

How Inflationary Gravitons Affect Gravitational Radiation

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ABSTRACT

We include the single graviton loop contribution to the linearized Einstein equation. Explicit results are obtained for one loop corrections to the propagation of gravitational radiation. Although suppressed by a minuscule loop-counting parameter, these corrections are enhanced by the square of the number of inflationary e-foldings. One consequence is that perturbation theory breaks down for a very long epoch of primordial inflation. Another consequence is that the one loop correction to the tensor power spectrum might be observable, in the far future, after the full development of 21cm cosmology.

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1 Introduction

Primordial inflation produces a vast ensemble of infrared gravitons [1,2]. If the tensor power spectrum for modes of co-moving wave number k is $\Delta_h^2(k)$, then the occupation number for each polarization of wave vector $\vec{k}$ is,

$$N(\eta, k) = \frac{\pi \Delta_h^2(k)}{64 G k^2} \times a^2(\eta) , \quad (1)$$

where $a(\eta)$ is the scale factor at conformal time η and G is Newton's constant. The aim of this paper is to study how these gravitons change the kinematics of other gravitons.

The background geometry of cosmology can be well described using spatially flat, conformal coordinates,

$$ds^2 = a^2(\eta) \left[-d\eta^2 + d\vec{x} \cdot d\vec{x} \right] \quad \implies \quad H \equiv \frac{a'}{a^2} , \quad \epsilon \equiv -\frac{H'}{aH^2} , \quad (2)$$

where $H(\eta)$ is the Hubble parameter and $\epsilon(\eta)$ is the first slow roll parameter. The special case of de Sitter ($\epsilon = 0$, constant H and $a(\eta) = -1/H\eta$) is often considered as a reasonable paradigm for inflation, and is attractive both because we possess analytic expressions for the graviton propagator [3,4] and because there is no mixing with whatever matter fields support inflation [5,6]. On this background the quantum-corrected, linearized Einstein equation takes the form,

$$\mathcal{D}^{\mu\nu\rho\sigma} h_{\rho\sigma}(x) - \int d^4 x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma} \right](x; x') h_{\rho\sigma}(x') = \frac{1}{2} \kappa T_{\text{lin}}^{\mu\nu}(x) , \quad (3)$$

where $\mathcal{D}^{\mu\nu\rho\sigma}$ is the gauge-fixed kinetic operator, $-i[{}^{\mu\nu}\Sigma^{\rho\sigma}](x; x')$ is the 1PI (one particle irreducible) 2-graviton function, $T_{\text{lin}}^{\mu\nu}(x)$ is the linearized stress-energy tensor, $\kappa^2 \equiv 16\pi G$ is the loop-counting parameter of quantum gravity and $h_{\mu\nu} \equiv (g_{\mu\nu} - a^2 \eta_{\mu\nu})/\kappa$ is the graviton field. In this paper we will show how to extract a fully renormalized result for the one loop graviton self-energy from an old, unregulated computation [7]. We then use this result to derive one loop corrections to the mode functions of dynamical gravitons.

In section 2 we derive a renormalized result for the part of the graviton self-energy which affects dynamical gravitons. Section 3 solves equation (3) for one loop corrections to transverse-traceless, spatial plane waves. Our conclusions comprise section 4. There we discuss the exciting possibility that the one loop correction we find might be observable.

2 Quantum Linearized Einstein Equation

The purpose of this section is to give an explicit expression for the linearized Einstein equation (3). We begin by defining the gauge-fixed kinetic operator $\mathcal{D}^{\mu\nu\rho\sigma}$, explaining how we represent the graviton self-energy, and working out $3+1$ decompositions of both. We then describe the process through which an old, unregulated computation of the graviton self-energy [7] can be used to recover most of the fully renormalized, Schwinger-Keldysh result. The section closes with a direct computation of the remaining contribution.

2.1 $3+1$ Decomposition

In $D = 3 + 1$ dimensions the gauge-fixed kinetic operator is [3,4],

$$\mathcal{D}^{\mu\nu\rho\sigma} = \frac{1}{2}\eta^{\mu(\rho}\eta^{\sigma)\nu}D_A - \frac{1}{4}\eta^{\mu\nu}\eta^{\rho\sigma}D_A + 2a^4H^2\delta_0^{(\mu}\eta^{\nu)(\rho}\delta^{\sigma)}_0, \quad (4)$$

where the kinetic operator of a massless, minimally coupled scalar is,

$$D_A = -a^2\left[\partial_0^2 + 2aH\partial_0 - \nabla^2\right]. \quad (5)$$

The $3+1$ components of $\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}$ are,

$$\mathcal{D}^{00\rho\sigma}h_{\rho\sigma} = \frac{1}{4}D_A(h_{00} + h_{kk}) - 2a^4H^2h_{00}, \quad (6)$$

$$\mathcal{D}^{0i\rho\sigma}h_{\rho\sigma} = -\frac{1}{2}D_B h_{0i}, \quad (7)$$

$$\mathcal{D}^{ij\rho\sigma}h_{\rho\sigma} = \frac{1}{2}D_A\left[h_{ij} + \frac{1}{2}\delta_{ij}(h_{00} - h_{kk})\right], \quad (8)$$

where D_B is the kinetic operator of a conformally coupled scalar,

$$D_B = -a^2\left[\partial_0^2 + 2aH\partial_0 - \nabla^2 + 2a^2H^2\right]. \quad (9)$$

On a cosmological background [2] the graviton self-energy can be expressed as a sum of 21 tensor differential operators $[\mu\nu\mathcal{D}^{\rho\sigma}]$ acting on scalar functions of η , η' and $\|\vec{x} - \vec{x}'\|$ [8],

$$-i\left[\mu\nu\Sigma^{\rho\sigma}\right](x; x') = \sum_{i=1}^{21}\left[\mu\nu\mathcal{D}_i^{\rho\sigma}\right]\times T^i(x; x'). \quad (10)$$

By general tensor analysis the 21 basis tensors are constructed from δ^μ_0 , the spatial part of the Minkowski metric $\bar{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} + \delta^\mu_0 \delta^\nu_0$ and the spatial derivative operator $\bar{\partial}^\mu \equiv \partial^\mu + \delta^\mu_0 \partial_0$. Table 1 lists the $[\mu\nu\mathcal{D}_i^{\rho\sigma}]$.

i	$[\mu\nu\mathcal{D}_i^{\rho\sigma}]$	i	$[\mu\nu\mathcal{D}_i^{\rho\sigma}]$	i	$[\mu\nu\mathcal{D}_i^{\rho\sigma}]$
1	$\bar{\eta}^{\mu\nu}\bar{\eta}^{\rho\sigma}$	8	$\bar{\partial}^\mu\bar{\partial}^\nu\bar{\eta}^{\rho\sigma}$	15	$\delta^{(\mu}_0\bar{\partial}^{\nu)}\delta^\rho_0\delta^\sigma_0$
2	$\bar{\eta}^{\mu(\rho}\bar{\eta}^{\sigma)\nu}$	9	$\delta^{(\mu}_0\bar{\eta}^{\nu)(\rho}\delta^\sigma_0$	16	$\delta^\mu_0\delta^\nu_0\bar{\partial}^\rho\bar{\partial}^\sigma$
3	$\bar{\eta}^{\mu\nu}\delta^\rho_0\delta^\sigma_0$	10	$\delta^{(\mu}_0\bar{\eta}^{\nu)(\rho}\bar{\partial}^{\sigma)}$	17	$\bar{\partial}^\mu\bar{\partial}^\nu\delta^\rho_0\delta^\sigma_0$
4	$\delta^\mu_0\delta^\nu_0\bar{\eta}^{\rho\sigma}$	11	$\bar{\partial}^{(\mu}\bar{\eta}^{\nu)(\rho}\delta^\sigma_0$	18	$\delta^{(\mu}_0\bar{\partial}^{\nu)}\delta^{(\rho}_0\bar{\partial}^{\sigma)}$
5	$\bar{\eta}^{\mu\nu}\delta^{(\rho}_0\bar{\partial}^{\sigma)}$	12	$\bar{\partial}^{(\mu}\bar{\eta}^{\nu)(\rho}\bar{\partial}^{\sigma)}$	19	$\delta^{(\mu}_0\bar{\partial}^{\nu)}\bar{\partial}^\rho\bar{\partial}^\sigma$
6	$\delta^{(\mu}_0\bar{\partial}^{\nu)}\bar{\eta}^{\rho\sigma}$	13	$\delta^\mu_0\delta^\nu_0\delta^\rho_0\delta^\sigma_0$	20	$\bar{\partial}^\mu\bar{\partial}^\nu\delta^{(\rho}_0\bar{\partial}^{\sigma)}$
7	$\bar{\eta}^{\mu\nu}\bar{\partial}^\rho\bar{\partial}^\sigma$	14	$\delta^\mu_0\delta^\nu_0\delta^{(\rho}_0\bar{\partial}^{\sigma)}$	21	$\bar{\partial}^\mu\bar{\partial}^\nu\bar{\partial}^\rho\bar{\partial}^\sigma$

Table 1: The 21 basis tensors used in expression (10). The pairs (3, 4), (5, 6), (7, 8), (10, 11), (14, 15), (16, 17) and (19, 20) are related by reflection.

Reflection invariance — $-i[\mu\nu\Sigma^{\rho\sigma}](x; x') = -i[\rho\sigma\Sigma^{\mu\nu}](x'; x)$ — relates the 7 pairs of $T^i(x; x')$ given in Table 2.

i	Relation	i	Relation
3, 4	$T^4(x; x') = +T^3(x'; x)$	14, 15	$T^{15}(x; x') = -T^{14}(x'; x)$
5, 6	$T^6(x; x') = -T^5(x'; x)$	16, 17	$T^{17}(x; x') = +T^{16}(x'; x)$
7, 8	$T^8(x; x') = +T^7(x'; x)$	19, 20	$T^{20}(x; x') = -T^{19}(x'; x)$
10, 11	$T^{11}(x; x') = -T^{10}(x'; x)$		

Table 2: Scalar coefficient functions in expression (10) which are related by reflection.

The factor $[\mu\nu\Sigma^{\rho\sigma}](x; x')h_{\rho\sigma}(x')$ in expression (3) has the 3 + 1 decomposition,

$$[{}^{00}\Sigma^{\rho\sigma}]h_{\rho\sigma} \longrightarrow iT^4h_{kk} + iT^{13}h_{00} + iT^{14}h_{0k,k} + iT^{16}h_{k\ell,k\ell}, \quad (11)$$

$$\begin{aligned}
\left[{}^{0i}\Sigma^{\rho\sigma}\right]h_{\rho\sigma} &\longrightarrow \frac{i}{2}\partial_i\left[T^6h_{kk}+T^{15}h_{00}+T^{18}h_{0k,k}+T^{19}h_{k\ell,k\ell}\right] \\
&\quad +\frac{i}{2}T^9h_{0i}+\frac{i}{2}T^{10}h_{ik,k}, \quad (12) \\
\left[{}^{ij}\Sigma^{\rho\sigma}\right]h_{\rho\sigma} &\longrightarrow i\delta_{ij}\left[T^1h_{kk}+T^3h_{00}+T^5h_{0k,k}+T^7h_{k\ell,k\ell}\right]+iT^2h_{ij} \\
&\quad +i\partial_{(i}\left[T^{11}h_{j)0}+T^{12}h_{j)k,k}\right]+i\partial_i\partial_j\left[T^8h_{kk}+T^{17}h_{00}+T^{20}h_{0k,k}+T^{21}h_{k\ell,k\ell}\right]. \quad (13)
\end{aligned}$$

In these relations we have sometimes exploited spatial transition invariance to partially integrate spatial derivatives from the coefficient functions $T^i(x; x')$ onto the graviton field.

2.2 The Quantum Correction

Let $S[g]$ represent the classical action of gravity, while $S_g[h, \bar{\theta}, \theta]$ stands for the ghost and gauge fixing action, and $\Delta S[g]$ is the counter-action. The one loop graviton self-energy can be expressed as the expectation value of the sum of four variational derivatives of these quantities,

$$\begin{aligned}
-i\left[{}^{\mu\nu}\Sigma^{\rho\sigma}\right](x; x') &= \left\langle \Omega \left| T^* \left[\left[\frac{i\delta S[g]}{\delta h_{\mu\nu}(x)} \right]_{hh} \left[\frac{i\delta S[g]}{\delta h_{\rho\sigma}(x')} \right]_{hh} + \left[\frac{i\delta S[g]}{\delta h_{\mu\nu}(x)} \right]_{\bar{\theta}\theta} \right. \right. \\
&\quad \times \left. \left[\frac{i\delta S[g]}{\delta h_{\rho\sigma}(x')} \right]_{\bar{\theta}\theta} + \left[\frac{i\delta^2 S[g]}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')} \right]_{hh} + \left[\frac{i\delta^2 \Delta S[g]}{\delta h_{\mu\nu}(x)\delta h_{\rho\sigma}(x')} \right]_1 \right| \Omega \right\rangle. \quad (14)
\end{aligned}$$

The various subscripts indicate how many graviton fields contribute and the T^* -ordering symbol signifies that any derivatives are taken after time ordering. Figure 1 gives the relevant Feynman diagrams.

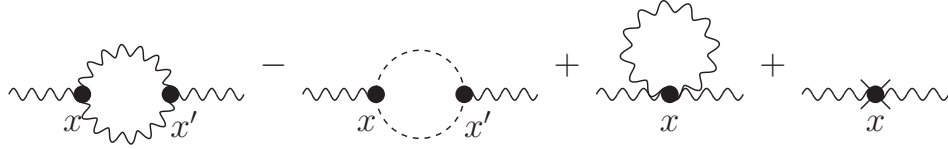


Figure 1: Diagrams contributing to the one loop graviton self-energy, shown in the same order, left to right, as the four contributions to (14). Graviton lines are wavy and ghost lines are dashed.

2.2.1 The $D = 4$ Result

To understand the unregulated result [7] it helps to consider how a computation of $-i[{}^{\mu\nu}\Sigma^{\rho\sigma}](x; x')$ would look in dimensional regularization. The

3-graviton and 4-graviton vertices take the general form [3,9]¹

$$\kappa a^{D-2} h \partial h \partial h \quad , \quad \kappa H a^{D-1} h h \partial h \quad , \quad (15)$$

$$\kappa^2 a^{D-2} h h \partial h \partial h \quad , \quad \kappa^2 H a^{D-1} h h h \partial h \quad . \quad (16)$$

Without worrying about indices (which come in a dizzying variety of different possibilities), contributions to the first two diagrams of Figure 1 have the nonlocal structure,

$$\kappa a^{D-2} \times \partial \partial' i \Delta(x; x') \times \partial \partial' i \Delta(x; x') \times \kappa a'^{D-2} \quad , \quad (17)$$

where $i \Delta(x; x')$ is a ghost or graviton propagator, and one derivative at each vertex could be exchanged for H times the appropriate scale factor. One should also bear in mind that if the external leg graviton field happens to be differentiated then minus its derivative is acted on the entire diagram. In contrast, the third diagram of Figure 1 is local,

$$\kappa^2 a^{D-2} \times \partial \partial' i \Delta(x; x') \times i \delta^D(x - x') \quad , \quad (18)$$

with the same stipulation about derivatives. The final diagram of Figure 1 is also local,

$$\frac{\kappa^2 a^{D-4}}{D-4} \times \partial^2 \partial'^2 \times i \delta^D(x - x') \quad , \quad (19)$$

where any number of the four derivatives could be exchanged for H times a scale factor.

The graviton and ghost propagators were defined by adding to the Lagrangian a functional whose D -dimensional extension is [3,4],

$$\mathcal{L}_{GF} = -\frac{a^{D-2}}{2} \eta^{\mu\nu} F_\mu F_\nu \quad , \quad F_\mu = \eta^{\rho\sigma} \left(h_{\mu\rho,\sigma} - \frac{1}{2} h_{\rho\sigma,\mu} + (D-2) a H h_{\mu\rho} \delta_\sigma^0 \right) . \quad (20)$$

The wonderful property of this gauge is that the gauge and graviton propagators are sums of constant tensor factors times scalar propagators,

$$i \left[{}_\mu \Delta_\rho \right] (x; x') = \bar{\eta}_{\mu\rho} \times i \Delta_A(x; x') - \delta_\mu^0 \delta_\rho^0 \times i \Delta_B(x; x') \quad , \quad (21)$$

$$i \left[{}_{\mu\nu} \Delta_{\rho\sigma} \right] (x; x') = \sum_{I=A,B,C} \left[{}_{\mu\nu} T_{\rho\sigma}^I \right] \times i \Delta_I(x; x') \quad . \quad (22)$$

¹Ghost-antighost-graviton vertices are similar to (15).

The constant tensor factors $[\mu\nu T_{\rho\sigma}^I]$ are,

$$[\mu\nu T_{\rho\sigma}^A] = 2\bar{\eta}_{\mu(\rho}\bar{\eta}_{\sigma)\nu} - \frac{2}{D-3}\bar{\eta}_{\mu\nu}\bar{\eta}_{\rho\sigma} \quad , \quad [\mu\nu T_{\rho\sigma}^B] = -4\delta^0_{(\mu}\bar{\eta}_{\nu)(\rho}\delta^0_{\sigma)} \quad , \quad (23)$$

$$[\mu\nu T_{\rho\sigma}^C] = \frac{2E_{\mu\nu}E_{\rho\sigma}}{(D-2)(D-3)} \quad , \quad E_{\mu\nu} \equiv (D-3)\delta^0_\mu\delta^0_\nu + \bar{\eta}_{\mu\nu} \quad . \quad (24)$$

The three scalar propagators are given in terms of the de Sitter length function $y(x; x') \equiv aa'H^2\Delta x^2$ and a function $A(y)$,

$$i\Delta_A(x; x') = A(y) + k \ln(aa') \quad k \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \quad , \quad (25)$$

$$i\Delta_B(x; x') = B(y) \equiv -\frac{[(4y-y^2)A'(y) + (2-y)k]}{2(D-2)} \quad , \quad (26)$$

$$i\Delta_C(x; x') = C(y) \equiv \frac{1}{2}(2-y)B(y) + \frac{k}{D-3} \quad . \quad (27)$$

The first derivative of $A(y)$ is [10, 11],

$$A'(y) = -\frac{H^{D-2}}{4(4\pi)^{\frac{D}{2}}} \left\{ \Gamma\left(\frac{D}{2}\right) \left(\frac{4}{y}\right)^{\frac{D}{2}} + \Gamma\left(\frac{D}{2}+1\right) \left(\frac{4}{y}\right)^{\frac{D}{2}-1} + \sum_{n=0}^{\infty} \left[\frac{\Gamma(n+\frac{D}{2}+2)}{\Gamma(n+3)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} - \frac{\Gamma(n+D)}{\Gamma(n+\frac{D}{2}+1)} \left(\frac{y}{4}\right)^n \right] \right\} \quad (28)$$

Because the y^n and $y^{n-\frac{D}{2}-2}$ terms cancel in $D=4$ dimensions they can only contribute when multiplied by an ultraviolet divergence.

Divergences only occur from taking a propagator to coincidence, that is, by setting $x'^\mu = x^\mu$. It follows that we can take the unregulated ($D=4$) limit in the first two diagrams of Figure 1 so long as $x'^\mu \neq x^\mu$. Of course that same stipulation makes the last two diagrams of Figure 1 vanish. Taking $D=4$ also results in a huge simplification of the ghost and graviton propagators,

$$i\left[\mu\Delta_{\rho}^{D=4}\right](x; x') = \frac{1}{4\pi^2} \left\{ \frac{\eta_{\mu\rho}}{aa'\Delta x^2} - \frac{1}{2}H^2 \ln(H^2\Delta x^2)\bar{\eta}_{\mu\rho} \right\} \quad , \quad (29)$$

$$i\left[\mu\nu\Delta_{\rho\sigma}^{D=4}\right](x; x') = \frac{1}{4\pi^2} \left\{ \frac{(2\eta_{\mu(\rho}\eta_{\sigma)\nu} - \eta_{\mu\nu}\eta_{\rho\sigma})}{aa'\Delta x^2} - H^2 \ln(H^2\Delta x^2) \left(\bar{\eta}_{\mu(\rho}\bar{\eta}_{\sigma)\nu} - \bar{\eta}_{\mu\nu}\bar{\eta}_{\rho\sigma} \right) \right\} \quad . \quad (30)$$

i	Coefficients $T_L^i(x; x')$ in expression (31)
1	$8a^2a'^2H^4 \times [\frac{4\Delta\eta^2}{\Delta x^6} + \frac{1}{\Delta x^4}] + 4a^3a'^3H^6 \times [\frac{4\Delta\eta^4}{\Delta x^6} - \frac{\Delta\eta^2}{\Delta x^4} + \frac{3}{\Delta x^2}]$
2	$-16a^2a'^2H^4 \times [\frac{4\Delta\eta^2}{\Delta x^6} + \frac{1}{\Delta x^4}] - 4a^3a'^3H^6 \times [\frac{8\Delta\eta^4}{\Delta x^6} + \frac{1}{\Delta x^2}]$
3	$8a^3a'^3H^6 \times [\frac{\Delta\eta^2}{\Delta x^4} - \frac{2}{\Delta x^2}] - 4a^3a'^2H^5 \times [\frac{4\Delta\eta^3}{\Delta x^6} - \frac{\Delta\eta}{\Delta x^4}]$
5	$16a^2a'^2H^4 \times \frac{\Delta\eta}{\Delta x^4} + 4a^3a'^2H^5 \times [\frac{2\Delta\eta^2}{\Delta x^4} + \frac{3}{\Delta x^2}]$
7	$-8a^2a'^2H^4 \times \frac{1}{\Delta x^2} - 2a^3a'^3H^6 \times \frac{\Delta\eta^2}{\Delta x^2} - 2a^3a'^2H^5 \times \frac{\Delta\eta}{\Delta x^2}$
9	$-96aa'H^2 \times [\frac{16\Delta\eta^4}{\Delta x^{10}} + \frac{12\Delta\eta^2}{\Delta x^8} + \frac{1}{\Delta x^6}] - 4a^2a'^2H^4 \times [\frac{24\Delta\eta^4}{\Delta x^8} + \frac{8\Delta\eta^2}{\Delta x^6} - \frac{1}{\Delta x^4}]$
10	$96aa'H^2 \times [\frac{2\Delta\eta^3}{\Delta x^8} + \frac{\Delta\eta}{\Delta x^6}] + 12a^2a'^2H^4 \times [\frac{4\Delta\eta^3}{\Delta x^6} + \frac{\Delta\eta}{\Delta x^4}] + a^3a'^3H^6 \times [\frac{8\Delta\eta^3}{\Delta x^4} - \frac{4\Delta\eta}{\Delta x^2}]$ $-8a^2a'H^3 \times [\frac{4\Delta\eta^2}{\Delta x^6} + \frac{1}{\Delta x^4}] - 4a^3a'^2H^5 \times [\frac{2\Delta\eta^2}{\Delta x^4} - \frac{1}{\Delta x^2}]$
12	$-8aa'H^2 \times [\frac{4\Delta\eta^2}{\Delta x^6} + \frac{1}{\Delta x^4}] - 2a^2a'^2H^4 \times [\frac{6\Delta\eta^2}{\Delta x^4} - \frac{9}{\Delta x^2}] + 4a^3a'^3H^6 \times \frac{\Delta\eta^2}{\Delta x^2}$
13	$-96aa'H^2 \times [\frac{16\Delta\eta^4}{\Delta x^{10}} + \frac{12\Delta\eta^2}{\Delta x^8} + \frac{1}{\Delta x^6}] + 4a^2a'^2H^4 \times [\frac{24\Delta\eta^4}{\Delta x^8} + \frac{56\Delta\eta^2}{\Delta x^6} + \frac{11}{\Delta x^4}]$ $+8a^3a'^3H^6 \times [\frac{4\Delta\eta^4}{\Delta x^6} - \frac{2\Delta\eta^2}{\Delta x^4} + \frac{3}{\Delta x^2}]$
14	$192aa'H^2 \times [\frac{2\Delta\eta^3}{\Delta x^8} + \frac{\Delta\eta}{\Delta x^6}] + 8a^2a'^2H^4 \times [\frac{4\Delta\eta^3}{\Delta x^6} - \frac{3\Delta\eta}{\Delta x^4}]$ $-16a^2a'H^3 \times [\frac{4\Delta\eta^2}{\Delta x^6} + \frac{1}{\Delta x^4}] - 16a^3a'^2H^5 \times [\frac{\Delta\eta^2}{\Delta x^4} + \frac{1}{\Delta x^2}]$
16	$-8aa'H^2 \times [\frac{4\Delta\eta^2}{\Delta x^6} + \frac{1}{\Delta x^4}] + 2a^2a'^2H^4 \times [-\frac{6\Delta\eta^2}{\Delta x^4} + \frac{1}{\Delta x^2}] - 2a^3a'^3H^6 \times \frac{\Delta\eta^2}{\Delta x^2}$ $+16a^2a'H^3 \times \frac{\Delta\eta}{\Delta x^4} + 6a^3a'^2H^5 \times \frac{\Delta\eta}{\Delta x^2}$
18	$-24aa'H^2 \times [\frac{4\Delta\eta^2}{\Delta x^6} + \frac{1}{\Delta x^4}] - 2a^2a'^2H^4 \times [\frac{6\Delta\eta^2}{\Delta x^4} + \frac{5}{\Delta x^2}]$
19	$8aa'H^2 \times \frac{\Delta\eta}{\Delta x^4} + 6a^2a'^2H^4 \times \frac{\Delta\eta}{\Delta x^2} - 4a^2a'H^3 \times \frac{1}{\Delta x^2}$

Table 3: Each tabulated term must be multiplied by $-\frac{\kappa^2}{64\pi^4}$.

Because one of the propagators in the nonlocal diagrams (17) might be undifferentiated, the functions $T^i(x; x')$ of equation (10) take the form,

$$T^i(x; x') \equiv T_N^i(x; x') + T_L^i(x; x') \times \ln(H^2 \Delta x^2). \quad (31)$$

where $T_L^i(x; x')$ (given in Table 3) and $T_N^i(x; x')$ (given in Table 4) are functions of a , a' , $\Delta\eta \equiv \eta - \eta'$ and inverse powers of the Poincaré interval $\Delta x^2 \equiv \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\varepsilon)^2$.

i	Coefficient $T_N^i(x; x')$ in expression (31)
1	$\frac{736}{\Delta x^8} - aa'H^2[\frac{616\Delta\eta^2}{\Delta x^8} + \frac{220}{\Delta x^6}] - a^2a'^2H^4[\frac{96\Delta\eta^4}{\Delta x^8} + \frac{812\Delta\eta^2}{\Delta x^6} + \frac{19}{\Delta x^4}] - a^3a'^3H^6[\frac{64\Delta\eta^4}{\Delta x^6} + \frac{22\Delta\eta^2}{\Delta x^4}]$
2	$\frac{1952}{\Delta x^8} - aa'H^2[\frac{416\Delta\eta^2}{\Delta x^8} + \frac{128}{\Delta x^6}] - a^2a'^2H^4[\frac{112\Delta\eta^2}{\Delta x^6} - \frac{56}{\Delta x^4}] + a^3a'^3H^6[\frac{32\Delta\eta^4}{\Delta x^6} + \frac{12\Delta\eta^2}{\Delta x^4}]$
3	$-184[\frac{8\Delta\eta^2}{\Delta x^{10}} + \frac{1}{\Delta x^8}] + 32aa'H^2[\frac{36\Delta\eta^4}{\Delta x^{10}} + \frac{14\Delta\eta^2}{\Delta x^8} - \frac{43}{\Delta x^6}]$ $-a^2a'^2H^4[\frac{288\Delta\eta^4}{\Delta x^8} - \frac{1228\Delta\eta^2}{\Delta x^6} + \frac{145}{\Delta x^4}] + 4a^3a'^3H^6[\frac{16\Delta\eta^4}{\Delta x^6} + \frac{5\Delta\eta^2}{\Delta x^4}]$ $-8aH[\frac{232\Delta\eta^3}{\Delta x^{10}} + \frac{203\Delta\eta}{\Delta x^8}] + 4a^2a'H^3[\frac{74\Delta\eta^3}{\Delta x^8} - \frac{319\Delta\eta}{\Delta x^6}] - a^3a'^2H^5 \times \frac{38\Delta\eta}{\Delta x^4}$
5	$\frac{368\Delta\eta}{\Delta x^8} - 32aa'H^2[\frac{9\Delta\eta^3}{\Delta x^8} - \frac{4\Delta\eta}{\Delta x^6}] + 4a^2a'^2H^4[\frac{8\Delta\eta^3}{\Delta x^6} - \frac{\Delta\eta}{\Delta x^4}]$ $+232aH[\frac{2\Delta\eta^2}{\Delta x^8} + \frac{1}{\Delta x^6}] - 2a^2a'H^3[\frac{28\Delta\eta^2}{\Delta x^6} - \frac{23}{\Delta x^4}] - a^3a'^2H^5 \times \frac{12\Delta\eta^2}{\Delta x^4}$
7	$-\frac{92}{\Delta x^6} + aa'H^2[\frac{24\Delta\eta^2}{\Delta x^6} + \frac{7}{\Delta x^4}] + a^2a'^2H^4[\frac{8\Delta\eta^2}{\Delta x^4} - \frac{3}{\Delta x^2}] + a^3a'^3H^6 \times \frac{\Delta\eta^2}{\Delta x^2}$ $-aH \times \frac{116\Delta\eta}{\Delta x^6} - a^2a'H^3 \times \frac{23\Delta\eta}{\Delta x^4} + a^3a'^2H^5 \times \frac{\Delta\eta}{\Delta x^2}$
9	$-16[\frac{488\Delta\eta^2}{\Delta x^{10}} + \frac{61}{\Delta x^8}] + 16aa'H^2[\frac{88\Delta\eta^4}{\Delta x^{10}} - \frac{10\Delta\eta^2}{\Delta x^8} - \frac{35}{\Delta x^6}] + 4a^2a'^2H^4[\frac{8\Delta\eta^4}{\Delta x^8} + \frac{2\Delta\eta^2}{\Delta x^6} - \frac{3}{\Delta x^4}]$
10	$\frac{976\Delta\eta}{\Delta x^8} - 16aa'H^2[\frac{8\Delta\eta^3}{\Delta x^8} - \frac{23\Delta\eta}{\Delta x^6}] - 2a^2a'^2H^4[\frac{16\Delta\eta^3}{\Delta x^6} - \frac{23\Delta\eta}{\Delta x^4}]$ $+a^3a'^3H^6 \times \frac{4\Delta\eta^3}{\Delta x^4} + aH \times \frac{64}{\Delta x^6} + 8a^2a'H^3[\frac{4\Delta\eta^2}{\Delta x^6} - \frac{5}{\Delta x^4}] - a^3a'^2H^5 \times \frac{4\Delta\eta^2}{\Delta x^4}$
12	$-\frac{488}{\Delta x^6} + \frac{32}{3}aa'H^2[\frac{\Delta\eta^2}{\Delta x^6} - \frac{1}{\Delta x^4}] - a^2a'^2H^4[\frac{10\Delta\eta^2}{\Delta x^4} - \frac{8}{\Delta x^2}] - a^3a'^3H^6 \times \frac{2\Delta\eta^2}{\Delta x^2}$
13	$16[\frac{336\Delta\eta^4}{\Delta x^{12}} + \frac{336\Delta\eta^2}{\Delta x^{10}} + \frac{63}{\Delta x^8}] + 4aa'H^2[\frac{336\Delta\eta^4}{\Delta x^{10}} + \frac{868\Delta\eta^2}{\Delta x^8} + \frac{409}{\Delta x^6}]$ $+a^2a'^2H^4[\frac{424\Delta\eta^4}{\Delta x^8} + \frac{144\Delta\eta^2}{\Delta x^6} + \frac{557}{\Delta x^4}] - 24a^3a'^3H^6[\frac{4\Delta\eta^4}{\Delta x^6} - \frac{\Delta\eta^2}{\Delta x^4}]$
14	$-672[\frac{8\Delta\eta^3}{\Delta x^{10}} + \frac{\Delta\eta}{\Delta x^8}] - 8aa'H^2[\frac{30\Delta\eta^3}{\Delta x^8} + \frac{107\Delta\eta}{\Delta x^6}] - 4a^2a'^2H^4[\frac{16\Delta\eta^3}{\Delta x^6} - \frac{7\Delta\eta}{\Delta x^4}]$ $-16aH[\frac{4\Delta\eta^2}{\Delta x^8} + \frac{35}{\Delta x^6}] - 2a^2a'H^3[\frac{68\Delta\eta^2}{\Delta x^6} + \frac{137}{\Delta x^4}] + a^3a'^2H^5 \times \frac{16\Delta\eta^2}{\Delta x^4}$
16	$4[\frac{84\Delta\eta^2}{\Delta x^8} + \frac{13}{\Delta x^6}] + aa'H^2[\frac{24\Delta\eta^2}{\Delta x^6} + \frac{127}{\Delta x^4}] + a^2a'^2H^4[\frac{6\Delta\eta^2}{\Delta x^4} + \frac{5}{\Delta x^2}]$ $-a^3a'^3H^6 \times \frac{3\Delta\eta^2}{\Delta x^2} - aH \times \frac{28\Delta\eta}{\Delta x^6} + a^2a'H^3 \times \frac{49\Delta\eta}{\Delta x^4} + a^3a'^2H^5 \times \frac{\Delta\eta}{\Delta x^2}$
18	$8[\frac{168\Delta\eta^2}{\Delta x^8} + \frac{29}{\Delta x^6}] + 4aa'H^2[\frac{12\Delta\eta^2}{\Delta x^6} - \frac{23}{\Delta x^4}] + a^2a'^2H^4 \times \frac{10\Delta\eta^2}{\Delta x^4}$
19	$-\frac{112\Delta\eta}{\Delta x^6} - 8aa'H^2 \times \frac{\Delta\eta}{\Delta x^4} - a^2a'^2H^4 \times \frac{\Delta\eta}{\Delta x^2} + aH \times \frac{50}{\Delta x^4}$
21	$\frac{14}{\Delta x^4}$

Table 4: Each of the tabulated terms must be multiplied by $-\frac{\kappa^2}{64\pi^4}$.

2.2.2 Recovering the Renormalized Result

We have developed a 4-step procedure for converting the unregulated expressions of Tables [3](#) and [4](#) to fully renormalized results:

1. Express $T_L^i(x; x')$ as derivatives acting on integrable functions;
2. Pass the multiplicative factor of $\ln(H^2\Delta x^2)$ through the derivatives;
3. Express the remainder $\Delta T_L^i(x; x')$ from step 2, plus $T_N^i(x; x')$, as derivatives acting on integrable functions and $1/\Delta x^4$; and
4. Renormalize the factors of $1/\Delta x^4$ from step 3.

We will provide details and explain the rationale for each step below. As an example, we also implement the steps on the crucial coefficient function $T^i(x; x')$ which controls corrections to the graviton mode function,

$$T_L^2(x; x') = -\frac{\kappa^2 \ln(H^2\Delta x^2)}{64\pi^4} \left\{ a^2 a'^2 H^4 \left[-\frac{64\Delta\eta^2}{\Delta x^6} - \frac{16}{\Delta x^4} \right] + a^3 a'^3 H^6 \left[-\frac{32\Delta\eta^4}{\Delta x^6} - \frac{4}{\Delta x^2} \right] \right\}, \quad (32)$$

$$T_N^2(x; x') = -\frac{\kappa^2}{64\pi^4} \left\{ \frac{1952}{3} \frac{1}{\Delta x^8} + aa' H \left[-\frac{416\Delta\eta^2}{\Delta x^8} - \frac{128}{3} \frac{1}{\Delta x^6} \right] + a^2 a'^2 H^4 \left[-\frac{112}{3} \frac{\Delta\eta^2}{\Delta x^6} + \frac{56}{\Delta x^4} \right] + a^3 a'^3 H^6 \left[\frac{32\Delta\eta^4}{\Delta x^6} + \frac{12\Delta\eta^2}{\Delta x^4} \right] \right\}. \quad (33)$$

The rationale for **Step 1** derives from the genesis of the logarithm contribution to the coefficient functions ([31](#)). The only way a factor of $\ln(H^2\Delta x^2)$ can survive from one of the ghost or graviton propagators ([29-30](#)) in a non-local contribution like ([17](#)), is for all the derivatives to act on the other propagator. It is this differentiated propagator, times the scale factors from the vertices, that contribute to $T_L^i(x; x')$. Hence we must be able to express each $T_L^i(x; x')$ as a sum of terms involving scale factors times a number of derivatives acting on one of three integrable expressions,

$$\frac{1}{\Delta x^2} \quad , \quad \frac{\Delta\eta}{\Delta x^2} \quad , \quad \frac{\Delta\eta^2}{\Delta x^2}. \quad (34)$$

Implementing **Step 1** on expression (32) for $T_L^i(x; x')$ gives,

$$T_L^2(x; x') = -\frac{\kappa^2 \ln(H^2 \Delta x^2)}{64\pi^4} \left\{ a^2 a'^2 H^4 \times -8\partial_0^2 \left(\frac{1}{\Delta x^2} \right) \right. \\ \left. + a^3 a'^3 H^6 \left[-4\partial_0^2 \left(\frac{\Delta \eta^2}{\Delta x^2} \right) + 20\partial_0 \left(\frac{\Delta \eta}{\Delta x^2} \right) - \frac{16}{\Delta x^2} \right] \right\}. \quad (35)$$

In **Step 2** we pass the overall multiplicative factor of $\ln(H^2 \Delta x^2)$ through the derivatives to multiply the three integrable functions (34). Implementing this on expression (35) for $T_L^i(x; x')$ gives,

$$T_L^2(x; x') = -\frac{\kappa^2}{64\pi^4} \left\{ a^2 a'^2 H^4 \times -8\partial_0^2 \left(\frac{\ln(H^2 \Delta x^2)}{\Delta x^2} \right) + a^3 a'^3 H^6 \left[-4\partial_0^2 \right. \right. \\ \left. \times \left(\frac{\Delta \eta^2 \ln(H^2 \Delta x^2)}{\Delta x^2} \right) + 20\partial_0 \left(\frac{\Delta \eta \ln(H^2 \Delta x^2)}{\Delta x^2} \right) - \frac{16 \ln(H^2 \Delta x^2)}{\Delta x^2} \right] \left. \right\} \\ - \frac{\kappa^2}{64\pi^4} \left\{ a^2 a'^2 H^4 \left[-\frac{96\Delta \eta^2}{\Delta x^6} - \frac{1}{\Delta x^4} \right] + a^3 a'^3 H^6 \times -\frac{48\Delta \eta^4}{\Delta x^6} \right\}. \quad (36)$$

We can at this stage identify six integrable functions, with a factor of $2\pi i$ extracted for future convenience,

$$2\pi i A_1 \equiv \frac{\ln(H^2 \Delta x^2)}{\Delta x^2}, \quad 2\pi i A_2 \equiv \frac{1}{\Delta x^2}, \quad (37)$$

$$2\pi i B_1 \equiv \frac{\Delta \eta \ln(H^2 \Delta x^2)}{\Delta x^2}, \quad 2\pi i B_2 \equiv \frac{\Delta \eta}{\Delta x^2}, \quad (38)$$

$$2\pi i C_1 \equiv \frac{\Delta \eta^2 \ln(H^2 \Delta x^2)}{\Delta x^2}, \quad 2\pi i C_2 \equiv \frac{\Delta \eta^2}{\Delta x^2}. \quad (39)$$

Implementing **Step 2** allows us to express $T_L^i(x; x')$ as a sum of logarithm terms involving A_1 , B_1 and C_1 , which we term “Group 1”, plus a “remainder” $\Delta T_L^i(x; x')$ which is devoid of logarithms. For example, expression (36) for $T_L^2(x; x')$ can be written as,

$$T_L^2 = -\frac{i\kappa^2}{32\pi^3} \left\{ -8a^2 a'^2 H^4 \partial_0^2 A_1 - 4a^3 a'^3 H^6 \left[\partial_0^2 C_1 - 5\partial_0 B_1 - 4A_1 \right] \right\} + \Delta T_L^2, \quad (40)$$

where the remainder term is,

$$\Delta T_L^2(x; x') = -\frac{\kappa^2}{64\pi^4} \left\{ a^2 a'^2 H^4 \left[-\frac{96\Delta \eta^2}{\Delta x^6} - \frac{1}{\Delta x^4} \right] + a^3 a'^3 H^6 \times -\frac{48\Delta \eta^4}{\Delta x^6} \right\}. \quad (41)$$

Step 3 begins by combining $T_N^i(x; x')$ with the remainder $\Delta T_L^i(x; x')$. For the case of $T^2(x; x')$ this consists of expressions (33) and (41),

$$T_N^2(x; x') + \Delta T_L^2(x; x') = -\frac{\kappa^2}{64\pi^4} \left\{ \frac{1952}{5\Delta x^8} + aa'H \left[-\frac{416\Delta\eta^2}{\Delta x^8} - \frac{128}{\Delta x^6} \right] \right. \\ \left. + a^2a'^2H^4 \left[-\frac{400}{3}\frac{\Delta\eta^2}{\Delta x^6} + \frac{40}{\Delta x^4} \right] + a^3a'^3H^6 \left[-\frac{16\Delta\eta^4}{\Delta x^6} + \frac{12\Delta\eta^2}{\Delta x^4} \right] \right\}. \quad (42)$$

The resulting expressions can harbor ultraviolet divergences. One can see this by once again extracting derivatives,

$$T_N^2(x; x') + \Delta T_L^2(x; x') \\ = -\frac{\kappa^2}{64\pi^4} \left\{ \partial^4 \left(\frac{61}{30\Delta x^4} \right) + aa'H \left[-\partial_0^2 \left(\frac{52}{3\Delta x^4} \right) + \partial^2 \left(\frac{10}{3\Delta x^4} \right) \right] + a^2a'^2H^4 \right. \\ \left. \times \left[-\partial_0^2 \left(\frac{50}{3\Delta x^2} \right) + \frac{220}{3\Delta x^4} \right] + a^3a'^3H^6 \left[-2\partial_0^2 \left(\frac{\Delta\eta^2}{\Delta x^2} \right) + 16\partial_0 \left(\frac{\Delta\eta}{\Delta x^2} \right) - \frac{12}{\Delta x^2} \right] \right\}. \quad (43)$$

In addition to the three integrable functions (34) one also encounters logarithmically divergent factors of $1/\Delta x^4$. We consign the ultraviolet finite factors of A_2 , B_2 and C_2 to “Group 2” and reserve the factors of $1/\Delta x^4$ for further analysis,

$$T_N^2 + \Delta T_L^2 = -\frac{i\kappa^2}{32\pi^3} \left\{ -\frac{50}{3}a^2a'^2H^4\partial_0^2A_2 - 2a^3a'^3H^6 \left[\partial_0^2C_2 - 8\partial_0B_2 + 6A_2 \right] \right\} \\ -\frac{\kappa^2}{64\pi^4} \left\{ \partial^4 \left(\frac{61}{30\Delta x^4} \right) - aa'H^2 \left[\frac{52}{3}\partial_0^2 - \frac{10}{3}\partial^2 \right] \frac{1}{\Delta x^4} + \frac{220}{3}a^2a'^2H^4 \left(\frac{1}{\Delta x^4} \right) \right\}. \quad (44)$$

Step 4 is devoted to reducing and renormalizing the factor of $1/\Delta x^4$. In the dimensionally regulated computation it would derive from $1/\Delta x^{2D-4}$, from which we extract a factor of ∂^2 ,

$$\frac{1}{\Delta x^4} \longrightarrow \frac{1}{\Delta x^{2D-4}} = \frac{\partial^2}{2(D-3)(D-4)} \left[\frac{1}{\Delta x^{2D-6}} \right]. \quad (45)$$

The factor of $1/\Delta x^{2D-6}$ this produces is integrable in $D = 4$ dimensions, so we could take the unregulated limit except for the explicit factor of $1/(D-4)$. We can add zero in the form of the propagator equation for a massless scalar

in flat space [10, 11],

$$\begin{aligned} \frac{1}{\Delta x^4} &\longrightarrow \frac{\partial^2}{2(D-3)(D-4)} \left[\frac{1}{\Delta x^{2D-6}} \right] \\ &= \frac{\partial^2}{2(D-3)(D-4)} \left[\frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}} \right] + \frac{\mu^{D-4} 4\pi^{\frac{D}{2}} i \delta^D(x-x')}{2(D-3)(D-4)\Gamma(\frac{D}{2}-1)}. \end{aligned} \quad (46)$$

We can take the unregulated limit of the nonlocal part of (46),

$$\frac{\partial^2}{2(D-3)(D-4)} \left[\frac{1}{\Delta x^{2D-6}} - \frac{\mu^{D-4}}{\Delta x^{D-2}} \right] \longrightarrow -\frac{\partial^2}{4} \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2} \right] \equiv -\frac{\partial^2}{4} [2\pi i A_3]. \quad (47)$$

We call these ultraviolet finite terms “Group 3A”.

The local divergence in the final part of expression (46) requires explanation. First, recall from the generic expression (17) that the nonlocal diagrams of Figure 1 inherit a factor of $(aa')^{D-2}$ from the two vertices. Now, note from expressions (25-28) that the first terms in the expansions of the dimensionally regulated ghost and graviton propagators (21-22) each have the form,

$$\frac{H^{D-2}\Gamma(\frac{D}{2}-1)}{(4\pi)^{\frac{D}{2}}} \left(\frac{4}{y}\right)^{\frac{D}{2}-1} = \frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}} \left(\frac{1}{aa'\Delta x^2}\right)^{\frac{D}{2}-1}. \quad (48)$$

This means that the scale factors have no dependence on D , even in the fully regulated expression. On the other hand, the counterterms — the final diagram in Figure 1 — each inherit a factor of a^D from the universal $\sqrt{-g}$. Hence there is always a slight mismatch between the primitive divergences of the nonlocal diagrams and the counterterms which cancel them. This mismatch results in a local $\ln(a)$ in the renormalized, unregulated limit,

$$\begin{aligned} \frac{\mu^{D-4} 4\pi^{\frac{D}{2}} i \delta^D(x-x')}{2(D-3)(D-4)\Gamma(\frac{D}{2}-1)} - \frac{a^{D-4} \mu^{D-4} 4\pi^{\frac{D}{2}} i \delta^D(x-x')}{2(D-3)(D-4)\Gamma(\frac{D}{2}-1)} \\ \longrightarrow -2\pi^2 i \times \ln(a) \delta^4(x-x'). \end{aligned} \quad (49)$$

These local terms are termed “Group 3B”.

2.2.3 The Schwinger-Keldysh Result

The factors of $\delta^4(x-x')$ in Group 3B are already real and causal, however, the various nonlocal factors (47) and (37-39) are not. This is because Feynman diagrams represent in-out matrix elements rather than true expectation

values. The Schwinger-Keldysh formalism [12–16] is a diagrammatic technique for computing true expectation values that is almost as simple to use as Feynman diagrams. Expectation values of the graviton field obey effective field equations which are nonlocal but real and causal [17–19].

It is simple to convert an in-out 1PI N -point function such as the graviton self-energy to the Schwinger-Keldysh formalism. Without giving the derivation, the rules are [20]:

- Each spacetime point is assigned a polarity $\pm$.
- One consequence is that every Feynman propagator $i\Delta(x; x')$ gives rise to four Schwinger-Keldysh propagators $i\Delta_{\pm\pm}(x; x')$. The $++$ propagator is the same as the Feynman propagator and the $--$ propagator is its complex conjugate. The $-+$ propagator is the free expectation of the field at x^μ times the field at x'^μ , and the $+-$ propagator is the free expectation value of the reverse-ordered product.
- Another consequence is that each vertex has a $\pm$ polarity. The $+$ vertices are the same as those of the in-out formalism while the $-$ vertices are complex conjugates.
- A final consequence is that every in-out 1PI N -point function gives rise to 2^N N -point functions in the Schwinger-Keldysh formalism.
- The factor of $[\mu\nu\Sigma^{\rho\sigma}](x; x')$ in the linearized quantum Einstein equation (3) is replaced by the sum of $[\mu\nu\Sigma_{++}^{\rho\sigma}](x; x')$, which is the same as the in-out result, and $[\mu\nu\Sigma_{+-}^{\rho\sigma}](x; x')$.
- On our simple background (2), one can infer the result for $[\mu\nu\Sigma_{+-}^{\rho\sigma}](x; x')$ from that for $[\mu\nu\Sigma^{\rho\sigma}](x; x')$ by dropping all the local contributions of Group 3B, multiplying the nonlocal terms by -1 , and converting the coordinate interval Δx^2 from

$$\Delta x_{++}^2(x; x') \equiv \left\| \vec{x} - \vec{x}' \right\|^2 - \left(|\eta - \eta'| - i\varepsilon \right)^2, \quad (50)$$

to

$$\Delta x_{+-}^2(x; x') \equiv \left\| \vec{x} - \vec{x}' \right\|^2 - \left(\eta - \eta' + i\varepsilon \right)^2. \quad (51)$$

Implementing these rules is straightforward. First, recall that the only dependence on the coordinate interval Δx^2 in the nonlocal results of Groups

1, 2, and 3A comes through the integrable functions A_{1-3} , B_{12} and C_{1-2} , which were defined in expressions (37-39) and (47). We can eliminate the factors of $1/\Delta x^2$ by extracting derivatives. For example, the $++$ and $+-$ versions of $2\pi i \times A_1$ are,

$$2\pi i \times A_1 = \frac{\ln(H^2 \Delta x_{+\pm}^2)}{\Delta x_{+\pm}^2} = \frac{\partial^2}{8} \left[\ln^2(H^2 \Delta x_{+\pm}^2) - 2 \ln(H^2 \Delta x_{+\pm}^2) \right]. \quad (52)$$

Because the scale factors and the derivatives are identical in the $++$ and $+-$ contributions, we need just consider differences of logarithms,

$$\ln(H^2 \Delta x_{++}^2) - \ln(H^2 \Delta x_{+-}^2) = 2\pi i \times \theta(\Delta\eta - r), \quad (53)$$

$$\ln^2(H^2 \Delta x_{++}^2) - \ln^2(H^2 \Delta x_{+-}^2) = 4\pi i \times \theta(\Delta\eta - r) \ln[H^2(\Delta\eta^2 - r^2)], \quad (54)$$

where $r \equiv \|\vec{x} - \vec{x}'\|$. The final reductions are,

$$A_1 \longrightarrow +\frac{\partial^2}{4} \left\{ \theta(\Delta\eta - r) \left[\ln[H^2(\Delta\eta^2 - r^2)] - 1 \right] \right\}, \quad (55)$$

$$B_1 \longrightarrow -\frac{\partial_0}{2} \left\{ \theta(\Delta\eta - r) \ln[H^2(\Delta\eta^2 - r^2)] \right\}, \quad (56)$$

$$C_1 \longrightarrow +\frac{\partial_0^2}{4} \left\{ \theta(\Delta\eta - r)(r^2 - \Delta\eta^2) \left[\ln[H^2(\Delta\eta^2 - r^2)] - 1 \right] \right\} \\ + \frac{1}{2} \theta(\Delta\eta - r) \ln[H^2(\Delta\eta^2 - r^2)], \quad (57)$$

$$A_2 \longrightarrow +\frac{\partial^2}{4} \left\{ \theta(\Delta\eta - r) \right\}, \quad (58)$$

$$B_2 \longrightarrow -\frac{\partial_0}{2} \left\{ \theta(\Delta\eta - r) \right\}, \quad (59)$$

$$C_2 \longrightarrow +\frac{\partial_0^2}{4} \left\{ \theta(\Delta\eta - r)(r^2 - \Delta\eta^2) \right\} + \frac{1}{2} \theta(\Delta\eta - r), \quad (60)$$

$$A_3 \longrightarrow +\frac{\partial^2}{4} \left\{ \theta(\Delta\eta - r) \left[\ln[\mu^2(\Delta\eta^2 - r^2)] - 1 \right] \right\}. \quad (61)$$

$$(62)$$

2.3 The 4-Point Contribution

The previous discussion concerned the two nonlocal diagrams of Figure 1 and the local counterterms needed to renormalize them. There are also finite

local contributions from the 3rd diagram. It derives from the 42 4-graviton interactions given in equation (4.1) of [9]. One connects two of the graviton fields to the external legs and then replaces the remaining two fields by graviton propagator. The procedure is tedious and we shall content ourselves with simply sketching it and giving the final result.

As an example we reduce the first of the 42 interactions,

$$\mathcal{S}_1 \equiv \frac{\kappa^2}{32} \int d^D x a^{D-2} h^2 h_{,\theta} h^{,\theta}, \quad (63)$$

where a comma denotes differentiation and the trace of the graviton field is $h \equiv h^\alpha_\alpha \equiv \eta^{\alpha\beta} h_{\alpha\beta}$. We first take the variational derivatives of the action integral with respect to $h_{\mu\nu}(x)$ and $h_{\rho\sigma}(x')$ as in expression (14),

$$\begin{aligned} \frac{i\delta^2 \mathcal{S}_1}{\delta h_{\mu\nu}(x) \delta h_{\rho\sigma}(x')} &= \frac{\kappa^2}{32} \eta^{\mu\nu} \eta^{\rho\sigma} \left\{ -\partial_\theta \left[2a^{D-2} h^\alpha_\alpha(x) h^\beta_\beta(x) \partial^\theta i\delta^D(x-x') \right] \right. \\ &\quad + 4a^{D-2} h^\alpha_\alpha(x) h^\beta_{\beta,\theta}(x) \partial^\theta i\delta^D(x-x') - 4\partial^\theta \left[a^{D-2} h^\alpha_\alpha(x) h^\beta_{\beta,\theta}(x) i\delta^D(x-x') \right] \\ &\quad \left. + 2a^{D-2} h^\alpha_{\alpha,\theta}(x) h^\beta_{\beta,\theta}(x) i\delta^D(x-x') \right\}. \quad (64) \end{aligned}$$

Now take the expectation value of the T^* -ordered product, which amounts to replacing the remaining two graviton fields of each term by the appropriate coincident (and sometimes differentiated) propagator,

$$\begin{aligned} \left\langle \Omega \left| T^* \left[\frac{i\delta^2 \mathcal{S}_1}{\delta h_{\mu\nu}(x) \delta h_{\rho\sigma}(x')} \right] \right| \Omega \right\rangle &= \frac{\kappa^2}{32} \eta^{\mu\nu} \eta^{\rho\sigma} \left\{ -\partial_\theta \left[2a^{D-2} \times i \left[{}^\alpha_\alpha \Delta^\beta_\beta \right] (x; x) \right. \right. \\ &\quad \times \partial^\theta i\delta^D(x-x') \left. \right] + 4a^{D-2} \times \partial'_\theta i \left[{}^\alpha_\alpha \Delta^\beta_\beta \right] (x; x') \Big|_{x'=x} \times \partial^\theta i\delta^D(x-x') - 4\partial^\theta \left[a^{D-2} \right. \\ &\quad \times i\delta^D(x-x') \times \partial'_\theta i \left[{}^\alpha_\alpha \Delta^\beta_\beta \right] (x; x') \left. \right] + 2a^{D-2} i\delta^D(x-x') \times \partial_\theta \partial'^\theta i \left[{}^\alpha_\alpha \Delta^\beta_\beta \right] (x; x') \left. \right\}. \quad (65) \end{aligned}$$

Finally, we express the tensor structure using the 21 basis tensors of Group 1,

$$\eta^{\mu\nu} \eta^{\rho\sigma} = \left(\bar{\eta}^{\mu\nu} - \delta^\mu_0 \delta^\nu_0 \right) \left(\bar{\eta}^{\rho\sigma} - \delta^\rho_0 \delta^\sigma_0 \right), \quad (66)$$

$$= \left[{}^{\mu\nu} \mathcal{D}_1^{\rho\sigma} \right] - \left[{}^{\mu\nu} \mathcal{D}_3^{\rho\sigma} \right] - \left[{}^{\mu\nu} \mathcal{D}_4^{\rho\sigma} \right] + \left[{}^{\mu\nu} \mathcal{D}_{13}^{\rho\sigma} \right]. \quad (67)$$

The coincidence limits of the three propagators which appear in the graviton propagator (22) are,

$$i\Delta_A(x; x) = k \left[-\pi \cot\left(\frac{\pi D}{2}\right) + 2 \ln(a) \right] \quad , \quad i\Delta_B(x; x) = -\frac{k}{D-2} \quad , \quad (68)$$

$$i\Delta_C(x; x) = \frac{k}{(D-2)(D-3)} \quad , \quad k \equiv \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \quad . \quad (69)$$

Note that only the undifferentiated A -type propagator is ultraviolet divergent in dimensional regularization. The undifferentiated A -type propagator is also the only way to get a factor of $\ln(a)$. First derivatives of coincident propagators are all finite,

$$\partial_\alpha i\Delta_A(x; x') \Big|_{x'=x} = aHk\delta_\alpha^0 \quad , \quad \partial_\alpha i\Delta_B(x; x') \Big|_{x'=x} = 0 = \partial_\alpha i\Delta_C(x; x') \Big|_{x'=x} \quad . \quad (70)$$

Mixed second derivatives are also finite,

$$\partial_\alpha \partial'_\beta i\Delta_A(x; x') \Big|_{x'=x} = -\left(\frac{D-1}{D}\right) kH^2 g_{\alpha\beta} \quad , \quad (71)$$

$$\partial_\alpha \partial'_\beta i\Delta_B(x; x') \Big|_{x'=x} = \frac{1}{D} kH^2 g_{\alpha\beta} \quad , \quad (72)$$

$$\partial_\alpha \partial'_\beta i\Delta_C(x; x') \Big|_{x'=x} = -\frac{2}{D(D-2)} kH^2 g_{\alpha\beta} \quad . \quad (73)$$

Note that all primitive contributions have factors of a^{D-2} , a^{D-1} or a^D . The counterterms which absorb ultraviolet divergences possess the very same dependence on a so renormalization engenders no finite factors of $\ln(a)$ the way it did for the nonlocal diagrams of expression (49). It does produce factors of $\ln(H/\mu)$ but we report only the $\ln(a)$ contributions to $iT^2(x; x')$,

$$\frac{\kappa^2 \ln(a)}{32\pi^2} \times 8a^2 H^2 (\partial_0 + 2aH) \partial_0 \delta^4(x - x') \quad . \quad (74)$$

We call this class of contributions “Group 4”.

3 The Effect on Gravitational Radiation

The purpose of this section is to solve (3) for $T_{\text{lin}}^{\mu\nu}(x) = 0$ to derive one loop corrections to the graviton mode function. We begin by deriving an

equation for the graviton mode function, then consider the one loop response to various sources. To simplify the computation we specialize the various source terms to late times and small wave numbers. Finally, we compute the source integrals and read off the late time result for the one loop correction.

Dynamical gravitons correspond to zero stress tensor and the wave form,

$$h_{\mu\nu}(x; \vec{k}) = \epsilon_{\mu\nu}(\vec{k})u(\eta, k)e^{i\vec{k}\cdot\vec{x}} \quad , \quad 0 = \epsilon_{\mu 0} = \epsilon_{ij}k_j = \epsilon_{ii} . \quad (75)$$

The graviton polarization tensors $\epsilon_{\mu\nu}(\vec{k})$ is identical to the usual ones from flat space. From their properties, and the $3+1$ decomposition (6.8) of the gauge-fixed kinetic operator, we find,

$$\mathcal{D}^{\mu\nu\rho\sigma}h_{\rho\sigma}(x; \vec{k}) = \epsilon^{\mu\nu}(\vec{k})e^{i\vec{k}\cdot\vec{x}} \times -\frac{1}{2}a^2 \left[\partial_0^2 + 2Ha\partial_0 + k^2 \right] u(\eta, k) . \quad (76)$$

The graviton wave form (75) implies $0 = h_{00} = h_{0i} = h_{ij,j}$. Using these relations in the $3+1$ decomposition (11.13) of the quantum source term implies,

$$\begin{aligned} \int d^4x' \left[{}^{\mu\nu}\Sigma^{\rho\sigma} \right] (x; x') h_{\rho\sigma}(x'; \vec{k}) \\ = \epsilon^{\mu\nu}(\vec{k})e^{i\vec{k}\cdot\vec{x}} \times \int d^4x' iT_{SK}^2(x; x')u(\eta', k)e^{-i\vec{k}\cdot(\vec{x}-\vec{x}')} . \end{aligned} \quad (77)$$

Equating the left hand side (76) with the right hand side (77) gives the graviton mode equation,

$$-\frac{1}{2}a^2 \left[\partial_0^2 + 2Ha\partial_0 + k^2 \right] u(\eta, k) = \int d^4x' iT_{SK}^2(x; x')u(\eta', k)e^{-i\vec{k}\cdot\Delta\vec{x}} . \quad (78)$$

The graviton mode equation (78) is valid to all orders, but we only possess one loop results for the coefficient function $T_{SK}^2(x; x')$. Hence we must solve the equation perturbatively. The 0th order mode function, and its asymptotic late time expansion, are,

$$u_0(\eta, k) = \frac{H}{\sqrt{2k^3}} \left[1 - \frac{ik}{Ha} \right] \exp \left[\frac{ik}{Ha} \right] \longrightarrow \frac{H}{\sqrt{2k^3}} \left[1 + \frac{k^2}{2H^2a^2} + \dots \right] . \quad (79)$$

The one loop correction we seek obeys the equation,

$$-\frac{1}{2}a^2 \left[\partial_0^2 + 2Ha\partial_0 + k^2 \right] u_1(\eta, k) = \int d^4x' iT_{SK}^2(x; x')u_0(\eta', k)e^{-i\vec{k}\cdot\Delta\vec{x}} \equiv S(\eta) . \quad (80)$$

Finally, let us consider the relation between possible late time behaviors for the quantum source $S(\eta)$ and the corresponding asymptotic late time form of the one loop mode function,

$$S = a^4 \ln(a) \implies u_1 \longrightarrow -\frac{\ln^2(a)}{3H^2} \quad , \quad S = a^4 \implies u_1 \longrightarrow -\frac{2\ln(a)}{3H^2} \quad , \quad (81)$$

$$S = a^3 \ln(a) \implies u_1 \longrightarrow +\frac{\ln(a)}{H^2 a} \quad , \quad S = a^3 \implies u_1 \longrightarrow +\frac{1}{H^2 a} \quad , \quad (82)$$

$$S = a^2 \ln(a) \implies u_1 \longrightarrow +\frac{\ln(a)}{H^2 a^2} \quad , \quad S = a^2 \implies u_1 \longrightarrow +\frac{1}{H^2 a^2} \quad . \quad (83)$$

Let us start with the local contributions of Groups 3B and 4,

$$S_{3B} = \frac{\kappa^2 \ln(a)}{32\pi^2} \left\{ -\frac{61}{30} \partial^4 u_0 + aH^2 \left[\frac{52}{3} \partial_0^2 - \frac{10}{3} \partial^2 \right] (au_0) - \frac{220}{3} a^4 H^4 u_0 \right\}, \quad (84)$$

$$S_4 = \frac{\kappa^2 \ln(a)}{32\pi^2} \left\{ 8a^2 H^2 \partial_0^2 u_0 + 16a^3 H^3 \partial_0 u_0 \right\}, \quad (85)$$

where $\partial^2 \rightarrow -(\partial_0^2 + k^2)$. Expression (85) was studied previously when making the Hartree approximation [21], but it is simple enough to evaluate both local contributions (84-85) using relations such as,

$$\partial_0 u_0(\eta, k) = \frac{H}{\sqrt{2k^3}} \times k^2 \eta e^{-ik\eta} \quad , \quad \partial_0^2 u_0(\eta, k) = \frac{H}{\sqrt{2k^3}} \left(k^2 - ik^3 \eta \right) e^{-ik\eta} \quad , \quad (86)$$

$$(\partial_0^2 + k^2)^2 u_0(\eta, k) = 0 \quad , \quad (\partial_0^2 + k^2) (au_0(\eta, k)) = 2a^3 H^2 u_0(\eta, k) \quad . \quad (87)$$

Hence (84-85) are,

$$S_{3B}(\eta) = \frac{\kappa^2 H^4}{32\pi^2} \times a^4 \ln(a) \times \left[-32 - \frac{52}{3} \frac{k^2}{a^2 H^2} \right] u_0(\eta, k) \quad , \quad (88)$$

$$S_4(\eta) = \frac{\kappa^2 H^4}{32\pi^2} \times a^4 \ln(a) \times \left[0 - \frac{8k^2}{a^2 H^2} \right] u_0(\eta, k) \quad . \quad (89)$$

However, to recover the leading late time behavior we can set $\partial_0^2 + k^2$ to just ∂_0^2 , and $u_0(\eta, k)$ to just $u_0(0, k)$,

$$S_{3B}(\eta) \longrightarrow \frac{\kappa^2 H^4}{4\pi^2} \times u_0(0, k) \times -4a^4 \ln(a) \quad , \quad (90)$$

$$S_4(\eta) \longrightarrow \frac{\kappa^2 H^4}{4\pi^2} \times u_0(0, k) \times 0 \quad . \quad (91)$$

Old results [22, 23] could be exploited to evaluate the nonlocal contributions from Groups 1, 2 and 3A exactly, the same way we did with the local contribution (88). However, if we are only interested in the leading behavior at late times, as in expression (90), we can replace $u_0(\eta', k)e^{-i\vec{k}\cdot\Delta\vec{x}}$ with $u_0(0, k)$, which means that ∂^2 degenerates to just $-\partial_0^2$. We also change ∂_0 to $-\partial_0'$ and partially integrate to act on any primed scale factors, or give zero if there are no primed scale factors. The source integrals for Groups 1, 2 and 3A are,

$$S_1 \longrightarrow \frac{\kappa^2}{32\pi^3} \times u_0(0, k) \times \int d^4x' \theta(\Delta\eta - r) \left\{ -144a^3a'^5H^8 \ln[H^2(\Delta\eta^2 - r^2)] \right. \\ \left. + [240a^2a'^6H^8 + 48a^3a'^5H^8 - 360a^3a'^7H^{10}(r^2 - \Delta\eta^2)] \right. \\ \left. \times [\ln[H^2(\Delta\eta^2 - r^2)] - 1] \right\}, \quad (92)$$

$$S_2 \longrightarrow \frac{\kappa^2}{32\pi^3} \times u_0(0, k) \times \int d^4x' \theta(\Delta\eta - r) \left\{ 500a^2a'^6H^8 - 72a^3a'^5H^8 \right. \\ \left. - 180a^3a'^7H^{10}(r^2 - \Delta\eta^2) \right\}, \quad (93)$$

$$S_{3A} \longrightarrow \frac{\kappa^2}{32\pi^3} \times u_0(0, k) \times \int d^4x' \theta(\Delta\eta - r) \left\{ [930aa'^7H^8 - 550a^2a'^6H^8] \right. \\ \left. \times [\ln[\mu^2(\Delta\eta^2 - r^2)] - 1] \right\}. \quad (94)$$

The next step is to perform the spatial integrations,

$$\int d^3x' \theta(\Delta\eta - r) = \frac{4\pi}{3} \Delta\eta^3 \theta(\Delta\eta), \quad (95)$$

$$\int d^3x' \theta(\Delta\eta - r) \ln[H^2(\Delta\eta^2 - r^2)] = \frac{8\pi}{3} \left[\ln(2H\Delta\eta) - \frac{4}{3} \right] \Delta\eta^3 \theta(\Delta\eta), \quad (96)$$

$$\int d^3x' \theta(\Delta\eta - r)(r^2 - \Delta\eta^2) = -\frac{8\pi}{15} \Delta\eta^5 \theta(\Delta\eta), \quad (97)$$

$$\int d^3x' \theta(\Delta\eta - r)(r^2 - \Delta\eta^2) \ln[H^2(\Delta\eta^2 - r^2)] \\ = -\frac{8\pi}{15} \left[2 \ln(2H\Delta\eta) - \frac{31}{15} \right] \Delta\eta^5 \theta(\Delta\eta). \quad (98)$$

We also change variables from η' to $a' = -1/H\eta'$,

$$\int_{\eta_i}^{\eta} d\eta' = \int_1^a \frac{da'}{Ha'^2}, \quad (99)$$

where the initial time $\eta_i = -1/H$ corresponds to unit scale factor. Note also that $a'H\Delta\eta = 1 - a'/a$. These reductions allow us to express (92-94) as,

$$S_1 \longrightarrow \frac{\kappa^2 H^4}{4\pi^2} u_0(0, k) \int_1^a da' \left\{ -48a^3 \left(1 - \frac{a'}{a}\right)^3 \left[\ln(2H\Delta\eta) - \frac{4}{3} \right] + \left[80a^2 a' + 16a^3 \right] \right. \\ \left. \times \left(1 - \frac{a'}{a}\right)^3 \left[\ln(2H\Delta\eta) - \frac{11}{6} \right] + 48a^3 \left(1 - \frac{a'}{a}\right)^5 \left[\ln(2H\Delta\eta) - \frac{23}{15} \right] \right\}, \quad (100)$$

$$S_2 \longrightarrow \frac{\kappa^2 H^4}{4\pi^2} u_0(0, k) \int_1^a da' \left\{ \left[\frac{250}{3} a^2 a' - 12a^3 \right] \left(1 - \frac{a'}{a}\right)^3 + 12a^3 \left(1 - \frac{a'}{a}\right)^5 \right\}, \quad (101)$$

$$S_{3A} \longrightarrow \frac{\kappa^2 H^4}{4\pi^2} u_0(0, k) \int_1^a da' \left[310aa'^2 - \frac{550}{3} a^2 a' \right] \left(1 - \frac{a'}{a}\right)^3 \left[\ln(2\mu\Delta\eta) - \frac{11}{6} \right]. \quad (102)$$

It remains to perform the temporal integrations. These can be done exactly but we give only the coefficients of the leading $a^4 \ln(a)$ dependence,

$$S_1 \longrightarrow \frac{\kappa^2 H^4}{4\pi^2} \times u_0(0, k) \times -4a^4 \ln(a), \quad (103)$$

$$S_2 \longrightarrow \frac{\kappa^2 H^4}{4\pi^2} \times u_0(0, k) \times 0, \quad (104)$$

$$S_{3A} \longrightarrow \frac{\kappa^2 H^4}{4\pi^2} \times u_0(0, k) \times 4a^4 \ln(a). \quad (105)$$

Combining these with the local results (90-91) we see that the late time asymptotic form of the total source is,²

$$S(\eta) \longrightarrow \frac{\kappa^2 H^4}{4\pi^2} \times u_0(0, k) \times -4a^4 \ln(a). \quad (106)$$

In view of expression (81) it follows that the asymptotic form of the one loop correction to the mode function is,

$$u_1(\eta, k) \longrightarrow \frac{\kappa^2 H^2}{4\pi^2} \times u_0(0, k) \times \frac{4}{3} \ln^2(a). \quad (107)$$

²It is interesting to note that the terms of Groups 3A and 3B, which ultimately descended from factors of $1/\Delta x^4$, cancel one another at leading order. It is also interesting that the nonzero contribution from Group 1 derives entirely from A_1 because the B_1 and C_1 contributions also cancel.

4 Epilogue

In section 2 of this paper we give a 4-step procedure for converting an old, unregulated computation [7] of the single graviton loop contribution to the self-energy $-i[\mu\nu\Sigma^{\rho\sigma}](x;x')$ to the fully renormalized result, with only one additional computation needed for the 4-point contribution. The technique laid out in Section 2 has been used to obtain results for all 21 of the coefficient functions $T^i(x;x')$ in the representation (110), and these will be reported in a subsequent work [24]. Here we give only the coefficient $T^2(x;x')$ which enters the equation (78) for the mode function $u(\eta,k)$ of plane wave gravitons (75). In section 3 we applied this result to derive the asymptotic late time form (107) for one loop corrections to $u(\eta,k)$.

There are two striking features about our result (107). First, it grows with time. We saw in section 3 that this growth derives entirely from the “tail” part of the graviton propagator, which is associated with the continual production of gravitons with Hubble-scale momenta. The physical interpretation is that the spin-spin coupling allows our test graviton to remain in interaction with these gravitons, long after its own kinetic energy has red-shifted to insignificance. The interaction produces a growth in the amplitude for the simple reason that scattering between very long wavelength and shorter wavelength particles tends to transfer momentum from the latter to the former. Even though the loop counting parameter $\kappa^2 H^2 \lesssim 10^{-10}$ is minuscule for actual inflation, this growth must eventually cause perturbation theory to break down if inflation persists long enough.

The second striking feature of our result is that it implies a potentially observable one loop correction to the tensor power spectrum,

$$\Delta_h^2(k) \simeq \frac{16\hbar G H^2(t_k)}{\pi c^3} \left\{ 1 + \frac{16\hbar G H^2(t_k)}{3\pi c^3} \times N_k^2 + O(G^2) \right\}, \quad (108)$$

where $H(t_k)$ is the value of the Hubble parameter at horizon crossing (defined by $a(t_k)H(t_k) = k$) and N_k is the number of e-foldings from horizon crossing to the end of inflation.³ No one knows how small the tree order power spectrum is, but if it is near the current upper bound of 10^{-10} [27] then the large factor of N_k^2 in (108) might allow the one loop correction to be resolved in the far future, after the full development of 21cm cosmology [28–30].

³Note that our result obeys the bound on loop corrections to the primordial power spectrum derived by the late Steven Weinberg [25, 26].

The great uncertainty in all this is the gauge issue. The graviton propagator requires gauge fixing, and how one accomplishes that can affect loop corrections. This is well known in flat space. For example, the numerical coefficient $\frac{61}{30}$ in the first term of equation (84) derives from flat space. When the graviton self-energy is re-computed in the most general Poincaré invariant gauge ($\mathcal{L} = -\frac{1}{2\alpha}F_\mu F^\mu$, with $F_\mu = \partial^\nu h_{\mu\nu} - \frac{\beta}{2}\partial_\mu h$), this numerical coefficient becomes [31],

$$\frac{61}{30} \longrightarrow \frac{1}{2}\alpha^2 + \frac{3}{4}\alpha - \frac{43}{60} + \frac{1}{6}\frac{(\alpha-3)^2}{(\beta-2)^4} - \frac{7}{4}\frac{(\alpha-3)}{(\beta-2)^3} - \frac{1}{12}\frac{(\alpha-51)}{(\beta-2)^2} + \frac{1}{12}\frac{(9\alpha-11)}{(\beta-2)}. \quad (109)$$

This particular term makes zero correction to $u(\eta, k)$, and the gauge dependence of the distinctly de Sitter terms which produce (107) is not known. Computations on de Sitter background are so difficult that all but one [32] of the graviton loop results [7,9,33-40] so far obtained have been made using the simplest gauge [3,4]. The single exception [32] was a re-computation of the vacuum polarization [37] in a 1-parameter family of de Sitter invariant gauges [41]. When that result was used to compute quantum gravitational corrections the photon field strength [42], the same asymptotic time dependence was found as for the simple, de Sitter breaking gauge [43], but with a different numerical coefficient, and no dependence at all on the gauge parameter. One must therefore assume a slight gauge dependence in cosmological solutions to the naive effective field equations.

John Donoghue has demonstrated how general relativity can be used as a low energy effective field theory on flat space background to derive quantum gravitational corrections to the long-range potentials carried by massless particles such as photons and gravitons [44,45]. The method is first to compute the scattering amplitude between two massive particles that exchange the massless field, then infer the exchange potential using inverse scattering theory. That is how gauge independent, graviton loop corrections were derived for the Newtonian [46,47] and Coulomb [48] potentials.

It has recently been shown that Donoghue’s S-matrix technique can be applied directly to produce gauge independent effective field equations, without computing cosmologically unobservable scattering amplitudes [49]. The new procedure employs position space versions of crucial identities Donoghue and collaborators derived to isolate the nonlocal and nonanalytic parts of scattering amplitudes that affect long-range potentials [45,50]. These “Donoghue Identities” reduce internal massive propagators to delta functions, which

causes higher-point contributions to 2-particle scattering to assume a form that can be regarded as corrections to the 1PI 2-point function of the massless exchange particle. The new technique was implemented at one loop order for quantum gravitational corrections to a massless scalar on flat space background to derive a result which is independent of the gauge parameters α and β [49]. The same technique has just been applied to quantum gravitational corrections to electrodynamics on flat space background [51], and strenuous efforts are underway to generalize it to de Sitter background [52]. The two flat space exercises demonstrate that gauge independent results typically show the same spacetime dependence as in a fixed gauge, but with different numerical coefficients. We expect that the same thing will be found on de Sitter background.

We should also comment on the breakdown of perturbation theory which must occur over the course of a very long period of inflation during which $\ln(a)$ becomes larger than $1/\kappa H$. This would be relevant to Λ -driven inflation in which inflation is driven by a positive cosmological constant, without any scalar inflaton, and the self-gravitation of inflationary gravitons gradually builds up to stop inflation [53, 54]. Of course one cannot trust our one loop result (107) when $\kappa^2 H^2 \ln^2(a) \sim 1$ because unknown higher loop corrections could be equally or even more important. What is needed is a way to sum up the leading logarithms at each order. Starobinsky has developed a stochastic formalism that accomplishes this for the similar large logarithms produced in scalar potential models [55, 56]. This technique captures large logarithms that arise from the “tail” part of the propagator [57], but it fails for large logarithms which have an ultraviolet origin, as can sometimes happen with the derivative interactions of quantum gravity [58]. The renormalization group was designed to sum up large logarithms from the ultraviolet, but it fails to recover large logarithms from the “tail” part of the propagator [59]. Our one loop result (107) is a purely “tail” effect, but it is not known what happens at higher loops, and one loop quantum gravitational corrections to electrodynamics are known to derive from both sources [60]. It would be interesting to try summing all large quantum gravitational logarithms by combining a variant of the renormalization group with some version of Starobinsky’s technique.

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