An Angle Included Optimal Power Flow (OPF) Model for Power Distribution Network Using Second Order Cone Programming (SOCP)

Md Mahmud-Ul-Tarik Chowdhury, Student Member, IEEE, Sukumar Kamalasadan, Senior Member, IEEE

2020 IEEE Industry Applications Society Annual Meeting | 978-1-7281-7192-0/20/$31.00 ©2020 IEEE | DOI: 10.1109/IAS44978.2020.9334785

***Abstract*—In recent years, penetration of distributed generation (DG) has increased rapidly in the power distribution network, which demands novel and optimal power flow solutions for improving the power system reliability, resiliency and better-operating conditions. In this paper, we propose a branch flow-based SOCP model for the optimal power flow of radial networks. In the proposed model, the voltage and current angles are relaxed and a quadratic equation is converted to a conic form for convexification. Though the angles are relaxed, a convex equation is also proposed to recover the angle which makes the solution efficient. In the simulation, we have shown that for radial networks the relaxation is exact when there are no upper bounds on loads. Also, theoretical analysis for the branch flow model is deduced for the convexification of the model. Finally, the proposed model was tested on two radial networks (modified 32 bus network and IEEE 123 bus network) and the simulation results are verified with the OpenDSS and MatPower software packages.**

***Index Terms*—Optimal power flow (OPF), convex relaxation, branch flow model, load flow, second-order conic programming (SOCP), power system management.**

I. Introduction

Optimal power flow (OPF) problem is very important for power systems to decide the optimal point of operation considering minimization of generation cost, power losses, voltage amplitude, phase angle fluctuations, and different other factors including line transmission capacity and voltage regulations [1]. In a modern distribution system, the integration of renewable energy-based distributed generation systems (DGs) has been growing rapidly and the structure of conventional distribution networks is changing because of increased penetration of renewable energy resources. Not only the renewable energy-based distributed generation sources but also new types of loads, such as electric vehicles, have motivated to look for novel and smart solutions by the utility industry for power flow solutions and optimization [2]. Though the bus injection model is used widely for power flow analysis and optimization, it focuses only on the nodal parameters and does not take into account the current and power flow across the branches [3]. The branch flow model emphasizes the branch parameters calculations and thus very suitable for distribution system power flow analysis. In this paper we propose the branch flow model for radial networks with high penetration of renewable energy-based DGs and also used the model for optimization of a network and to recover the bus angle difference across a branch from the optimization.

This work is supported in part by the NSF grant ECCS-1810174 awarded to the last author. The authors are with the Energy Production Infrastructure Center (EPIC), Department of Electrical Engineering, University of North Carolina at Charlotte, Charlotte, NC 28223 USA. Corresponding Author: S. Kamalasadan (Email: skamalas@uncc.edu).

Different types of optimization algorithms have been discussed in different papers [4]–[10]. Besides that, a power flow model in radial distribution networks was proposed in [11] using 3N equations for a network with N+1 buses with newly defined variables and using the Newton-Raphson method. Another simple and efficient model for radial distribution networks was proposed only by the evaluation of a simple algebraic expression of voltage magnitudes and no trigonometric functions in [12]. Using a simple and efficient algorithm another model was presented in [13] to solve radial power distribution networks, where the algebraic recursive expression of voltage magnitude and all other parameters for the network are stored in vector form. However, typical AC optimization is non-convex and non-convexity increases the complexity of the power flow equation solutions. On the other hand, with the increasing penetration of renewable energy-based distributed generation sources, the optimal power flow (OPF) model for power systems network require the convex formulation of power flow equations so that the problem can be solved in a feasible, fast and efficient way. The convex relaxation of OPF problem formulation had been proposed first in [14] for SOCP and in [15] for SDP, since then it has become an important research topic. Using SOCP relaxation for OPF a feasible and exact solution can be recovered and the solution is a global optimum of the original OPF problem if the quadratic and arctangent equalities both are within the constrains discussed in [16], [17]. Relaxing the quadratic equality constraints to inequality constraints, SOCP relaxation is exact for radial networks, if there are no upper bounds on the loads [18], [19]. Conditions for SDP in radial and mesh networks and the exactness related to the modeling of the capacity of a power line was discussed in [20]–[22].

For optimization in a radial power distribution system, our model is motivated by [3] and [23], [24]. In [3] a second-order cone programming (SOCP) based optimization and angle recovery algorithms have been discussed in both radial and mesh networks. The optimal placement and sizing of switched capacitors in distribution circuits for Volt/VAR control are discussed in [23], [24] which can be treated as a particular relaxation for the branch flow model considering only the amplitude of voltage and current flow in the distribution network.

In the proposed approach, at first, the voltage and current angle are eliminated for converting the non-convex equations into convex equations and the quadratic equality is converted into an inequality constraint by the means of conic relaxation for the OPF. Also, a convex equation has been derived for recovering the bus voltage angle difference, from which the bus angle will be calculated for a radial distribution network. The main contributions of the proposed work is that

- The architecture provides an exact solution of the OPF problem
- It can recover the angles and the recovered angles are exact
- The approach can be used for both cost and loss minimization
- The approach is scalable and computationally feasible.
- The approach considers multiple DER penetration based OPF.

The paper is organized as follows. Section II discusses the theoretical background including the OPF methodology and relaxation framework. Section III discusses the model implementation methods and results and Section IV concludes the paper.

II. Theoretical Background

A. Branch Flow Model

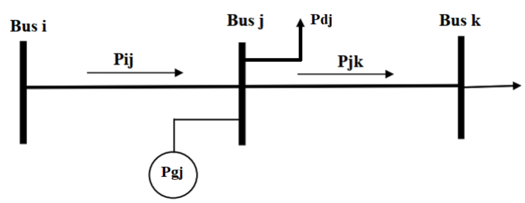

Fig. 1. Branch flow model including PEVs.

In this paper, L is denoted for the set of connected branches in the network and i, j, and k denote the bus indexes. For the line from bus i to bus j, real power flow, reactive power flow and squared of the magnitude of current flow denotes as P_{ij}, Q_{ij} and l_{ij} respectively. S_{ij} is the apparent power flow. P_{gj} and Q_{gj} are the power injected at bus j when P_{dj} and Q_{dj} are the real and reactive power demand at bus j. Considering the power in a line as

$$S_{ij} = V_i I_{ij}^* \tag{1}$$

$$V_i - V_j = Z_{ij} I_{ij} \tag{2}$$

The power flow between buses can be represented as

$$S_{g_j} - S_{d_j} = \sum_{k:j\longrightarrow k} S_{jk} - \sum_{i:i\longrightarrow j} (S_{ij} - Z_{ij}|I_{ij}|^2) + y_j^*|V_j|^2 \tag{3}$$

$$i, j, k \in N$$
$$[(i,j),(j,k)] \in L$$

$$V_j = V_i - \frac{Z_{ij}S_{ij}^*}{V_i^*} \tag{4}$$

Taking the magnitude squared in (4),

$$|V_j|^2 = |V_i|^2 + |Z_{ij}^2||I_{ij}|^2 - (Z_{ij}S_{ij}^* + Z_{ij}^*S_{ij}) \tag{5}$$

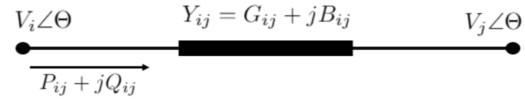

Fig. 2. Line model.

Based on the line model in Fig. 2, the real and reactive power flow across a line between two buses i and j can be expressed as following:

$$P_{ij} = G_{ij}V_i^2 - G_{ij}V_iV_j\cos(\Theta_{ij}) - B_{ij}V_iV_j\sin(\Theta_{ij}) \tag{6}$$

$$Q_{ij} = -B_{ij}V_i^2 + B_{ij}V_iV_j\cos(\Theta_{ij}) - G_{ij}V_iV_j\sin(\Theta_{ij}) \tag{7}$$

Then from (1),

$$|V_i|^2 = \frac{P_{ij}^2 + Q_{ij}^2}{|I_{ij}|^2} \tag{8}$$

B. Optimal Power Flow

The aim of optimal power flow is to supply the demand in a whole network in a way, so that all of the physical laws of power flow are satisfied, while some constraints are imposed. In this paper mainly four objective functions as represented in (9)-(12) is considered.

Line loss minimization,

$$min[\sum_{(i,j)\in L} r_{ij}l_{ij}] \tag{9}$$

or, generation cost minimization,

$$min[\sum_{(i)\in N} c_iP_{g_i}] \tag{10}$$

or, minimization in bus voltage difference,

$$min[\sum_{(i,j)\in N} \alpha_i(u_i - u_j)] \tag{11}$$

or

$$min[\sum_{(i,j)\in L} r_{ij}l_{ij} + \sum_{(i)\in N} c_iP_{g_i} + \sum_{(i)\in N} \alpha_iu_i] \tag{12}$$

In addition to the equations $9) - (12)$ the following constraints have been included to satisfy realistic power balance.

$$\begin{aligned} P_{g_i}^{min} &\leq P_{g_i} \leq P_{g_i}^{max} \\ Q_{g_i}^{min} &\leq Q_{g_i} \leq Q_{g_i}^{max} \\ P_{d_i}^{min} &\leq P_{d_i} \leq P_{d_i}^{max} \\ Q_{d_i}^{min} &\leq Q_{d_i} \leq Q_{d_i}^{max} \\ \Theta_{ij}^{min} &\leq \Theta_{ij} \leq \Theta_{ij}^{max} \\ u_i^{min} &\leq u_i \leq u_i^{max} \\ l_{ij}^{min} &\leq l_{ij} \leq l_{ij}^{max} \end{aligned} \tag{13}$$

C. Relaxation and convexification for SOCP OPF

Convex relaxation encloses the non-convex space in a feasible convex space for the power flow equations. The solution from this convexed space bounds the optimal objective value of the parent non-convex space [7]. A lower bound and a upper boundary for minimization and maximization is provided by relaxations. For convexification of the equations (1)-(7), new variables have been introduced which can be defined as:

$|I_{ij}|^2 = l_{ij}$
$|V_j|^2 = u_j$ and $|V_i|^2 = u_i$

Then from (3), we can get

$$Pg_j - Pd_j = \sum_{k:j\longrightarrow k} P_{jk} - \sum_{i:i\longrightarrow j} (P_{ij} - r_{ij}l_{ij}) + g_j u_j \tag{14}$$

$$Qg_j - Qd_j = \sum_{k:j\longrightarrow k} Q_{jk} - \sum_{i:i\longrightarrow j} (Q_{ij} - x_{ij}l_{ij}) + b_j u_j \tag{15}$$

$$u_j = u_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) + (r_{ij}^2 + x_{ij}^2)l_{ij} \tag{16}$$

As (8) is non-convex we need to represent it in a cone. This can be derived as

$$\begin{aligned} 4u_i l_{ij} &\geq (2P_{ij})^2 + (2Q_{ij})^2 \\ (u_i + l_{ij})^2 - (u_i - l_{ij})^2 &\geq (2P_{ij})^2 + (2Q_{ij})^2 \\ u_i + l_{ij} &\geq \sqrt{(2P_{ij})^2 + (2Q_{ij})^2 + (u_i - l_{ij})^2} \\ u_i + l_{ij} &\geq \left\| \begin{matrix} 2P_{ij} \\ 2Q_{ij} \\ u_i - l_{ij} \end{matrix} \right\| \end{aligned} \tag{17}$$

Further with $(6) * B_{ij} + (7) * G_{ij}$, we get,

$$-V_i V_j \sin(\Theta_{ij})(G_{ij}^2 + B_{ij}^2) = (B_{ij}P_{ij} + G_{ij}Q_{ij}) \tag{18}$$

Equation (18) is non-convex but if we relax $V_i V_j \approx 1$ and $\sin(\Theta_{ij}) \approx \Theta_{ij}$, where Θ_{ij} is the bus voltage angle difference between the bus i and bus j, we get

$$\Theta_{ij} = -\frac{B_{ij}P_{ij} + G_{ij}Q_{ij}}{B_{ij}^2 + G_{ij}^2} \tag{19}$$

Using (19), the angle difference across a line in the network can be calculated. For radial networks, taking the substation bus as reference, the bus voltage angle for all of the buses in the network can be recovered from the calculated angle difference.

III. Model Implementation and Result Evaluation

The proposed model has been tested for several cases in the Matlab platform using MOSEK solver. MOSEK solver can solve convex models, thus the convexity of the proposed SOCP model is also validated in this environment. The model is tested on a modified 32 bus system with DGs and for IEEE 123 bus system for the base case and also for the cases where 10 %, 30 % and 50 % loads are supplied by DGs. Then the results were validated from Matpower and OpenDSS simulation software packages. The simulation shows that the results are very close for all test cases. In this paper, first, the test case is performed on a modified 32-bus system which is a radial distribution system [25]. The system is 12.66 kV and contains 33 buses and 32 lines.

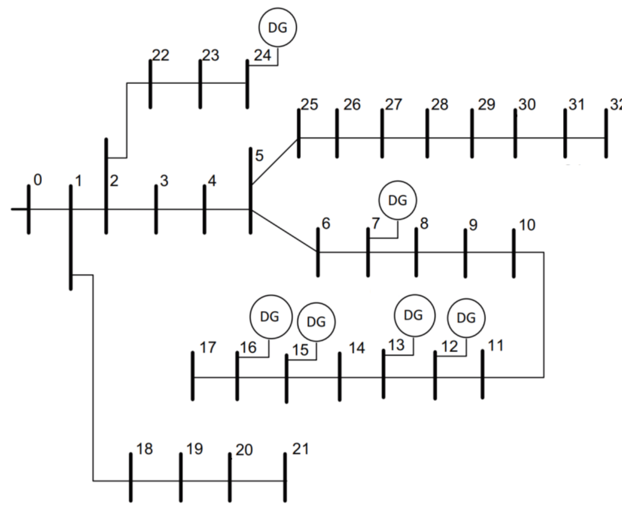

Fig. 3. Modified 32-bus distribution network.

Comparison among the simulation results for MatPower Power Flow (MP_PF), MatPower Optimal Power Flow (MP_OPF) and Second-Order Conic Programming Optimal Power Flow (SOCP_OPF) is shown in Table II. The voltage profile from the SOCP model and MatPower NLP model is shown in Fig. 4, while the objective is generation cost minimization. The change in the voltage profile is shown in Fig. 5 in SOCP-OPF with the change in the objective function.

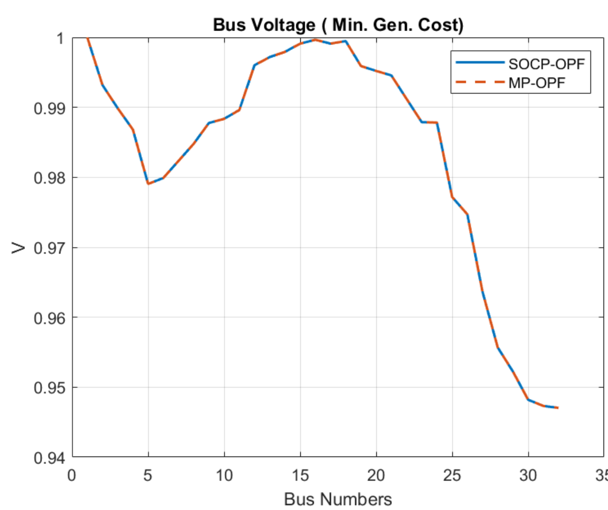

Fig. 4. Voltage profile comparison between MatPower and SOCP OPF.

Fig. 6 is showing the recovered bus voltage angle difference in the network from the proposed SOCP OPF model and com-

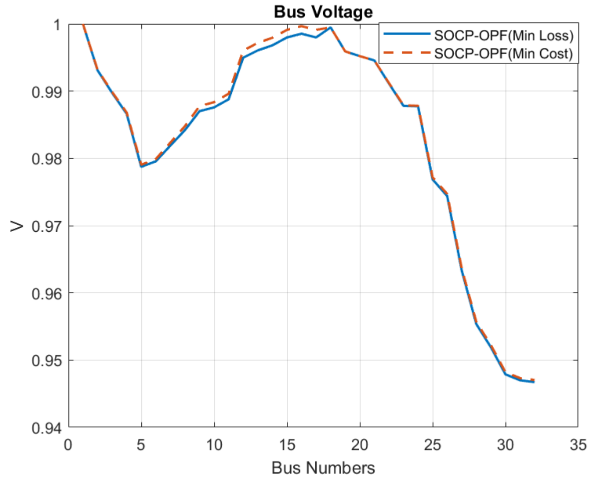

Fig. 5. Voltage profile for SOCP OPF for different objective functions.

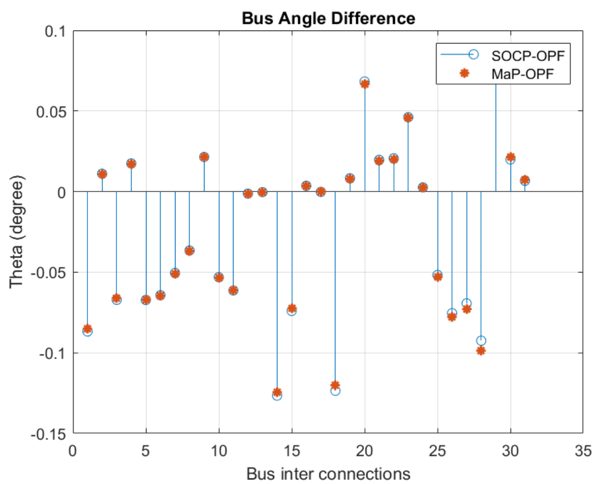

Fig. 6. Bus voltage angle difference in 32 bus network.

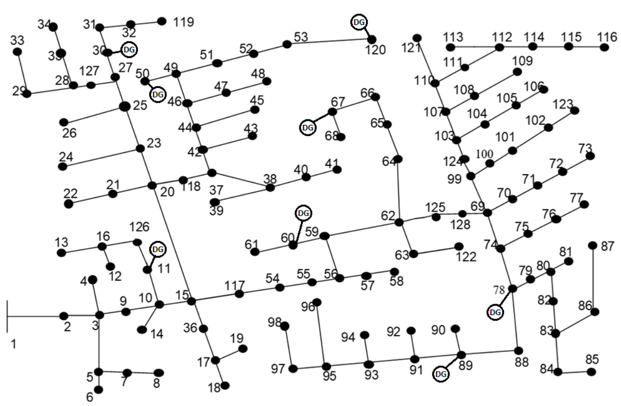

Fig. 7. IEEE 123 bus network with 10% DGs.

pared it with the MatPower result. The angle difference calculated in the SOCP-OPF is almost the same as the MatPower NLP, which validates the approximation for the convexification in (19). The loss in the 32 bus network has been shown for different cases in Table I . The convergence times and the voltage mismatch values for the different algorithms are also included in this Table. For the 32 bus network, maximum and minimum limit and supply from the connected DGs at different buses and cost coefficients are provided in Table II.

It can be seen from Table I that the % loss using the proposed approach is less when compared to the power flow and NLP OPF in Matpower. Also, the computational time for the proposed architecture is less when compared to other benchmark software. It can also be noticed that the bus voltage mismatch is within the limits and very small when compared to benchmark software. As can be seen from Table II, the power flow balance is very close to the benchmark software indicating the optimal solution is close to actual values.

TABLE I
MODIFIED 32 BUS SYSTEM.

	MP_PF	MP_OPF (min cost)	SOCP_OPF (min cost)	SOCP_OPF (min loss)
Time	.08 Sec	.92 Sec	.61 Sec	.53 Sec
% Loss	4.74 %	1.85 %	1.78 %	1.77 %
Gen cost			319.72	320.21
Bus voltage mismatch	MP_OPF (mc) vs SOCP_OPF(mc)		SOCP_OPF (mc) vs SOCP_OPF (ml)	
	.0003 %		0.045 %	
$[mc^* =$ min cost$]$ $[ml^* =$ min loss$]$				

A. Scalability Analysis

Besides the 32 bus radial network, the proposed model has been also tested in IEEE 123 bus system (Fig. 7) for the base case as well as for when 10% , 30% and 50% DGs of the total load is connected in the network. The total load, generation, % of the loss from base case power flow and % of loss for different percents of DGs connected in the network have been shown in Table III. For the 123 bus network, the voltage mismatch between the SOCP-OPF and the OPenDSS results and convergence time is shown in Table IV. Voltage profile for the base case, 10% DG and 30% DG are shown in Fig. 8, 9 and 10 respectively.

Including (19) in the optimization model, the bus voltage angle difference can be calculated for the 123 bus system. For a radial network, taking the substation bus as the reference, the other bus voltage angle is determined from the angle difference value. The angle difference value from the proposed model has been compared with the angle difference from the OpenDSS in Fig. 11, 12 and 13. The results retrieved from the proposed SOCP model are very similar to the OpenDSS values. The small discrepancy in the angle difference is due to the approximations from the (19) for making it convex. But from the comparison of the results with the OpenDSS, the approximation is validated, when the network is a radial distribution network and the bus angle difference across a line is small. Besides, as the proposed model is convex, it acquires less time for converging than compared to NLP based power flow or optimal power flow.

TABLE II
MODIFIED 32 BUS SYSTEM FLOW COMPARISONS

						MP_PF		MP_OPF		SOCP_OPF	
Bus No.	Pg(max)	Pg(min)	Qg(max)	Qg(min)	C	Pg	Qg	Pg	Qg	Pg	Qg
1	10	-	10	-10	90	3.90	2.43	2.03	1.50	2.03	1.50
7	0.35	0.100	0.25	00	79	0.00	0.00	0.35	0.25	0.35	0.25
12	0.30	0.075	0.20	00	87	0.00	0.00	0.30	0.20	0.30	0.20
13	0.32	0.000	0.00	00	70	0.00	0.00	0.32	0.00	0.32	0.00
15	0.08	0.075	0.20	00	92	0.00	0.00	0.75	0.20	0.075	0.20
16	0.30	0.300	0.00	00	70	0.00	0.00	0.30	0.00	0.30	0.00
24	0.41	0.100	0.20	00	81	0.00	0.00	0.41	0.20	0.41	0.20

TABLE III
IEEE 123 BUS SYSTEM PERFORMANCE COMPARISONS

123 Bus System	Total Load [Pd(MW)]	Total Gen SOCP [Pg(MW)]	% Loss SOCP (min loss)	% Loss SOCP (min cost)	% Loss (PF)	Cost SOCP (min loss)	Cost SOCP (min cost)
Base Case	1.1633	1.2017	3.20 %	3.20 %	3.20 %	110.55	110.55
10% DG	1.1633	1.1892	2.18 %	2.18 %	2.75 %	109.15	109.15
30% DG	1.1633	1.1836	1.71 %	1.71 %	2.71 %	108.44	108.44
50% DG	1.1633	1.1782	1.26 %	1.26 %	2.64 %	107.74	107.74

TABLE IV
IEEE 123 BUS SYSTEM FLOW COMPARISONS

123 Bus System	Voltage Mismatch (min loss)	Voltage Mismatch (min cost)	SOCP Time (min loss)	SOCP Time (min cost)	PF Time
Base Case	0.015 %	0.015 %	0.39 sec	0.28 sec	0.73 sec
10% DG	0.026 %	0.026 %	0.39 sec	0.33 sec	0.80 sec
30% DG	0.073 %	0.073 %	0.41 sec	0.36 sec	0.77 sec
50% DG	0.065 %	0.065 %	0.39 sec	0.36 sec	0.81 sec

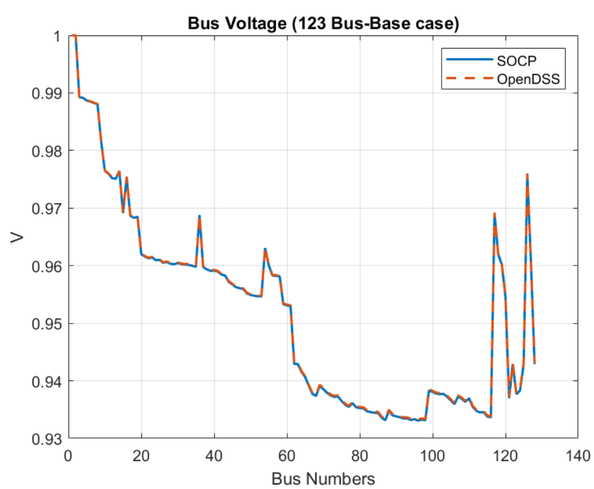

Fig. 8. Voltage profile in 123 bus network base case.

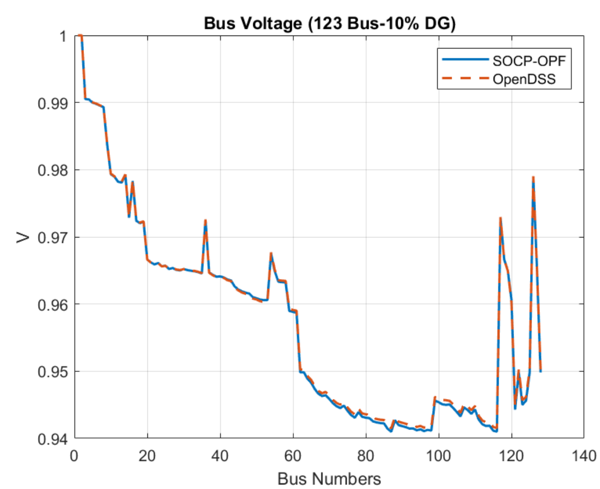

Fig. 9. Voltage profile in 123 bus network with 10% DGs.

IV. CONCLUSION

In this paper, we have proposed and evaluated a branch flow-based SOCP model that mainly concerns on branch current and branch power flows instead of nodal injections. We have also demonstrated the implementation and result evaluation in modified 32 Bus radial network and IEEE 123 Bus system. Our results confirm that, for radial networks, the model guarantees a globally optimal solution. The computational efficiency and time for convergence of the proposed algorithm are also improved. Furthermore, in this paper, we have proposed a convex equation to recover the angle from the optimal power flow model. Including this angle in angle cyclic constraints can enable the proposed SOCP based OPF model to work in a mesh network as well.

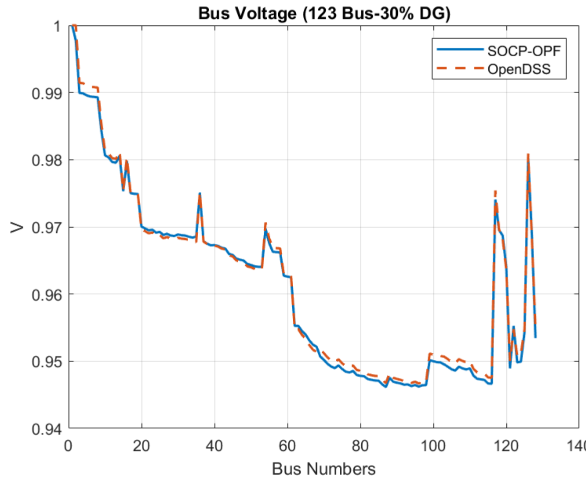

Fig. 10. Voltage profile in 123 bus network with 30% DGs.

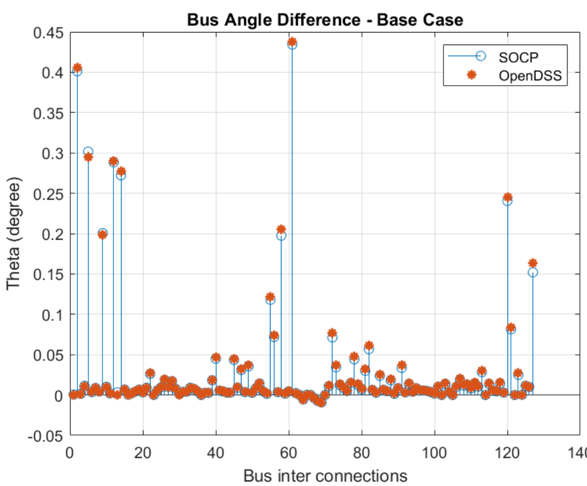

Fig. 11. Bus voltage angle difference in 123 bus network base case.

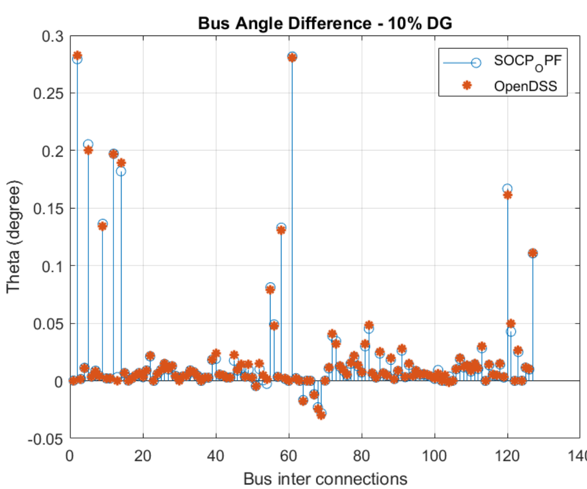

Fig. 12. Bus voltage angle difference in 123 bus network with 10% DGs.

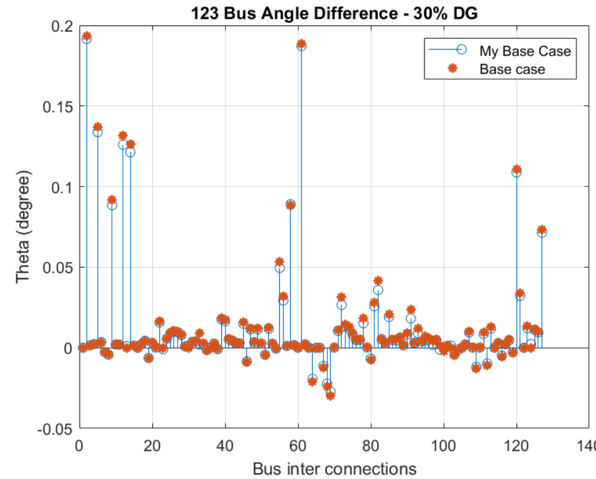

Fig. 13. Bus voltage angle difference in 123 bus network with 30% DGs.

References

[1] Z. Tian and W. Wu, "Recover feasible solutions for socp relaxation of optimal power flow problems in mesh networks," *IET Generation, Transmission Distribution*, vol. 13, no. 7, pp. 1078–1087, 2019.

[2] P. Mahat, Z. Chen, B. Bak-Jensen, and C. L. Bak, "A simple adaptive overcurrent protection of distribution systems with distributed generation," *IEEE Transactions on Smart Grid*, vol. 2, no. 3, pp. 428–437, Sep. 2011.

[3] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification—part i," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2554–2564, 2013.

[4] M. Huneault and F. D. Galiana, "A survey of the optimal power flow literature," *IEEE Transactions on Power Systems*, vol. 6, no. 2, pp. 762–770, 1991.

[5] J. A. Momoh, R. Adapa, and M. E. El-Hawary, "A review of selected optimal power flow literature to 1993. i. nonlinear and quadratic programming approaches," *IEEE Transactions on Power Systems*, vol. 14, no. 1, pp. 96–104, 1999.

[6] H. H. Happ, "Optimal power dispatcha comprehensive survey," *IEEE Transactions on Power Apparatus and Systems*, vol. 96, no. 3, pp. 841–854, 1977.

[7] D. K. Molzahn and I. A. Hiskens, *A Survey of Relaxations and Approximations of the Power Flow Equations*, 2019.

[8] B. D. Biswas, S. Moghadasi, S. Kamalasadan, and S. Paudyal, "Integrated transmission systems convex optimal power flow considering security constraints," in *2019 North American Power Symposium (NAPS)*, 2019, pp. 1–6.

[9] S. Moghadasi and S. Kamalasadan, "Voltage security cost assessment of integrated ac-dc systems using semidefinite programming," in *2016 IEEE Power Energy Society Innovative Smart Grid Technologies Conference (ISGT)*, 2016, pp. 1–5.

[10] ——, "Optimal fast control and scheduling of power distribution system using integrated receding horizon control and convex conic programming," *IEEE Transactions on Industry Applications*, vol. 52, no. 3, pp. 2596–2606, 2016.

[11] A. Gómez Expósito and E. Romero Ramos, "Reliable load flow technique for radial distribution networks," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 1063–1069, 1999.

[12] D. Das, D. Kothari, and A. Kalam, "Simple and efficient method for load flow solution of radial distribution networks," *International Journal of Electrical Power Energy Systems*, vol. 17, no. 5, pp. 335 – 346, 1995. [Online]. Available: http://www.sciencedirect.com/science/article/pii/0142061595000500

[13] R. Ranjan, "Simple and efficient computer algorithm to solve radial distribution networks," *Electric Power Components and Systems - ELECTR POWER COMPON SYST*, vol. 31, pp. 95–107, 01 2003.

[14] R. A. Jabr, "Radial distribution load flow using conic programming," *IEEE Transactions on Power Systems*, vol. 21, no. 3, pp. 1458–1459, 2006.

[15] X. Bai, H. Wei, K. Fujisawa, and Y. Wang, "Semidefinite programming for optimal power flow problems," *International Journal of Electrical Power & Energy Systems*, vol. 30, no. 6-7, pp. 383–392, 2008.

[16] R. A. Jabr, "A conic quadratic format for the load flow equations of

meshed networks," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 2285–2286, 2007.

[17] R. A. Jabr,, "Optimal power flow using an extended conic quadratic formulation," *IEEE Transactions on Power Systems*, vol. 23, no. 3, pp. 1000–1008, 2008.

[18] M. Farivar, C. R. Clarke, S. H. Low, and K. M. Chandy, "Inverter var control for distribution systems with renewables," in *2011 IEEE International Conference on Smart Grid Communications (SmartGridComm)*, 2011, pp. 457–462.

[19] M. Farivar, R. Neal, C. Clarke, and S. Low, "Optimal inverter var control in distribution systems with high pv penetration," in *2012 IEEE Power and Energy Society General Meeting*, 2012, pp. 1–7.

[20] S. H. Low, "Convex relaxation of optimal power flow—part i: Formulations and equivalence," *IEEE Transactions on Control of Network Systems*, vol. 1, no. 1, pp. 15–27, 2014.

[21] ——, "Convex relaxation of optimal power flow—part ii: Exactness," *IEEE Transactions on Control of Network Systems*, vol. 1, no. 2, pp. 177–189, 2014.

[22] R. Madani, S. Sojoudi, and J. Lavaei, "Convex relaxation for optimal power flow problem: Mesh networks," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 199–211, 2015.

[23] M. E. Baran and F. F. Wu, "Optimal capacitor placement on radial distribution systems," *IEEE Transactions on Power Delivery*, vol. 4, no. 1, pp. 725–734, 1989.

[24] M. Baran and F. F. Wu, "Optimal sizing of capacitors placed on a radial distribution system," *IEEE Transactions on Power Delivery*, vol. 4, no. 1, pp. 735–743, 1989.

[25] M. E. Baran and F. F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing," *IEEE Transactions on Power Delivery*, vol. 4, no. 2, pp. 1401–1407, 1989.