathcal{P}_1$. Next, we characterize the minimal solution of problem $\mathcal{P}_1$. Lastly, we develop an algorithm to obtain a solution to $\mathcal{P}_1$, and we assess its computational complexity.

We start by introducing the notion of *generic rank*, which allows us to provide conditions for structural state and input observability of continuous-time switched LTI systems with unknown inputs.

Definition 5 (*Generic Rank*)**.** The generic rank (g-rank) of an $n_1 \times n_2$ structural matrix $\bar{M}$ is

$$g - rank(\bar{M}) = \max_{M \in [\bar{M}]} rank(M),$$

where $[\bar{M}] = \{M \in \mathbb{R}^{n_1 \times n_2} : M_{i,j} = 0 \text{ if } \bar{M}_{ij} = 0\}$. ∘

Next, we introduce several graph-theoretical and algebraic definitions required for defining the conditions for structural state and input observability of switched LTI systems with unknown inputs.

A *directed graph* associated with any structural system matrix $\bar{M}$ is constructed in the following manner. A directed graph is written as $\mathcal{G}(\bar{M}) = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ denotes the set of vertices (or nodes) such that $\mathcal{V} = \mathcal{M}_x$, and $\mathcal{E}$ denotes the (directed) edges between the vertices in the graph such that $\mathcal{E} = \mathcal{E}_{\mathcal{M}_x,\mathcal{M}_x} = \{(m_j, m_i) : \bar{M}_{i,j} \neq 0\}$. For a specific time t' such that $\sigma(t') = k$, we associate the system in (1a) and (1b) with a system digraph $\mathcal{G} \equiv \mathcal{G}(\bar{A}_k, \bar{F}_k, \bar{Q}_k, \bar{\mathbb{I}}_n^{\mathcal{J}_x}, \bar{\mathbb{I}}_p^{\mathcal{J}_d}) = (\mathcal{V}, \mathcal{E}^k)$, where $\mathcal{V} = \mathcal{X} \cup \mathcal{D} \cup \mathcal{Y}$, $\mathcal{X} = \{x_1, \dots, x_n\}$, $\mathcal{D} = \{d_1, \dots, d_p\}$, and $\mathcal{Y} = \{y_1, \dots, y_n\}$ are the state, unknown input, and output vertices, respectively. Furthermore, we have that $\mathcal{E}^k = \mathcal{E}^k_{\mathcal{X},\mathcal{X}} \cup \mathcal{E}^k_{\mathcal{D},\mathcal{X}} \cup \mathcal{E}^k_{\mathcal{D},\mathcal{D}} \cup \mathcal{E}_{\mathcal{X},\mathcal{Y}} \cup \mathcal{E}_{\mathcal{D},\mathcal{Y}}$, where $\mathcal{E}^k_{\mathcal{X},\mathcal{X}} = \{(x_j, x_i) : \bar{A}_k(i,j) \neq 0\}$, $\mathcal{E}^k_{\mathcal{D},\mathcal{X}} = \{(d_j, x_i) : \bar{F}_k(i,j) \neq 0\}$, $\mathcal{E}^k_{\mathcal{D},\mathcal{D}} = \{(d_j, d_i) : \bar{Q}_k(i,j) \neq 0\}$, $\mathcal{E}_{\mathcal{X},\mathcal{Y}} = \{(x_j, y_i) : \bar{\mathbb{I}}_n^{\mathcal{J}_x}(i,j) \neq 0\}$, and $\mathcal{E}_{\mathcal{D},\mathcal{Y}} = \{(d_j, y_i) : \bar{\mathbb{I}}_p^{\mathcal{J}_d}(i,j) \neq 0\}$ are the state, input, and output edges, respectively.

Next, we introduce a mathematical operator, which plays a key role in presenting the conditions for structural state and input observability of switched LTI systems with unknown inputs.

Definition 6 (*Union of Structural Matrices*)**.** The mathematical operator $\vee$ is an entry-wise operation such that a structural matrix $\bar{A} = \bigvee_{k=1}^m \bar{A}_k = \bar{A}_1 \vee \bar{A}_2 \vee \cdots \vee \bar{A}_m$ has a non-zero entry at (i,j) if at least one of the matrices $\bar{A}_k$ has a non-zero entry in that same location (i,j), and $\bar{A}(i,j) = 0$, otherwise. ∘

With this definition, we introduce the directed graphs $\mathcal{G}\left(\bigvee_{k=1}^m \bar{A}'_k\right)$ and $\mathcal{G}\left(\bigvee_{k=1}^m \bar{A}'_k, \bar{C}'\right)$. More specifically, $\mathcal{G}\left(\bigvee_{k=1}^m \bar{A}'_k\right) = (\mathcal{X}', \mathcal{E}_{\mathcal{X}',\mathcal{X}'})$ where $\mathcal{E}_{\mathcal{X}',\mathcal{X}'} = \{(x'_j, x'_i) : \bigvee_{k=1}^m \bar{A}'_k(i,j) \neq 0\}$. In addition, $\mathcal{G}\left(\bigvee_{k=1}^m \bar{A}'_k, \bar{C}'\right) = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \mathcal{X}' \cup \mathcal{Y}'$ and $\mathcal{E} = \mathcal{E}_{\mathcal{X}',\mathcal{X}'} \cup \mathcal{E}_{\mathcal{X}',\mathcal{Y}'}$ such that $\mathcal{E}_{\mathcal{X}',\mathcal{X}'} = \{(x'_j, x'_i) : \bigvee_{k=1}^m \bar{A}'_k(i,j) \neq 0\}$ and $\mathcal{E}_{\mathcal{X}',\mathcal{Y}'} = \{(x'_j, y'_i) : C'_{i,j} \neq 0\}$. We next introduce the necessary and sufficient conditions for structural state and input observability for continuous-time switched LTI systems with unknown inputs.

Theorem 7 (*Necessary and Sufficient Conditions for Structural State and Input Observability*)**.** *A continuous-time switched LTI system with unknown inputs in (1a), (1b), and (1c′) is structurally state and input observable* *if and only if* *the next two conditions hold:*

(i) $\mathcal{G}\left(\bigvee_{k=1}^m \bar{A}'_k, \bar{C}'\right)$ *has all state vertices that access at least one output vertex;*
(ii) *g-rank* $\left([\bar{A}'_1; \dots; \bar{A}'_m; \bar{C}']\right) = n + p$,

where, for $k \in \mathbb{M}$ *and the matrices* $\bar{A}'_k$ *and* $\bar{C}'$ *are defined as* $\bar{A}'_k = \begin{bmatrix} \bar{Q}_k & \mathbf{0} \\ \bar{F}_k & \bar{A}_k \end{bmatrix}$ *and* $\bar{C}' = [\,\bar{D}\ \bar{C}\,]$. ∘

Remark 8. Consider a switching signal that ensures the structural observability of the switched linear continuous-time systems. The order of transitions of system modes does not influence its structural observability. This property comes from the fact that:

- the "$\vee$" operation, in condition (i) Theorem 7, is commutative;
- a permutation of the matrices, in condition (ii) Theorem 7, yields the same g-rank. ⋄

Next, we define a few other important graph-theoretic concepts. A *bipartite graph* denoted as $\mathcal{B}$ associates a matrix M of dimension $n_1 \times n_2$ to two vertex sets $\mathcal{V}_r = \{1, \dots, n_1\}$ and $\mathcal{V}_c = \{1, \dots, n_2\}$, which are the set of row and column vertices, respectively. The connections in the matrix M relate to the connections between vertex sets $\mathcal{V}_r$ and $\mathcal{V}_c$ by an edge set $\mathcal{E}_{\mathcal{V}_c,\mathcal{V}_r} = \{(v_{c_j}, v_{r_i}) : M_{ij} \neq 0\}$ thereby allowing the bipartite graph of matrix $\bar{M}$ to be written as $\mathcal{B}(\mathcal{V}_c, \mathcal{V}_r, \mathcal{E}_{\mathcal{V}_c,\mathcal{V}_r})$. A *matching* is a collection of edges where the beginning vertex is different from the ending vertex for all edges in the set and there are no two edges in the set that have any of the same vertices. A *maximum matching* is the matching that has the maximum number of edges among all possible matchings. A *weighted bipartite graph* of a matrix M, denoted as $\mathcal{B}((\mathcal{V}_c, \mathcal{V}_r, \mathcal{E}_{\mathcal{V}_c,\mathcal{V}_r}), w)$, has weights $w : \mathcal{E}_{\mathcal{V}_c,\mathcal{V}_r} \to \mathbb{R}$ associated with the edges in the bipartite graph. Finding the maximum matching such that the sum of the weights is minimized in the weighted bipartite graph is called the *minimum weight maximum matching* (MWMM).

Now, we must introduce the notions of a strongly connected component and non-accessible states. Let $\mathbb{Z}_{\geq 0}$ denote the set of non-negative integers. First, we define a *path* of size $l \in \mathbb{Z}_{\geq 0}$ as a sequence of vertices, $p_s = (v_1, v_2, \dots, v_l)$, where the vertices do not repeat, $v_i \neq v_j$ for $i \neq j$, and (v_i, v_{i+1}) is an edge of the directed graph for $i = 1, \dots, l-1$. A *subgraph* denoted by $\mathcal{G}(\mathcal{V}', \mathcal{E}')$ is a subset of vertices $\mathcal{V}' \subset \mathcal{V}$ and its corresponding edges $\mathcal{E}' \subset \mathcal{E}$ of a particular graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. A *connected component* is any subgraph with paths that connect any two vertices in the subgraph. A connected component is said to be a *strongly connected component* (SCC) if the subgraph is maximal meaning there is no other subgraph that contains the maximal subgraph. A *sink SCC* is a strongly connected component that is connected to an output vertex. A *source SCC* is a strongly connected component that is connected to an input vertex. A *target-SCC* is a strongly connected component that does not have any outgoing edges.

We note that every digraph can be represented as a directed acyclic graph (DAG), where each node in the DAG represents an SCC in the digraph. Finally, a *non-accessible state* is one that does not have a path to an output vertex (either measuring a state or input).

We present graph-theoretic conditions for structural state and input observability of continuous-time switched LTI systems with unknown inputs.

Corollary 9. *A switched LTI continuous-time system (1) is structurally observable if and only if the next two conditions hold:*

(i) there exists an edge from one state variable of each target-SCC of $\mathcal{G}\left(\bigvee_{k=1}^{m}\bar{A}'_k\right)$ to an output variable of $\mathcal{G}\left(\bigvee_{k=1}^{m}\bar{A}'_k, \bar{C}'\right)$;
(ii) $\mathcal{B}\left([\bar{A}'_1; \ldots; \bar{A}'_m; \bar{C}']\right)$ has a maximum matching of size $n+p$;

where, for $k \in \mathbb{M}$, the matrices $\bar{A}'_k$ and $\bar{C}'$ are defined as $\bar{A}'_k = \begin{bmatrix} \bar{Q}_k & \mathbf{0} \\ \bar{F}_k & \bar{A}_k \end{bmatrix}$ and $\bar{C}' = \begin{bmatrix} \bar{D} & \bar{C} \end{bmatrix}$. ○

In the following remark, we outline the computational complexity in which we can verify the conditions of Corollary 9.

Remark 10. We can verify the two conditions in Corollary 9 in $O((m(n+p))^2)$, where n is the number of state variables, p is the number of unknown inputs, and m is the number of modes (Section 3.3, Liu, Lin, and Chen (2013)). We notice that the number of variables required to be measured is always less than or equal to $n+p$. ◇

With the graph-theoretic conditions for structural state and input observability enumerated, we introduce Algorithm 1. Briefly, the algorithm finds the minimum set of state and input variables to ensure that the conditions of Corollary 9 are satisfied. First, the algorithm finds the maximum collection of variables that satisfy the condition of Corollary 9 by constructing the MWMM of $\mathcal{B}([\bar{A}'_1; \ldots; \bar{A}'_m; \bar{T}])$, where $\bar{T}$ has as many rows as target-SCCs, and the non-zero column entries of $\bar{T}$ specify the indices of the augmented states that make up each target-SCC. Furthermore, weights are considered on the edges of the bipartite graph such that all edge weights are zero unless the edges connect to a vertex established by $\bar{T}$ at which the weight is set to one. If there is an edge in the MWMM that has a weight of one, then the index of the column vertex connecting the edge is the same index of the augmented state variable that satisfies both conditions in Corollary 9. The algorithm then proceeds to find the minimum set of variables from the maximum collection that still ensure the conditions of Corollary 9.

In the next result, we show that Algorithm 1 finds the minimum set of states and inputs to ensure structural state and input observability.

Theorem 11. *Algorithm 1 is sound, i.e., it provides a solution to $\mathcal{P}_1$, and the computational complexity of Algorithm 1 is $O((m(n+p)+\alpha)^{\varsigma})$, where $\varsigma < 2.373$ is the exponent of the best known computational complexity of performing the product of two square matrices (Alman & Williams, 2021).* ○

Remark 12. The computational complexity presented in Theorem 11 might not be amenable for ensuring the sensor placement for very large systems. Nonetheless, there are some particular classes of systems for which algorithms with lower computational complexity can be devised—see Reed, Ramos, Bogdan, and Pequito (2021) for further details. ◇

Algorithm 1 Dedicated solution to $\mathcal{P}_1$

1: **Input**: A structural switched LTI system with $\mathbb{M} = \{1, \ldots, m\}$ modes described by $\{\bar{A}_1, \ldots, \bar{A}_m, \bar{F}_1, \ldots, \bar{F}_m, \bar{Q}_1, \ldots, \bar{Q}_m\}$, where $\bar{A}_k \in \{0, \star\}^{n\times n}$, $\bar{F}_k \in \{0, \star\}^{n\times p}$, $\bar{Q}_k \in \{0, \star\}^{p\times p}$, $\forall k \in \{1, \ldots, m\}$
2: **Output**: Output $\bar{C} = \bar{\mathbb{I}}_n^{\mathcal{J}_x}$ and $\bar{D} = \bar{\mathbb{I}}_p^{\mathcal{J}_d}$, where $\mathcal{J} = \mathcal{J}_x \cup \mathcal{J}_d$, $\mathcal{J}_d = \{i \in \mathcal{J} : i \leq p\}$, and $\mathcal{J}_x = \{i \in \mathcal{J} : i > p\}$
3: **Set** $\bar{A}'_k = \begin{bmatrix} \bar{Q}_k & \mathbf{0} \\ \bar{F}_k & \bar{A}_k \end{bmatrix}$
4: **Compute** the α target-SCCs of $\mathcal{G}\left(\bigvee_{k=1}^{m}\bar{A}'_k\right) = (\mathcal{X}', \mathcal{E}'_{\mathcal{X}',\mathcal{X}})$, denoted by $\{\mathcal{S}_1, \ldots, \mathcal{S}_\alpha\}$
5: **Build** the bipartite graph $\mathcal{B}([\bar{A}'_1; \ldots; \bar{A}'_m; \bar{T}]) = (\mathcal{V}_c, \mathcal{V}_r, \mathcal{E}_{\mathcal{V}_c,\mathcal{V}_r})$, where $\bar{T} \in \{0, \star\}^{(n+p)\times\alpha}$ and $\bar{T}_{i,j} = \star$ if $x'_i \in \mathcal{S}_j$ and $\bar{T}_{i,j} = 0$, otherwise. We denote the rows of matrix $\bar{A}'_k$ by $\{r_1^k, \ldots, r_{n+p}^k\}$, and the rows of $\bar{T}$ by $\{t_1, \ldots, t_\alpha\}$.
6: **Set** the weight of the edges $e \in \mathcal{E}_{\mathcal{V}_c,\mathcal{V}_r}$ to $\begin{cases} 1, & \text{if } e \in (\{t_1, \ldots, t_\alpha\} \times \mathcal{V}_c) \cap \mathcal{E}_{\mathcal{V}_c,\mathcal{V}_r} \\ 0, & \text{otherwise} \end{cases}$.
7: **Find** a MWMM $\mathcal{M}'$ of the bipartite graph constructed in Step 5, with the cost of the edges defined in Step 6.
8: **Set** the column vertices associated with $\bar{T}$ belonging to $\mathcal{M}'$, i.e., $\mathcal{J}' = \{i : (t_j, c_i) \in \mathcal{M}', j \in \{1, \ldots, \alpha\}$ and $c_i \in \mathcal{V}_c\}$ and $\mathcal{T} = \{j : (t_j, c_i) \in \mathcal{M}', j \in \{1, \ldots, \alpha\}$ and $c_i \in \mathcal{V}_c\}$
9: **Set** $\mathcal{J}'' = \{1, \ldots, n+p\} \setminus \{i \in \{1, \ldots, n+p\} : (r_j^k, c_i) \in \mathcal{M}', k \in \{1, \ldots, m\}, j \in \{1, \ldots, n+p\}\}$
10: **Set** $\mathcal{J}'''$ to contain one and only one index of a state variable from each target-SCC in $\{\mathcal{S}_s : s \in \{t_1, \ldots, t_\alpha\} \setminus \mathcal{T}\}$
11: **Set** $\mathcal{J} = \mathcal{J}' \cup \mathcal{J}'' \cup \mathcal{J}'''$
12: **Set** $\mathcal{J}_d = \{i \in \mathcal{J} : i \leq p\}$, and $\mathcal{J}_x = \{i \in \mathcal{J} : i > p\}$

4. Real-world example

In this section, we find the minimum sensor placement for a real-world example from power systems by considering the IEEE 5-bus system (Ramos et al., 2013), which has three generators and two loads. Through linearization, we can model this system as a continuous-time switched LTI system with unknown inputs by considering two modes. The union of the two modes are shown in Fig. 1. One mode is the working system, and the second mode contains a fault that disconnects generator 1 to load 1, which corresponds to the connection between x_{14} and x_{10} being eliminated. The unknown inputs d_1 and d_2 capture the unknown amount of load consumed by loads 1 and 2, respectively. Table 1 describes the states and unknown inputs of the network. The shaded rows in the table correspond to the unknown inputs. The variables/nodes that are not listed in the table but appear in the system digraph correspond to the internal variables that connect the different bus, generators, and loads. The blue nodes correspond to load 1. The orange nodes correspond to load 2. The green nodes correspond to generator 1. The red nodes correspond to generator 2. The gray nodes correspond to generator 3.

Since the system possesses nodal dynamics on all the inputs and states, we only need to perform steps 1–4 and steps 10–12 of Algorithm 1 to find the minimum set of dedicated sensors to achieve structural observability, which involves only finding the target-SCCs. We start by finding the union of the modes—see Fig. 1. Next, we augment the system and relabel it with the state $x' = [d^{\intercal}\ x^{\intercal}]^{\intercal}$—see Step 3 of Algorithm 1. With the system properly combined and augmented, we continue by finding the target-SCCs of $\mathcal{G}(\bigvee_{k=1}^{m}\bar{A}'_k)$—see Step 4 of Algorithm 1. We find that there are 3 SCCs, which are outlined in dashed polygons depicted in Fig. 1. We also find that there is 1 target-SCCs, which is outlined in a blue dashed polygon in Fig. 1, implying that $\alpha = 1$. Next, we find a single state in the target-SCCs and add its index to $\mathcal{J}$—see Step 10 of Algorithm 1. Hence, $\mathcal{J}_x = \{12\}$ or $\mathcal{J}_x = \{10\}$—see Step 12 of Algorithm 1. These solutions correspond to measuring

Table 1
States and Unknown Inputs for IEEE 5-bus system.

Description	Node
Frequency of G_1	x_1
Turbine output mechanical power of G_1	x_2
Steam valve opening position of G_1	x_3
Frequency of G_2	x_4
Turbine output mechanical power of G_2	x_5
Steam valve opening position of G_2	x_6
Frequency of G_3	x_7
Turbine output mechanical power of G_3	x_8
Steam valve opening position of G_3	x_9
Unknown uncertainty L_1	d_1
Load consumed by L_1	x_{10}
Unknown uncertainty of L_2	d_2
Load consumed by L_2	x_{12}

the load consumed from either of the two loads in the considered power grid example, which matches the physical intuition for the power grid.

5. Conclusions

In this paper, we investigated the structural state and input observability of continuous-time switched LTI systems under unknown inputs. To this end, we derived necessary and sufficient conditions for the structural state and input observability of continuous-time switched LTI systems. These conditions can be verified in polynomial-time, more precisely in $O((m(n+p))^2)$, where n is the number of state variables, p is the number of unknown inputs, and m is the number of modes. Additionally, adopting a novel feed-forward analysis, we addressed the minimum sensor placement for these systems by designing an algorithm with a computational complexity of $O((m(n+p)+\alpha)^{2.373})$, where α is the number of target strongly connected components. Finally, we applied our algorithm to find the minimum sensor placement to a real-world example in power systems to illustrate our results.

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Appendix

Proof of Theorem 7. The continuous-time switched LTI system with unknown inputs described in (1a), (1b), and (1c′) may be re-written as the following augmented continuous-time switched LTI system where the new augmented system is $x' = [d^\intercal \ x^\intercal]^\intercal$,

$$\dot{x}'(t) = \underbrace{\begin{bmatrix} Q_{\sigma(t)} & \mathbf{0} \\ F_{\sigma(t)} & A_{\sigma(t)} \end{bmatrix}}_{A'_{\sigma(t)}} x'(t) \text{ and } y'(t) = \underbrace{\begin{bmatrix} D & C \end{bmatrix}}_{C'_{\sigma(t)}} x'(t). \quad (2)$$

Moreover, let $\mathbb{M} = \{1, \dots, m\}$ be the ordered finite set of modes where the function $\sigma(t)$ is constant in each mode. Then, we have that $A'_k = \begin{bmatrix} Q_k & \mathbf{0} \\ F_k & A_k \end{bmatrix}$ and $C' = \begin{bmatrix} D & C \end{bmatrix}$., $\forall k \in \mathbb{M}$. Therefore, when the system in (1a), (1b), and (1c′) is structurally state and input observable, it is equivalent to when the system in (2) is structurally state observable. Interestingly, despite the fact that observability and controllability are not dual in general for switched LTI systems, in Theorem 4 in Meng (2006), the authors showed that switched LTI systems are dual in the case of circulatory switching (see Definition 3 in Meng (2006)). From Remark 3 in Liu et al. (2013), it readily follows that, in the context of structural switched LTI systems, the order of the switches does not play a role in attaining structural controllability. Therefore, in particular, it follows that structural controllability can be attained in the case of circulatory switching. As such, we can leverage Theorem 4 in Meng (2006) and invoke duality between structural controllability and structural (state) observability. Hence, by Theorem 3 of Pequito and Pappas (2017), the system in (2) is structurally observable whenever the conditions (*i*) and (*ii*) hold.

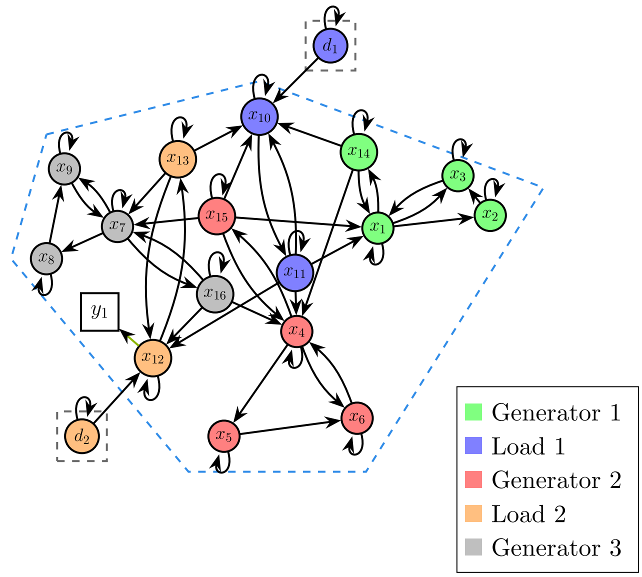

Fig. 1. This figure shows the union of the two modes of the continuous-time system with unknowns for the IEEE 5-bus system. The SCCs are outlined by two dotted black rectangles, and the target-SCCs is outlined by a dotted blue polygon. The minimum output sensor and its placement is shown by y_1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Proof of Corollary 9. First, we construct the augmented system (2) from the original system (1). Second, we need to ensure that the conditions in Theorem 7 are satisfied. When the digraph $\mathcal{G}\left(\bigvee_{k=1}^{m} \bar{A}'_k, \bar{C}'\right)$ has no non-accessible output vertices, it is equivalent to the existence of an edge from a state variable in each target-SCC $\mathcal{G}\left(\bigvee_{k=1}^{m} \bar{A}'_k\right)$ to an output vertex of $\mathcal{G}\left(\bigvee_{k=1}^{m} \bar{A}'_k, \bar{C}'\right)$. Thus, condition (*i*) is equivalent to condition (*i*) of Theorem 7. Subsequently, we recall the result from Commault, Dion, and van der Woude (2002), which states that for $\bar{M} \in \{0, \star\}^{n_1 \times n_2}$, when the g-rank$(\bar{M}) = \min\{n_1, n_2\}$, it is equivalent to when there exists a maximum matching of $\mathcal{B}(\bar{M})$ of size $\min\{n_1, n_2\}$. Hence, by the previous result, condition (ii) is equivalent to condition (ii) of Theorem 7.

Proof of Theorem 11. To address the problem $\mathcal{P}_1$, we augment the system in (1a) and (1b) to be written as in (2) where $x' = [d^\intercal\ x^\intercal]$. With this augmented system, Algorithm 1 constructs three minimum sets of dedicated outputs that combine to satisfy the two conditions outlined in Theorem 7, which guarantee structural state and input observability. Minimality of the combined sets is ensured as we use maximum matchings to build the three sets.

Subsequently, we use Algorithm 1 with the structural switched LTI system with $\mathbb{M} = \{1, \dots, m\}$ modes described by the matrices $\{\bar{A}'_1, \dots, \bar{A}'_m\}$, where the matrices $\bar{A}'_k$ are defined as $\bar{A}'_k = \begin{bmatrix} \bar{Q}_k & \mathbf{0} \\ \bar{F}_k & \bar{A}_k \end{bmatrix}, \forall k \in \mathbb{M}$.

First, we observe that $\mathcal{J}'$ comprises of a minimum set of dedicated outputs, which maximizes the g-rank$([\bar{A}'_1; \dots; \bar{A}'_m; \mathbb{I}^{\mathcal{J}'}_{(n+p)}])$, where $\mathbb{I}^{\mathcal{J}'}_{(n+p)}$ is a diagonal matrix whose entries in $\mathcal{J}'$ are nonzero. Concatenating $[\bar{A}'_1; \dots; \bar{A}'_m]$ with $\mathbb{I}^{\mathcal{J}'}_{(n+p)}$ increases the generic rank by $|\mathcal{J}'|$ and produces dedicated outputs assigned to state variables in distinct target-SCCs. In fact, $\mathcal{B}([\bar{A}'_1; \dots; \bar{A}'_m; \mathbb{I}^{\mathcal{J}'}_{(n+p)}])$ yields a MWMM $\mathcal{M}$ with weight 0 and size $|\mathcal{M}|$. Hence, by the result from Commault et al. (2002) used in the proof of Corollary 9, it follows that g-rank$([\bar{A}'_1; \dots; \bar{A}'_m; \mathbb{I}^{\mathcal{J}'}_{(n+p)}]) = |\mathcal{M}|$.

Next, a MWMM $\mathcal{M}'$ of $\mathcal{B}([\bar{A}'_1; \dots; \bar{A}'_m; \bar{T}])$ has size $|\mathcal{M}'|$. This corresponds to an increase in g-rank$([\bar{A}'_1; \dots; \bar{A}'_m; \mathbb{I}^{(\mathcal{J}'\cup\mathcal{J}'')}_{(n+p)}])$ from g-rank$([\bar{A}'_1; \dots; \bar{A}'_m; \mathbb{I}^{\mathcal{J}'}_{(n+p)}])$ of $|\mathcal{M}'| - |\mathcal{M}|$. Observe that, by the construction of the matrix $\bar{T}$, we have that $\mathbb{I}^{\mathcal{J}''}_{(n+p)}$ corresponds to dedicated outputs assigned to state variables in distinct target-SCCs. This means that $|\mathcal{J}''|$ target-SCCs will have outgoing edges to different outputs of the system digraph. This is necessary to satisfy condition (i) of Theorem 7 but may not be sufficient.

Therefore, we have to finally consider a third set, $\mathcal{J}'''$, to ensure that condition (i) is fulfilled. In other words, there might still be target-SCCs that are not accounted for by state variables indexed in $\mathcal{J}' \cup \mathcal{J}''$, which we account for in $\mathcal{J}'''$.

By minimizing the number of additional dedicated outputs $\mathbb{I}^{\mathcal{J}''}_{(n+p)}$, in step 8, we satisfy condition (ii) in Theorem 7 since g-rank$([\bar{A}'_1; \dots; \bar{A}'_m; \mathbb{I}^{(\mathcal{J}'\cup\mathcal{J}'')}_{(n+p)}]) = n + p$. Additionally, the set $\mathcal{J}'''$ of minimum extra dedicated outputs, found in step 9, ensures that there are not state vertices that do not access at least one output vertex in $\mathcal{G}\left(\bigvee_{k=1}^{m} \bar{A}'_k,\ \mathbb{I}^{\mathcal{J}}_{(n+p)}\right)$, where $\mathcal{J} = \mathcal{J}' \cup \mathcal{J}'' \cup \mathcal{J}'''$, thereby fulfilling condition (i) of Theorem 7. Notice that $\mathbb{I}^{\mathcal{J}''}_{(n+p)}$ are not assigned to previously assigned target-SCCs, as they would have been considered in $\mathbb{I}^{\mathcal{J}'}_{(n+p)}$.

Consequently, by the construction, setting $\mathcal{J} = \mathcal{J}' \cup \mathcal{J}'' \cup \mathcal{J}'''$ in step 10 yields a solution $\mathbb{I}^{\mathcal{J}}_{(n+p)}$ that is minimal, ensuring both conditions of Corollary 9. Notice that the produced solution easily translates to the original problem $\mathcal{P}_1$ solution by setting the originals $\bar{C} = \mathbb{I}^{\mathcal{J}_x}_n$ and $\bar{D} = \mathbb{I}^{\mathcal{J}_d}_p$, where $\mathcal{J}_x = \{i \in \mathcal{J} : i > p\}$ and $\mathcal{J}_d = \{i \in \mathcal{J} : i \leq p\}$.

The computational complexity of Algorithm 1 comes from the step with the highest computational cost (step 6) since the remaining steps of the algorithm have lower complexity. The computational complexity of step 6 can be solved by resorting to the Hungarian algorithm (Kuhn, 1955) that finds a MWMM of $\mathcal{B}([\bar{A}'_1; \dots; \bar{A}'_m; \bar{T}])$ in $O(\max\{|\mathcal{V}_r|, |\mathcal{V}_c|\}^{\varsigma})$, where $\mathcal{V}_r$ and $\mathcal{V}_c$ are defined in step 5, and $\varsigma < 2.373$ is the exponent of the best known computational complexity of performing the product of two square matrices. Since $|\mathcal{V}_c| \leq |\mathcal{V}_r|$, this results in a computational cost of $O(|\mathcal{V}_r|^{\varsigma}) = O((m(n + p) + \alpha)^{\varsigma})$.

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Emily A. Reed received a B.S. degree in Electrical and Computer Engineering with honors research and global engineering distinction from The Ohio State University and a M.S. in Electrical Engineering from the University of Southern, where she is currently a Ph.D. student.

Emily is interested in designing and analyzing novel control strategies, algorithms, and machine learning tools to better understand, predict, and control complex dynamical networks.

Emily has received several fellowships including the National Science Foundation Graduate Research Fellowship, the National Defense Science and Engineering Graduate Fellowship, the USC Annenberg merit fellowship, one of USC's most prestigious fellowships, and the Qualcomm USC Women in Science and Engineering merit fellowship. Emily was nominated as a Best Student Paper Finalist at the 42nd Annual International Virtual Conferences of the IEEE Engineering in Medicine and Biology Society in conjunction with the 43rd Annual Conference of the Canadian Medical and Biological Engineering in July 2020.

Guilherme Ramos received a B.Sc. (2011) and M.Sc. (2013) in applied mathematics, and a Ph.D. in information security (2018) from the Instituto Superior Técnico, Lisbon, Portugal. From 2018 to 2020, he was a postdoctoral researcher with the Institute for Systems and Robotics at IST (ISR/IST). From 2020 to 2022 he was a postdoctoral researcher at the Department of Electrical and Computer Engineering (DEEC), Faculty of Engineering, University of Porto (FEUP).

In the 2nd semester of 2022, he was an invited assistant professor at the Department of Computer Science, Faculdade de Ciências, Universidade de Lisboa (FCUL), Lisbon and an integrated researcher at LASIGE (FCUL). Currently, he is an assistant professor at the Department of Computer Science, Instituto Superior Técnico, Universidade de Lisboa (IST).

His areas of interest include structural systems, control systems, security, information theory, and ranking and recommendation systems.

Paul Bogdan (Senior Member, IEEE) received the Ph.D. degree from Carnegie Mellon University. He is a Jack Munushian Early Career Chair associate professor with the Ming Hsieh Department of Electrical and Computer Engineering, University of Southern California. His research interests include the cyber–physical systems, control of complex time-varying networks, modeling and analysis of biological systems and swarms, new control algorithms for dynamical systems exhibiting multifractal characteristics, modeling biological or molecular communication, fractal mean field games, machine learning, artificial intelligence, performance analysis and design methodologies for manycore systems. His work has been recognized with a number of awards and distinctions, including the 2019 Defense Advanced Research Projects Agency (DARPA) Directors Fellowship, the 2018 IEEE CEDA Ernest S. Kuh Early Career Award, 2017 DARPA Young Faculty Award, 2017 Okawa Foundation Award, 2015 National Science Foundation CAREER Award, 2012 A.G. Jordan Award from Carnegie Mellon University for an outstanding Ph.D. thesis and service, several best paper awards, including the 2013 Best Paper Award from the 18th Asia and South Pacific Design Automation Conference, 2012 Best Paper Award from the Networks-on-Chip Symposium, 2012 D.O. Pederson Best Paper Award from IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, 2012 Best Paper Award from the International Conference on Hardware/Software Codesign and System Synthesis, and the 2009 Roberto Rocca Ph.D. Fellowship.

Sérgio Pequito received a B.Sc. and M.Sc. in applied mathematics from Instituto Superior Técnico (IST) at University of Lisbon, in 2007 and 2009, respectively. Afterwards, he obtained a dual Ph.D. from IST and Carnegie Mellon University, in electrical and computer engineering, with specialization in control theory. He is an associate professor of automatic control in the Department of Information Technology, Uppsala University. Pequito's research consists of understanding the global qualitative behavior of large-scale systems from their structural or parametric descriptions and provides a rigorous framework for the design, analysis, optimization, and control of large scale systems. Currently, his interests span to neuroscience and biomedicine, where dynamical systems and control theoretic tools can be leveraged to develop new analysis tools for brain dynamics towards effective personalized medicine and improve brain–computer and brain–machine–brain interfaces. Pequito was awarded the best student paper finalist in the 48th IEEE Conference on Decision and Control (2009) and the 2016 O. Hugo Schuck Award in the Theory Category by the American Automatic Control.