TOPOLOGY OPTIMIZATION DESIGN OF STRUCTURES BASED ON EIGENFREQUENCY MATCHING

Daniel Giraldo-Guzmán(1) **Clifford Lissenden(2)** **Parisa Shokouhi(2)** **Mary Frecker(1)**

(1) Department of Mechanical Engineering
Pennsylvania State University
University Park, PA

(2) Department of Engineering Science and Mechanics
Pennsylvania State University
University Park, PA

ABSTRACT

We demonstrate the design of resonating structures using a density-based topology optimization approach, which requires the eigenfrequencies to match a set of target values. To develop a solution, several optimization modules are implemented, including material interpolation models, penalization schemes, filters, analytical sensitivities, and a solver. Moreover, common challenges in topology optimization for dynamic systems and their solutions are discussed. In this study, the objective function is to minimize the error between the target and actual eigenfrequency values. The finite element method is used to compute the eigenfrequencies at each iteration. To solve the optimization problem, we use the sequential linear programming algorithm with move limits, enhanced by a filtering technique. Finally, we present a resonator design as a case study and analyze the design process with different optimization parameters.

Keywords: Topology Optimization, Eigenfrequency Matching, Resonator Design, Wave Propagation Control

1. INTRODUCTION

Structural optimization has become an indispensable tool in simulation-based designs through size, shape, material, and topology optimization techniques [1,2]. An important branch of this discipline focuses on the structural dynamic response to vibration, either to prevent failure or enhance dynamic structural properties. The applications are diverse, including control of wind-induce vibrations in tall buildings with optimized tuned mass dampers [3], control of eigenfrequency distribution in micro-mechanical resonator design [4], optimal configuration of a Hard Disk Drive (HDD) arm to prevent bending by constraining the mass moment of inertia [5], and in general, vibration control of structures by passive, active, semi-active and hybrid schemes [6]. One of the most commonly used methods in these diverse applications is topology optimization [7], which provides a systematic structural design approach.

The Topology Optimization Method (TOM) is a computational method to devise complex layouts in various applications such as micro-robots, structural aerodynamics, sound radiation, or nano-optical crystals [8]. Initially intended to solve structural design problems [9], the applications of TOM are now extended to diverse multi-physics problems in acoustics, fluids, optics, electromagnetics, materials, among others [10,11]. An example is the structural vibration control design, including either active or passive control mechanisms [12-14], where the dynamic response of the structure constitutes an essential part of the optimization problem formulation. This class of problems has motivated the introduction of the TOM in applications that involve optimization of eigenvalues and eigenvectors. Supported by various developments in the calculation of eigenvalues and eigenvectors derivatives [15], the seminal work of Díaz and Kikuchi [16] introduced the homogenization method to solve these kinds of problems. Since then, studies have been focusing on maximizing eigenfrequencies while preventing localized modes [17-19], tracking mode shapes and mode order switching [20], proposing optimized shapes for sound radiation [21], additive manufacturing [22], turbomachinery components [23], material design [24], and many others.

The three principal design requirements of a topology optimization problem involving eigenfrequencies are [25]: (i) maximizing the first eigenfrequency of the structure [17], usually to reduce low-frequency vibrations [19,26,27], (ii) generating a gap in between eigenfrequencies or around a certain frequency [28], typically used to prevent vibrations around a resonance frequency [29], and (iii) matching the eigenfrequencies of the structure with a reference set of desired frequencies [25], which is the focus of this paper. Maximizing eigenfrequencies requires the user to decide whether or not to follow the position of a specified eigenvalue and its mode shape, due to the order changes a mode can experience during the optimization process [30]. To overcome this problem, a mean-eigenvalue formulation [25] can be used to consider multiple eigenfrequencies of the structure. To generate a gap, it is necessary to take the difference between two adjacent eigenfrequencies or to take the ratio between the squared eigenfrequencies [28]. To address the problem of modes switching positions, a double-bound formulation is used to ensure that all the eigenfrequencies are greater or less than the bounds defining the gap. Lastly, the eigenfrequency matching approach has been found in few works. The general formulation to match eigenfrequencies was proposed by Ma *et al*. [25]. Maeda *et al*. [31] introduced this formulation to the Homogenization Design Method (HDM) for topology optimization. Then, Yamasaki *et al*. [32] explored the use of level-set methods for minimum compliance, maximization of eigenfrequencies and eigenfrequency matching. Nishizu *et al*. [33] used the same formulation to find topologies for damaged structures characterized with Non-Destructive Evaluation (NDT), and finally, Bernal *et al*. [34] used this approach to create topology optimized structures to propose a material characterization technique by impulse excitation.

The overarching goal of this study is to find topologically optimized resonators for a resonant meta-surface, i.e., to design resonating structures that can be coupled to the surface of a host structure (waveguide) and control the wave propagation in the waveguide. In search of resonating structures that fulfill a predefined set of requirements, including resonance at a particular frequency or set of frequencies, the TOM has the potential to be used as the design tool. The objective of this work is to propose a design methodology for resonating structures based on eigenfrequency matching using a density-based TOM. Our investigation focuses on developing the necessary combination of techniques to solve a general topology optimization problem, where a structure is to be designed to have a predefined set of eigenfrequencies.

The remainder of this paper is organized in three sections: (i) the problem formulation is presented as a general problem that minimizes the error between a reference set of eigenfrequencies and the actual eigenfrequencies, (ii) the solution methods section presents in detail the programming technique, sensitivity derivation, interpolation models, and filters used in topology optimization. Finally, (iii) the results section demonstrates a case study serving as a reference solution, for which a parametric study is conducted. A final discussion is presented to summarize our findings.

2. OPTIMIZATION PROBLEM

To develop a design methodology, it is necessary to generalize the optimization problem. To do so, the optimization problem formulation describes a relation between the eigenfrequencies of a structure and a reference set of predefined eigenfrequencies. The idea is to systematically modify the structure's eigenfrequencies until the desired match is achieved. However, simultaneously matching multiple eigenvalues is rather challenging; therefore, the idea is to get as close as possible to the reference set of values with minimum error.

Here, the objective function is defined as the minimization of the error between a set of target eigenfrequencies g_q and the actual eigenfrequencies of the structure f_q. An L_2-norm measures the cumulative relative error summed over all the modes. A set of scalar weights w_q is used to control the influence of each mode. The optimization problem formulation is shown in Equation (1):

$$
\begin{aligned}
&\min_{\boldsymbol{\rho}} \left[\sum_{q=1}^{m} w_q \left(\frac{f_q - g_q}{g_q} \right)^2 \right]^{\frac{1}{2}} \\
&\text{s.t. } \textstyle\sum_{e=1}^{N_e} \rho_e V_e - V_{max} \leq 0 \\
&0 < \rho_{min} \leq \rho_e \leq 1 \\
&\big([K] - \lambda_q [M]\big)\{\Phi_q\} = 0
\end{aligned}
\qquad (1)
$$

where f_q and λ_q are the q^{th} eigenfrequency and eigenvalue, respectively, g_q is the q^{th} reference eigenfrequency, m is the total number of modes considered, w_q is the q^{th} weighting coefficient, and N_e is the number of finite elements. Since ρ_e and V_e are the pseudo-density and volume of element e, respectively, the factor $\rho_e V_e$ is the effective volume of such element. In addition, V_{max} is maximum volume allowed, ρ_{min} is the minimum allowed pseudo-density used to prevent numerical problems, and $[K]$ and $[M]$ are the global stiffness and mass matrices, respectively. Finally, Φ_q is the q^{th} eigenvector corresponding to the q^{th} eigenvalue λ_q.

3. SOLUTION METHODS

To solve a topology optimization problem, different approaches have been used, *e.g.*, optimality criteria, moving asymptotes, sequential programming, genetic algorithms, or level-set methods [35,36]. Among these methods, linear programming uses highly efficient and reliable algorithms to solve non-linear problems [37]. Specifically, Sequential Linear Programming (SLP) is a popular method in structural optimization to deal with the nonlinear nature of complex problems, in essence, because of its simplicity and the possibility to use linear solvers, *e.g.*, Simplex [38]. However, a topology optimization problem is by definition a binary problem, where the design variables should be either zero or one, representing void or solid material. Therefore, a relaxation of the solution space is necessary to allow for intermediate values between 0 and 1, mathematically allowing for intermediate material properties between solid and void [39]. To describe these intermediate material properties, it is necessary to introduce a material interpolation model. Two material models will be analyzed in this work, starting with the most popular model called Solid Isotropic Material with Penalization (SIMP) [40], and an alternative model, the Rational Approximation of Material Properties (RAMP) [41]. Moreover, it is necessary to deal with other known problems in density-based topology optimization such as checkerboard solutions, mesh dependency, numerical instabilities, and lack of solutions. To address these issues, filters or "Regularization Schemes" have been widely used [42,43]. In this work, the projection filter [44] is used.

This section is divided in three subsections: Sequential Linear Programming (SLP), Material Interpolation Models and, Filters for Topology Optimization. The first subsection presents our strategy to solve this non-linear optimization problem including the details of the sensitivity analysis. Next, two material interpolation models are presented and compared. Lastly, different filters commonly used in topology optimization are introduced.

3.1 Sequential Linear Programming (SLP)

The SLP algorithm is used to solve non-linear optimization problems, linearizing them through a first-order Taylor series around the current design variables [45]. Thereby, a linear optimization problem can be solved using efficient algorithms for this type of problem (*e.g.,* Simplex method). A linearized optimization problem is then expressed as:

$$\min_{\boldsymbol{\rho}} \nabla \boldsymbol{f}(\rho_0)^T \boldsymbol{\rho}$$

where ρ_0 is the linearization point. Note that this formulation does not include the constant terms in the Taylor series, as they do not contribute to the optimal solution. The linearized optimization problem presented in Equation (1) involves taking the gradient of the objective function, a process known as sensitivity analysis. Thus:

$$\min_{\boldsymbol{\rho}} \frac{\partial}{\partial \rho_k} \left[\sum_{q=1}^{m} w_q \left(\frac{f_q - g_q}{g_q} \right)^2 \right]^{1/2} \rho_k \qquad (2)$$

Therefore, the linearized objective function simplifies to:

$$\min_{\boldsymbol{\rho}} \left[\sum_{q=1}^{m} w_q \left(\frac{f_q - g_q}{g_q} \right)^2 \right]^{-1/2} \left[\sum_{q=1}^{m} \frac{w_q \left(f_q - g_q \right)}{4\pi g_q^2 \sqrt{\lambda_q}} \frac{\partial \lambda_q}{\partial \rho_k} \right] \rho_k \qquad (3)$$

where:

$$\frac{\partial \lambda_q}{\partial \rho_k} = \frac{\Phi_q^T \left(\frac{\partial [K]}{\partial \rho_k} - \lambda_q \frac{\partial [M]}{\partial \rho_k} \right) \Phi_q}{\Phi_q^T [M] \Phi_q}$$

3.1.1 Move Limits

The numerical efficiency of the SLP method often depends on an appropriate choice of the move limits [46]. As part of the SLP implementation in this work, a heuristic criterion for the move limits was developed. At each iteration, the slope of the objective function for the preceding three iterations is analyzed to determine the trend of the function. This trend is then used to decide on stretching or shrinking the limit values in order to allow taking larger or smaller steps. This move limits criterion was tailored to improve performance, stability, and convergence.

Figure 1 presents six possible objective function trends considering three iteration points (two slope changes). Depending on whether the objective is increasing or decreasing, the move limits change to prevent the optimization from progressing in the undesired direction. In the case of the objective going in the desired direction (gradually decreasing), the limit values are "relaxed" to allow larger steps in the search direction. In the other case, *i.e.,* the objective going in the undesired direction, the limits shrink to the smallest values to prevent further progression in that direction. A special case occurs when the objective function is oscillating as depicted in Figure 1 (right-hand side plots), which is a common observation near convergence. In this case, the move limits shrink to control the optimal search direction by taking smaller steps. Although the move limits values must be tuned according to the problem; in this work, the limits were set in a range from 0.02 to 0.16 with a continuity scheme that gradually decreases the step size as the optimization goes on.

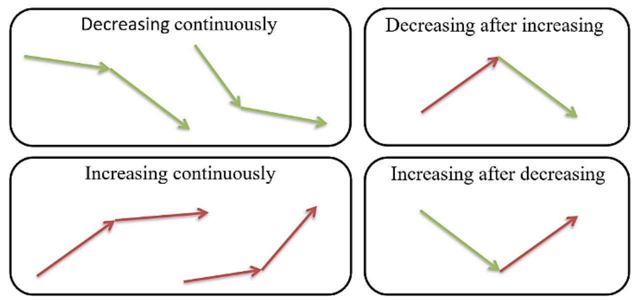
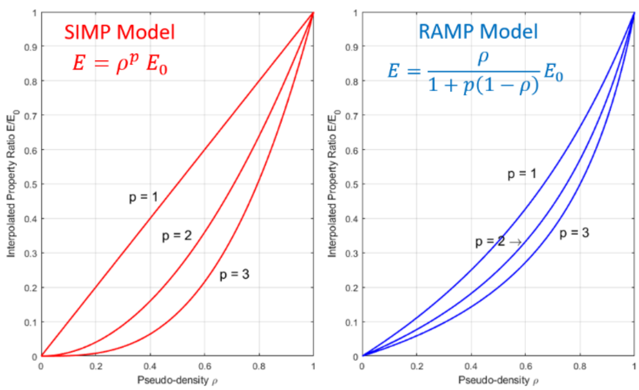


FIGURE 1: OBJECTIVE FUNCTION BEHAVIOR FOR THREE ITERATIONS ILLUSTRATED AS TWO SLOPE CHANGES (DIRECTION OF ARROWS).

3.2 Material Interpolation Models

The topology optimization method defines the design variables as the pseudo-densities ρ_e assigned to every finite element [39], representing the existence or absence of solid material. This is a discrete (binary) problem by definition, but it can be relaxed into a continuous problem by allowing the pseudo-densities to take intermediate values from 0 to 1 [11] using material interpolation models such as the SIMP or the RAMP models. The SIMP model, originally introduced by Bendsøe and Rozvany et al. [40, 47], is the most popular model [7]. Stolpe and Svanberg [41] proposed an interpolation scheme; the RAMP model, which is shown to improve the probability of obtaining an approximately solid-void solution. The RAMP model can also overcome the so-called "localized modes" problem [27, 48]. Both models are used in this work to compare their solutions, stability, and convergence.

The parameter that controls the behavior of both interpolation models is the penalization factor p. This factor modifies the concavity of the interpolation scheme as shown in Figure 2. These graphs show the relationship between the input pseudo-density (before penalization) and the output pseudo-density (after penalization). Note that the interpolated pseudo-density represents the material property ratio E/E_0.



(a) (b)

FIGURE 2: MATERIAL INTERPOLATION SCHEMES COMPARISON BETWEEN (a) SIMP AND (b) RAMP MODELS FOR DIFFERENT PENALIZATION FACTORS

3.3 Filters in Topology Optimization

Filters in topology optimization are used to mathematically transform a group of parameters or variables, mainly, to prevent problems associated with the density-based approach for topology optimization [43]. Some of these problems are mesh dependency, checkerboard patterns, numerical instability, lack of solution, or simply poor structural connectivity. An extensive development of filtering techniques can be found in the literature; however, in this work, we use a density filter [49] in combination with a Heaviside function [50]; called the Projection filter. This filter promotes solid-void structures while preserving the volume after filtering and improving stability [44].

Figure 3 shows a typical diagram for topology optimization, specifying the sequence of different filtering operations commonly used, namely, Density filters [49], Heaviside filters [44, 50], Sensitivity filters [42], and Average-Weighted Spatial filter (AWS) [51]. Although the designer is free to select the necessary filters and it is possible to use all the filters at once, the modification of the solution space could lead to numerical problems. Note that bypassing any of the filters does not affect the optimization loop.

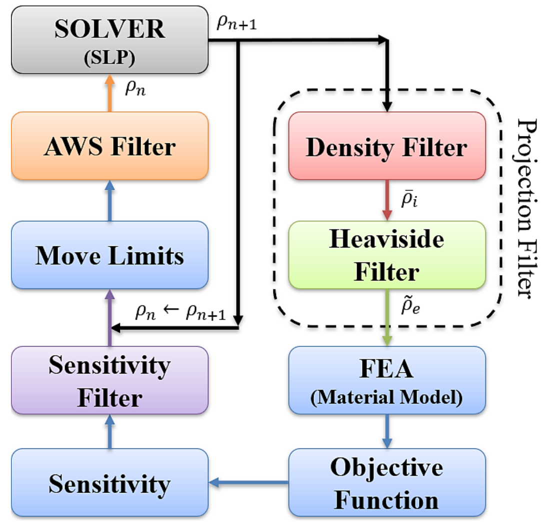


FIGURE 3: OPTIMIZATION ALGORITHM.

4. RESULTS AND DISCUSSION

Here, the optimization problem will be formulated for a case study. We start with an initial (reference) solution followed by a parametric study i.e., some of the parameters will be modified to analyze their influence on the solution.

This section is divided into three subsections: Optimization Problem, Reference Solution Topology, and Variation of Parameters. In the first subsection, the topology optimization case study will be presented in detail, followed by an initial solution to the problem along with the specification of the setup parameters and its analysis. Lastly, a variation of some key parameters used in the reference solution will be performed to discuss their implications on the solution.

4.1 Optimization Problem

The optimization problem consists of designing a structure such that its natural frequencies (eigenfrequencies) are as close as possible to a reference set of eigenfrequencies defined beforehand. To specify this problem, the reference set of frequencies will be introduced along with its corresponding original structure. Then, a design domain is defined as the available space to design the optimized structure along with the material properties used.

4.1.1 Target Eigenfrequencies

Inspired by the four-arm resonator design presented by Hakoda et al. [52], a set of eigenfrequencies is obtained from a simplified cross-shape structure (see Figure 4), composed of a central cube of 4×4×4 mm^3 with four arms, each of 4×4×18 mm^3. This constitutes a four-arm structure confined with a volume of 40×40×4 mm^3 total. The only boundary conditions are defined at the bottom surface of the central cube, where all degrees of freedom are constrained.

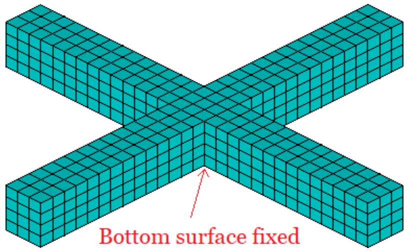


FIGURE 4: STRUCTURE ILLUSTRATION TO GENERATE THE REFERENCE EIGENFREQUENCIES.

The first 20 eigenfrequencies of the structure in Figure 4 are presented in Table 1. These eigenfrequencies were computed in the commercial software ANSYS, using the material properties presented in Table 2. As expected, there are some repeated eigenfrequencies. This is due to the symmetry of the structure's geometry, which leads to some resonance modes having the same eigenfrequencies (eigenvalues) but different mode shapes (eigenvectors).

TABLE 1: TARGET EIGENFREQUENCIES

Mode	Frequency (kHz)	Mode	Frequency (kHz)
1	7.522	11	18.560
2	7.626	12	18.560
3	7.626	13	33.741
4	8.775	14	33.741
5	8.775	15	35.811
6	8.823	16	36.280
7	8.825	17	40.949
8	9.291	18	40.949
9	18.560	19	41.126
10	18.560	20	47.181

TABLE 2: MATERIAL PROPERTIES

Property	Value
Young's Modulus	69 [GPa]
Density	2700 [kg/m^3]
Poisson's Ratio	0.33

4.1.2 Design Domain

We choose a design domain with a similar cuboid-shape volume as the structure of Figure 4. The shape is purposefully intended to be similar to the reference in order to test the design methodology. Since the global solution is known and contained within the design domain, the optimization solution should propose a similar topology, verifying the design methodology of resonating structures based on matching eigenfrequencies. Moreover, using a similar domain volume results in the eigenfrequencies within the same range as the target set. Figure 5 shows the design domain with a mesh discretization of 31×31×3 elements, for a total of 4096 nodes, and 12288 degrees of freedom. Note that all results presented were obtained using the same material properties (Table 2).

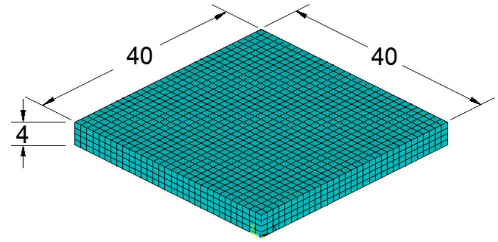


FIGURE 5: DESIGN DOMAIN DEFINITION.

4.2 Reference Solution Topology

To illustrate the analysis of different configurations, we first solve an initial case as a baseline, after which the variation of parameters will be performed in the next section. This baseline solution represents only one possible solution obtained using a select set of parameters, which does not guarantee the global optimization solution has been achieved. The parameters used to obtain the baseline solution are given in Table 3.

TABLE 3: OPTIMIZATION PARAMETERS FOR REFERENCE SOLUTION

	Value
Material Interpolation Model	SIMP with p = 3
Maximum Volume	40%
Minimum Volume	20%
Starting guess point	$\rho_e = 0.4$ for all e
Reference Eigenfrequencies	From Table 1
Material properties	From Table 2
Weighting coefficients (w_q)	w_q=1, for all q
Filter used	Projection Filter
Density filter radius	6 [mm]
Heaviside filter Beta value	Continuously increasing

The baseline solution presented in Figure 6 shows the optimized topology at iteration 610, where the objective function reached its minimum value, *i.e.*, 0.299. Note that the resultant topology is different from the reference structure (Figure 4), which suggest that the optimization has found a different distribution of material to match the eigenfrequencies. This topology, as well as the cross in Figure 4, is composed of arms, which confirms the importance of this design feature to generate the target set of eigenfrequencies. Moreover, the symmetric shape with respect to the mid-planes suggests that the optimization could be defined with symmetry constraints; a condition that may not be true for all cases, as will be shown in subsequent results.

Figure 6 shows a topology with a grayscale colormap, where black elements are solid, white elements are void and the others have intermediate density definition. Figure 7 shows the corresponding postprocessed topology, where all the elements have been completely defined using thresholding.

To obtain this image, the software developed by Zegard and Paulino [53] was used. Specifically, all pseudo-densities over 40% are turned into solid material, while the others become void to create a well-defined structure.

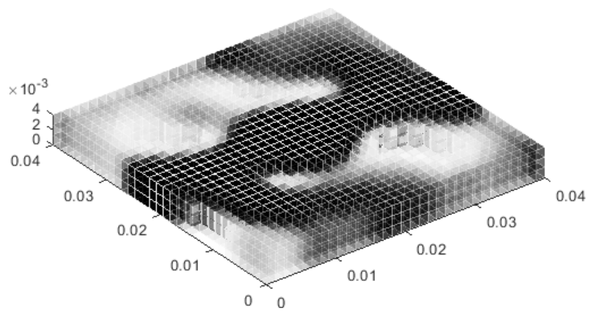


FIGURE 6: REFERENCE OPTIMIZED TOPOLOGY.

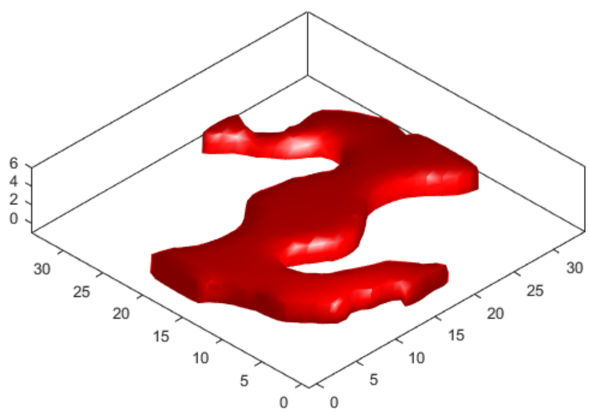


FIGURE 7: POST-PROCESSED REFERENCE TOPOLOGY.

Although the topology is not same as the target cross (Figure 4), its eigenfrequencies are close enough to fulfil the design objective, validating the optimized solution. To confirm the effectiveness of this solution, Figure 8 shows the comparison between the original set of eigenfrequencies reported in Table 1, with the actual eigenfrequencies of the optimized topology (Figure 6). The good agreement between the two sets of eigenfrequencies is evident, with a relative error of 4.48%. Analyzing the optimized topology and its vibrational response, it is interesting to note that the design methodology can generate non-intuitive structures given a design requirement, in this case, a reference set of eigenfrequencies.

Finally, Figure 9 presents the objective function evolution throughout the optimization process showing that the objective function is oscillating. The objective function decreases and oscillates until iteration 400, after which it keeps oscillating around a value of ~0.4. The minimum value was reached at iteration 610 long before reaching the maximum iteration limit of 1000. After iteration 610, a few sets of oscillations appear but the objective function does not show further improvement nor apparent convergence; however, as shown in figures 6 and 7, the resulting structure is well defined and, as confirmed by Figure 8, the optimization has adequately matched the eigenfrequencies.

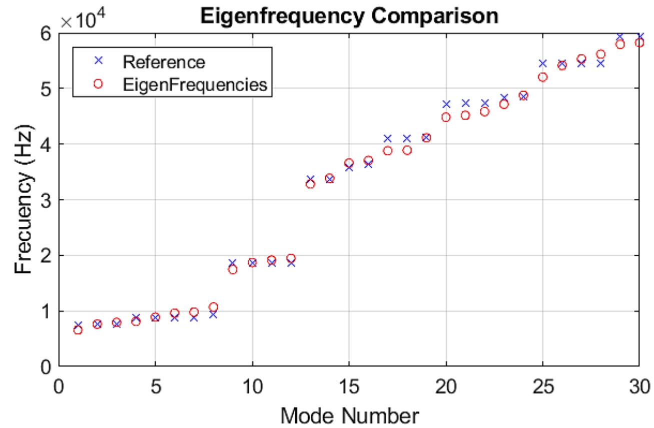


FIGURE 8: OPTIMIZED EIGENFREQUENCIES FOR THE REFERENCE SOLUTION (RED CIRCLES) VERSUS THE TARGET SET OF EIGENFREQUENCIES (BLUE CROSSES) FROM TABLE 1.

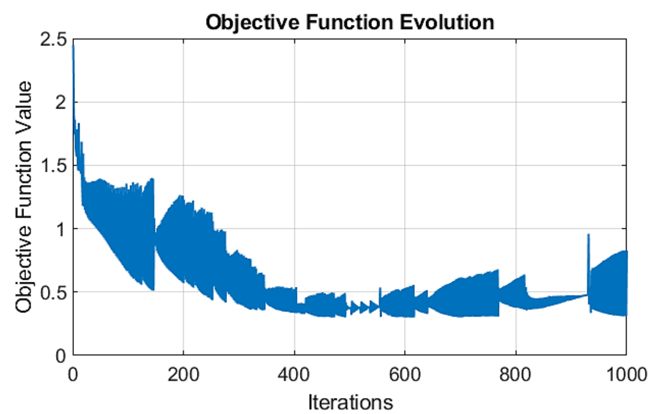


FIGURE 9: OBJECTIVE FUNCTION EVOLUTION FOR THE REFERENCE SOLUTION.

4.3 Parametric Study

Many parameters are involved in the definition of an optimization process. Some of these parameters are shown in Table 3 for the baseline solution. In this section, some of these parameters are varied to study their effect on the optimization solution and discuss associated issues. Specifically, we analyze the influence of the choice for the material interpolation model (SIMP vs. RAMP) as well as the penalization factor p (Figure 2).

4.3.1 Material Interpolation Model

Here, instead of SIMP, the problem is solved using the RAMP model while maintaining all the other parameters shown in Table 3 the same. Note that the penalization factor p remains the same, even when the interpolation curve differs due to the choice of a different model (see Figure 2).

Figure 10 shows the optimized topology obtained using the RAMP model. The new topology is similar to the previous result (Figure 6) and exhibits a similar geometrical symmetry except that the structure in Figure 10 has "disconnected" members on both extremities of the arms. In other words, the optimization has resulted in a low-density material in those locations that still connects the members through what can be interpreted as "very soft" bridges. However, when the post-processing threshold is applied, that material is removed. This is a known problem in topology optimization, which is often addressed with filters, although it is not always possible to prevent such undesirable solutions.

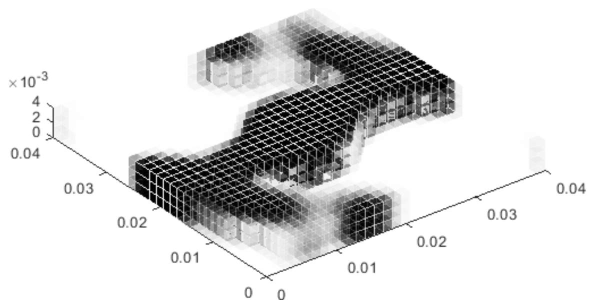


FIGURE 10: OPTIMIZED TOPOLOGY OBTAINED WITH THE RAMP INTERPOLATION MODEL.

For comparison purposes, Figure 11 shows the objective function evolution when optimizing using the RAMP model. Note the differences between Figure 9 and Figure 11. Although still oscillating, the objective function is more stable and converges to a lower value. In this case, a minimum value of 0.214 was reached at iteration 823. The oscillatory behavior near the end suggests that the optimization is jumping back and forth around a local minimum; therefore, converging asymptotically around the optimal value.

While the solution using the RAMP model shows a more stable decreasing objective function with a lower optimized value, the topology has disconnected members. If these members are discarded, a lower volume structure could be obtained but its dynamic response may deviate from the optimized solution. One the other hand, if the gap between the disconnected members and the main structure is filled with solid material, the final topology will be similar to the structure shown in Figure 6, suggesting that both models can achieve similar solutions.

4.3.2 Penalization Factors

When using either the SIMP or RAMP model, a user-defined factor p is set to control the interpolation curvature. As shown in Table 3, the penalization factor used to obtain the topologies shown in Figure 6 and Figure 10 is p = 3, a commonly used value in structural topology optimization. Here, we study the influence of this parameter on the solution while keeping all the other parameters the same.

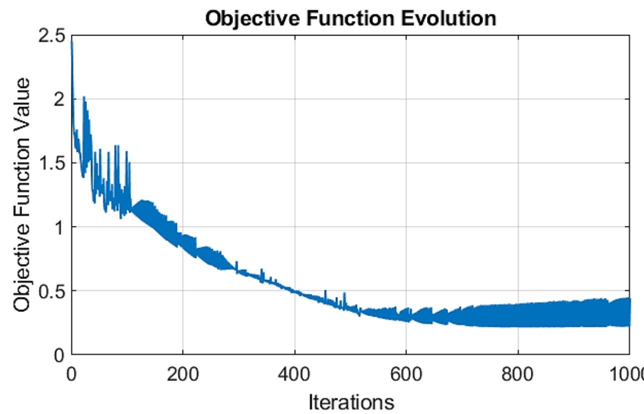


FIGURE 11: OBJECTIVE FUNCTION EVOLUTION OBTAINED WITH THE RAMP INTERPOLATION MODEL.

Figure 12 shows the topology obtained using SIMP and setting $p = 2$. The resulting topology is substantially different from the previous once; the symmetry is no longer present, and it does not share the characteristics of an arm-composed structure. In other words, this solution is highly dependent on the penalization factor p due to the resulting modification in the pseudo-densities.

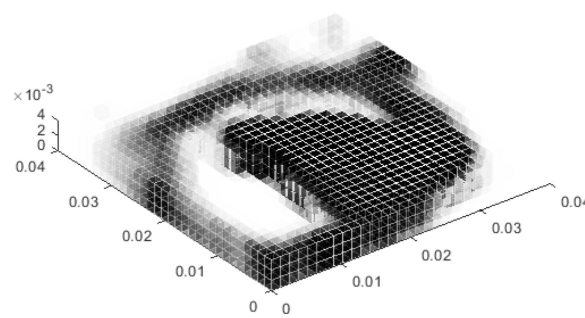


FIGURE 12: OPTIMIZED TOPOLOGY OBTAINED WITH SIMP AND PENALIZATION FACTORS $p = 2$.

It is important to note that so far, the penalization factors used are the same for mass and stiffness. In the literature a suggested combination of penalization factors to prevent localized modes is $p = 3$ for stiffness and $p = 1$ for mass [17]. However, in this work, using such combination of factors leads to numerical problems without yielding a solution. Similar issues are found when using larger penalization factors, *e.g.*, $p = 6$, and using unitary factors, *i.e.*, $p = 1$. The reason for numerical problems with $p = 6$ is the lack of relaxation of the optimization problem, while the reason for problems when $p = 1$ is unknown. When using the RAMP model with $p = 2$, the solution is stable and similar to previous results, as expected. Therefore, the RAMP model was used to explore different combinations of penalization factors.

Figure 13 shows a topology obtained using the RAMP model with penalization factors $p = 3$ for stiffness and $p = 1$ for mass. It is clear that the RAMP model can generate topologies even when the penalization factor induces instability in convergence during the optimization process. Although the topology is not symmetric, it has a distribution that resembles the original structure (Figure 4) the most, compared to all the other topologies previously presented. Moreover, it does not include disconnected members. However, the corresponding objective function (Figure 14), reaches a minimum of 0.415 at iteration 974, which is considerably higher compared to the objective function values obtained when both penalization factors were set to $p = 3$; as shown in Figures 9 and 11.

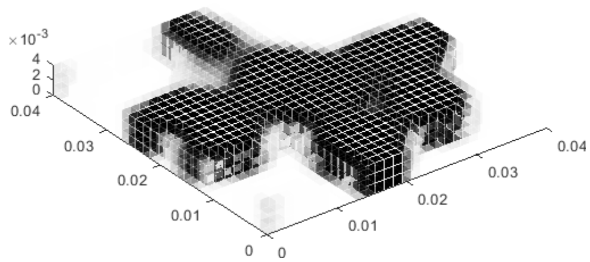


FIGURE 13: OPTIMIZED TOPOLOGY. PENALIZATION FACTORS $p = 3$ FOR STIFFNESS, $p = 1$ FOR MASS.

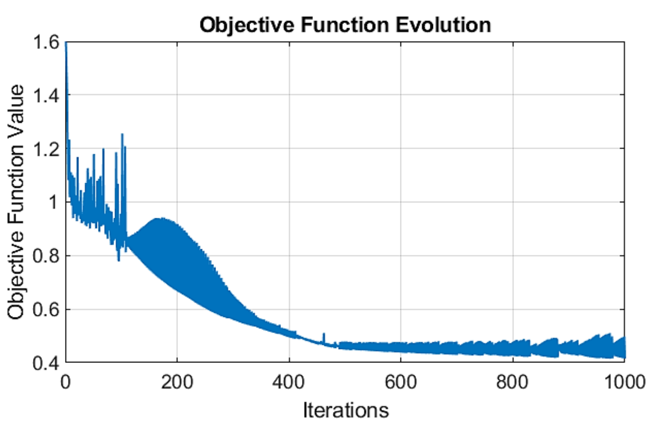


FIGURE 14: OBJECTIVE FUNCTION EVOLUTION CORRESPONDING TO FIGURE 13 TOPOLOGY .

5. SUMMARY AND CONCLUSION

The design of resonating structures can be defined as a structural optimization problem considering their dynamic behavior, requiring that their eigenfrequencies match a target set of values. This paper presents a design methodology for structures under such conditions illustrated with a case study. It is shown how to use topology optimization to design a resonating structure having a given set of eigenfrequencies. We demonstrate how to tailor the optimized topology using some optimization capabilities, such as volume constraints or minimum feature size of structural members.

It is demonstrated how the choice of initial parameters leads to different optimized topologies, which grants the designer control over structural features. This approach to structural design is fundamentally different from other optimization techniques that use eigenfrequencies to create gaps in their frequency response or to maximize their eigenfrequencies. Moreover, this approach allows the designer to tailor the frequency response of a structure and its maximum or minimum volume, while fulfilling physical and mechanical constraints. Having a strategy to use eigenfrequencies as the optimization target is especially useful to design resonating structures such as musical instruments, acoustic meta-materials, vibration control devices, sensors and actuators, energy harvesting devices, and many other applications where the frequency response of the structure plays an important role in its performance.

On the design of structures with topology optimization is important to recognize the effects of using different interpolation models, *i.e.*, the SIMP model or the RAMP model. As discussed in subsection 4.3, the RAMP demonstrates a more stable optimization process, lower values regarding the objective function, numerical stability for different initial parameters, and a clear definition of topologies. However, the SIMP model can achieve good results anyway, which opens the possibility to use either model accordingly to the problem characteristics. In addition to this, it was evidenced an influence on the solution and its convergence by modifying the move limits. As discussed in subsection 3.1, sequential linear programming is a powerful solver, but it needs the move limits criteria to be set according to the problem's non-linearity. The new heuristic approach proposed in this work was proven to analyze and control the optimization behavior appropriately to prevent instabilities due to the highly non-linearity of the eigenfrequency matching optimization problem.

On the one side, the design of structures for specified eigenfrequencies presents a powerful tool to control the modal response of the structure. On the other side, the lack of control over the eigenvectors, *i.e.*, modal shapes, reveals a weakness of this particular design methodology. Having control over the mode shapes (eigenvectors) would allow tracking the modal order switching that occurs during the optimization and it would provide an additional design tool to define particular dynamic responses. However, this is an extension of this work that would be implemented in the future using modal shape cross-correlation assurance criteria. Additional future developments include the exploration of harmonic response optimization, improvements in computational efficiency, better move limits criteria, and techniques to prevent localized modes.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the support of the National Science Foundation under Grant No. 1934527. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

Computations for this research were performed on the Pennsylvania State University's Institute for Computational and Data Sciences' Roar supercomputer.

REFERENCES

[1] Herskovits, J., 2012. Advances in structural optimization, Vol. 25. Springer Science & Business Media.

[2] Guo, X., and Cheng, G.-D., 2010. "Recent development in structural design and optimization". Acta Mechanica Sinica, 26(6), pp. 807–823.

[3] Huang, M., Tse, K., Chan, C. M., and Lou, W., 2011. "Integrated structural optimization and vibration control for improving wind-induced dynamic performance of tall buildings". International Journal of Structural Stability and Dynamics, 11(06), pp. 1139–1161.

[4] He, W., Bindel, D., and Govindjee, S., 2012. "Topology optimization in micromechanical resonator design". Optimization and Engineering, 13(2), pp. 271–292.

[5] Kim, W., Song, Y. H., and Kim, J. E., 2012. "Topology optimization of actuator arms in hard disk drives for reducing bending resonance-induced off-tracks". Structural and Multidisciplinary Optimization, 46(6), pp. 907–912.

[6] El-Khoury, O., and Adeli, H., 2013. "Recent advances on vibration control of structures under dynamic loading". Archives of Computational Methods in Engineering, 20(4), pp. 353–360.

[7] Zargham, S., Ward, T. A., Ramli, R., and Badruddin, I. A., 2016. "Topology optimization: a review for structural designs under vibration problems". Structural and Multidisciplinary Optimization, 53(6), pp. 1157–1177.

[8] Sigmund, O., and Bendsøe, M. P., 2004. "Topology optimization–from airplanes to nanooptics". In BRIDGING from technology to society: DTU 1829-2004-175 ar. Technical University of Denmark, pp. 40–51.

[9] Bendsøe, M. P., and Kikuchi, N., 1988. "Generating optimal topologies in structural design using a homogenization method". Computer methods in applied mechanics and engineering, 71(2), pp. 197–224.

[10] Maute, K., and Sigmund, O., 2013. "Topology optimization approaches: A comparative review". Structural and Multidisciplinary Optimization, 6.

[11] Eschenauer, H. A., and Olhoff, N., 2001. "Topology optimization of continuum structures: a review". Appl. Mech. Rev., 54(4), pp. 331–390.

[12] Zhang, X., and Kang, Z., 2013. "Topology optimization of damping layers for minimizing sound radiation of shell structures". Journal of Sound and Vibration, 332(10), pp. 2500–2519.

[13] Sun, H., Yang, Z., Li, K., Li, B., Xie, J., Wu, D., and Zhang, L., 2009. "Vibration suppression of a hard disk driver actuator arm using piezoelectric shunt damping with a topology optimized pzt transducer". Smart Materials and Structures, 18(6), p. 065010.

[14] Guzman, D. G., Silva, E. C., and Rubio, W. M., 2020. ´ "Topology optimization of piezoelectric sensor and actuator layers for active vibration control". Smart Materials and Structures, 29(8), p. 085009.

[15] Mills-Curran, W. C., 1988. "Calculation of eigenvector derivatives for structures with repeated eigenvalues". AIAA journal, 26(7), pp. 867–871.

[16] Díaz, A. R., and Kikuchi, N., 1992. "Solutions to shape and topology eigenvalue optimization problems using a homogenization method". International Journal for Numerical Methods in Engineering, 35(7), pp. 1487–1502.

[17] Pedersen, N. L., 2000. "Maximization of eigenvalues using topology optimization". Structural and multidisciplinary optimization, 20(1), pp. 2–11.

[18] Yoon, G. H., 2010. "Maximizing the fundamental eigenfrequency of geometrically nonlinear structures by topology optimization based on element connectivity parameterization". Computers & structures, 88(1-2), pp. 120–133.

[19] Li, Z., Shi, T., and Xia, Q., 2017. "Eliminate localized eigenmodes in level set based topology optimization for the maximization of the first eigenfrequency of vibration". Advances in Engineering Software, 107, pp. 59–70.

[20] Kim, T. S., and Kim, Y. Y., 2000. "Mac-based mode tracking in structural topology optimization". Computers & Structures, 74(3), pp. 375–383.

[21] Luo, J., and Gea, H. C., 2003. "Optimal stiffener design for interior sound reduction using a topology optimization based approach". J. Vib. Acoust., 125(3), pp. 267–273.

[22] Cheng, L., Liang, X., Belski, E., Wang, X., Sietins, J. M., Ludwick, S., and To, A., 2018. "Natural frequency optimization of variable-density additive manufactured lattice structure: theory and experimental validation". Journal of Manufacturing Science and Engineering, 140(10).

[23] Rindi, A., Meli, E., Boccini, E., Iurisci, G., Corbo, S., and Falomi, S., 2016. "Static and modal topology optimization of turbomachinery components". Journal of Engineering for Gas Turbines and Power, 138(11).

[24] Ma, Z.-D., Kikuchi, N., Pierre, C., and Raju, B., 2006. "Multidomain topology optimization for structural and material designs". Journal of Applied Mechanics.

[25] Ma, Z.-D., Cheng, H.-C., and Kikuchi, N., 1994. "Structural design for obtaining desired eigenfrequencies by using the topology and shape optimization method". Computing Systems in Engineering, 5(1), pp. 77–89.

[26] Ma, Z.-D., Kikuchi, N., and Cheng, H.-C., 1995. "Topological design for vibrating structures". Computer methods in applied mechanics and engineering, 121(1-4), pp. 259– 280.

[27] Zhang, X., and Kang, Z., 2016. "Vibration suppression using integrated topology optimization of host structures

and damping layers". Journal of Vibration and Control, 22(1), pp. 60–76.
[28] Jensen, J. S., and Pedersen, N. L., 2006. "On maximal eigenfrequency separation in two-material structures: the 1d and 2d scalar cases". Journal of Sound and Vibration, 289(4-5), pp. 967–986.
[29] Du, J., and Olhoff, N., 2007. "Topological design of freely vibrating continuum structures for maximum values of simple and multiple eigenfrequencies and frequency gaps". Structural and Multidisciplinary Optimization, 34(2), pp. 91–110.
[30] Ma, Z.-D., Kikuchi, N., Cheng, H.-C., and Hagiwara, I., 1995. "Topological optimization technique for free vibration problems". Journal of Applied Mechanics.
[31] Maeda, Y., Nishiwaki, S., Izui, K., Yoshimura, M., Matsui, K., & Terada, K. (2006). Structural topology optimization of vibrating structures with specified eigenfrequencies and eigenmode shapes. International Journal for Numerical Methods in Engineering, 67(5), 597-628.
[32] Yamasaki, S., Nishiwaki, S., Yamada, T., Izui, K., & Yoshimura, M. (2010). A structural optimization method based on the level set method using a new geometry-based re-initialization scheme. International journal for numerical methods in engineering, 83(12), 1580-1624.
[33] Nishizu, T., Takezawa, A., & Kitamura, M. (2017). Eigenfrequecy-based damage identification method for non-destructive testing based on topology optimization. Engineering Optimization, 49(3), 417-433.
[34] Bernal, J. D. B., Silva, E. C., and Rubio, W. M., 2019. "Characterization of effective young's modulus for fused deposition modeling manufactured topology optimization designs". The International Journal of Advanced Manufacturing Technology, 103(5), pp. 2879–2892.
[35] Rojas-Labanda, S., and Stolpe, M., 2015. "Benchmarking optimization solvers for structural topology optimization". Structural and Multidisciplinary Optimization, 52(3), pp. 527–547.
[36] van Dijk, N. P., Maute, K., Langelaar, M., and Van Keulen, F., 2013. "Level-set methods for structural topology optimization: a review". Structural and Multidisciplinary Optimization, 48(3), pp. 437–472.
[37] Haftka, R. T., and Gurdal, Z., 2012. ¨ Elements of structural optimization, Vol. 11. Springer Science & Business Media.
[38] Lamberti, L., and Pappalettere, C., 2003. "Move limits definition in structural optimization with sequential linear programming. part i: Optimization algorithm". Computers & structures, 81(4), pp. 197–213.
[39] Bendsøe, M. P., and Sigmund, O., 1999. "Material interpolation schemes in topology optimization". Archive of applied mechanics, 69(9), pp. 635–654.
[40] Bendsøe, M. P., 1989. "Optimal shape design as a material distribution problem". Structural optimization, 1(4), pp. 193–202.
[41] Stolpe, M., and Svanberg, K., 2001. "An alternative interpolation scheme for minimum compliance topology optimization". Structural and Multidisciplinary Optimization, 22(2), pp. 116–124.
[42] Sigmund, O., and Petersson, J., 1998. "Numerical instabilities in topology optimization: a survey on procedures dealing with checkerboards, mesh-dependencies and local minima". Structural optimization, 16(1), pp. 68–75.
[43] Sigmund, O., 2007. "Morphology-based black and white filters for topology optimization". Structural and Multidisciplinary Optimization, 33(4-5), pp. 401–424.
[44] Xu, S., Cai, Y., and Cheng, G., 2010. "Volume preserving nonlinear density filter based on Heaviside functions". Structural and Multidisciplinary Optimization, 41(4), pp. 495–505.
[45] Rao, S. S., 2019. Engineering optimization: theory and practice. John Wiley & Sons.
[46] Lamberti, L., and Pappalettere, C., 2000. "Comparison of the numerical efficiency of different sequential linear programming-based algorithms for structural optimization problems". Computers & Structures, 76(6), pp. 713–728.
[47] Rozvany, G. I., Zhou, M., and Birker, T., 1992. "Generalized shape optimization without homogenization". Structural optimization, 4(3-4), pp. 250–252.
[48] Rong, J. H., Tang, Z. L., Xie, Y. M., and Li, F. Y., 2013. "Topological optimization design of structures under random excitations using SQP method". Engineering Structures, 56, pp. 2098–2106.
[49] Bruns, T. E., and Tortorelli, D. A., 2001. "Topology optimization of non-linear elastic structures and compliant mechanisms". Computer methods in applied mechanics and engineering, 190(26-27), pp. 3443–3459.
[50] Guest, J. K., Prevost, J. H., and Belytschko, T., 2004. ´ "Achieving minimum length scale in topology optimization using nodal design variables and projection functions". International journal for numerical methods in engineering, 61(2), pp. 238–254.
[51] Cardoso, E. L., and Fonseca, J. S. O., 2003. "Complexity control in the topology optimization of continuum structures". Journal of the brazilian society of mechanical sciences and engineering, 25(3), pp. 293–301.
[52] Hakoda, C., Lissenden, C. J., & Shokouhi, P. (2019). Clamping resonators for low-frequency S0 lamb wave reflection. Applied Sciences, 9(2), 257.
[53] Zegard, T., and Paulino, G. H., 2016. "Bridging topology optimization and additive manufacturing". Structural and Multidisciplinary Optimization, 53(1), pp. 175–192.