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Large Logarithms from Quantum Gravitational Corrections to a Massless, Minimally Coupled Scalar on de Sitter

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ABSTRACT

We consider single graviton loop corrections to the effective field equation of a massless, minimally coupled scalar on de Sitter background in the simplest gauge. We find a large temporal logarithm in the approach to freeze-in at late times, but no correction to the feeze-in amplitude. We also find a large spatial logarithm (at large distances) in the scalar potential generated by a point source, which can be explained using the renormalization group with one of the higher derivative counterterms regarded as a curvature-dependent field strength renormalization. We discuss how these results set the stage for a project to purge gauge dependence by including quantum gravitational corrections to the source which disturbs the effective field and to the observer who measures it.

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1 Introduction

One of the most profound predictions of primordial inflation is that the accelerated expansion literally rips long wavelength quanta out of the vacuum [1]. This is what produced the tensor power spectrum $\Delta^2_h(k)$ [2]. The occupation number $N(\eta, k)$ of a single polarization of wave number $\vec{k}$ at conformal time η is simply staggering,

$$N(\eta, k) = \frac{\pi \Delta^2_h(k)}{64 G k^2} \times a^2(\eta) \; , \tag{1}$$

where G is Newton's constant and $a(\eta)$ is the scale factor, which we remind the reader grows exponentially rapidly in co-moving time.

The tensor power spectrum is the primary, tree order signal of the production of inflationary gravitons. However, there must be secondary, loop effects from the interactions of these gravitons with each other and with other particles. Among these effects are:

1. The self-gravitation between inflationary gravitons may slow the expansion rate [3, 4];

2. Inflationary gravitons correct the linearized Einstein equation [5, 6], which enhances the field strength of gravitational radiation [7, 8] and has the potential to change the force of gravity [9];

3. Inflationary gravitons correct the linearized Dirac equation [10, 11], which enhances the field strength of fermions [12];

4. Inflationary gravitons correct the field equations for a massless, minimally coupled scalar [13], but make no significant change in the field strength of scalar radiation [14];

5. Inflationary gravitons correct Maxwell's equation [15], which enhances the field strength of electromagnetic radiation [16] and makes significant changes to the response to charges and currents at large distances and late times [17]; and

6. Inflationary gravitons correct the field equation for a massless, conformally coupled scalar [18–20], but do not make significant changes, either in the propagation of dynamical scalars or in the scalar exchange potential [21].

No one doubts that a classical ensemble of gravitational radiation would change kinematics and forces; this is the basis for the proposal to detect gravitational radiation using the timing of pulsars [22,23]. However, graviton propagators do require gauge fixing, and it has been argued that the apparent effects of inflationary gravitons are artifacts of the gauge [24–31]. These doubts persist in spite of the fact that similar effects derive from loops of massless, minimally coupled scalars [9,32], which experience the same growth (1) as inflationary gravitons and require no gauge fixing.

A technique has been developed for removing gauge dependence from effective field equations by including quantum gravitational correlations with the source which disturbs the effective field and the observer who detects it [33]. The procedure is to build the same diagrams (Figure 1 gives two examples) that would go into an S-matrix element, and then simplify them with a series of relations derived by Donoghue [34–36] to capture the infrared physics. In the end only vestigial traces of the source and observer remain, and each of the simplified diagrams can be regarded as a correction to the 1PI 2-point function in the linearized, effective field equation.

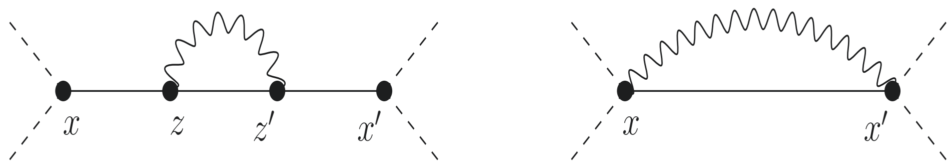

Figure 1: The left diagram shows how the self-mass (3) contributes to massive scalar scattering. The diagram on the right gives the contribution from graviton correlations between the vertices. Solid lines represent the massless scalar, wavy lines represent the graviton, and dashed lines are massive scalars.

It is worthwhile describing this for a massless, minimally coupled scalar on flat space background ($g_{\mu\nu} \equiv \eta_{\mu\nu} + \sqrt{16\pi G}\, h_{\mu\nu}$) in the 2-parameter family of covariant gauge fixing functions,

$$\mathcal{L}_{GF} = -\frac{1}{2\alpha}\eta^{\mu\nu}F_\mu F_\nu \qquad , \qquad F_\mu = \eta^{\rho\sigma}\Bigl(h_{\mu\rho,\sigma} - \frac{\beta}{2}h_{\rho\sigma,\mu}\Bigr) \; . \tag{2}$$

The renormalized self-mass is [33],

$$-iM^2(x;x') \;=\; \mathcal{C}_0(\alpha,\beta) \times \frac{G\partial^6}{4\pi^3}\Biggl[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Biggr] \quad , \quad \Delta x^2 \equiv (x-x')^2 \; , \tag{3}$$

$$\mathcal{C}_0(\alpha,\beta) \;=\; +\frac{3}{4} - \frac{3}{4}\times\alpha - \frac{3}{2}\times\frac{1}{\beta-2} + \frac{3}{4}\times\frac{(\alpha-3)}{(\beta-2)^2} \; . \tag{4}$$

Now imagine quantum gravitational corrections to the scattering of two massive scalars by the exchange of such a massless scalar. Figure 1 shows two of the many diagrams which contribute. After applying the Donoghue Identities, each of these contributions can be regarded as a correction to the self-mass, having the same spacetime dependence as (3) but multiplied by different gauge dependent coefficients. Table 1 lists each contribution, and one can see that the sum is indeed independent of α and β.

i	1	α	$\frac{1}{\beta-2}$	$\frac{(\alpha-3)}{(\beta-2)^2}$	Description
0	$+\frac{3}{4}$	$-\frac{3}{4}$	$-\frac{3}{2}$	$+\frac{3}{4}$	scalar exchange
1	0	0	0	$+1$	vertex-vertex
2	0	0	0	0	vertex-source,observer
3	0	0	$+3$	-2	vertex-scalar
4	$+\frac{17}{4}$	$-\frac{3}{4}$	0	$-\frac{1}{4}$	source-observer
5	-2	$+\frac{3}{2}$	$-\frac{3}{2}$	$+\frac{1}{2}$	scalar-source,observer
Total	$+3$	0	0	0	

Table 1: The gauge dependent factors $C_i(\alpha,\beta)$ for each contribution to the invariant scalar self-mass-squared, and their gauge-independent sum. Figure 1 shows the $i=0$ and $i=1$ diagrams.

In position space all diagrams consist of products of (possibly differentiated) massive and massless propagators (from the scalar and the graviton), $i\Delta_m(x;x')$ and $i\Delta(x;x')$, respectively. All the 3-point and 4-point diagrams can be reduced to 2-point form by applying the Donoghue Identities [33],

$$i\Delta_m(x;y)i\Delta(x;x')i\Delta(y;x') \longrightarrow \frac{i\delta^D(x-y)}{2m^2}\Big[i\Delta(x;x')\Big]^2 , \tag{5}$$

$$\partial^x_\mu i\Delta(x;x')\partial^\mu_y i\Delta(y;y')i\Delta_m(x;y)i\Delta_m(x';y') \longrightarrow \frac{i\delta^D(x-y)i\delta^D(x'-y')}{2m^2}\Big[i\Delta(x;x')\Big]^2 , \tag{6}$$

$$\partial^x_\mu i\Delta(x;y')\partial^\mu_y i\Delta(y;x')i\Delta_m(x;y)i\Delta_m(x';y') \longrightarrow -\frac{i\delta^D(x-y)i\delta^D(x'-y')}{2m^2}\Big[i\Delta(x;x')\Big]^2 . \tag{7}$$

Any 2-point contribution so obtained can be regarded as a correction to the self-mass through a trivial identity based the massless propagator equation $\partial^2 i\Delta(x;x') = i\delta^D(x-x')$,

$$f(x;x') = -\int d^D z \, i\Delta(x;z) \int d^D z' \, i\Delta(x';z') \times \partial_z^2 \partial_{z'}^2 f(z;z') \; . \qquad (8)$$

The procedure just described has been implemented on flat space background for scalars [33] and for electromagnetism [37]. Generalizing it to de Sitter will be challenging because the Hubble parameter H permits more varied spacetime dependence than (3) on the dimensionless product $H^2 \Delta x^2$. Our program is therefore to find the simplest venue for implementing the gauge purge on de Sitter background, and then check gauge independence using the family of de Sitter breaking gauges analogous to (2) [38]. In addition to simplicity, we require a system for which the potentially gauge dependent computation shows big effects, because there is little point to removing gauge dependence from a small or null effect. Previous studies have revealed that graviton corrections to massless, conformally coupled scalars are simple but do not engender significant effects [20, 21]. In this paper we show that the massless, minimally coupled scalar provides the system we seek.

In section 2 of this paper we compute the single graviton loop contribution to the self-mass $-iM^2(x;x')$ of a massless, minimally coupled scalar on de Sitter background. Section 3 uses this result to quantum-correct the linearized effective scalar field equation. Solving this equation reveals no significant 1-loop correction to the field strength of scalar radiation, but a large logarithmic correction to the scalar exchange potential. We also explain the large logarithm using a version of the renormalization group. Our conclusions comprise section 4.

2 Graviton Loop Contribution to $-iM^2(x;x')$

The purpose of this section is to compute the 1-graviton loop contribution to the 1PI (one-particle-irreducible) 2-point function of a massless, minimally coupled scalar on de Sitter background. We begin by precisely defining $-iM^2(x;x')$, analytically and diagrammatically, and by giving the required Feynman rules. We next employ dimensional regularization to evaluate first the simplest diagram and then the more complicated one. The section closes with a discussion of renormalization.

2.1 Feynman Rules

The bare Lagrangian in D spacetime dimensions is,

$$\mathcal{L} = \frac{[R-(D-2)\Lambda]\sqrt{-g}}{16\pi G} - \frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} \qquad , \qquad \Lambda \equiv (D-1)H^2 \; , \tag{9}$$

where G is Newton's constant and Λ is the cosmological constant. We define the graviton field $h_{\mu\nu}(x)$ as a perturbation of the conformally rescaled metric,

$$g_{\mu\nu}(x) \equiv a^2(\eta)\widetilde{g}_{\mu\nu}(x) \equiv a^2(\eta)\Big[\eta_{\mu\nu} + \kappa h_{\mu\nu}\Big] \qquad , \qquad a(\eta) \equiv -\frac{1}{H\eta} \; , \tag{10}$$

where $\kappa^2 \equiv 16\pi G$ is the loop-counting parameter and $\eta < 0$. Our signature is spacelike, and we employ an overlined tensor to denote the suppression of its temporal components,

$$\overline{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta^0_{\ \mu}\delta^0_{\ \nu} \qquad , \qquad \overline{\partial}_\mu \equiv \partial_\mu - \delta^0_{\ \mu}\partial_0 \; . \tag{11}$$

The 1-graviton loop contribution to the scalar self-mass can be represented as the in-out matrix element of variations of the action $S[g,\phi]$ and the counterterm action $\Delta S[g,\phi]$ (discussed in subsection 2.4),

$$-iM^2(x;x') = \Big\langle \Omega^{\rm out}\Big| T^*\Bigg\{ \Big[\frac{i\delta S[g,\phi]}{\delta\phi(x)}\Big]_{h\phi} \times \Big[\frac{i\delta S[g,\phi]}{\delta\phi(x')}\Big]_{h\phi} + \Big[\frac{i\delta^2 S[g,\phi]}{\delta\phi(x)\delta\phi(x')}\Big]_{hh} + \Big[\frac{i\delta^2 \Delta S[g,\phi]}{\delta\phi(x)\delta\phi(x')}\Big]_{1}\Bigg\}\Big|\Omega^{\rm in}\Big\rangle . \tag{12}$$

The T^*-ordering symbol in expression (12) indicates that derivatives are taken outside time-ordering; the subscripts of square-bracketed variations indicate how many perturbative fields contribute to the 1-loop result. The associated diagrams are shown in Figure 2.

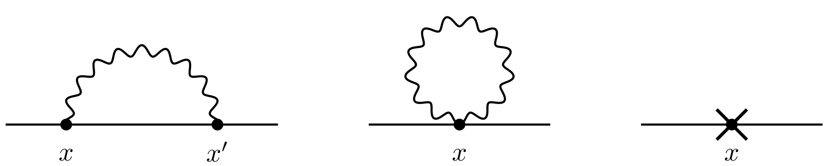

Figure 2: 1-graviton loop contributions to $-iM^2(x;x')$ corresponding to the three terms of expression (12). Graviton lines are wavy, scalar lines are straight and counterterms are denoted by a cross.

The propagators mostly depend on the de Sitter length function $y(x;x')$,

$$y(x;x') \equiv aa'H^2 \Delta x^2(x;x') \equiv aa'H^2 \Bigl[\Bigl\Vert \vec{x} - \vec{x}' \Bigr\Vert^2 - \Bigl(|\eta - \eta'| - i\epsilon\Bigr)^2\Bigr] . \qquad (13)$$

The scalar propagator is [41, 42],

$$i\Delta_A(x;x') = A(y) + k \ln(aa') \qquad , \qquad k \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} , \qquad (14)$$

where the derivative of $A(y)$ is,

$$A'(y) = -\frac{H^{D-2}}{4(4\pi)^{\frac{D}{2}}} \Biggl\{ \Gamma\Bigl(\frac{D}{2}\Bigr) \Bigl(\frac{4}{y}\Bigr)^{\frac{D}{2}} + \Gamma\Bigl(\frac{D}{2} + 1\Bigr) \Bigl(\frac{4}{y}\Bigr)^{\frac{D}{2}-1} + \sum_{n=0}^{\infty} \Biggl[\frac{\Gamma(n + \frac{D}{2} + 2)}{\Gamma(n+3)} \Bigl(\frac{y}{4}\Bigr)^{n - \frac{D}{2} + 2} - \frac{\Gamma(n+D)}{\Gamma(n + \frac{D}{2} + 1)} \Bigl(\frac{y}{4}\Bigr)^n \Biggr] \Biggr\} . \qquad (15)$$

The undifferentiated result can be inferred from the coincidence limit,

$$i\Delta_A(x;x) = k \Bigl[-\pi \cot\Bigl(\frac{D\pi}{2}\Bigr) + 2 \ln(a) \Bigr] . \qquad (16)$$

Our gauge fixing term is a de Sitter breaking analog of (2) for $\alpha = \beta = 1$ [39, 40],

$$\mathcal{L}_{GF} = -\frac{1}{2} a^{D-2} \eta^{\mu\nu} F_\mu F_\nu \quad , \quad F_\mu = \eta^{\rho\sigma} \Bigl[h_{\mu\rho,\sigma} - \frac{1}{2} h_{\rho\sigma,\mu} + (D-2) a H h_{\mu\rho} \delta^0_{\ \sigma} \Bigr] . \qquad (17)$$

In this gauge the graviton propagator is the sum of three constant tensor factors times scalar propagators,

$$i\Bigl[{}_{\mu\nu}\Delta_{\rho\sigma}\Bigr](x;x') = \sum_{I=A,B,C} \Bigl[{}_{\mu\nu} T^I_{\rho\sigma}\Bigr] \times i\Delta_I(x;x') . \qquad (18)$$

The A-type propagator is the same as the scalar propagator (14). The B-type and C-type propagators are for minimally coupled scalars with masses $M_B^2 = (D-2)H^2$ and $M_C^2 = 2(D-3)H^2$, which can be expressed as,

$$i\Delta_B = -\frac{[(4y - y^2) A'(y) + (2-y) k]}{2(D-2)} \quad , \quad i\Delta_C = \frac{(2-y) i\Delta_B}{2} + \frac{k}{D-3} . \qquad (19)$$

And the constant tensor factors $[{}_{\mu\nu}T^I_{\rho\sigma}]$ are,

$$\Big[{}_{\mu\nu}T^A_{\rho\sigma}\Big] = 2\overline{\eta}_{\mu(\rho}\overline{\eta}_{\sigma)\nu} - \frac{2}{D-3}\overline{\eta}_{\mu\nu}\overline{\eta}_{\rho\sigma} \quad , \quad \Big[{}_{\mu\nu}T^B_{\rho\sigma}\Big] = -4\delta^0{}_{(\mu}\overline{\eta}_{\nu)(\rho}\delta^0{}_{\sigma)} \; , \tag{20}$$

$$\Big[{}_{\mu\nu}T^C_{\rho\sigma}\Big] = \frac{2E_{\mu\nu}E_{\rho\sigma}}{(D-2)(D-3)} \quad , \quad E_{\mu\nu} \equiv (D-3)\delta^0{}_{\mu}\delta^0{}_{\nu} + \overline{\eta}_{\mu\nu} \; . \tag{21}$$

Here and henceforth, parenthesized indices are symmetrized.

A hatted 2nd rank tensor denotes the trace-reversal,

$$\widehat{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h \; . \tag{22}$$

Trace-reversing a single index of the graviton propagator gives,

$$i\Big[{}_{\widehat{\mu\nu}}\Delta_{\rho\sigma}\Big] = 2\eta_{\mu(\rho}\eta_{\sigma)\nu}i\Delta_A + 4\delta^0{}_{(\mu}\overline{\eta}_{\nu)(\rho}\delta^0{}_{\sigma)}(i\Delta_A - i\Delta_B) - \frac{2\delta^0{}_{\mu}\delta^0{}_{\nu}E_{\rho\sigma}}{D-3}(i\Delta_A - i\Delta_C) \; . \tag{23}$$

This form is desirable because the noncovariant tensor factors multiply differences, $(i\Delta_A - i\Delta_B)$ and $(i\Delta_A - i\Delta_C)$, which are only logarithmically singular at coincidence. Trace-reversing on both indices gives,

$$i\Big[{}_{\widehat{\mu\nu}}\Delta_{\widehat{\rho\sigma}}\Big] = \Big[2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \eta_{\mu\nu}\eta_{\rho\sigma}\Big]i\Delta_A$$
$$+4\delta^0{}_{(\mu}\overline{\eta}_{\nu)(\rho}\delta^0{}_{\sigma)}(i\Delta_A - i\Delta_B) - 2\Big(\frac{D-2}{D-3}\Big)\delta^0{}_{\mu}\delta^0{}_{\nu}\delta^0{}_{\rho}\delta^0{}_{\sigma}(i\Delta_A - i\Delta_C) \; . \tag{24}$$

2.2 The Primitive 4-Point Contribution

We might call the middle diagram of Figure 2 $-iM^2_4(x;x')$. From expression (12) we see that it involves the in-out matrix element of,

$$\Big[\frac{i\delta^2 S[g,\phi]}{\delta\phi(x)\delta\phi(x')}\Big]_{hh} = i\partial_\mu\Big[\sqrt{-g(x)}\,g^{\mu\nu}(x)\partial_\nu\delta^D(x-x')\Big]_{hh} \; , \tag{25}$$

$$= i\kappa^2\partial_\mu\Big[a^{D-2}\Big(\widehat{h}^{\mu\rho}(x)h^\nu{}_\rho(x) - \frac{1}{4}\eta^{\mu\nu}\widehat{h}^{\rho\sigma}(x)h_{\rho\sigma}(x)\Big)\partial_\nu\delta^D(x-x')\Big] \; . \tag{26}$$

The matrix element involves the single trace-reversed propagator (23),

$$-iM^2_4 = i\kappa^2\partial_\mu\Bigg\{a^{D-2}\Bigg(i\Big[{}^{\widehat{\mu\rho}}\Delta^\nu{}_\rho\Big](x;x) - \frac{1}{4}\eta^{\mu\nu}i\Big[{}^{\widehat{\rho\sigma}}\Delta_{\rho\sigma}\Big](x;x)\Bigg)\partial_\nu\delta^D(x-x')\Bigg\}. \tag{27}$$

It can be reduced to give,

$$-iM^2_4(x;x') = i\kappa^2 \partial_\mu \Biggl\{ a^{D-2} \Biggl(\eta^{\mu\nu} \Bigl[-\frac{(D-5)D}{4}\, i\Delta_A(x;x) + \frac{1}{2}\Bigl(\frac{D-4}{D-3}\Bigr) k \Bigr] + \delta^\mu_{\ 0}\delta^\nu_{\ 0} \Bigl[D i\Delta_A(x;x) + \frac{(D-1)(D-4)}{(D-2)(D-3)} k \Bigr] \Biggr) \partial_\nu \delta^D(x-x') \Biggr\}, \tag{28}$$

where we used the coincidence limits of $i\Delta_B$ and $i\Delta_C$ inferred from (19).

2.3 The Primitive 3-Point Contribution

The left hand diagram of Figure 2 might be called $-iM^2_3(x;x')$. From expression (12) we see that it involves the product of two first variations,

$$\Bigl[\frac{i\delta S[g,\phi]}{\delta\phi(x)} \Bigr]_{h\phi} = i\partial_\mu \Bigl[\sqrt{-g}\, g^{\mu\nu} \partial_\nu \phi \Bigr]_{h\phi} = -i\kappa \partial_\mu \Bigl[a^{D-2} \widehat{h}^{\mu\nu} \partial_\nu \phi \Bigr] \; . \tag{29}$$

The 3-point contribution involves the twice trace-reversed propagator (24),

$$-iM^2_3(x;x') = -\kappa^2 \partial_\mu \partial'_\rho \Bigl\{ (aa')^{D-2} i \Bigl[{}^{\widehat{\mu\nu}} \Delta^{\widehat{\rho\sigma}} \Bigr](x;x')\, \partial_\nu \partial'_\sigma i\Delta_A(x;x') \Bigr\}. \tag{30}$$

Expression (24) suggests a natural decomposition into four parts,

$$-iM^2_{3A} \equiv -\kappa^2 \partial\!\cdot\!\partial' \Bigl\{ (aa')^{D-2} i\Delta_A \!\times\! \partial\!\cdot\!\partial' i\Delta_A \Bigr\} \; , \tag{31}$$

$$-iM^2_{3B} \equiv -\kappa^2 \partial^\mu \partial'^\rho \Bigl\{ (aa')^{D-2} i\Delta_A \!\times\! (\partial_\rho \partial'_\mu \!-\! \partial_\mu \partial'_\rho) i\Delta_A \Bigr\} \; , \tag{32}$$

$$-iM^2_{3C} \equiv -\kappa^2 \partial^\mu \partial'^\rho \Bigl\{ (aa')^{D-2} (i\Delta_A \!-\! i\Delta_B) \times \Bigl[\overline{\eta}_{\mu\rho} \partial_0 \partial'_0 \!-\! \delta^0_{\ \mu} \overline{\partial}_\rho \partial'_0 \!+\! \delta^0_{\ \rho} \overline{\partial}_\mu \partial_0 \!-\! \delta^0_{\ \mu} \delta^0_{\ \rho} \nabla^2 \Bigr] i\Delta_A \Bigr\} \; , \tag{33}$$

$$-iM^2_{3D} \equiv 2\Bigl(\frac{D-2}{D-3}\Bigr) \kappa^2 \partial_0 \partial'_0 \Bigl\{ (aa')^{D-2} (i\Delta_A \!-\! i\Delta_C) \partial_0 \partial'_0 i\Delta_A \Bigr\} \; . \tag{34}$$

The terms inside the curly brackets of expression (31) are quadratically divergent, whereas the curly brackets of (32-34) are only logarithmically divergent. This means we only need a few terms of $i\Delta_A(x;x')$,

$$i\Delta_A(x;x') = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}} (aa')^{\frac{D}{2}-1}} \Biggl\{ \frac{1}{\Delta x^{D-2}} + \frac{D(D-2)}{8(D-4)} \frac{aa' H^2}{\Delta x^{D-4}} + \dots \Biggr\} + k \Biggl\{ -\pi \cot\Bigl(\frac{D\pi}{2}\Bigr) + \ln(aa') + \dots \Biggr\}. \tag{35}$$

The 4-step procedure for reducing expressions (31-34) is,

1. Act the two inner derivatives on $i\Delta_A$;[1]
2. Multiply by the scalar propagators from the graviton propagator, and retain only those terms which are nonzero in the unregulated limit;
3. Extract derivatives from the quadratically divergent terms to reach a logarithmically divergent form,
$$\frac{1}{\Delta x^{2D-2}} = \frac{\partial^2}{2(D\!-\!2)^2}\Big[\frac{1}{\Delta x^{2D-4}}\Big] \qquad ;\text{and} \tag{36}$$
4. Localize the divergence and take the unregulated limit on the remainder using the flat space propagator equation [41],
$$\frac{1}{\Delta x^{2D-4}} = \frac{\partial^2}{2(D\!-\!3)(D\!-\!4)}\Big[\frac{1}{\Delta x^{2D-6}}\Big] \; , \tag{37}$$
$$= \frac{\mu^{D-4} 4\pi^{\frac{D}{2}} i\delta^D(x\!-\!x')}{2(D\!-\!3)(D\!-\!4)\Gamma(\frac{D}{2}\!-\!1)} + \frac{\partial^2}{2(D\!-\!3)(D\!-\!4)}\Big[\frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}}\Big], \tag{38}$$
$$= \frac{\mu^{D-4} 4\pi^{\frac{D}{2}} i\delta^D(x\!-\!x')}{2(D\!-\!3)(D\!-\!4)\Gamma(\frac{D}{2}\!-\!1)} - \frac{1}{4}\partial^2\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] + O(D\!-\!4) \; . \tag{39}$$

Before applying the 4-step procedure on $-iM^2_{3B}(x;x')$ it is useful to expand the traces over μ and ρ,
$$-iM^2_{3B}(x;x') = -\kappa^2(\partial_0\!+\!\partial'_0)\partial_i\Big\{(aa')^{D-2} i\Delta_A(x;x')(\partial_0\!+\!\partial'_0)\partial_i i\Delta_A(x;x')\Big\} \; . \tag{40}$$
A similar expansion of $-iM^2_{3C}(x;x')$ gives,
$$-iM^2_C = \kappa^2\nabla^2\Big\{(aa')^{D-2}(i\Delta_A\!-\!i\Delta_B)\partial_0\partial'_0 i\Delta_A\Big\} + \kappa^2\partial_0\partial_i\Big\{(aa')^{D-2} \times(i\Delta_A\!-\!i\Delta_B)\partial'_0\partial_i i\Delta_A\Big\} + \kappa^2\partial'_0\partial_i\Big\{(aa')^{D-2}(i\Delta_A\!-\!i\Delta_B)\partial_0\partial_i i\Delta_A\Big\} +\kappa^2\partial_0\partial'_0\Big\{(aa')^{D-2}(i\Delta_A\!-\!i\Delta_B)\nabla^2 i\Delta_A\Big\}. \tag{41}$$
Also note that each divergence is proportional to one of two constants,

[1]Note that this can produce a delta function when acting on the most singular term,
$$\partial_\alpha\partial'_\beta\Big[\frac{1}{\Delta x^{D-2}}\Big] = \frac{\delta^0{}_\alpha\delta^0{}_\beta 4\pi^{\frac{D}{2}} i\delta^D(x\!-\!x')}{\Gamma(\frac{D}{2}\!-\!1)} + (D\!-\!2)\Big[\frac{\eta_{\alpha\beta}}{\Delta x^D} - \frac{D\Delta x_\alpha\Delta x_\beta}{\Delta x^{D+2}}\Big] \; .$$

$$A_0 \equiv \kappa^2 k\pi \mathrm{cot}\Bigl(\frac{D\pi}{2}\Bigr) \qquad , \qquad A_1 \equiv \frac{\kappa^2 H^2}{4\pi^{\frac{D}{2}}} \frac{\mu^{D-4}\Gamma(\frac{D}{2})}{(D\!-\!3)(D\!-\!4)} \; . \tag{42}$$

Now apply the 4-step procedure to find,

$$-iM^2_{3A}(x;x') = \Bigl[A_0 + \tfrac{3}{4}A_1\Bigr]\partial^\mu\Bigl[a^2\partial_\mu i\delta^D(x\!-\!x')\Bigr] + \frac{\kappa^2 H^2 \partial\!\cdot\!\partial'}{64\pi^4}\Bigl\{aa'\partial_0^2\Bigl[\frac{1}{\Delta x^2}\Bigr] + \tfrac{3}{2}aa'\partial^2\Bigl[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Bigr] + 4a^2a'^2H^2\Bigl[\frac{\ln(\frac{1}{4}H^2\Delta x^2)\!+\!1}{\Delta x^2}\Bigr] + \tfrac{1}{2}a^2a'^2H^2\partial_0^2\Bigl[\ln^2(\tfrac{1}{4}H^2\Delta x^2)\!+\!3\ln(\tfrac{1}{4}H^2\Delta x^2)\Bigr]\Bigr\}, \tag{43}$$

$$-iM^2_{3B}(x;x') = \tfrac{1}{2}A_1\partial_i\Bigl[a^2\partial_i i\delta^D(x\!-\!x')\Bigr] - \frac{\kappa^2 H^2\nabla^2}{64\pi^4}\Bigl\{aa'\partial^2\Bigl[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Bigr] + 12a^2a'^2H^2\Bigl[\frac{\ln(\frac{1}{4}H^2\Delta x^2)\!+\!\frac{3}{2}}{\Delta x^2}\Bigr] + \tfrac{1}{2}a^2a'^2H^2\Delta\eta^2\partial^2\Bigl[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Bigr] + 4a^3a'^3H^4\Delta\eta^2\Bigl[\frac{\ln(\frac{1}{4}H^2\Delta x^2)\!+\!\frac{3}{2}}{\Delta x^2}\Bigr]\Bigr\}, \tag{44}$$

$$-iM^2_{3C}(x;x') = A_1\Bigl\{-\tfrac{1}{4}\partial_i\Bigl[a^2\partial_i i\delta^D(x\!-\!x')\Bigr] + \tfrac{1}{4}(D\!-\!1)\partial_0\Bigl[a^2\partial_0 i\delta^D(x\!-\!x')\Bigr]\Bigr\} + \frac{\kappa^2H^2\nabla^2}{64\pi^4}\Bigl\{\tfrac{1}{2}aa'\partial^2\Bigl[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Bigr] + 6aa'\partial_0^2\Bigl[\frac{\ln(\frac{1}{4}H^2\Delta x^2)\!+\!2}{\Delta x^2}\Bigr] + \tfrac{1}{2}a^2a'^2H^2\partial^2\Bigl[\ln^2(\tfrac{1}{4}H^2\Delta x^2)\!+\!\tfrac{1}{2}\ln(\tfrac{1}{4}H^2\Delta x^2)\Bigr] - a^2a'^2H^2\partial_0^2\Bigl[\ln^2(\tfrac{1}{4}H^2\Delta x^2) + \tfrac{7}{2}\ln(\tfrac{1}{4}H^2\Delta x^2)\Bigr] + a^3a'^3H^4\Bigl[\ln^2(\tfrac{1}{4}H^2\Delta x^2)\!+\!3\ln(\tfrac{1}{4}H^2\Delta x^2)\Bigr] + \tfrac{1}{2}a^3a'^3H^4\partial_0^2\Bigl(\Delta x^2\Bigl[\ln^2(\tfrac{1}{4}H^2\Delta x^2)\!+\!\ln(\tfrac{1}{4}H^2\Delta x^2)\!-\!1\Bigr]\Bigr)\Bigr\} + \frac{\kappa^2H^2\partial_0\partial_0'}{64\pi^4}\Bigl\{\tfrac{3}{2}aa'\partial^2\Bigl[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Bigr] - 2aa'\nabla^2\Bigl[\frac{\ln(\frac{1}{4}H^2\Delta x^2)\!+\!2}{\Delta x^2}\Bigr] - \tfrac{3}{2}a^2a'^2H^2 \times\partial^2\ln(\tfrac{1}{4}H^2\Delta x^2) + \tfrac{1}{2}a^2a'^2H^2\nabla^2\Bigl[\ln^2(\tfrac{1}{4}H^2\Delta x^2)\!+\!3\ln(\tfrac{1}{4}H^2\Delta x^2)\Bigr]\Bigr\}, \tag{45}$$

$$-iM^2_{3D}(x;x') = A_1\partial_0\Bigl[a^2\partial_0 i\delta^D(x\!-\!x')\Bigr] + \frac{\kappa^2H^2\partial_0\partial_0'}{64\pi^4}\Bigl\{2aa'\partial^2\Bigl[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Bigr] + 8aa'\partial_0^2\Bigl[\frac{\ln(\frac{1}{4}H^2\Delta x^2)\!+\!2}{\Delta x^2}\Bigr]\Bigr\}. \tag{46}$$

2.4 Renormalization

Reducing the 4-point contribution (28) to the 3-point form (43-46) gives,

$$-iM^2_4(x;x') = \tfrac{1}{4}D(D-5)A_0\partial^\mu\Bigl[a^2\partial_\mu i\delta^D(x-x')\Bigr] - DA_0\partial_0\Bigl[a^2\partial_0 i\delta^D(x-x')\Bigr] \; . \tag{47}$$

We can now sum the divergent parts from expressions (43-47),

$$-iM^2_{\rm div}(x;x') = \Bigl[\tfrac{1}{4}(D-1)(D-4)A_0 + A_1\Bigr]\partial^\mu\Bigl[a^2\partial_\mu i\delta^D(x-x')\Bigr] + \Bigl[-DA_0 + \tfrac{1}{4}(D+4)A_1\Bigr]\partial_0\Bigl[a^2\partial_0 i\delta^D(x-x')\Bigr] \; . \tag{48}$$

Recall that A_0 and A_1 were defined in (42).

The 1-loop divergences (48) are canceled by three counterterms,

$$\Delta\mathcal{L} = -\frac{1}{2}\alpha_1 \square\phi\square\phi\sqrt{-g} - \frac{1}{2}\alpha_2 R\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\alpha_3 R\partial_0\phi\partial_0\phi g^{00}\sqrt{-g} \; . \tag{49}$$

Hence the final diagram of Figure 2 is,

$$-iM^2_{\rm ctm}(x;x') \equiv \Bigl[\frac{i\delta^2\Delta S}{\delta\phi(x)\delta\phi(x')}\Bigr]_1 = -\alpha_1\partial^\mu\partial'^\rho\Biggl[(aa')^{D-2}\partial_\mu\partial'_\rho\Bigl(\frac{i\delta^D(x-x')}{a^D}\Bigr)\Biggr] + \alpha_2\partial^\mu\Bigl[Ra^{D-2}\partial_\mu i\delta^D(x-x')\Bigr] - \alpha_3\partial_0\Bigl[Ra^{D-2}\partial_0 i\delta^D(x-x')\Bigr] \; , \tag{50}$$

where the Ricci scalar is $R = D(D-1)H^2$. Comparison between expressions (48) and (50) implies,

$$\alpha_1 = 0 \; , \tag{51}$$

$$\alpha_2 R = -\frac{\kappa^2 H^2\mu^{D-4}}{4\pi^{\frac{D}{2}}}\Biggl\{\frac{\Gamma(D)}{\Gamma(\frac{D}{2})}\frac{(D-4)}{16}\pi\cot\Bigl(\frac{D\pi}{2}\Bigr) + \frac{\Gamma(\frac{D}{2})}{(D-3)(D-4)}\Biggr\}, \tag{52}$$

$$\alpha_3 R = -\frac{\kappa^2 H^2\mu^{D-4}}{4\pi^{\frac{D}{2}}}\Biggl\{\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})}\frac{D}{4}\pi\cot\Bigl(\frac{D\pi}{2}\Bigr) - \frac{(D+4)\Gamma(\frac{D}{2})}{4(D-3)(D-4)}\Biggr\}. \tag{53}$$

The vanishing of α_1 is an artifact of $\alpha = \beta = 1$ gauge in the flat space limit (3). The counterterms proportional to α_2 and α_3 vanish in the flat space limit and their potential gauge dependence is not known.

Combining the primitive divergence with the counterterms and taking the unregulated limit gives,

$$-iM^2_{\rm div}(x;x') - iM^2_{\rm ctm}(x;x') = -\frac{\kappa^2 H^2}{4\pi^2}\partial^\mu\Bigl[a^2\ln(a)\partial_\mu i\delta^4(x-x')\Bigr] + \frac{\kappa^2H^2}{2\pi^2}\partial_0\Bigl[a^2\ln\Bigl(\frac{4\mu^2 a}{H^2}\Bigr)\partial_0 i\delta^4(x-x')\Bigr] + O(D-4) \; . \tag{54}$$

The renormalized self-mass comes from adding these local terms to the non-local parts of (43-46), and then simplifying the sum,

$$-iM^2_{\rm ren}(x;x') = -\frac{\kappa^2 H^2}{4\pi^2}\partial^\mu\Big[a^2\ln(a)\partial_\mu i\delta^4(x-x')\Big] + \frac{\kappa^2 H^2}{2\pi^2}\partial_0\Big[a^2\ln\Big(\frac{4\mu^2 a}{H^2}\Big)\partial_0 i\delta^4(x-x')\Big] + \frac{\kappa^2 H^2\partial_0\partial_0'}{64\pi^4}\Bigg\{aa'\partial_0\partial_0'\Big[\frac{1}{\Delta x^2}\Big] + 2aa'(\partial_0\partial_0'+\nabla^2)\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] - 2aa'(4\partial_0\partial_0'+\nabla^2)\Big[\frac{\ln(\frac14 H^2\Delta x^2)+2}{\Delta x^2}\Big]\Bigg\} + \frac{\kappa^2 H^2\nabla^2}{64\pi^4}\Bigg\{-2aa'(\partial_0\partial_0'+\nabla^2)\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] - 6aa'\partial_0\partial_0'\Big[\frac{\ln(\frac14 H^2\Delta x^2)+\frac{11}{6}}{\Delta x^2}\Big] + \frac14 a^2 a'^2 H^2(\partial_0\partial_0'-\nabla^2)\ln(\mu^2\Delta x^2) + \frac{15}{4}a^2a'^2H^2\partial_0\partial_0'\ln(\tfrac14 H^2\Delta x^2) - a^2a'^2H^2\nabla^2\Big[\frac32\ln^2(\tfrac14 H^2\Delta x^2) + \frac54\ln(\tfrac14 H^2\Delta x^2)\Big]\Bigg\}. \quad (55)$$

3 The Linearized Effective Field Equation

The purpose of this section is to use the renormalized self-mass (55) to quantum-correct the linearized effective field equation and then solve this equation for scalar radiation and for the scalar exchange potential. We begin by explaining the Schwinger-Keldysh formalism that is used to produce a causal and real effective field equation. The equation is then solved perturbatively, first for scalar radiation and then for the exchange potential. The section closes by using the renormalization group to explain the latter.

3.1 Schwinger-Keldysh Formalism

The linearized effective field equation is,

$$\sqrt{-g}\,\Box\phi(x) = \partial^\mu\Big[a^2\partial_\mu\phi(x)\Big] \equiv \mathcal{D}\phi(x) = J(x) + \int d^4x' M^2(x;x')\phi(x')\ , \quad (56)$$

where $J(x)$ is the source. Substituting expression (55) for the self-mass results in an equation with three peculiar properties:

- It isn't local because $M^2_{\rm ren}(x;x')$ fails to vanish for $x'^\mu \neq x^\mu$;

- It isn't causal because $M^2_{\rm ren}(x;x')$ fails to vanish for x'^μ outside the past light-cone of x^μ; and

- It isn't real because $M^2_{\rm ren}(x;x')$ has a nonzero imaginary part.

Effective field equations are unavoidably nonlocal but the other two properties derive from $-iM^2_{\rm ren}(x;x')$ representing an in-out amplitude rather than a true expectation value. Of course that is what the Feynman rules produce, and it is exactly the right thing for scattering amplitudes. However, the "in" and "out" vacua disagree due to the very cosmological particle production (1) whose effect we seek to study, and causality precludes the S-matrix from being an observable. It is therefore more sensible to study the evolution of the expectation value of $\phi(x)$ in the presence of a state which was empty in the distant past. The Schwinger-Keldysh formalism provides a diagrammatic procedure for computing this which is almost as simple to use as the Feynman rules [43–47]. This expectation value obeys the Schwinger-Keldysh effective field equations, which are both causal and real [48–50].

It is straightforward to convert the in-out effective field equations to the in-in equations of the Schwinger-Keldysh formalism. The rules are [51]:

- End points of lines in the diagrammatic formalism have $\pm$ polarizations, resulting in four propagators and 2^N 1PI N-point functions;

- The $++$ propagator is the same as the Feynman propagator, and the $--$ propagator is its complex conjugate;

- The $+-$ and $-+$ propagators are homogeneous solutions of the propagator equation, which are obtained from the Feynman propagator by changing the $i\epsilon$ in the conformal coordinate interval from (13) to,

$$\Delta x^2_{+-} \equiv \|\vec{x}-\vec{x}'\|^2 - (\eta-\eta'+i\epsilon)^2 \; , \qquad (57)$$

$$\Delta x^2_{-+} \equiv \|\vec{x}-\vec{x}'\|^2 - (\eta-\eta'-i\epsilon)^2 \qquad ; \text{and} \qquad (58)$$

- Vertices carry only a single polarity, so all their lines are either $+$ or $-$, with the $+$ vertices being the same as those of the Feynman rules and the $-$ vertices being their complex conjugates.

The term "$M^2(x;x')$" in the Schwinger-Keldysh effective field equation (56) is $M^2_{++}(x;x') + M^2_{+-}(x;x')$. It is real because the $\pm$ vertex at x'^μ results in a relative minus sign, and because $\Delta x^2_{+-} = \Delta x^2_{++}$ for $\eta < \eta'$ whereas

$\Delta x^2_{+-} = (\Delta x^2_{++})^*$ for $\eta > \eta'$. To see causality one first eliminates inverse powers of Δx^2,

$$\frac{1}{\Delta x^2} = \tfrac{1}{4}\partial^2 \ln(\mu^2 \Delta x^2) \; , \tag{59}$$

$$\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2} = \tfrac{1}{8}\partial^2 \Bigl[\ln^2(\mu^2\Delta x^2) - 2\ln(\mu^2\Delta x^2)\Bigr] \; . \tag{60}$$

Now note that differences of powers of $++$ and $+-$ logarithms are proportional to $\theta(\Delta\eta - \Delta r)$, where $\Delta\eta \equiv \eta - \eta'$ and $\Delta r \equiv \|\vec{x} - \vec{x}'\|$,

$$\ln(\mu^2\Delta x^2_{++}) - \ln(\mu^2\Delta x^2_{+-}) = 2\pi i\theta(\Delta\eta - \Delta r) \; , \tag{61}$$

$$\ln^2(\mu^2\Delta x^2_{++}) - \ln^2(\mu^2\Delta x^2_{+-}) = 4\pi i\theta(\Delta\eta - \Delta r)\ln[\mu^2(\Delta\eta^2 - \Delta r^2)] \; . \tag{62}$$

In converting expression (55) to Schwinger-Keldysh form we will employ the notation $\Theta \equiv \theta(\Delta\eta - \Delta r)$ to achieve a more compact form,

$$\begin{aligned}
M^2_{\rm SK}(x;x') = {} & \frac{\kappa^2 H^2}{4\pi^2}\Biggl\{\partial^\mu\Bigl[a^2\ln(a)\partial_\mu\Bigr] - 2\partial_0\Bigl[a^2\ln\Bigl(\frac{4\mu^2 a}{H^2}\Bigr)\partial_0\Bigr]\Biggr\}\delta^4(x-x') \\
& + \frac{\kappa^2 H^2 \partial_0\partial_0'}{128\pi^3}\Biggl\{-2aa'(\partial_0\partial_0' + \nabla^2)\partial^2\Bigl[\Theta\ln[\mu^2(\Delta\eta^2 - \Delta r^2)]\Bigr] + 9aa'\partial_0\partial_0'\partial^2\Theta \\
& \qquad + 4aa'\nabla^2\partial^2\Theta + 2aa'(4\partial_0\partial_0' + \nabla^2)\partial^2\Bigl[\Theta\ln[\tfrac{1}{4}H^2(\Delta\eta^2 - \Delta r^2)]\Bigr]\Biggr\} \\
& + \frac{\kappa^2 H^2 \nabla^2}{128\pi^3}\Biggl\{2aa'(\partial_0\partial_0' + \nabla^2)\partial^2\Bigl[\Theta\ln[\mu^2(\Delta\eta^2 - \Delta r^2)]\Bigr] + 3aa'\partial_0\partial_0'\partial^2\Theta \\
& \qquad - 2aa'\nabla^2\partial^2\Theta + 6aa'\partial_0\partial_0'\partial^2\Bigl[\Theta\ln[\tfrac{1}{4}H^2(\Delta\eta^2 - \Delta r^2)]\Bigr] - 16a^2a'^2H^2\partial_0\partial_0'\Theta \\
& \qquad + 6a^2a'^2H^2\nabla^2\Theta + 12a^2a'^2H^2\nabla^2\Bigl[\Theta\ln[\tfrac{1}{4}H^2(\Delta\eta^2 - \Delta r^2)]\Bigr]\Biggr\} .
\end{aligned} \tag{63}$$

3.2 The Scalar Mode Function

Scalar radiation corresponds to $J(x) = 0$ and solutions take the form,

$$\phi(x) = u(\eta, k)e^{i\vec{k}\cdot\vec{x}} \qquad , \qquad k \equiv \|\vec{k}\| \; . \tag{64}$$

The spatial exponential can be factored out using translation invariance,

$$\mathcal{D}u(\eta,k) \equiv -a^2\Big[\partial_0^2 + 2aH\partial_0 + k^2\Big]u(\eta,k) = \int d^4x' M^2_{\rm SK}(x;x')u(\eta',k)e^{-i\vec{k}\cdot\Delta\vec{x}}. \tag{65}$$

Here any spatial derivatives in the self-mass are replaced by $\partial_i \longrightarrow ik_i$.

Because we only have the 1-loop self-mass, equation (65) must be solved perturbatively ($u = u_0 + u_1 + \dots$) in powers of the loop-counting parameter $\kappa^2 \equiv 16\pi G$. The tree order solution is,

$$u_0(\eta,k) = \frac{H}{\sqrt{2k^3}}\Big[1 - \frac{ik}{aH}\Big]\exp\Big[\frac{ik}{aH}\Big] \xrightarrow{a\to\infty} \frac{H}{\sqrt{2k^3}}\Big[1 + \frac{k^2}{2a^2H^2} + \dots\Big] \; , \tag{66}$$

and it is useful to note,

$$\partial_0 u_0(\eta,k) = \frac{H}{\sqrt{2k^3}}\Big[-\frac{k^2}{aH}\Big]\exp\Big[\frac{ik}{aH}\Big] \quad \Longrightarrow \quad (\partial_0^2 + k^2)\Big[a\partial_0 u_0(\eta,k)\Big] = 0 \; . \tag{67}$$

Relation (67) means that the 2nd and 3rd lines of expression (63) for $M^2_{\rm SK}$ make no contribution to the 1-loop correction,

$$\begin{aligned}\mathcal{D}u_1(\eta,k) = \frac{\kappa^2H^2}{4\pi^2}\Big\{-3a^3H\partial_0 u_0(\eta,k) + 2\ln\Big(\frac{4\mu^2 a}{H^2}\Big)a^2k^2u_0(\eta,k)\Big\} \\ -\frac{\kappa^2H^4k^2a}{64\pi^3}\Bigg\{2(\partial_0^2\!+\!k^2)\!\int\! d^4x'\Theta\ln[\mu^2(\Delta\eta^2\!-\!\Delta r^2)]a'^3u_0(\eta',k)e^{-i\vec{k}\cdot\Delta\vec{x}} \\ +(3\partial_0^2\!-\!2k^2)\!\int\! d^4x'\Theta a'^3u_0(\eta',k)e^{-i\vec{k}\cdot\Delta\vec{x}}\!+\!6\partial_0^2\!\int\! d^4x'\Theta\ln[\tfrac14 H^2(\Delta\eta^2\!-\!\Delta r^2)] \\ \times a'^3u_0(\eta',k)e^{-i\vec{k}\cdot\Delta\vec{x}} + a(8\partial_0^2\!-\!3k^2)\!\int\! d^4x'\Theta a'^2u_0(\eta',k)e^{-i\vec{k}\cdot\Delta\vec{x}} \\ -6ak^2\!\int\! d^4x'\Theta\ln[\tfrac14 H^2(\Delta\eta^2\!-\!\Delta r^2)]a'^2u_0(\eta',k)e^{-i\vec{k}\cdot\Delta\vec{x}}\Bigg\}. \end{aligned} \tag{68}$$

In reaching this form we have also used,

$$(\partial_0^2\!+\!k^2)\Big[au_0(\eta,k)\Big] = 2a^3H^2u_0(\eta,k) \; . \tag{69}$$

Equation (68) has the general form,

$$-a^2\Big[\partial_0^2 + 2aH\partial_0 + k^2\Big]u_1(\eta,k) = S(\eta) \; . \tag{70}$$

It is important to understand the relation between the asymptotic late time form of the source $S(\eta)$ and the late time form it induces in $u_1(\eta,k)$,

$$S = a^4 H^2 \ln(a) \Longrightarrow u_1 \rightarrow -\tfrac{1}{6}\ln^2(a) \quad , \quad S = a^4 H^2 \Longrightarrow u_1 \rightarrow -\tfrac{1}{3}\ln(a) \; , \tag{71}$$

$$S = a^3 H^2 \ln(a) \Longrightarrow u_1 \rightarrow +\frac{\ln(a)}{2a} \quad , \quad S = a^3 H^2 \Longrightarrow u_1 \rightarrow +\frac{1}{2a} \; , \tag{72}$$

$$S = a^2 H^2 \ln(a) \Longrightarrow u_1 \rightarrow +\frac{\ln(a)}{2a^2} \quad , \quad S = a^2 H^2 \Longrightarrow u_1 \rightarrow +\frac{1}{2a^2} \; . \tag{73}$$

One can see that the leading asymptotic form of the local source terms on the first line of (68) is,

$$\frac{\kappa^2 H^2}{4\pi^2}\Bigl\{-3a^3 H\partial_0 u_0 + 2\ln\Bigl(\frac{4\mu^2 a}{H^2}\Bigr)a^2 k^2 u_0\Bigr\} \longrightarrow \frac{\kappa^2 H^2}{4\pi^2} \times 2a^2\ln(a)k^2 u_0(0,k) \; . \tag{74}$$

Because all the nonlocal terms contain at least one factor of k^2, we can suppress higher powers of k^2, which carry factors of $1/a^2$. This simplifies the integrations,

$$-\frac{\kappa^2 H^4 k^2 u_0(0,k) a}{64\pi^3}\Biggl\{2\partial_0^2\!\int\! d^4x'\Theta\ln[\mu^2(\Delta\eta^2-\Delta r^2)]a'^3 + 3\partial_0^2\!\int\! d^4x'\Theta a'^3$$
$$+6\partial_0^2\!\int\! d^4x'\Theta\ln[\tfrac{1}{4}H^2(\Delta\eta^2-\Delta r^2)]a'^3 + 8a\partial_0^2\!\int\! d^4x'\Theta a'^2\Biggr\}$$
$$\longrightarrow \frac{\kappa^2 H^2}{4\pi^2} \times \Bigl[\ln\Bigl(\frac{H}{2\mu}\Bigr)\Bigr] + \frac{5}{4}\Bigr] a^2 k^2 u_0(0,k) \; . \tag{75}$$

In view of (73), expressions (74) and (75) imply,

$$u_1(\eta,k) \longrightarrow \frac{\kappa^2 H^2}{4\pi^2} \times \frac{k^2\ln(a)}{a^2 H^2} \times u_0(0,k) \; . \tag{76}$$

Hence 1-loop graviton corrections do not change the constant freeze-in value of the mode function (66), but they do slow down the rate of approach to this constant.

3.3 The Response to a Point Source

The scalar exchange potential corresponds to $J(x) = K a\delta^3(\vec{x})$. The solution has the form $\Phi(\eta,r)$, so this system is fundamentally 2-dimensional, unlike the 1-dimensional problem of the mode function $u(\eta, k)$. The tree order response is [52],

$$\Phi_0(t,r) = \frac{KH}{4\pi}\Bigl\{-\frac{1}{aHr} + \ln\Bigl(Hr + \frac{1}{a}\Bigr)\Bigr\} \xrightarrow{a\to\infty} \frac{HK}{4\pi}\Bigl\{\ln(Hr) - \frac{1}{2a^2H^2r^2} + \dots\Bigr\} . \tag{77}$$

Derivatives of $\Phi_0(\eta, r)$ are,

$$\partial_0\Phi_0(\eta,r) = \frac{KH^2}{4\pi}\Bigl\{\frac{1}{Hr} - \frac{1}{Hr+\frac{1}{a}}\Bigr\} \quad , \quad \partial_0^2\Phi_0(\eta,r) = -\frac{KH^3}{4\pi}\frac{1}{(Hr+\frac{1}{a})^2} , \tag{78}$$

$$\nabla^2\Phi_0(\eta,r) = \frac{K\delta^3(\vec{x})}{a} + \frac{1}{a^2}\partial_0\Bigl[a^2\partial_0\Phi_0\Bigr]. \tag{79}$$

In addition to $\mathcal{D}\Phi_0 = Ka\delta^3(\vec{x})$, two useful consequences are,

$$\partial^2\Bigl[a\Phi_0\Bigr] = K\delta^3(\vec{x}) - 2a^3H^2\Phi_0 \qquad , \qquad \partial^2\Bigl[a\partial_0\Phi_0\Bigr] = -HKa\delta^3(\vec{x}) . \tag{80}$$

The second identity of (80) can be used to perform the spatial integrations in the 1-loop sources induced by the 2nd and 3rd lines of (63),

$$\begin{aligned}
\mathcal{D}\Phi_1(\eta,r) = \frac{\kappa^2H^2}{4\pi^2}\Bigl\{Ka\ln(a)\delta^3(\vec{x}) - 3a^3H\partial_0\Phi_0 - 2\ln\Bigl(\frac{4\mu^2a}{H^2}\Bigr)\partial_0\Bigl[a^2\partial_0\phi_0\Bigr]\Bigr\} \\
+\frac{\kappa^2H^3K\partial_0}{128\pi^3}\Biggl\{-2a\partial^2\!\int_{\eta_i}^{\eta-r}\!d\eta' a'\ln[\mu^2(\Delta\eta^2-r^2)] + a(4\nabla^2-9\partial_0^2)\!\int_{\eta_i}^{\eta-r}\!d\eta' a' \\
+2a(\nabla^2-4\partial_0^2)\!\int_{\eta_i}^{\eta-r}\!d\eta' a'\ln[\tfrac{1}{4}H^2(\Delta\eta^2-r^2)]\Biggr\} + \frac{\kappa^2H^2\nabla^2}{128\pi^3}\Biggl\{2a\partial^2\!\int\! d^4x'\Theta \\
\times\ln[\mu^2(\Delta\eta^2-\Delta r^2)]\partial'^2[a'\Phi_0(x')] - a(2\nabla^2+3\partial_0^2)\!\int\! d^4x'\Theta\partial'^2[a'\Phi_0(x')] \\
-6a\partial_0^2\!\int\! d^4x'\Theta\ln[\tfrac{1}{4}H^2(\Delta\eta^2-\Delta r^2)]\partial'^2[a'\Phi_0(x')] + a^2H^2(16\partial_0^2+6\nabla^2)\!\int\! d^4x'\Theta \\
\times a'^2\Phi_0(x') + 12a^2H^2\nabla^2\!\int\! d^4x'\Theta\ln[\tfrac{1}{4}H^2(\Delta\eta^2-\Delta r^2)]a'^2\Phi_0(x')\Biggr\}.
\end{aligned} \tag{81}$$

Each of the source terms on the right hand side of (81) can be evaluated, at least for late times, and then its contribution to $\Phi_1(\eta, r)$ can be derived by integrating against the retarded Green's function associated with the differential operator $\mathcal{D} \equiv \partial^\mu a^2 \partial_\mu$,

$$G_{\rm ret}(x;x') = -\frac{1}{4\pi}\Bigl\{\frac{\delta(\Delta\eta - \Delta r)}{aa'\Delta r} + H^2 \theta(\Delta\eta - \Delta r)\Bigr\} \; . \qquad (82)$$

However, it turns out that only the first source term in equation (81) makes a significant contribution at late times,

$$\int d^4x' G_{\rm ret}(x;x') \times \frac{\kappa^2 H^2}{4\pi^2} K a' \ln(a') \delta^3(\vec{x}') \\ = -\frac{\kappa^2 H^3 K}{32\pi^3}\Bigl\{\ln^2\Bigl(Hr + \frac{1}{a}\Bigr) - \frac{2\ln(Hr + \frac{1}{a})}{aHr}\Bigr\} \; . \qquad (83)$$

The easiest way to see that the other source terms in equation (81) do not contribute at late times is by changing the time variable to a and the space variable to aHr, and then extracting a factor of $a^4 H^2$ from both sides of equation (81). The left hand side becomes,

$$\mathcal{D} = a^4 H^2 \Bigl[-a^2 \frac{\partial^2}{\partial a^2} - 4a \frac{\partial}{\partial a} + \frac{1}{a^2 H^2}\frac{\partial^2}{\partial r^2} + \frac{2}{a^2 H^2 r}\frac{\partial}{\partial r}\Bigr] \; . \qquad (84)$$

For an example of the right hand side, consider the second of the nonlocal source terms,

$$\frac{\kappa^2 H^3 K \partial_0}{128\pi^3}\Bigl\{a(4\nabla^2 - 9\partial_0^2)\int_{\eta_i}^{\eta - r} d\eta' a'\Bigr\} = \frac{\kappa^2 H^2 K \partial_0}{128\pi^3}\Bigl\{a(4\nabla^2 - 9\partial_0^2)\ln\Bigl(\frac{1}{Hr + \frac{1}{a}}\Bigr)\Bigr\}, \quad (85)$$

$$= a^4 H^2 \times \frac{\kappa^2 H^3 K}{128\pi^3}\Bigl\{-\frac{16}{aHr} + \frac{16}{aHr + 1} + \frac{3}{(aHr+1)^2} - \frac{10}{(aHr+1)^3}\Bigr\}. \quad (86)$$

The part of expression (86) inside the curly brackets goes like $1/(aHr)^2$ at late times, which corresponds to a late time contribution to $\Phi_1(\eta, r)$ of the same strength according to (84). The strongest nonlocal source goes like $\ln(a)/aHr$. Hence the leading ($a \gg 1$, $aHr \gg 1$) form comes from (83),

$$\Phi_1(t,r) \longrightarrow -\frac{\kappa^2 H^2}{8\pi^2}\ln(Hr) \times \frac{HK}{4\pi}\ln(Hr) \; . \qquad (87)$$

3.4 Renormalization Group Explanation

The $h\partial\phi\partial\phi$ interaction of gravity with our scalar is very similar to the $A\partial B\partial B$ interaction of a nonlinear sigma model which was recently studied [53]. It was shown that the leading inflationary logarithms of that model could all be explained by combining a variant of Starobinsky's stochastic formalism [54, 55], based on curvature-dependent effective potentials, with a variant of the renormalization group, based on curvature-dependent field strength renormalizations. The $\phi \to \phi +$ constant shift symmetry of our Lagrangian (9) precludes there being any effective potential for ϕ but it does seem possible to identify a curvature-dependent field strength renormalization among the three 1-loop counterterms of expression (49),

$$\Delta\mathcal{L} = -\frac{1}{2}\alpha_1 \Box\phi\Box\phi\sqrt{-g} - \frac{1}{2}\alpha_2 R\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\alpha_3 R\partial_0\phi\partial_0\phi g^{00}\sqrt{-g} \; . \tag{88}$$

This is the α_2 counterterm, which can be viewed as a field strength renormalization of the original Lagrangian (9) with $\delta Z = \alpha_2 R$. Just as for A-B nonlinear sigma model of Ref. [53], the higher derivative counterterm proportional to α_1 plays no role because it cannot be viewed as a renormalization of the bare Lagrangian. Nor can the noncovariant counterterm proportional to α_3, whose existence is partially due to our use of the simple, de Sitter breaking gauge [39,40], and partly to the time-ordering of interactions which seems unavoidable in the Schwinger-Keldysh formalism [56].

If we accept the α_2 counterterm as a field strength renormalization, and employ our result (52) for $\alpha_2 R$, the associated γ function is,

$$Z = 1 + \alpha_2 R + O(\kappa^4) \qquad \Longrightarrow \qquad \gamma \equiv \frac{\partial \ln(Z)}{\partial \ln(\mu^2)} = -\frac{\kappa^2 H^2}{8\pi^2} + O(\kappa^4) \; . \tag{89}$$

The exchange potential $\Phi(\eta, r)$ represents an integral of the 1PI 2-point function, so the Callan-Symanzik equation for it should read,

$$\Big[\frac{\partial}{\partial \ln(\mu)} - 2\gamma\Big]\Phi(t, r) = 0 \; . \tag{90}$$

If we replace the scale parameter μ by r, it will be seen that equation (90) exactly explains the leading 1-loop logarithm from the known tree order result (77),

$$\frac{\partial \Phi_1}{\partial \ln(r)} = -2 \times \frac{\kappa^2 H^2}{8\pi^2} \times \Phi_0 \quad \Longrightarrow \quad \Phi_1(t, r) \longrightarrow -\frac{\kappa^2 H^3 K}{32\pi^3} \ln^2(Hr) \; . \tag{91}$$

No similar explanation can be given for the late time correction (76) to the mode function. This seems to be because $u_1(\eta, k)$ is not a leading logarithm effect; indeed, it vanishes at late times. Replacing μ by a, or any other time variable, is also problematic because most of the time dependence of the mode function comes in the form of k/aH.

4 Conclusions

Our long term goal is to establish the reality of large loop corrections from inflationary gravitons (1) by purging the effective field equations of gauge dependence. The Introduction described a procedure which has already been implemented on flat space background for massless, minimally coupled scalars [33], and for electromagnetism [37]. We plan to generalize this procedure to de Sitter background and, for that purpose, we have sought the simplest system which exhibits large graviton loop corrections before the gauge purge. Although conformally coupled scalars are very simple, neither their mode function nor their exchange potential shows a large correction at 1-loop order [21]. The next simplest system is the massless, minimally coupled scalar which we analyzed in this paper. We found that there is no large correction to the 1-loop mode function (76), but the 1-loop exchange potential (87) does experience such a correction. Our main conclusion is therefore that the massless, minimally coupled scalar (9) is the system we have been seeking, and its exchange potential is the proper thing to study.

Section 2 employed dimensional regularization to compute the fully renormalized 1-loop graviton contribution to the scalar self-mass (55). Although we discovered some small mistakes in a previous computation [14], which do not alter the conclusion of that work that there are no growing secular corrections to the mode function, our biggest improvement is the use of a simple representation which is not burdened with cumbersome de Sitter invariant inverse differential operators. In section 3 we used the result to quantum-correct the linearized, Schwinger-Keldysh effective field equation (56). Specializing this equation to scalar radiation gave relation (68) for the 1-loop mode function, whose asymptotic late time solution is (76). We find no correction to the freeze-in amplitude of the mode function, but we do find a large temporal logarithm correction to the approach to freeze-in. Specializing (56) to the response to a point source gave relation (81) for the 1-loop exchange potential, whose asymptotic late time solution is (87). We find a large spatial logarithm correction to the exchange potential. It is significant that this correction derives entirely from the first of the source terms on the right hand side of equation (81). Of course that means we can focus on just this term when carrying out the gauge purge, which is a huge simplification and justifies the effort put into this study.

In section 3.4 we used the renormalization group to explain the large logarithm in the 1-loop exchange potential (87). This is significant for two reasons:

- It is the first time a large logarithm from inflationary gravitons has been explained using the renormalization group; and
- It ties the appearance of a large inflationary logarithm to the existence of the α_2 counterterm in (49).

The second point is relevant to the continuing controversy over the reality of graviton-induced logarithms because it means that the absence of inflationary logarithms at ℓ-loop order requires that divergences proportional to $(GR)^\ell \partial_\mu \phi \partial_\nu \phi g^{\mu\nu} \sqrt{-g}$ must vanish, for all ℓ, and in the absence of any symmetry argument. That seems to strain credulity.

Before closing we should adumbrate the subsequent steps in our program, its relation to the manner in which one combines Green's functions to produce an S-matrix, and our expectations for the fate of large logarithms. Our aim is to purge gauge dependence from the linearized effective field equations of massless fields. The procedure was described in some detail in the Introduction, and it has been explicitly carried out on flat space background for gravtion corrections to the massless, minimally coupled scalar [33] and for graviton corrections to electromagnetism [37]. What one does is to consider exactly the same Green's functions that would contribute to the scattering amplitude for the exchange of a massless particle between two massive particles. These Green's functions will include 2-point, 3-point and 4-point diagrams of the massive particle. Instead of going on-shell in momentum space (which would not be observable in cosmology) one applies a series relations derived by Donoghue [34–36] which reduce the 3-point and 4-point diagrams to 2-point form, without disturbing the long-range part of the amplitude. The flat space forms of these relations were given in equations (5-7), and we propose to make the most straightforward generalizations to de Sitter background. The next step is using the propagator equation for the massless field to regard the various 2-point diagrams as contributions to a gauge-independent 1PI 2-point function of the massless field, as per equation (8). We then use this 1PI 2-point function to quantum-correct the linearized effective field equation of the massless field. Gauge independence can be checked by repeating the computation in the 2-parameter family of simple, de Sitter-breaking gauges for which the graviton propagator has been derived [38]. Our expectation is that whatever large logarithmic corrections occurred with the gauge-dependent computation in the simplest gauge [39,40] will persist in the gauge-independent computation but with possibly different numerical coefficients.

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