S-Parameter Extraction Methodology in FDTD for Nano-Scale Optical Interconnects

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***Abstract*—We propose a methodology for extracting scattering parameters of optical interconnects comprised of dielectric slab waveguides. The proposed methodology is demonstrated by calculating the TE mode attenuation and phase delay of an example dielectric slab in 2D FDTD. Results show good correlation between the proposed FDTD-based methodology and analytic solutions. Work is currently under way to extend the proposed methodology to dielectric waveguides exhibiting stochastic surface roughness.**

***Index Terms*—Attenuation, Dielectric slab, FDTD, Optical Interconnects, Phase delay, S-parameters, Waveguides.**

I. Introduction

In the characterization of arbitrary structures, it is often useful to utilize the scattering parameters (S-parameters) which are nearly universal in design applications in which signal power integrity is a concern. In low frequency applications, interconnects may be approximated as lossless, but as frequency increases to the mid-infrared regime, such approximations may no longer be feasible.

Naturally, it would be useful to have a low-cost method for characterizing arbitrarily structured optical interconnects. The finite-difference time-domain (FDTD) method is a powerful technique [2] that may be employed for this purpose. Although, there exists some FDTD-based methods of S-parameter extraction for conductive rectangular waveguides based on modal decomposition [3], it would appear that few FDTD-based S-parameter extraction methods have been developed for characterization of optical interconnects.

In this paper, we propose a methodology for extracting S-parameters of optical interconnects comprised of dielectric slab waveguides. This methodology is then tested for characterizing the expected attenuation and phase delay of the dielectric slab waveguide.

II. Formulation

A. Derivation of Attenuation and Phase Coefficients from Modal Solutions of Dielectric Slab Waveguides

There are multiple methods for characterizing networks. Two crucial pieces of information to have are the rate of attenuation and the phase shift between two arbitrary points within a network. Here, the attenuation and phase coefficients (α and β, respectively) are calculated for a dielectric slab waveguide. The dielectric slab chosen here is assumed to operate in the transverse electric (TE) mode to the z-direction (TE^z). We assume $\frac{\partial}{\partial y} = 0$ and the electric field of the form expressed by (1). Propagation in the waveguide is assumed to be along $\hat{z}$, with field components defined by equations (56)-(61) in [4].

$$E_y(x, z) = E_{y0}\Phi(x)e^{-(\alpha+j\beta)z} \tag{1}$$

The ratio of this expression when evaluated at $z = 0$ and at $z = \ell$, where ℓ is the distance between measurement points in (m), results in (2), where the modal amplitude function cancels, leaving only the exponential term and the ratio itself.

$$\frac{E_y(x, \ell)}{E_y(x, 0)} = \frac{E_{y0}\Phi(x)e^{-(\alpha+j\beta)\ell}}{E_{y0}\Phi(x)} = e^{-(\alpha+j\beta)\ell} \tag{2}$$

The complex logarithm, defined in (3), is then applied to this ratio, and the real and imaginary components are separated. Each component is simplified, resulting in expressions for α and β, where the units of α are (Np/m) and the units of β are (rad/m).

$$\text{Log}(z) = \ln|z| + j\arg(z) \tag{3}$$

$$\alpha = -\frac{1}{\ell}\ln\left|\frac{E_y(x, \ell)}{E_y(x, 0)}\right| \tag{4a}$$

$$\beta = -\frac{\arg(E_y(x, \ell)) - \arg(E_y(x, 0))}{\ell} \tag{4b}$$

B. Derivation of Attenuation Coefficient from S-Parameters of a Simple 2-Port Network

While (4a) is clearly useful for modal solutions in dielectric waveguides, its usefulness can be expanded by replacing modal measurements with s-parameters. In this case, it is easiest to simply use the natural logarithm $\ln()$ rather than the complex log function $\text{Log}()$. To start, (5) takes only the magnitude squared of (2) and represents the modal solutions as total fields evaluated at the ports.

$$\frac{|E_y(x, \ell)|^2}{|E_y(x, 0)|^2} = \frac{|E_2^T|^2}{|E_1^T|^2} = e^{-2\alpha\ell} = A \tag{5}$$

To obtain the most general case, it is assumed that the total fields are comprised of a non-zero incident and non-zero

This work was funded, in part, by the NSF; Award #1816542 [1]

978-1-6654-4442-2/21/$31.00 ©2021 IEEE

reflected wave, i.e. $E_p^T = E_p^+ + E_p^-$, where p is either 1 or 2, and each component is complex-valued. The magnitude squared of complex-valued functions is easily represented by multiplying the component with its complex conjugate. This step is performed in (6), where the complex conjugate operator is designated by *.

$$A = \frac{|E_2^+ + E_2^-|^2}{|E_1^+ + E_1^-|^2} = \frac{(E_2^+ + E_2^-)(E_2^+ + E_2^-)^*}{(E_1^+ + E_1^-)(E_1^+ + E_1^-)^*} \tag{6}$$

After several simplifying steps, we obtain the expression (7) or A which is dependent only on S-parameters. Solving (7) for α results in an attenuation coefficient in terms of S-parameters only. This is shown in (8), where α again has units of (Np/m), and ℓ is the distance between ports 1 and 2.

$$A = \frac{|1 + S_{11}|^2 |S_{12}|^2}{|S_{11} + S_{12}S_{21}|^2} = e^{-2\alpha\ell} \tag{7}$$

$$\alpha = -\frac{1}{\ell} \ln \left| \frac{(1 + S_{11})(S_{12})}{S_{11} + S_{12}S_{21}} \right| \tag{8}$$

C. Methodology for S-Parameter Measurement using FDTD

To compare the attenuation coefficient expressions (4a) and (8), the 2D FDTD method is used [2]; the geometry for the FDTD simulations is shown in Figure 1. The analytic model being used is the TE^z mode as given by the field components in equations (56)-(61) in [4].

In the FDTD setup, an infinite line source (ILS) is used to excite the fields, and the computational space is divided into the primary computational domain and the perfectly matched layer (PML) region [5]. The refractive index n is defined as $n = \sqrt{\epsilon/\epsilon_0}$, and the finite dimension of the waveguide is $\delta = 2d$, where d is the half-width or half-height. The refractive indices correspond to the silicon/silicon dioxide interfaces. Finally, field data are collected over time at ports 1 and 2.

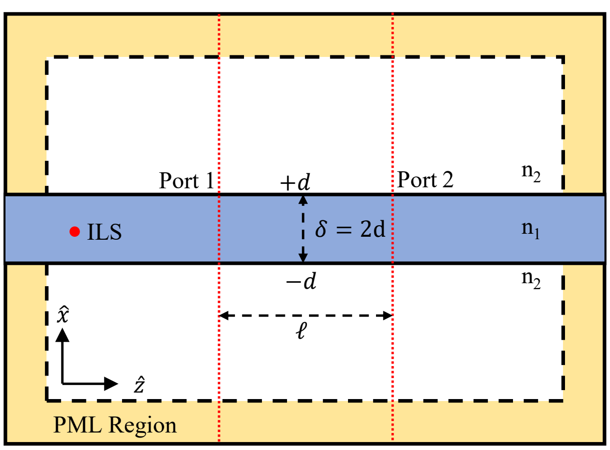

Fig. 1. FDTD simulation geometry, ILS: Infinite Line Source, $n_1 = 3.5$, $n_2 = 1.5$, Dashed box: primary computational space and the PML region boundary, δ is the width or height of the waveguide, ℓ is the distance between port 1 and port 2, and the dotted red lines are the locations of ports 1 and 2.

The collection of electric field data for the S-parameters is somewhat more involved. For this, the task is split into four individual simulations. These are represented visually in Figure 2, where the waveguide is depicted by the blue region bounded by the solid horizontal black lines, the dashed box is the boundary between the PML region and the main computational domain, and the single red dot is the location of the ILS. *Sim 1* collects incident field data equivalent to E_1^+. *Sim 2* collects total field data (E_1^T, E_2^T) with the assumption $E_2^+ = 0$. These data will be used for the calculation of S_{11} and S_{21}. *Sim 3* is similar to *Sim 1* but for E_2^+. Then, *Sim 4* collects field data similar to *Sim 2* but with the assumption that $E_1^+ = 0$ this time. Likewise, *Sim 3* and *Sim 4* are used for the calculation of S_{22} and S_{12}.

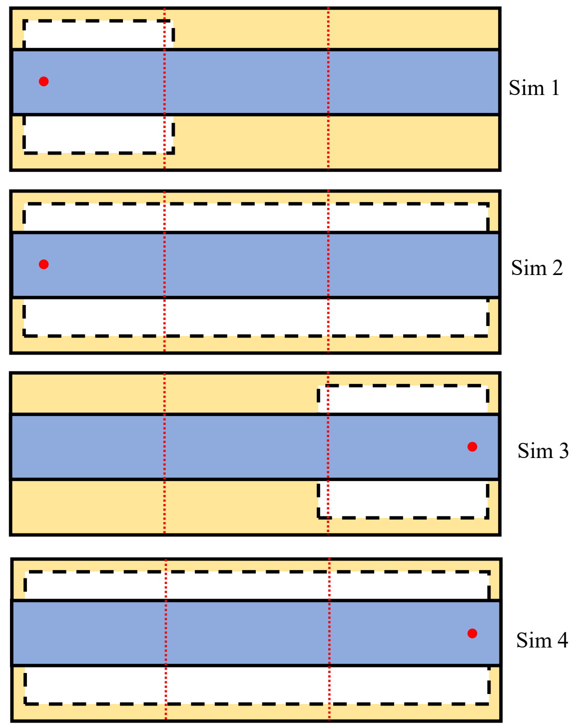

Fig. 2. S-parameter measurement setup. Field data are collected over time at ports 1 and 2 as defined in Figure 1. The dashed box represents the boundary between the primary computational domain and the PML region.

D. Model Verification and Wave Impedance

Dividing (56) with (59) in [4] gives a general analytic solution for wave impedance in the TE mode. The methodology for calculating auxiliary variables in this solution is from section D-C in [4].

To verify the FDTD model used, a setup similar to *Sim 2* in Figure 2 is used. The electric and magnetic field intensities are measured just outside the core region, and it can be seen that E_z and H_x are most similar to (56) and (59) from [4]. Taking the imaginary component of E_y/H_z gives the FDTD solution for wave impedance.

In order to calculate the wave impedance Z_w, we must calculate the phase constant β for the underlying structure; thus, β computed from FDTD through (4b) may be compared to β computed through the effective index method (EIM) as presented in section D-C in [4].

III. Results and Discussion

Starting with model verification, Figure 3 shows the comparison between the analytic solution for wave impedance Z_w and its FDTD counterpart. Here, and in other figures, the frequency $f = 194.8$ (THz) is highlighted for its potential use in optical interconnects [6]. At this frequency there is very good correlation between the FDTD model and the analytic solution. From this data it can be reasonably concluded that the FDTD model simulates the physical structure accurately. The fields are measured in FDTD at the point $-d$ at port 2 on Figure 1.

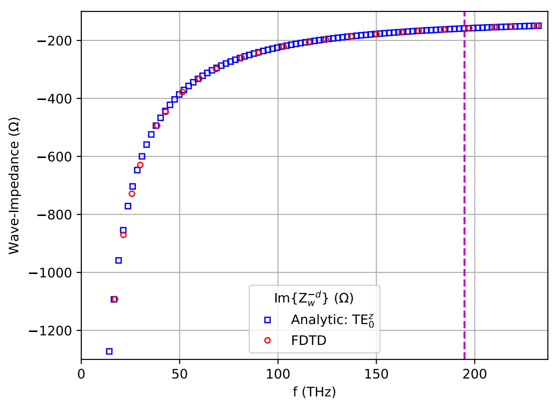

Fig. 3. Correlation of wave impedance on the lower edge of the dielectric slab from FDTD vs. analytic solution.

Beyond just the wave impedance correlation, β can also be compared between analytic and FDTD methods; this is shown in Figure 4. Here, it is clearly shown that the use of (4b) for the calculation of phase delay lines up well with the effective index method of calculating β analytically; note, there is only a very small offset in the comparison.

With an accurate model in hand, the field data are collected according to the methodology. In this manner, sample field data in the FDTD setup is shown in Figure 5 and Figure 6. The results shown in these figures are the electric and magnetic total field values evaluated at ports 1 and 2, according to *Sim 2* of Figure 2.

Figure 7 shows the attenuation coefficient as a function of frequency using both the simple modal method as well as the S-parameter method; note the error is negligible, as shown in Figure 8.

The overall S-parameters are shown in Figure 9 and Figure 10. As expected from a smooth dielectric waveguide structure in 2D FDTD, the attenuation is very small compared to the size of the simulation space, and power is almost fully transferred from port 1 to port 2. Additionally, S_{11} and S_{22} are negligibly small. Furthermore, the various methods for calculating the attenuation coefficient are nearly perfectly aligned.

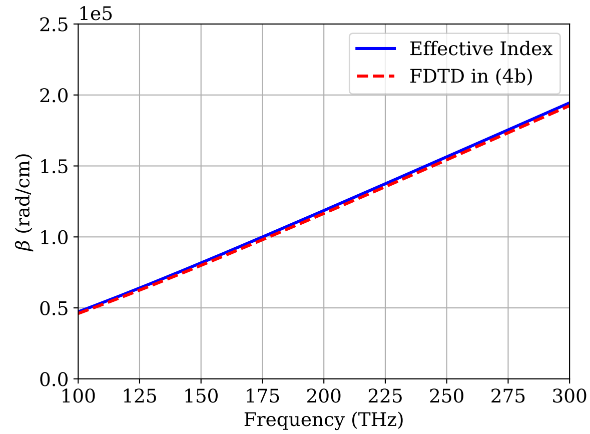

Fig. 4. Correlation of β values using the effective index method (EIM) and the FDTD fields through (4b).

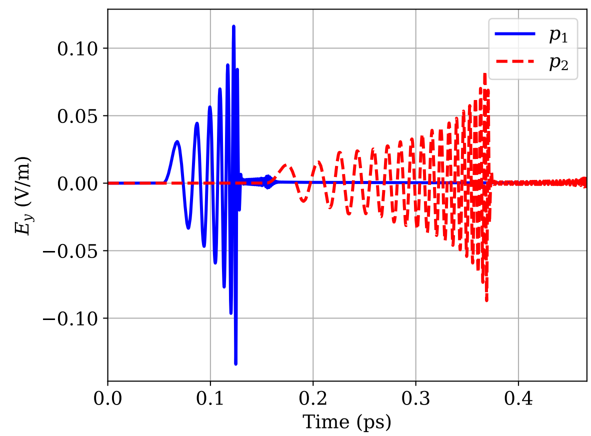

Fig. 5. Electric field waveform samples at ports 1 and 2.

IV. Conclusion

A novel FDTD-based methodology for extraction of S-parameters for optical interconnects was proposed. Using the proposed methodology, the attenuation and the phase delay were calculated for a dielectric slab waveguide operating in the mid-infrared regime. The results of the FDTD simulations were then shown to correlate well with known analytic solutions. Currently, work is underway for FDTD-based characterization of optical interconnects exhibiting stochastic surface roughness.

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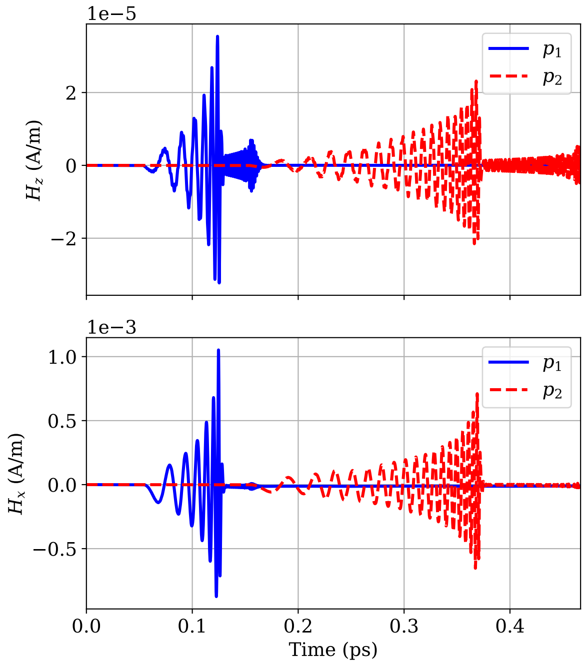

Fig. 6. Magnetic field waveform samples at ports 1 and 2.

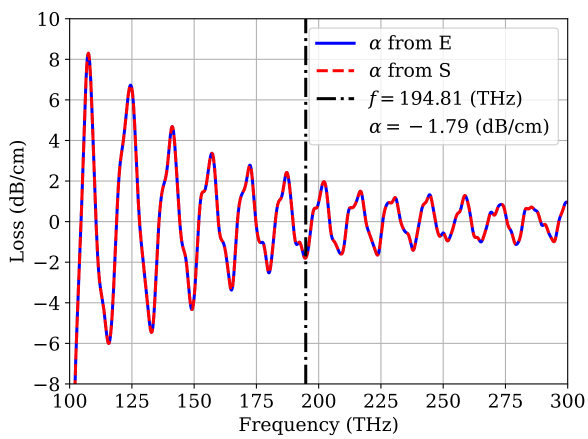

Fig. 7. Comparison of α using (4a) vs. (8), including a numerical evaluation at $f = 194.81$ (THz).

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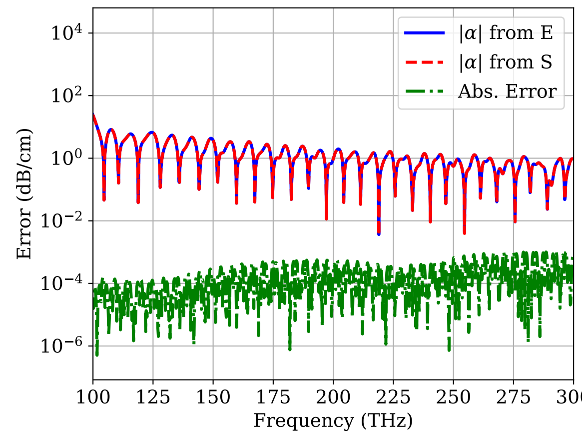

Fig. 8. Magnitude of the absolute error (green) between the two methods presented here is compared with the magnitude of α using both methods.

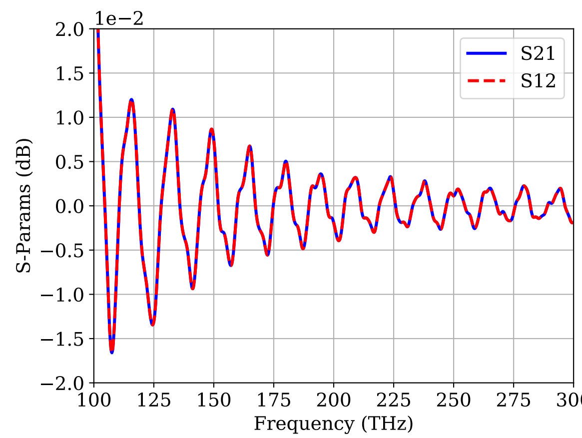

Fig. 9. S-parameter cross-terms, showing the inherent symmetry of the structure.

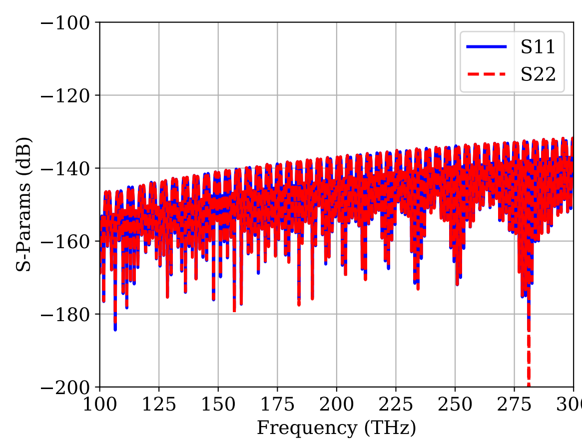

Fig. 10. S-parameter self-terms, showing the low-loss nature of the structure.