Variance-Based Sensitivity Analysis of Λ-type Quantum Memory

Kai Shinbrough and Virginia O. Lorenz
Department of Physics, University of Illinois at Urbana-Champaign, 1110 W. Green St., Urbana, IL 61801, USA
IQUIST, University of Illinois at Urbana-Champaign, 1101 W. Springfield Ave., Urbana, IL 61801, USA
kais@illinois.edu

Abstract: We examine the sensitivity of Λ-type optical quantum memories to experimental fluctuations using a variance-based analysis. The results agree with physical interpretations of quantum memory protocols, and are important for practical implementations.

The ability to store and retrieve photonic quantum states on demand—optical quantum memory—is a critical enabling technology for quantum applications [1]. In the ideal case, an optical quantum memory is capable of storing single-photon quantum states and retrieving them on demand with identical efficiency every time it is used. A quantum memory with large shot-to-shot fluctuations in storage efficiency and large drift over time, for example, is significantly less useful than a memory with smaller fluctuations and drift. Fluctuations and drift in experimental parameters are inevitably present in physical implementations, but it is unclear how variation in these input parameters affects memory efficiency. Here we quantitatively address the sensitivity of Λ-type quantum memory to changes in experimental parameters for the first time. We provide a variance-based sensitivity analysis borrowed from classical systems [2, 3], which sheds light on not only the sensitivity of an individual physical quantum memory implementation with device-specific fluctuations and drift, but also on the intrinsic sensitivity of different physical Λ-type quantum memory protocols.

We consider and numerically solve the standard Maxwell-Bloch equations describing the mapping from an optical state to a collective atomic excitation, described in detail in our previous work [4]. We partition the ex-

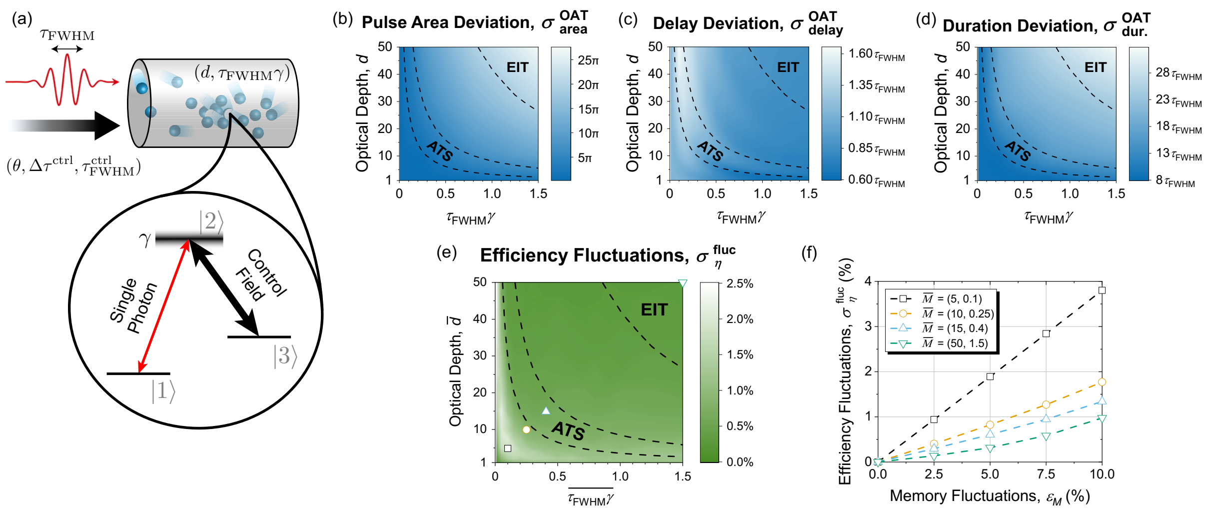

Fig. 1. (a) Schematic of Λ-type, atomic ensemble quantum memory. (b-d) Contour plots showing the amount of deviation required to reduce memory efficiency by $\sim 1/e^2$ for each control field parameter, with darker shades corresponding to smaller deviations required (larger memory sensitivity). The EIT protocol is most sensitive to delay, whereas the ATS protocol is most sensitive to pulse area and duration. (e) Memory efficiency fluctuations for 5% fluctuations in optical depth d and relative linewidth $\tau_{\mathrm{FWHM}}\gamma$ around center values $\overline{d}$ and $\overline{\tau_{\mathrm{FWHM}}\gamma}$. The largest memory efficiency fluctuations occur below the ATS region, in the so-called "absorb-then-transfer" regime [4]. (f) Memory efficiency fluctuations vs fluctuations of d and $\tau_{\mathrm{FWHM}}\gamma$ for indicated center values $(\overline{d}, \overline{\tau_{\mathrm{FWHM}}\gamma})$, showing close to linear dependence and that the less adiabatic absorb-then-transfer and ATS protocols are more sensitive to fluctuations than the more adiabatic EIT protocol.

perimental parameters of interest into two categories: the memory parameters $\mathscr{M} = (d, \tau_{\mathrm{FWHM}}\gamma)$, which depend on internal properties of a given physical memory, and the control field parameters $\mathscr{G} = (\theta, \Delta\tau^{\mathrm{ctrl}}, \tau^{\mathrm{ctrl}}_{\mathrm{FWHM}})$, which are external. Here d is the optical depth of the atomic ensemble, τ_{FWHM} is the signal photon duration (evaluated at full width at half maximum, FWHM), γ is the Λ-system excited state decay rate [see Fig. 1(a)], and θ, $\Delta\tau^{\mathrm{ctrl}}$, and $\tau^{\mathrm{ctrl}}_{\mathrm{FWHM}}$ refer to control field pulse area, delay relative to the signal field, and duration, respectively.

The sensitivity of classical systems is a much-discussed subject with well-established theoretical and numerical tools [2, 3]. The simplest method for determining a system's sensitivity to long timescale changes in its input parameters (i.e., drift) is to vary each parameter one-at-a-time (OAT), and to measure the resulting variance in the system's performance. In the case of Λ-type quantum memory, we take the memory efficiency η as the primary indicator of a quantum memory's performance and we consider the control field parameters $g_i \in \mathscr{G}$ as the system's input parameters, such that the variance in parameter g_i weighted by η is

$$V_i^{\mathrm{OAT}} = V_{g_i}[g_i, \eta(\mathscr{M};\mathscr{G})|g^0_{j\neq i}], \tag{1}$$

where $g^0_{j\neq i}$ implies that each parameter $g_{j\neq i}$ is held constant at the point where the efficiency reaches a maximum, $\mathscr{G}^0$, while g_i is varied. The square root of this variance, $\sigma_i^{\mathrm{OAT}} = \sqrt{V_i^{\mathrm{OAT}}}$, represents how far parameter g_i can be varied before the memory efficiency drops to approximately $1/e^2$ of its optimal value. Results of this calculation for the parameter range and optimum values discussed in Ref. [4] are shown in Fig. 1(b)-(d) for control field pulse area, delay, and duration, respectively. Regions of smaller σ_i^{OAT} correspond to larger memory sensitivity in parameter g_i, and vice versa. An important result here is that the memory sensitivity depends on the physical quantum memory protocol employed. The electromagnetically-induced-transparency (EIT) protocol appears to be most sensitive to changes in delay, whereas the Autler-Townes-Splitting (ATS) protocol is most sensitive to control field pulse area and duration. These results agree with physical interpretation of the protocols, where EIT memories rely on careful timing of the control field such that a transparency window is opened and closed at the correct times, whereas ATS memories rely on the use of exactly 2π pulse area control fields that have the same duration as the signal fields they store.

Another critical measure is the memory sensitivity to short timescale fluctuations in the atomic ensemble. We assume a generic noise model for each memory parameter where a random 2D fluctuation ζ_M is drawn stochastically from a normal distribution $P(\zeta_M) \propto e^{-|\zeta_M|^2/(2\varepsilon_M^2)}$ with standard deviation ε_M. In this case, the memory parameters fluctuate around the central points $\overline{\mathscr{M}}$ and we assume the control field parameters are fixed at $\mathscr{G}^0$. We evaluate the variance in memory efficiency over many trials, $V_M^{\mathrm{fluc}} = V_{\zeta_M}[\eta(\mathscr{M};\mathscr{G}^0)]$, which leads to a standard deviation measure of the memory efficiency fluctuations, $\sigma_\eta^{\mathrm{fluc}} = \sqrt{V_M^{\mathrm{fluc}}}$, shown in Fig. 1(e) for $\varepsilon_M = 5\%$. In this case, the largest memory efficiency fluctuations occur below the ATS region, in the so-called "absorb-then-transfer" regime [4]. In Fig. 1(f) we vary the magnitude of the memory parameter fluctuations, ε_M, and evaluate the fluctuations in memory efficiency for a few fixed values of $\overline{\mathscr{M}}$ marked by the symbols in Fig. 1(e). This result shows close to linear dependence of the efficiency fluctuations on memory parameter fluctuations, and shows that the points corresponding to less adiabatic absorb-then-transfer and ATS protocols are more sensitive to fluctuations than the more adiabatic EIT protocol.

Future work includes evaluating first- and higher-order Sobol' variances that go beyond OAT analysis [3], and investigation of memory sensitivity to the full shape of non-Gaussian, kernel-optimized control fields [5]. Combined with the results above, this will provide a complete picture of the sensitivity of Λ-type quantum memory to experimental error. These results are useful for evaluating the robustness of a given practical quantum memory implementation, and for determining which experimental parameters are most important to ensure stability, as well as for evaluating the relative sensitivity of different physical quantum memory regimes and protocols.

We gratefully acknowledge helpful discussion provided by Bryan K. Clark, Benjamin D. Hunt, and Donny R. Pearson; support from NSF Grants No. 1640968, No. 1806572, No. 1839177, and No. 1936321; and support from NSF Award DMR1747426.

References

1. A. I. Lvovsky, B. C. Sanders, and W. Tittel, Nat. Photonics **3**, 706–714 (2009).
2. D. G. Cacuci, *Sensitivity & uncertainty analysis, volume 1: Theory*, vol. 1 (CRC press, 2003).
3. A. Saltelli, M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, and S. Tarantola, *Global sensitivity analysis: the primer* (John Wiley & Sons, 2008).
4. K. Shinbrough, B. D. Hunt, and V. O. Lorenz, "Optimization of broadband Λ-type quantum memory using gaussian pulses," Phys. Rev. A **103**, 062418 (2021).
5. A. V. Gorshkov, A. André, M. Fleischhauer, A. S. Sørensen, and M. D. Lukin, "Universal approach to optimal photon storage in atomic media," Phys. Rev. Lett. **98**, 123601 (2007).