

"I use IBL in this course" may say more about an instructor's beliefs than about their teaching

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Abstract

Inquiry-Based learning (IBL) is a fairly well-known term in the United States (US) describing a range of student-centered or active pedagogical approaches in mathematics. However, the 'big tent' definitions of IBL mean that there is much variation in IBL users' instructional practices, variation which we set out to codify. Cluster analysis of self-reported data from a survey of postsecondary calculus instructors across the US reveals three instructional profiles among self-declared users of IBL: (a) heavy users of group work; (b) users of a variety of mixed approaches; and (c) heavy users of didactic lecture. The instructional profile of this third group is indistinguishable from that of calculus instructors who report never having heard of IBL. We further investigate the relationship of these instructional profiles to certain beliefs about teaching and learning. All groups agree that inquiry supports learning; the groups who spend a minority of time in didactic lecture disagree with statements that there are benefits to lecturing. Implications for research and adoption of IBL are discussed.

Keywords Instructional practices · Calculus · STEM · Individual characteristics

Introduction

Inquiry forms of instruction have been part of broader educational reforms going back at least to the work of John Dewey (Artigue & Blomhøj, 2013). One manifestation of this push in postsecondary mathematics education in the United States (US) is Inquiry Based Learning (IBL), a broadly construed student-centered approach to instruction which has evolved over several decades and is supported by a broad coalition of mathematics instructors (Haberler et al., 2018). Through the work of this

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coalition, which includes *The Academy of Inquiry Based Learning* and the *Inquiry-Based Learning Special Interest Group of the Mathematical Association of America*, IBL has emerged as the predominant "active learning" instructional approach in undergraduate mathematics courses. Their work in promoting and supporting the adoption of IBL is backed by educational research that has indicated positive learning outcomes for active learning in general (e.g., Freeman et al., 2014) and IBL specifically (e.g., Laursen et al., 2014). It also aligns with calls from US professional mathematician societies (CBMS, 2016; Saxe & Braddy, 2015) as well as international government bodies (e.g., Rocard 2007).

While there have been some shifts toward increased student engagement, adoption of active learning techniques in postsecondary mathematics courses has not been widespread. The majority of students taking mathematics in US postsecondary contexts receive traditional didactic lectures, in which they primarily take notes, listen to the instructor speak, and watch the instructor solve example problems (Apkarian & Kirin, 2017). Given the pervasiveness of didactic lecture in postsecondary US mathematics, the research community needs to better understand how and why individuals have taken up active learning, or not. Equally important however, we must also consider what individuals are actually doing in their classrooms when they say they are using evidence-based instructional practices such as IBL.

Questions around how instructors are implementing IBL are especially pertinent given how broadly IBL has been traditionally defined and how similar the tenets of IBL are to other active learning approaches (Ernst et al., 2017; Rasmussen & Kwon, 2007; Rasmussen et al., 2017; Walczyk & Ramsey, 2003). Thus, the way in which IBL is implemented by individual instructors may be largely dependent on variations in instructor beliefs about teaching and learning. While IBL is fairly well-known and promoted among mathematicians (Johnson, 2019), there are open questions about whether IBL is being implemented in consistent ways (Stains & Vickrey, 2016). The variety in implementations of IBL lends it to being conceived of as a spectrum of related activities for a common goal, as opposed to a rigid instructional format. Thus, we do not know if IBL implementation translates into a distinct set of instructional practices within active learning, or even if IBL implementation translates into a set of instructional practices distinct from teacher-centered instructional approaches. The goal of our paper is to examine whether self-identified IBL implementers have a clearly definable set of instructional practices and beliefs that are distinct from those of non-IBL implementers. In this paper we focus on single-variable calculus instructors and address the following research questions:

- 1. What are the instructional practice profiles among instructors of undergraduate calculus courses who implement IBL in their classes, and how does this compare to the instructional practices of those who have never heard of IBL?
- 2. What beliefs about teaching and learning, if any, differ between IBL users and those who have never heard of IBL? If there are differences, to what extent do they relate to variation in instructional practice profile?

Literature Review

Inquiry-Based learning

IBL makes for a particularly interesting context for investigating instructional practice and beliefs given its familiarity with US mathematics instructors (see results) and the open-endedness in which it is discussed in the literature. For instance, in describing IBL Ernst et al., (2017) make several references to the fact that IBL is a "big tent [...] that comes in many shapes and sizes," with classes that "can look very different" (p. 571). While there is no set format for instruction, there are guiding principles for IBL:

The core idea is that students are engaged in an apprenticeship into the practice of mathematics. Students actively participate in contributing their mathematical ideas to solve problems, rather than applying teacher-demonstrated techniques to similar exercises. The instructor facilitates student progress, ensuring that the class can move toward increasingly sophisticated ways of thinking. (Yoshinobu & Jones, 2012, p. 307)

This core idea shifted somewhat as the community has evolved, with the establishment of "the twin pillars of deep engagement in rich mathematics and opportunities to collaborate" (Ernst et al., 2017, p. 571).

In the US, the earliest conceptualizations of IBL were predominantly focused on the first pillar: promoting student learning and self-sufficiency through loosely guided independent inquiry (Haberler et al., 2018). With regards to this pillar, deep student engagement in rich mathematics, students are to make significant contributions to the curriculum progression and math concept development. Students may not be provided answers or methods to follow ahead of time but instead have to wrestle with ideas before making conclusions; the goal of the instructor is to get the students to work on activities that are novel and challenging to them. As the community grew, the idea of student-student collaboration became a (broadly, though not universally) accepted aspect of IBL (Ernst et al., 2017). This second pillar, collaboration, involves students working in groups, as a class, and/or individually in order to learn how to effectively communicate mathematics and deepen their understanding (e.g., individuals going to present a proof for the class to evaluate); the goal of an instructor is to help students engage with other students' thinking and facilitate cross-talk among students. In conjunction, these two widely acknowledged principles are enough to set IBL apart as a distinct pedagogical approach, and thus we used this established pair of principles to characterize IBL to our study participants (Table 1, Methods section).

In just the last couple of years conceptualization of IBL shifted again, this time with the addition of two new pillars, "(c) instructor focus on **student thinking**, and (d) instructor focus on **equity**" (Academy of Inquiry Based Learning, 2021). These two new pillars (which are presented in addition to deep engagement in rich mathematics and opportunities to collaborate), were put forward by Laursen & Rasmussen (2019) in their conceptualization of Inquiry-Based Mathematics Education, a conceptualization which coordinates IBL with Inquiry-Oriented (IO) approaches; they have since been adopted by many (but not all) proponents of IBL in the US. The third pillar, focusing on student thinking, draws heavily from the IO approaches, grounded

in Realistic Mathematics Education, while the fourth is an aspirational goal intended to guide future pedagogical activity rather than to describe current practice.

The group Academy of Inquiry Based Learning (AIBL) is one of the major contributors of IBL's spread across the US. This group runs conferences, offers workshops, hosts repositories of materials, and runs an academic journal all with the aim of growing the IBL community and spreading student-centered learning. Thus, while it is reasonable to assume that AIBL is having a major impact of the implementation of IBL across the US, they are not the only organization promoting IBL. Therefore, it is unknown the extent to which the IBL community as a whole has embraced the four pillars put forward by AIBL. Furthermore, even if there was broad support for these pillars, the community still affirms the "big tent" mentality, with a reduced emphasis on prescribed, specific pedagogical choices for implementing IBL. Generally speaking, however, IBL is usually associated with student-centered instruction and activities such as working in small groups, presentations, whole-class discussions, and decreased use of instructor-centered activities such as lecturing and solving problems (Hayward et al., 2016; Kogan & Laursen, 2014; Laursen et al., 2014). The variety in conceptualizations and implementations of IBL lends itself to be conceived of a spectrum of related activities, as opposed to a rigid instructional format. Given the broad conceptualization of IBL and how innovative instructional practices are then taken up (Scanlon et al., 2019; Stains & Vickrey, 2017), implementation of IBL is likely to vary widely between instructors; this variation is the focus of our first research question.

Beliefs influencing Instructional Practice

One determining factor in how variations in teaching practices arise is instructors' beliefs. As Leatham (2006) notes, there are many views on how beliefs are defined, with the term often being closely associated or even grouped together with other terms, such as 'conceptions', 'knowledge', and 'views.' In order to sidestep these complexities, Leatham holds that:

Of all the things we believe, there are some things that we "just believe" and other things that we "more than believe – we know." Those things we "more than believe" we refer to as knowledge and those things we "just believe" we refer to as beliefs. (p. 92)

In this paper, we adopt the term 'beliefs' to refer to all of Leatham's "set of things we believe" regardless of the strength with which they are held. Thus, we do not attempt to parse what an instruct may "just believe" (i.e., beliefs) from what they "more than believe" (i.e., knowledge). As argued by Johnson et al. (2019) "differentiating between beliefs and knowledge does not give additional insight into instructional decision-making" (p. 3). Adopting a similar stance here allows us to focus on relationships between instructors' beliefs and their instructional practices, rather than the nuances of various strengths and/or substantiation of beliefs.

Prior research has identified how beliefs, and specifically beliefs about teaching and learning, impact instructional practice (Hoyles, 1992; Leatham, 2007; Johnson

et al., 2018; Johnson et al., 2019; Philipp 2007; Shultz, 2022; Speer, 2008; Sztajn, 2003). In terms of beliefs about teaching, a pair of studies looking at factors influencing upper-division mathematics instruction (Johnson et al., 2018; Johnson et al., 2019) suggested that beliefs about lecture (as an instructional practice) is a powerful indicator of instructional practice. As reported by Johnson et al. (2018) "unsurprisingly, positive beliefs about lecturing were held much more strongly by Lecturers than Non-Lecturers, with particular significance for both *I think lecture is the best way to teach* and *I think lecture is the only way to teach that allows me to cover the necessary content*" (p. 273). However, they also found that 64% of their respondents who thought lecture was *not* the best way to teach lectured anyway. These findings suggest that a negative disposition towards lecture may be a necessary, but not sufficient, belief for those interested in adopting IBL.

In terms of beliefs about learning, Shultz (2022) suggested that "believing students should struggle" was somewhat predictive of using several kinds of active learning, including group work. Similarly, Johnson et al. (2019) found that:

"Extensive lecturers not only agreed with the statement that lecture was the best way to teach, they also reported the highest mean agreement with the statement, *I believe students learn better if I first explain the material to them and then they work to make sense of the ideas themselves.* Limited lecturers, on the other hand, were the only group that disagreed with that statement. Instead, they showed the highest mean agreement with the statements *I think students learn best if they do mathematical work in class* and *I think students learn better when they struggle with the material prior to me explaining the material to them*" (p. 13).

However, even those who reported the least amount of class time spent lecturing also reported strong agreement with the statement "*I think students learn better when they struggle with the material prior to me explaining the material to them*"; in fact, in both Johnson et al. (2018) and Johnson et al. (2019) all of the groups of instructors (e.g., lecturers and non-lecturers; extensive, moderate, and limited lecturers) reported agreement with that statement. Thus, we conjecture that, among mathematics instructors as a whole, there may be a shared belief that learning mathematics requires active participation on the student's part – perhaps whether that active engagement should occur during, or outside of, class time is the actual driving belief for instructional decision-making.

Taken together, there is reason to believe that we will find variation in the ways in which IBL is implemented and that these variations may be related to differing beliefs about teaching and learning. By looking at those who reported to use IBL, and those who have reportedly never heard of IBL before, we expect to identify variation within the IBL community and establish some comparisons between IBL and non-IBL instructors.

Methods

Survey overview and key items

This work is part of a larger project aimed at understanding factors impacting the uptake of research-based instructional practices in chemistry, mathematics, and physics courses at the postsecondary level. Informed by related research in undergraduate STEM education (e.g., Apkarian et al., 2019; Johnson et al., 2018; Gibbons et al., 2018; Henderson & Dancy, 2009; Lund & Stains, 2015; Walter et al., 2016) the project developed a web-based survey. The survey is an amalgamation and adaptation of items from validated instruments which gathered information about instructors' general instructional practices (Landrum et al., 2017; Walter et al., 2016); beliefs about teaching, learning, and students (Aragón et al., 2018; Chan & Elliott, 2004; Dweck et al., 1995; Johnson et al., 2018; Meyers et al., 2007; Tollerud, 1990); and departmental climate and culture (Walter et al., 2021). The survey also included a section on demographic questions related to both professional and personal identities and roles (see Yik et al., 2022 for additional information about the survey). Survey participants were instructed to consider one particular, recently taught course (in mathematics, a single-variable calculus course) when responding.

In this report we focus on data from a subset of the items related to instructional practice and instructor's beliefs about teaching, and learning. Our analysis incorporated two items about instructional practice. The first (Table 1, Question A) asked instructors to report the percentage of in-class time (in a typical week) students spent in four kinds of activity: working individually, working in small groups, participating in whole-class discussions, and listening to the instructor lecture or solve problems. This question has been used repeatedly in other large-scale studies of instructional practice, when individual observations are impractical (e.g., the Characteristics of Successful Programs in College Calculus study; NSF, DRL REESE #0910240). The second (Table 1, Question B) asked instructors to rate their familiarity with IBL on a five-point scale from "I have never heard of this" to "I currently use it in this course to some extent." This item mimics one used on an earlier study of instructional practices among physicists, which this larger project builds on (Henderson & Dancy, 2009).

As mentioned, several items on the survey were written specifically to capture instructor beliefs about teaching, learning, and students, with many of these items also appearing in similar instructor surveys administered in prior studies (see above). In identifying which survey items to use in our analysis, we looked for survey items with statements that asked for agreement/disagreement, items that started with statements like "I think," and for items that have appeared (either verbatim or with slight modifications) in other research articles investigating the relationship between instructor's beliefs and their pedagogical practice. This includes items such as "*I think lecture is the best way to teach" and "I think students learn better when they struggle with the material prior to me explaining the material to them"* (Johnson at al., 2018). Items were selected for inclusion if they seemed to fall within a broad conceptualization of Leatham's (2006) beliefs, ranging from 'things we just believe' and 'things we more than believe.' Items were then condensed to reduce repetition. This resulted in seven belief-type items relevant for the undergraduate context (Tables 1,

Question C) to be rated on a six-point Likert scale from strongly disagree to strongly agree. Three of these items, labeled 1, 3, 5 in Tables 1, have also been used by Johnson et al., 2018 in their study of the beliefs and instructional practices of Abstract Algebra instructors. Items 2 and 6 were based on items that had be used by the Characteristics of Successful Programs in College Calculus study (NSF, DRL REESE #0910240). Items 4 and 7 came from Chan & Elliot (2004), in their study of Hong Kong teacher education students.

The survey was distributed electronically in Spring 2019, and 3769 instructors of introductory STEM courses at 851 postsecondary institutions across the US responded. Of these, 1349 were instructors of undergraduate single-variable calculus courses; of these, 963 responded to all of the target items. We further reduced the dataset using Question B (Table 1) to include only 366 respondents: those we refer to as IBL users (289 instructors who report *currently using* IBL in their calculus course) and non-IBL (nIBL) users (77 instructors who report *never having heard of* IBL). The former group was selected to assess instructional practices used by self-declared IBL instructors; the latter as a sort of "control" or foil group for contextualizing instructional profiles of IBL-users.

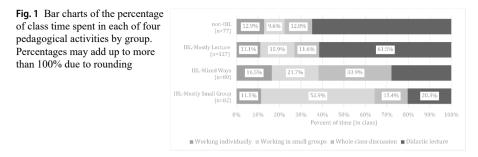
Data Analysis

Our goal was to investigate how different or similar were the instructional practices of calculus instructors implementing IBL in their classes. Specifically, we wanted to classify the 289 self-reported IBL-users based on their reported breakdown of how class time is spent. To accomplish this task, we conducted cluster analysis to identify homogeneous groups of participants and the four clustering variables were percent of time spent in each of the in-class student activities: working individually; working in small groups; participating in whole class discussion; and listening to the instructor lecture or solve problems (i.e., didactic lecture).

As finding the number of clusters is rarely clear (Borgen & Barnett, 1987), we determined the optimal number of clusters using a combination of hierarchical and non-hierarchical methods. Hierarchical cluster analysis using the Centroid Clustering method with Euclidean Distance as a measure of distance (Hayenga & Corpus, 2010) revealed that the respondents could be clustered into three distinct groups. As cluster analysis solutions can be unstable, we used a *k*-means clustering method to confirm the clusters (Wang & Biddle, 2001). The *k*-means cluster profiles agreed with those obtained from hierarchical cluster analysis, thus supporting the three-cluster solution. The nIBL group was incorporated into additional analyses as a fourth instructor group, as a benchmark for comparison.

We used one-way analysis of variance (ANOVA) to examine the effect of instructor group membership on the four in-class student activities and post hoc Tukey HSD testing to assess the effect size between groups. Finally, we examined each group's responses to seven Likert statements related to beliefs about teaching and learning (Table 1, Question C) using Tukey's HSD test for pairwise comparisons.

Table 1 Text of survey questions used in this analysis. (A) Participants selected the percentage of class time spent in each of four activities from drop-down menus of $0, 5$, $10, \ldots, 95$ 100; responses which did not total to 100 were removed. (B) Participants selected one of five options describing their level of awareness or experience with IBL, for which a description was provided. There were 12 such questions, about different named research based instructional strategies. (C) Participants indicated their level of agreement with seven statements on a six-point scale. Statements 1–4 are positive statements about lecture; statements $5-7$ are positive statements about inquiry as a part of learning. These statements were presented to participants a part of a larger set and in a scrambled order	ctivities from drop-down menus of 0, 5, el of awareness or experience with IBL, (C) Participants indicated their level of ive statements about inquiry as a part of
 [A] During a typical week, what proportion of time during regular class meet- [] Working individually ings (i.e., lecture sections) do students spend doing the following? (Answers [] Participating in whole-class discussions must total 100.) [] Listening to the instructor lecture or solve problems 	ems
[B] Please indicate your awareness and (if applicable) usage of I. I have never heard of this Inquiry-Based Learning (IBL): A broad range of empirically validated teaching 2. I know the name, but not much more methods which emphasize (a) deeply engaging students and (b) providing stu- ation as the provent of the network of the	course
 [C] To what extent do you agree/disagree with the following statements? Strongly disagree, disagree, slightly disagree, slightly agree, agree, strongly agree 1. I think lecture is the best way to teach. 2. Students learn best from lectures, provided they are clear and well-organized. 3. I think lecture is the only way to teach that allows me to cover the necessary content. 4. The major role of a teacher is to transmit knowledge to students. 5. I think students learn better when they struggle with the ideas prior to me explaining the material to them. 6. Making unsuccessful attempts is a natural part of problem-solving. 7. Learning means students have ample opportunities to explore, discuss, and express their ideas. 	



Results

Types of IBL users

The *k*-means cluster analysis of the instructional practices of self-declared IBL users identified three distinct groups of practitioners. Based on their instructional profiles (Fig. 1), we refer to the three clusters of IBL-users as MSG (mostly small group work), MW (mixed ways), and ML (mostly lecture). The MSG group consists of 82 instructors and report the highest average of class time spent in small-group work (53%). The MW group, of 80 instructors, does not have a dominant instructional activity. ML is the largest group, including 127 instructors, and this group reports the highest average of class time spent in didactic lecture (62%). We compare these instructional profiles to those of respondents who report never having heard of IBL (nIBL, 77 instructors). As shown in Fig. 1, the nIBL group reports a class time distribution that is similar to that of the ML group of IBL users.

For each of the four instructional activities queried on the survey, group membership has a statistically significant main effect on the amount of time spent in that activity (illustrated in Fig. 2). The effect of group on individual work time is statistically significant but corresponds to a small effect size, F(3,362)=4.07, p<0.01, $\eta^2=0.03$; there is a large effect of group on working in small groups F(3,362)=231, p<0.001, $\eta^2=0.66$, whole class discussion F(3,362)=73.32, p<0.001, $\eta^2=0.38$, and didactic lecture F(3,362)=196.9, p<0.001, $\eta^2=0.62$.

As all four ANOVA tests revealed statistically significant main effects of group membership, we used Tukey's post hoc HSD test to investigate which pairwise differences contribute to these effects and are of a meaningful effect size (Table 2). Individual work time is the least variable activity across the four groups of instructors – the largest effects correspond to differences of about 5% of class time, which is insignificant in practice. The mean time spent in whole class discussion is indistinguishable between the MSG, ML, and nIBL groups, but the MW group spends significantly more time on this activity than any other; this is a large effect of roughly 20% of class time. The ML and nIBL groups report the most time spent in lecture, with large significant differences of 34–46% of class time between those two and the MSG and MW groups; a medium effect separates MSG and MW by 7.6% of class time. Small group work is the in-class activity which most clearly separates all four groups, with a large significant difference (30–43% of class time) separating MSG

	MSG – MW	MSG – ML	MSG – nIBL	MW – ML	MW – nIBL	ML – nIBL
Individual work	-5.0^{*} $d=0.5_{[Med]}$	ns	ns	5.4** d=0.6 _[Med]	ns	ns
Small group work	31.2^{***} $d=2.9_{[Lrg]}$	37.0*** d=3.2 _[Lrg]	43.3*** d=3.4 _[Lrg]	5.8**	12.1***	-6.3** d=0.5 _[Med]
Whole class discussion	-18.6^{***} $d=1.5_{[Lrg]}$	ns	ns	22.4^{***} $d=2.0_{[Lrg]}$	21.1^{***} $d=1.5_{[Lrg]}$	ns
Listening to lecture	-7.6^{**} $d=0.7_{[Med]}$	-41.2*** d=3.3 _[Lrg]	-44.5*** d=2.4 _[Lrg]	-33.6***	-36.9***	ns

 Table 2
 Results of Tukey HSD post hoc testing at 95% family-wise confidence intervals between instructor groups for each of the four pedagogical activities. Cells show difference of group means

*p<0.05, **p<0.01, ***p<0.001; differences which were not statistically significant (ns) are omitted. Effect sizes calculated for significant differences using Cohen's d: 0.2<Small<0.5<Medium<0.8<Large.

from the other three groups; medium-large effects separate the MW group from the ML group from the nIBL group.

These pairwise comparisons confirm the impression that the ML group is similar in their instructional practice to the nIBL group; the only statistically significant difference is in their use of small group work, with a medium effect corresponding to a 6.3% difference in the average time for this activity, which may also be insignificant in practice. These two groups are separated from the other groups by their high usage of didactic lecture strategies in their course, while MW and MSG report lecturing for (on average) less than one-third of their class time. The distinguishing characteristic of MSG is, of course, small group work, while the key feature of MW is the 33% of class time spent in whole class discussion, double the average time spent by instructors in the other groups.

Individual beliefs about teaching and learning

We explore each instructor group's responses to the seven Likert-type (agree/disagree) items from the survey, related to beliefs about teaching and learning (Table 1, Question C). Four of these statements are associated with didactic lecture being "good," related to practical necessity (i.e., content coverage) as well as views of transmission models of learning. The other three statements describe ideas about learning consistent with inquiry approaches, particularly that – in order to learn – students should explore mathematics, make mistakes, and discuss ideas prior to direct instruction. Figure 2 shows the mean of each group's response to each of these seven items; Table 3 provides the results of Tukey's HSD test for pairwise differences, allowing us to see which of the differences are significant and consider practical implications.

We first describe the results for statements that inquiry supports learning (#5–7, Table 1, Question C). All groups' means are above 3.5 for all three items, meaning that (on average) all four groups agree that students' learning benefits from exploring mathematics and making a few mistakes along the way. The majority of these pairwise comparisons do not provide evidence of a difference in the means (i.e., are not significant). For item #5 (students should struggle *before* direct instruction), the nIBL group agree less strongly than the other three groups: ML (-0.49, p<0.05,

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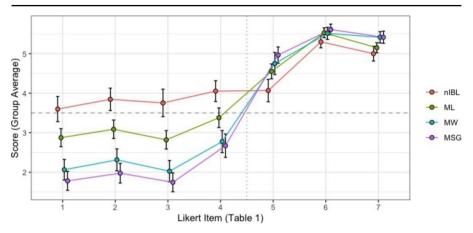


Fig. 2 Mean and 95% CI of the mean of each instructional group's responses (vertical axis) to each of the seven selected Likert items (horizontal axis). Scores correspond to a six-point Likert scale, 1=Strongly disagree; 6=Strongly agree; there was no neutral option. The horizontal line at 3.5 corresponds to a "neutral" average response on the Likert scale. The vertical line separates the four positive statements about lecture and the three positive statements about learning through inquiry. Group order from top to bottom: nIBL (no exposure to IBL; n=77), ML (IBL-mostly lecture; n=127), MW (IBL-mixed ways; n=80), MSG (IBL-mostly small group; n=82)

d=0.41 (small effect)); MW (-0.69, p<0.001, d=0.55 (medium effect)); and MSG (-0.90, p<0.001, d=0.81 (large effect)). For the same item, there is a small effect of MSG compared to ML (0.4, p<0.05, d=0.41)); there is no evidence of differences between the other group pairs. There is even less separation for items #6 and #7, with small-medium effect sizes corresponding to differences of less than half a point on the Likert scale and all of which indicate slightly less agreement by the nIBL group compared to MSG and/or MW (see Table 3 for details).

For items #1–4, i.e., items related to positive views of lecture, the interpretation is more complex. With these items, only the nIBL group has consistently average responses above 3.5, or "in agreement" with the statements; all groups of self-identified IBL users have means which indicate disagreement with the statements (on item #4, the ML group's average is very close to neutral). On these items the MSG and MW groups are indistinguishable and disagree most strongly with the statements compared to the ML and nIBL groups; on each item, MSG and MW are significantly lower in their average score than both ML and nIBL and these differences are medium-large on all but item #4 (the teacher's major role is to transmit knowledge). The ML group's averages are distinct from nIBL on all items, scoring between them and the lower MSG and MW groups.

Discussion

Instructional profiles

With regards to our first research question, our objective was to investigate "styles" of instructional practice among self-identified IBL practitioners, using those who have

	MSG	MSG –	MSG –	MW –	MW –	ML –
	MW	ML	nIBL	ML	nIBL	nIBL
1. I think lecture is the best way to teach	ns	-1.1^{***} $d=0.9_{[Lrg]}$	-1.8*** d=1.5 _[Lrg]	-0.8*** d=0.7 _[Med]	-1.5*** d=1.2 _[Lrg]	-0.7*** d=0.5 _[Med]
2. Students learn best from lectures, provided they are clear and well-organized	ns	-1.1^{***} $d=0.9_{[Lrg]}$	-1.9*** d=1.6 _[Lrg]	-0.8^{***} $d=0.6_{[Med]}$	-1.5*** d=1.2 _[Lrg]	-0.8^{***} $d=0.6_{[Med]}$
3. I think lecture is the only way to teach that allows me to cover the necessary content	ns	-1.1*** d=0.9 _[Lrg]	-2.0*** d=1.5 _[Lrg]	-0.8*** d=0.6 _[Med]	-1.7*** d=1.3 _[Lrg]	-0.9*** d=0.6 _[Med]
4. The major role of a teacher is to transmit knowledge to students	ns	-0.7^{***} $d=0.5_{[Med]}$		-0.6^{**} $d=0.5_{[Sm]}$	-1.3*** d=1.1 _[Lrg]	-0.7^{**} $d=0.5_{[Med]}$
5. I think students learn better when they struggle with the ideas prior to me explaining the material to them	ns	0.4* d=0.4 _[Sm]	0.9*** d=0.8 _[Lrg]	ns	0.7*** d=0.6 _[Med]	0.5^* $d=0.4_{[Sm]}$
6. Making unsuccessful attempts is a natural part of problem-solving	ns	ns	0.3^* $d=0.5_{[Sm]}$	ns	ns	ns
7. Learning means students have ample opportunities to explore, discuss, and express their ideas	ns	ns	0.4** d=0.5 _[Med]	ns	0.4** d=0.6 _[Med]	ns

 Table 3 Results of Tukey HSD testing at 95% family-wise confidence intervals. Cells show difference of the means of groups

Adjusted *p<0.05, **p<0.01, ***p<0.001; differences which were not statistically significant (ns) are omitted.

Effect sizes calculated for significant differences using Cohen's d; values here rounded to one decimal place: 0.2 < Small < 0.5 < Medium < 0.8 < Large

never heard of IBL as a comparison group. Out of 366 post-secondary mathematics instructors investigated here, 289 (79%) reported that they currently use IBL to some extent in their single-variable calculus course¹. As noted previously, IBL is generally conceptualized as a spectrum of related activities for a common goal, as opposed to a rigid instructional format – for example, there are no real guidelines for the *frequency* with which students should encounter inquiry activities in an IBL classroom. Thus, we were not necessarily expecting reports of highly similar teaching practice. Indeed, our cluster analysis revealed three instructional profiles among IBL-users: those who heavily leverage small group work (MSG, n=82), those who engage in multiple strategies (MW, n=80), and those who primarily lecture (ML, n=127). This final group spends slightly more time using small group work than those who have never heard of IBL (nIBL, n=77), but are otherwise indistinguishable in terms of their general pedagogical practice. Thus, in answer to our first research question, there *is* significant variation in what it means to teach using IBL, and this variation includes teaching using primarily lecture strategies.

¹ That 30% of our participants declare that they use IBL suggests increased usage of the approach in the last five years. Though not directly comparable, university mathematics departments reported IBL was in use in 3% of their Calculus 1 courses (Apkarian, N., & Kirin, D. 2017).

More than half of the self-identified IBL-users (MW and MSG) teach using primarily non-lecture strategies that are consistent with the tenets of IBL, in that they provide opportunities for collaborative student engagement with mathematics during class time. However, a large minority (44%) teach in much the same way as those who have never heard of IBL, employing lecture (on average) for more than 60% of class time. These instructors may be engaging in inquiry-style activities for short periods of time, incorporating the principles of IBL into their lectures (e.g., interactive lecture) and/or out-of-class activities (e.g., recitation sections, homework) and thus be qualitatively distinct from traditional models of didactic lecture. Regardless, this approach is not consistent with the format of IBL that has most prominently been linked to improved student outcomes, in which "about 60% of class time in IBL courses was spent on student-centered activities such as small group work, student presentation of problems at the board, or whole-class discussion" (Kogan & Laursen, 2014, p. 185). We also note that the IBL-ML group's instructional profile (and that of the nIBL group) is not as lecture-intensive as the comparison group in Kogan and Laursen's work, in which "over 85% of class time consisted of the instructor talking" (2014, p. 185).

Beliefs and practice

Our second research question asked about variation in instructor beliefs toward teaching and learning across the different instructional groups. Specifically, we investigated instructors' level of agreement with four positive statements about lecture and three statements aligned with inquiry approaches to learning. We interpreted the level of agreement or disagreement with these statements as an indication of what instructors believed about how teaching and learning occurs.

We expected that those who lecture more (the ML and nIBL groups) would have higher levels of agreement with the pro-lecture statements compared to those who do not lecture (MW and MSG), and this was borne out by our analyses. Furthermore, those who have not heard of IBL had the strongest average agreement with the prolecture statements and were the only group whose average response fell on the side of "agreement" (though only just). While all IBL groups' averages fell on the side of "disagreement," the ML averages were close to neutral while the MW and MSG are indistinguishable and suggest more disagreement with these statements. This suggests an alignment between beliefs toward lecture and in-class pedagogical choices.

Within each instructor group, responses to the first three pro-lecture statements were fairly consistent, as were the between-group differences. These items included a generic "lecture is the best way to teach" statement, a statement tied explicitly to learning ("students learn best from lectures"), and a statement related to practicalities ("the only way to teach that allows me to cover the necessary content"). These are the three statements which explicitly used the word "lecture," potentially contributing to the consistency in response. The fourth positive statement about lecture was less explicit ("the major role of a teacher is to transmit knowledge to students") and was intended to detect transmission beliefs about learning. Responses to this item had smaller between-group effect sizes but follow the same pattern. This suggests that some instructors do not strongly believe in lecture as an effective teaching strategy

yet still hold some transmissive views of learning. It may be that these instructors believe there are better ways to learn than through lecture, but that lecture still does result in learning. It might also reflect nuance on our part which was too far from everyday connotations of the language (Krosnick, 1999). For instance, it may be that there were differences in how instructors were interpreting terms such as "lecture" and "transmit knowledge".

At an intuitive level, we may have expected that those who lecture more (ML and nIBL) would agree less with the statements supporting inquiry as a component of learning compared to those who spend more time in student-centered activities (MW and MSG). Instead we found, just as Johnson et al. (2018) and Johnson et al. (2019) did, that average responses were similar across all groups – and all groups' averages suggest high levels of agreement with these inquiry related statements. The ML and nIBL groups, when it is possible to detect differences, show lower agreement than MW and MSG, but these differences only have practical significance in item #5: "students learn better when they struggle with the ideas prior to me explaining the material to them." For this item, the nIBL group agreed a bit less than the three IBL groups; this may be one item where we see the historical discovery-focused roots of IBL (Haberler et al., 2018) separating those who participate in IBL from those who do not, although the distinction is small.

Generally, we see these results as suggesting agreement across instructors – familiar with IBL or not – that struggle, mistakes, exploration, and discussion are productive components of learning to do mathematics. Yet these shared beliefs do not translate into common in-class pedagogy. We specify *in-class* pedagogy to acknowledge that learning – and particularly inquiry – can and does occur outside the classroom as well as within it; instructors with shared beliefs about inquiry may disagree about when and how they should support that inquiry.

Limitations and Implications

We do acknowledge some limitations with this study. Perhaps the most important being that we relied on self-reports of instructional practice. There is research that indicates self-reports of instructional practice can vary for what is captured through observational protocols (e.g., Ebert-May et al., 2011). However, this concern may be somewhat assuaged by the descriptive nature of the survey questions used here (i.e., percentage of class time spent with students working in small groups as opposed to qualitative or holistic statements about instruction). As reported by Smith et al., (2014) when university instructors were asked how often they used instructional practices such as lecturing and holding small group discussions they were generally able to do so in a way that matched descriptive observational protocols. Thus, "faculty members may be able to more accurately estimate the time they use specific learning strategies rather than whether or not broad instructional strategies, such as cooperative learning, are frequently used in their courses" (Smith et al., 2014, p. 632). Thus, while it may not be the case that these instructors were reporting exact percentages for their different teaching practices (e.g., 20% vs. 25% of class time spent lecturing), we do feel reasonably confident that their self-reported practices were representative enough to give a general sense for how they spend class time. Additionally, our analysis of the reported instructional practices did not depend on highly accurate reports; we note but do not rely on small differences in usage of class time. The analysis we conducted looked to identify groups based on trends across multiple items related to their instructional practice – and we were able to identify three groups that exhibited significantly different trends in their reported instructional profiles (e.g., we distinguish 60–65% vs. 20–30% of class time spent in lecture). So, while we are not able to make conclusive claims about the exact percentage of class time that any one instructor had their students work in small groups, we are confident in saying that we have identified three distinct groups of self-declared IBL users – one of which that reports spending class time in ways that more closely align with those who have never heard of IBL and predominately lecture.

The other main limitation we see with this study is also tied to use of survey responses, in particular we cannot be certain that all of our respondents interpreted the survey items in the same way. For instance, it is possible that instructors have different ideas about what constitutes a "whole-class discussion." Additionally, such differences in interpretation may be influenced by experience with professional development – such as those offered through workshops promoting IBL. Lastly, this study is inextricably tied to the US context in which the study took place. This includes the undergraduate mathematics context in which Calculus is taught and the ways in which IBL has been formulated and propagated.

Even with these limitations, this data clearly points to the fact that IBL is implemented across the US with very little consistency with regards to instructional practice. While this study was conducted entirely within a US context, the conclusion that IBL has significant variability in its implementation does match with other results coming out of European studies. Most notably, Engeln et al., (2013) presented results that investigated the implementation of IBL in 12 different European countries. In their analysis they found that only 8% of the teachers in their study reported classroom practices that would indicate that these teachers "seem to apply IBL in their daily lessons" (p. 832), whereas 41% of these teachers' responses were characterized as indicating that "some elements regularly that are an important part of IBL" (p. 832), and the remaining 51% do not report daily instructional practices aligned with IBL. While Engeln et al.'s study does provide a snapshot of the prevalence of IBL instructional practices – and different instructional profiles in these countries in relation to IBL – our study here adds another layer of analysis, investigating the instructional profiles of those who state that they are currently using IBL in their classes.

Given that we were investigating the reported instructional practice of those who indicated implementing IBL, the wide variety of instructional profiles found was surprising. The wide variety presented here – ranging from mostly lecture to mostly small group – may be related to factors not considered in this analysis. For instance, it may be that large class sizes may be a driving factor for why some IBL instructors continue to spend the majority of their class time lecturing. In this way, they may be "implementing IBL" in a way that best suits their instructional context but is not consistent with the stated principles of IBL – and might not be recognized as IBL by an outsider or education researcher. However, given that the two groups of instructors who report spending the majority of their class time lecturing also report more favor-

able beliefs about lecture, we do not think the variability in instructional practices found can be wholly attributed to instructional context.

Instead, given the results of our analysis on instructors' beliefs, we suggest that instructors who say they are using IBL, and are thus declaring membership in the IBL community, may be doing so based on an alignment of beliefs – as opposed to an alignment of instructional practices. While more research can be carried out to test this hypothesis, it is suggestive that where we saw the most alignment among IBL instructors, while also seeing separation between IBL and non-IBL instructors, was with regards to the belief that lecture is *not* the best way to teach Calculus 1. This differentiation between IBL users and those who had never heard of IBL before did not hold for all the belief statements. In general, all four of the instructor groups reported agreement with beliefs that inquiry supports learning, regardless of their reported inclass pedagogical activities.

Concluding thoughts

IBL was by far the most commonly reported research based instructional strategy in use among our sample of 1349 undergraduate single-variable calculus instructors in the US (overall, 338 out of 1180 who responded to the item; 28.6%). We suspect that the "big-tent" ethos of IBL is successfully creating a welcoming and inclusive environment, bringing together many instructors to discuss and innovate their pedagogy in ways which may not have occurred otherwise. The scale and reach of the IBL community is impressive and it is likely spearheading a shift in undergraduate mathematics instruction toward more student-centered approaches that are advocated for by researchers and professionals both in and outside the US (e.g., CBMS, 2016; Freeman et al., 2014; Rocard, 2007; Saxe & Braddy, 2015). Had we stopped with this first finding, we might have concluded that instructional practice in the US is rapidly shifting to incorporate inquiry practices. Instead, we investigated further the in-class activities of self-identified IBL classes.

Given our findings, we suggest caution when using the term IBL to describe instruction - for researchers and practitioners. This is particularly critical for those seeking to study or leverage the effects of IBL instruction on student experiences and/ or outcomes. For researchers, our findings imply that it would be a mistake to assume a consistency between IBL courses or even to assume that in all "IBL classrooms" the majority of class time is spent in student-centered activities. For practitioners interested in achieving similar student learning outcomes as those reported by the IBL community and research reports, it may be that careful considerations about what exactly is happening in these classes needs more attention. There are many ways to incorporate the principles of IBL into a course, we see evidence that these different ways are happening regularly in Calculus 1, and thus we should not assume that students' experience of different implementations will lead to consistent outcomes. This catch-all terminology and the associated challenges with linking practices to student outcomes has been documented for the broader term "active learning" (Lombardi & Shipley, 2021), and we suggest a similar situation in mathematics education research for the term IBL.

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