Collaborative Beamforming for Agents with Localization Errors

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***Abstract*—We consider a group of agents that estimate their locations in an environment through sensor measurements and aim to transmit a message signal to a client via collaborative beamforming. Assuming that the localization error of each agent follows a Gaussian distribution, we study the problem of forming a reliable communication link between the agents and the client that achieves a desired signal-to-noise ratio (SNR) at the client with minimum variability. In particular, we develop a greedy subset selection algorithm that chooses only a subset of the agents to transmit the signal so that the variance of the received SNR is minimized while the expected SNR exceeds a desired threshold. We show the optimality of the proposed algorithm when the agents' localization errors satisfy certain sufficient conditions that are characterized in terms of the carrier frequency.**

I. Introduction

Collaborative beamforming is a communication technique in which a group of agents, e.g., mobile robots each of which is equipped with an antenna, transmit a common message signal such that the transmitted signals superpose coherently at the client, i.e., intended receiver [1]–[3]. In many scenarios the agents are distributed in an environment, and they estimate their positions using sensor measurements. For example, the agents may be autonomous ground vehicles carrying out a task in the environment and their localization algorithm may result in an error associated with position [4]–[6]. The localization error translates to a phasing error as the agents form a beam, which results in an imperfect superposition of the transmitted signals at the client. In such scenarios, it is important to design beamforming algorithms that exploit the statistical knowledge of the localization errors and satisfy certain quality-of-service (QoS) requirements at the client. Assuming that the localization error of each agent follows a Gaussian distribution, this paper concerns the development of a simple greedy algorithm which minimizes the variance of the signal-to-noise ratio (SNR) received by the client subject to the constraint that the expected SNR exceeds a desired threshold.

Traditionally, the design problem associated with collaborative beamforming is to choose a complex scalar gain for each agent to multiply the transmitted signal so that the resulting beampattern satisfies certain properties [7]. Ideally, with no localization error, the geometry may be used to derive the optimal beamformer, and we refer to this as the perfect channel state information (CSI) case. When no CSI is available, an iterative algorithm that updates the agents' beamforming gains according to a feedback from the client can be employed [8]. In the case of imperfect CSI, a number of algorithms based on semi-definite programs (SDPs) have been proposed to ensure that the SNR received by the client is above a threshold with desired probability [9]–[11]. Similar conic optimization-based formulations are common in the robust beamforming literature [12]–[14]. While SDP formulations provide a powerful technique to improve the QoS at the client despite imperfect CSI, they are computationally expensive and do not scale well with the number of agents.

In this paper, we approach the beampattern design problem from a discrete optimization perspective and develop an algorithm to choose a subset of agents to transmit the message signal to the client. Given a group of agents with associated localization errors, we seek a subset of agents to form a beam which achieves the desired QoS requirements without requiring feedback from the client. To the best of our knowledge, this paper is the first one to employ discrete optimization techniques for mitigating the effects of localization errors in collaborative beamforming. In the proposed method, we first fix the beamforming gain of each agent such that, for a given subset of agents, the resulting beampattern maximizes the expected SNR at the client. We then develop a sorting-based greedy algorithm to choose a subset of agents that achieves an SNR with minimum variance among the ones whose expected SNR exceeds a desired threshold. We show that the greedy algorithm globally minimizes the variance of the SNR at the client when the agents' localization errors are below a certain threshold characterized in terms of the carrier frequency. In numerical simulations, we illustrate that the greedy algorithm synthesizes beamformers orders of magnitude faster than SDP-based approaches and employs less number of agents to satisfy the desired QoS requirements at the client.

II. System Model

We consider a group of N agents each of which is equipped with a single ideal isotropic antenna with a constant transmit power P. The agents' goal is to transmit a common message signal $m(t)\in\mathbb{R}$ to a stationary client carrying a single antenna.

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A. Communication Channel

We assume that (i) the transmitted message $m(t)$ propagates in free space with no reflection or scattering, (ii) the client is located in the far-field region, and (iii) there is no mutual coupling between the agents' antennas. We note that, in addition to free space propagation, these assumptions may also hold at longer wavelengths even when propagating through a complex environment. As experimentally validated in [15], significant signal penetration through obstacles is possible at low-VHF frequencies. Moreover, in urban-type scenarios, communication at low-VHF band yields improved penetration and reduced multipath as experimentally validated in [16].

The communication between the agents and the client takes place over a narrowband wireless channel which is represented by a complex scalar gain $h_i{:=}a_i e^{j\eta_i}$ where a_i is the *known* channel gain and η_i is the *unknown* channel phase due to the relative positions of the agents and the client. Finally, we assume that the local oscillators of all agents are time- and frequency-synchronized. Frequency synchronization may be achieved with a separate short-range radio protocol [17], [18], and although we do not consider it further in this paper, timing error may result in beamformer phasing error that could be folded into our approach.

B. Collaborative Transmission Model

A subset $\mathcal{S}{\subseteq}[N]$ of agents collectively form a distributed array to transmit the message signal $m(t)$ to the stationary client. Each agent $i{\in}\mathcal{S}$ transmits the signal $s_i(t){:=}\sqrt{P}e^{j\delta_i}m(t)$ where $\sqrt{P}$ is the amplitude of the transmission, and δ_i is a phase adjustment performed by the agent. Then, the signal received by the client is

$$r(t|\mathcal{S}) := \sqrt{P}\sum_{i\in\mathcal{S}} a_i e^{j(\delta_i+\eta_i)}m(t) + n(t) \tag{1}$$

where $n(t)$ is the additive white Gaussian noise. Assuming that the local oscillators are time-synchronized, the phase offset η_i of the signal $m(t)$ at the location of the client relative to a signal transmitted by an agent located at $\vec{r}_i{\in}\mathbb{R}^3$ (in Cartesian coordinates) is [19]

$$\eta_i := \frac{2\pi f_c}{C}\langle \vec{r}_i, \vec{r}_c\rangle. \tag{2}$$

In (2), $\vec{r}_c{\in}\mathbb{R}^3$ is the unit vector pointing in the *known* direction of the client, f_c is the carrier frequency, C is the speed of light, and $\langle\cdot,\cdot\rangle$ is the inner product of two vectors.

In this paper, we assume that the agents' local positions $\{\vec{r}_i : i \in [N]\}$ are not exactly known. In particular, for a given $i{\in}[N]$, we assume that $\vec{r}_i{\sim}\mathcal{N}(\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$ where $\boldsymbol{\mu}_i{\in}\mathbb{R}^3$ and $\boldsymbol{\Sigma}_i{\in}\mathbb{R}^{3\times 3}$ are, respectively, the known mean and the known covariance of the Gaussian distribution. This assumption is reasonable in practice because the first and second order statistics of robotic pose estimates are typically easy to obtain, e.g., using LIDAR scans. We also assume that $\vec{r}_i$ and $\vec{r}_j$ are independent for $i,j{\in}[N]$ such that $i{\neq}j$.

For a given subset $\mathcal{S}{\subseteq}[N]$ and phase adjustment parameters δ_i for all $i{\in}[N]$, the *array factor* [3] is given by

$$F(\mathcal{S},\delta) := \left|\sum_{i\in\mathcal{S}} e^{j(\delta_i+\eta_i)}\right| \tag{3}$$

where $\delta{=}[\delta_i|i{\in}\mathcal{S}]$ is the vector of phase adjustments. Under the assumption that $a_i{\approx}a_j$ for all $i,j{\in}[N]$, the magnitude of the array factor is proportional to the square root of the SNR received by the client [8]. We note that, for free space propagation, it holds that $a_i{=}a_j$ for all $i,j{\in}[N]$ when the distances between the agents and the client are equal to each other. Let the *total phase* be $\Phi_i{:=}\delta_i+\eta_i$. The square of the the array factor yields the *beamforming gain* $G(\mathcal{S},\delta)$ [3] that is proportional to the received SNR and given by

$$G(\mathcal{S},\delta) := \sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{S}}\cos\Big(\Phi_i-\Phi_j\Big). \tag{4}$$

The beamforming gain (4) is a fundamental quantifier of the quality of a communication link, which we seek to optimize as a function of the set of agents that transmit the message signal.

III. Problem Statement

Since SNR is proportional to the beamforming gain (4), when the relative phase offsets η_i are known, the agents may form an optimal communication link with the client by maximizing the beamforming gain, i.e., selecting a pair $(\mathcal{S}^\star,\delta^\star)$ such that

$$(\mathcal{S}^\star,\delta^\star) \in \underset{\substack{\mathcal{S}\subseteq[N],\\ \delta\in[0,2\pi)^N}}{\operatorname{argmax}}\ G(\mathcal{S},\delta). \tag{5}$$

The beamforming gain can be maximized by choosing $\mathcal{S}^\star{=}[N]$ and setting $\delta_i^\star{=}{-}\eta_i$ for all $i{\in}\mathcal{S}$. To see this, recall that the total phase $\Phi_i{=}\delta_i+\eta_i$, and note that

$$G(\mathcal{S},\delta) = \sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{S}}\cos\Big(\Phi_i-\Phi_j\Big) \le N^2. \tag{6}$$

The upper bound in (6) is attained if and only if $\mathcal{S}{=}[N]$ and $\Phi_i{=}\Phi_j$ for all $i,j{\in}\mathcal{S}$, *i.e.*, when the total phases are aligned.

In this paper, we focus on a scenario where the exact value of the relative phase offsets η_i are not known due to the agents' localization errors. We are interested in finding a subset of agents that, with high probability, forms a reliable communication link with the client through beamforming. The formal problem statement is as follows.

Problem 1: (Subset selection) For a constant $\Gamma{\in}\mathbb{R}$, and the fixed phase adjustment parameters $\delta{=}\widetilde{\delta}$ where, for all $i{\in}[N]$, $\widetilde{\delta}_i{:=}{-}\mathbb{E}[\eta_i]$, find a subset $\mathcal{S}^\star$ such that

$$\mathcal{S}^\star \in \underset{\mathcal{S}\subseteq[N]}{\operatorname{argmin}} \quad \operatorname{Var}\Big(G(\mathcal{S},\widetilde{\delta})\Big) \tag{7a}$$

$$\text{subject to:} \quad \mathbb{E}\Big[G(\mathcal{S},\widetilde{\delta})\Big] \ge \Gamma. \tag{7b}$$

Note that the vector $\delta{=}[\delta_i|i{\in}\mathcal{S}]$ of phase adjustment parameters is not a design variable in the subset selection problem.

The rationale behind the choice of δ is that for any given subset $\mathcal{S}\subseteq[N]$, $\mathbb{E}[G(\mathcal{S},\delta)]$ is maximized by choosing $\delta=\widetilde{\delta}$. We also remark that each agent $i\in[N]$ needs only its own local position information, i.e., distribution of η_i, to set $\delta=\widetilde{\delta}$; hence, each agent can individually adjust its beamforming phase. Finally, since δ is not a variable in the considered problem, for simplicity, we define $\overline{G}(\mathcal{S}):=G(\mathcal{S},\widetilde{\delta})$.

Remark: One can argue that, when $G(\mathcal{S},\delta)$ is a random variable, a reasonable objective may be to maximize the expected beamforming gain, i.e., selecting a pair $(\overline{\mathcal{S}},\overline{\delta})$ such that

$$(\overline{\mathcal{S}},\overline{\delta}) \in \underset{\substack{\mathcal{S}\subseteq[N],\\ \delta\in[0,2\pi)^N}}{\operatorname{argmax}}\ \mathbb{E}\Big[G(\mathcal{S},\delta)\Big]. \tag{8}$$

It can be shown that $\mathbb{E}[G(\mathcal{S},\delta)]$ is maximized by choosing $\overline{\mathcal{S}}=[N]$ and $\overline{\delta}=\widetilde{\delta}$. Although including all the agents in the beamforming maximizes the *expected* gain, due to the random phase errors, this approach may actually decrease the probability that the beamforming gain exceeds a certain threshold, which in turn reduces the reliability of the communication link. In the subset selection problem, considering the variance of the beamforming gain as a risk measure, we aim to find a subset of agents that achieves a desired level of gain while increasing the reliability of the link. Such a formulation is widely used in risk-sensitive optimization models [20], [21].

IV. Statistical Properties of the Beamforming Gain

In this section, we derive the explicit forms of the expected value $\mathbb{E}[\overline{G}(\mathcal{S})]$ and the variance $\mathrm{Var}(\overline{G}(\mathcal{S}))$ of the beamforming gain $\overline{G}(\mathcal{S})$. Recall that, in the subset selection problem, we fix the phase adjustment parameters δ_i by setting $\delta_i=-\mathbb{E}[\eta_i]$. Hence, we have $\Phi_i=\eta_i-\mathbb{E}[\eta_i]$. Recall also that $\eta_i=2\pi f_c\ \langle\vec{r}_i,\vec{r}_c\rangle/C$ and $\vec{r}_i\sim\mathcal{N}(\boldsymbol{\mu}_i,\boldsymbol{\Sigma}_i)$. Consequently, we have $\Phi_i\sim\mathcal{N}(0,\gamma_i)$ where

$$\gamma_i := \frac{4\pi^2 f_c^2}{C^2}\Big\langle \vec{r}_c, \boldsymbol{\Sigma}_i\vec{r}_c\Big\rangle. \tag{9}$$

We refer to γ_i as *the effective error variance* in the localization of the ith agent. Note that γ_i can be also interpreted as the variance of the phase for a Gaussian distributed CSI.

Lemma 1: Let $v_i:=\exp(-\gamma_i)$. The expected value and the variance of the beamforming gain $\overline{G}(\mathcal{S})$ are, respectively,

$$\mathbb{E}\Big[\overline{G}(\mathcal{S})\Big] = |\mathcal{S}| + \sum_{i\in\mathcal{S}}\sum_{\substack{j\in\mathcal{S}\\ j\neq i}}\sqrt{v_i v_j}, \tag{10}$$

$$\begin{aligned}\mathrm{Var}\Big(\overline{G}(\mathcal{S})\Big) &= \sum_{i\in\mathcal{S}}\sum_{\substack{j\in\mathcal{S}\\ j\neq i}}\Big(1-v_i v_j\Big)^2 \\ &\quad + 2\sum_{i\in\mathcal{S}}\sum_{\substack{j\in\mathcal{S}\\ j\neq i}}\sum_{\substack{k\in\mathcal{S}\\ k\neq i\\ k\neq j}}\Big(1-v_i\Big)^2\sqrt{v_j v_k}.\end{aligned} \tag{11}$$

We provide a proof for the above result in [22, Lemma 1]. The proof exploits the equivalence $\mathbb{E}[\exp(tX)]=\exp(t\mu-\sigma^2t^2/2)$ where $X\sim\mathcal{N}(\mu,\sigma)$ and the independence of $\vec{r}_i$ and $\vec{r}_j$ for $i\neq j$ to obtain the explicit forms.

V. Agent Selection Under Localization Errors

In this section, we propose a greedy algorithm to solve the subset selection problem and provide sufficient conditions for its optimality. We assume that the problem in (7a)-(7b) has a feasible solution. For a given instance, the validity of this assumption can be easily verified by checking whether $\mathbb{E}[\overline{G}([N])]\geq\Gamma$ due to the following result.

Proposition 1: For any $\mathcal{S}\subseteq\mathcal{S}'\subseteq[N]$, we have $\mathbb{E}[\overline{G}(\mathcal{S})]\leq\mathbb{E}[\overline{G}(\mathcal{S}')]$.

The above result follows immediately from the fact that $\mathbb{E}[\overline{G}(\mathcal{S})]$ is a sum of nonnegative terms; hence, adding an element to the subset can only increase the sum.

The greedy algorithm, shown in Algorithm 1, first sorts the agents' effective error variances γ_i, defined in (9), in ascending order. Initializing the output set $\mathcal{S}$ to the empty set, it then iteratively adds the agent with the next lowest effective error to the output set until the constraint $\mathbb{E}[\overline{G}(\mathcal{S})]\geq\Gamma$ is satisfied. The sorting operation can be performed in $\mathcal{O}(N\log(N))$ for an array of length N [23].

Algorithm 1 Greedy subset selection

Input: γ_i for all $i\in[N]$, $\Gamma\in\mathbb{R}$.
Sort γ_i such that $\gamma_{i_1}\leq\gamma_{i_2}\leq\ldots\leq\gamma_{i_N}$.
$\mathcal{S}:=\emptyset$, $k:=1$
while $\mathbb{E}[\overline{G}(\mathcal{S})]<\Gamma$ **do**
    $\mathcal{S}:=\mathcal{S}\cup\{i_k\}$, $k:=k+1$
return $\mathcal{S}$.

We now present the main result of this paper, sufficient conditions on the set $\{\gamma_i : i\in[N]\}$ for which the greedy algorithm returns an optimal solution to the problem in (7a)-(7b). Let *the total effective error variance of a subset* $\mathcal{S}\subseteq[N]$ be measured by the function $V:2^{[N]}\to\mathbb{R}$ where $V(\mathcal{S}):=\sum_{i\in\mathcal{S}}\gamma_i$. Consider the problem of choosing a subset $\mathcal{S}'\subseteq[N]$ that satisfies the constraint in (7b) and has the minimum total effective error variance, i.e.,

$$\mathcal{S}' \in \arg\min_{\mathcal{S}\subseteq[N]} \quad V(\mathcal{S}) \tag{12a}$$

$$\text{subject to:} \quad \mathbb{E}\Big[\overline{G}(\mathcal{S})\Big] \geq \Gamma. \tag{12b}$$

The next result, together with Proposition 1, implies that the greedy algorithm yields an optimal solution to the problem in (12a)-(12b).

Proposition 2: For any $K\in\mathbb{N}$ such that $K\leq N$, we have

$$\arg\min_{\substack{\mathcal{S}\subseteq[N]:\\ |\mathcal{S}|=K}} V(\mathcal{S}) = \arg\max_{\substack{\mathcal{S}\subseteq[N]:\\ |\mathcal{S}|=K}} \mathbb{E}\Big[\overline{G}(\mathcal{S})\Big].$$

The above result follows from the fact that the derivative of the expected beamforming gain $\mathbb{E}[\overline{G}(\mathcal{S})]$ with respect to γ_i, where $i\in\mathcal{S}$, is always negative. It can be shown that the optimization problems in (7a)-(7b) and (12a)-(12b) are not equivalent in general. Hence, including the agents with

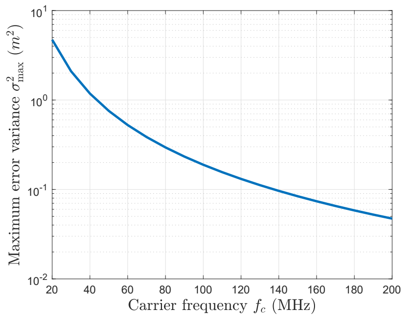

Fig. 1: Maximum localization error variance $\sigma^2_{\max}$ allowed for the optimality of the greedy algorithm as a function of the carrier frequency f_c. Note that the localization error tolerance is relaxed at lower frequencies (longer wavelengths).

minimum total effective error variance in beamforming is, in general, not optimal to solve the subset selection problem. However, there are certain sufficient conditions, which are formalized below, under which such a strategy becomes optimal.

Theorem 1: For a given set $\{\gamma_i : i\in[N]\}$ of effective error variances, let $\gamma_{i_1}\leq\gamma_{i_2}\leq...\leq\gamma_{i_N}$ where $i_k\in[N]$. A solution to the problem in (12a)-(12b) is also a solution to the problem in (7a)-(7b) if either one of the following conditions hold:

(C1) $\mathbb{E}[\overline{G}(\mathcal{S})]\geq\Gamma$ where $\mathcal{S}=\{i_1, i_2\}$,

(C2) $\gamma_{i_N}\leq 0.83$.

We provide a proof for the above result in [22, Theorem 1]. The main idea in the proof is to show that the derivative of $\mathrm{Var}(\overline{G}(\mathcal{S}))$ with respect to $\max_{i\in\mathcal{S}}\gamma_i$ is positive. Condition **(C1)** follows from the fact that, when $|\mathcal{S}|\leq 2$, the derivative is always positive. Condition **(C2)** follows from the fact that, when $\gamma_{i_N}\leq 0.83$, the derivative is positive regardless of the size of the set $\mathcal{S}$. For such γ_{i_N}, the subset with minimum total effective error variance is the one that minimizes the variance of the beamforming gain; hence, the problems in (12a)-(12b) and (7a)-(7b) become equivalent when **(C2)** holds.

Theorem 1 states that if all the agents have "small" effective error variances, then the greedy algorithm returns an optimal solution to the subset selection problem. In particular, it follows from Theorem 1 that a sufficient condition for optimality characterized by the carrier frequency is

$$\max_{i\in[N]} \left\langle \vec{r}_c, \mathbf{\Sigma}_i \vec{r}_c \right\rangle \leq \frac{0.83C^2}{4\pi^2 f_c^2}.$$

For example, suppose that $\Sigma_i=\sigma_i^2 I_{3\times 3}$, where $I_{3\times 3}$ is the identity matrix, and let $\sigma^2_{\max}:=\max_i \sigma_i^2$. Then, we have $\sigma^2_{\max}\leq\frac{0.83C^2}{4\pi^2 f_c^2}$ as the sufficient condition **(C2)**. In Figure 1, we graphically illustrate the trade-off between the carrier frequency f_c and the maximum variance $\sigma^2_{\max}$ under which the greedy algorithm is optimal. Note that as f_c increases (resulting in shorter wavelength), condition **(C2)** requires smaller position error variance, whereas longer wavelengths increase the error tolerance. For example, at lower VHF frequencies, e.g., f_c=40 MHz for which the effective wavelength is $\lambda_c=C/f_c\approx 7.5$ meters, the agents are allowed to have localization error variance up to 1 square meter. Hence, for this frequency range, the position error tolerance can easily be achieved with existing localization algorithms [4], [24].

VI. Numerical Simulations

In this section, we present numerical simulations to demonstrate the performance of the proposed greedy algorithm. We generate 100 subset selection problem instances each of which consists of N=50 agents. We set f_c=40 MHz and $\mathbf{\Sigma}_i=\sigma_i^2 I_{3\times 3}$ for all $i\in[N]$, and generate the variances σ_i^2 uniformly randomly from the interval $\sigma_i^2\in[0, 0.8C^2/(4\pi^2 f_c^2)]$ so that the condition **(C2)** is satisfied.

We compare the performance of the beamformer syntesized by the greedy algorithm with a beamformer $\mathbf{w}^\star\in\mathbb{C}^N$ where

$$\mathbf{w}^\star \in \arg\min_{\mathbf{w}\in\mathbb{C}^N} \ \|\mathbf{w}\|_2^2 \tag{13a}$$

$$\text{subject to: } \ \mathbb{E}[\mathbf{w}^H\mathbf{H}\mathbf{w}] \geq \Gamma \tag{13b}$$

$$\forall i \in [N], \ \ |\mathbf{w}(i)|^2 \leq 1. \tag{13c}$$

The matrix $\mathbf{H}\in\mathbb{C}^{N\times N}$ is $\mathbf{H}=\mathbf{h}\mathbf{h}^H$ where $\mathbf{h}^H=[h_1, h_2, \ldots, h_N]$, and $\mathbf{w}(i)$ is the i^{th} element of $\mathbf{w}$. The constraint in (13c) ensures that $\mathbf{w}(i)=\sqrt{P}e^{j\delta_i}$ for some $P\leq 1$. One can solve the problem in (13a)-(13c) *exactly* by solving its corresponding semi-definite program (SDP) [7], [14]. Note that the beamformer $\mathbf{w}^\star$ minimizes the total power of the antenna array while ensuring that the expected beamforming gain exceeds the desired threshold Γ. Therefore, it represents a solution to a convex relaxation of the problem $\min_{\mathcal{S}\subseteq[N]}|\mathcal{S}|$ subject to $\mathbb{E}[\overline{G}(\mathcal{S})]\geq\Gamma$, which is a risk-neutral version of the subset selection problem. For given $\mathbf{w}^\star$, we let the corresponding optimal subset be $\mathcal{S}^\star=\{i\in[N] : |\mathbf{w}^\star(i)| > \epsilon\}$ where $\epsilon=10^{-2}$.

Figure 2 illustrates the performance comparison of the beamformer synthesized by the greedy algorithm, i.e., greedy beamformer, and the SDP-based beamformer $\mathbf{w}^\star$. For both beamformers, Figure 2 (left) shows the normalized variance of the beamforming gain, i.e., $\mathrm{Var}(\overline{G}(\mathcal{S}^\star))/\mathrm{Var}(\overline{G}([N]))$, versus the expected beamforming gain threshold $\Gamma=k\mathbb{E}[\overline{G}([N])]$ where $k\in[0,1]$. As can be seen from the figure, simple greedy beamformer achieves a similar performance to the complex SDP-based beamformer. Note that the variance of the SDP-based beamformer is smaller than the variance of the greedy beamformer for some values of Γ. We observe such a result since the problem in (13a)-(13c) is a convex relaxation of the subset selection problem. In general, a beamformer $\mathbf{w}^\star$ may attain a variance value that is strictly smaller than the optimal value of the subset selection problem. Figure 2 (middle) demonstrates the trade-off between the normalized beamforming gain threshold Γ and the size of the optimal subset $\mathcal{S}^\star$. As can be seen from the figure, for $k<1$, the greedy beamformer employs strict subsets of the agent group $[N]$ where N=50, whereas the SDP-based beamformer includes all the agents to the beamforming for all $k>0$. Hence, in a sense, the greedy algorithm improves the capabilities of the agent group since it allows the utilization of the agents that are not part of beamforming for other purposes in general.

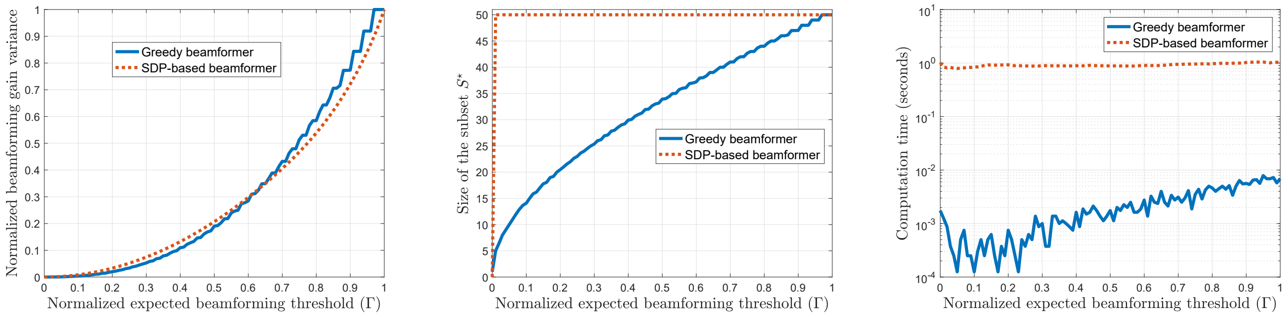

Fig. 2: Performance comparison of the greedy beamformer with an SDP-based beamformer. For a given expected gain threshold Γ, beamforming gains of both beamformers have similar variability (left). However, the greedy beamformer achieves its performance by employing strict subsets of the agent group when possible (middle) and is synthesized orders of magnitude faster than the SDP-based beamformer.

Finally, Figure 2 (right) demonstrates that one can synthesize the greedy beamformer orders of magnitude faster than the SDP-based beamformer.

VII. Conclusions

We considered a group of agents with localization errors that aim to transmit a message signal to a client via beamforming. To ensure the reliability of the communication link despite the agents' localization errors, we developed a greedy algorithm that chooses only a subset of agents to transmit the signal. We derived a bound on the maximum localization error variance allowed for the optimality of the greedy algorithm and showed that the bound becomes stricter as the carrier frequency increases. We showed that for lower VHF frequencies, e.g., around 40 MHz, if the agents' localization error variances are less than 1 square meter, then the subset returned by the greedy algorithm globally minimizes the variance of the SNR received by the client. Future research will focus on developing algorithms that either locally or globally minimizes the variance of the SNR received by the client for all carrier frequencies.

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