This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2022.3217023

Decentralized Distributed Convex Optimal Power Flow Model For Power Distribution System Based on Alternating Direction Method of Multipliers

Biswajit Dipan Biswas, *Student Member, IEEE,* Md Shamim Hasan, *Student Member, IEEE,* and Sukumar Kamalasadan, *Senior Member, IEEE*

***Abstract*—This paper proposes a fully decentralized distributed convex optimal power flow model for inverter-based distributed energy resources (DERs) integrated electric distribution networks based on Semi-Definite Programming (SDP) and alternating direction method of multipliers (ADMM) namely (SDP D-ADMM). The proposed approach is based on the SDP relaxed branch flow model of distribution networks within an auto-tuned accelerated decentralized ADMM architecture. The approach is based on dividing the power grid network into subproblems representing individual areas by interchanging minimum network information. In the proposed model the requirement of a central processor is also waived thus making the proposed approach more robust toward cyber-attacks. The effectiveness and scalability of the proposed method are validated by implementing modified IEEE 123 and IEEE 8500 bus systems with different levels of DER penetration. It has been observed that the proposed architecture outperforms other distributed optimization variants in terms of accuracy, global optimality, scalability, and computational time.**

***Index Terms*—Optimal power flow (OPF), distribution networks (DN), alternating direction method of multipliers (ADMM), decentralized optimization, semidefinite programming (SDP).**

Nomenclature

Vectors

- β_a Vectors of lagrangian multiplier in area a
- $\mathbf{d}_a^k$ Vectors of dual residual in area a after iteration k
- $\mathbf{r}_a^k$ Vectors of primal residual in area a after iteration k
- $\mathbf{x}_a$ Vectors of control variables in area a

Parameters

- $\bar{l}^{ij}$ Upper limit of current magnitude squared between buses i and j
- ϵ Threshold value of error for convergence
- ρ_a^t Local penalty parameter of area a at iteration t
- $\underline{P}_{a,G,i}, \bar{P}^{a,G,i}$ Lower and upper limit of active power generation at bus i in area a
- $\underline{P}_{ij}, \bar{P}^{ij}$ Lower and upper limit of active power flow between buses i and j
- $\underline{Q}_{a,G,i}, \bar{Q}^{a,G,i}$ Lower and upper limit of reactive power generation at bus i in area a
- $\underline{Q}_{ij}, \bar{Q}^{ij}$ Lower and upper limit of reactive power flow between buses i and j
- $\underline{v}_{a,i}, \bar{v}^{a,i}$ Lower and upper limit of voltage magnitude squared at bus i in area a
- $P_{a,D,i}, Q_{a,D,i}$ Active and reactive demand at bus i in area a
- $P_{D,i}, Q_{D,i}$ Active and reactive demand at bus i
- r_{ij}, x_{ij} Resistance and reactance of branch between bus i and j
- V_{ref} Voltage magnitude of reference bus

This work is supported in part by the U.S. Department of Energy's Office of Energy Efficiency and Renewable Energy (EERE) under the Solar Energy Technologies Office Award Number DE-EE0008774, National Science Foundation grant ECCS-1810174, and National Science Foundation grant ECCS-2001732. Corresponding Author: Biswajit Dipan Biswas, University of North Carolina at Charlotte, Email: bbiswas@uncc.edu

Sets

- a, b Indices of areas, from 1 to R
- E Sets of branches
- E^a Sets of branches in area a
- G Sets of generator buses
- G^a Sets of generator buses in area a
- i, j, k Indices of buses, from 1 to N
- N Sets of buses in the whole network
- N^a Sets of buses in area a

Variables

- $\hat{w}_a^t$ Local updated auxiliary variable of area a at iteration t
- $I_{a,ij}, l_{a,ij}$ Current magnitude between bus i and j in area a and the square of it
- I_{ij}, l_{ij} Current magnitude between bus i and j and the square of it
- $P_{a,G,i}, Q_{a,G,i}$ Active and reactive power generation at bus i in area a
- $P_{G,i}, Q_{G,i}$ Active and reactive power generation at bus i
- $S_{a,ij}, P_{a,ij}, Q_{a,ij}$ Apparent, active and reactive power flow between bus i and j in area a
- S_{ij}, P_{ij}, Q_{ij} Apparent, active and reactive power flow between bus i and j
- $V_{a,i}, v_{a,i}$ Voltage magnitude at bus i in area a and the square of it
- V_i, v_i Voltage magnitude at bus i and the square of it
- w_a^t Local auxiliary variable of area a at iteration t

I. Introduction

OPTIMAL Power Flow (OPF) was first introduced by Carpentier in 1962 [1] as method to manage the power flow with specific objectives. The objective of OPF is to minimize a specific objective function such as generation cost, distribution line losses, or voltage deviation. OPF was first formulated on transmission networks that consists of traditional generation resources in an effort to find cost optimal solution for power generation. A detailed study on optimal power flow can be found in [2]–[9]. With surge in popularity of renewable generation like photovoltaics, wind turbine, and energy storage systems, it has been challenging to the distribution network operators to maintain the stability of power distribution system operation considering overall objectives of economic operation. Thus, formulating OPF methodologies for distribution networks with high penetration of distributed energy resources (DER) has drawn a significant amount of interest for power system researchers [10]–[12]. In contrary to the transmission networks, the distribution networks usually consist of thousands of buses with highly unbalanced loading and different R/X ratio making the OPF problem more complex for power distribution networks.

OPF problem is an NP-hard problem and non-convex in nature. Thus, when the system size increases the OPF problem becomes a massive computational burden for the optimization solver. Also, due to the complexity and dimensionality issues, the solver most likely fails to provide the global optimal solution [13]–[15]. To handle computational complexity and non-linearity, several approximations and relaxation methods have been adopted such as Linear approximation, and Convexification based relaxation and approximation. Among them, convex optimization methods are a very popular and feasible approach. There are different relaxation approaches for convexification of the problem such as SemiDefinite Programming (SDP) [16]–[18], Second Order Cone Programming (SOCP) [19], [20], and Chordal Relaxation [21]. The convex optimization approaches can provide the global optimal solution if the relaxation is tight but when the system size and variables increase along with the dynamics, the convex optimization solvers may also fail to converge. As an effective way to ensure convergence, the distributed approaches are becoming the point of interest for the researchers [22]. There are other advantages to distributed approaches too such as, the ability to access the controllers through an aggregator as opposed to a centralized approach where the whole network's information should be available to the central controller to access. For example, sometimes the distribution networks contain large power grid nodes and different parts of the network are owned by different utilities. In such situations, information exchange becomes difficult due to confidentiality issues. So the distributed approaches can share only very little information related to the adjacent buses of the consensus region and the privacy of the information should be ensured.

On the other hand, the distributed approaches can be more computationally expensive than the centralized approaches since each subsystem is solving partial OPF and through iterative adjustment, the global optimum is achieved. For the past few years, researchers have explored different methodologies for distributed OPF. The most known methods are Auxiliary Problem Principal (APP), Analytical Target Cascading (ATC), Optimality Condition Decomposition (OCD), Alternating Direction Method of Multipliers (ADMM) [23]–[26] and other miscellaneous approaches. Among these approaches, ADMM is a very well-suited method to implement distributed convex optimization problems. ADMM was first introduced in the 1970s as a combination of benefits of dual decomposition and augmented lagrangian method for constraint optimization [27], [28]. The ADMM method has different variants based on the formulation such as consensus ADMM, Proximal Jacobian ADMM, and Fast or Accelerated ADMM [29]. All of these approaches are applicable to formulate the OPF problem for power systems. Commonly, in ADMM-based distributed OPF formulation, the problem is modeled on region-based or component-based subproblems. After each iteration, individual subproblems share the information regarding the adjoining buses or components. This information can be referred to as "public information". The convergence of the whole program depends on the convergence of the solution for boundary bus variables.

In most of the ADMM-based distributed approaches, there exists a central coordinator which collects the boundary bus information after each iteration and exchanges the information among the partitions [30]. These connections are prone to both physical and cyber-attacks. If any of the communication links got attacked the whole problem may lose the convergence. Since in distributed approaches communication structure plays a vital role, the communication delay time has a significant effect on the total solving time. For example, if a single loop of communication between connecting buses takes $50ms$, then for a period of 1000 iterations, costing around 0.8 minute as a delay. Thus, reducing the number of iterations is a prime objective of distributed approach [31]. The speed of convergence in ADMM-based formulations significantly depends on the right choice of penalty parameter [32]. The value of the penalty parameter is initialized at the beginning of the problem and stays constant throughout the iterations [33], [34]. However, if the penalty parameter can be updated by looking at the primal and dual residuals update, the convergence will be faster. All these motivations behind the formulation of the proposed decentralized method of solving OPF for the highly penetrated radial distribution network.

In this paper, for the first time, a convex optimization-based distributed optimization framework is proposed for a power distribution system that is fully decentralized, scalable, can converge faster, and ensures a globally optimum solution. The approach is compared with other state-of-the-art and the feasibility is evaluated on the real-life feeders. The main contribution of the proposed architecture is as follows. In [35] authors have proposed an SDP-based ADMM framework. Compared to that work and state-of-the-art the main innovation and contribution of the proposed approach are as follows.

- A fully decentralized approach is proposed to solve the distributed OPF. By removing the necessity of a central processor, the communication topology has been made more secure and robust.
- The update process of the consensus variables is accelerated to guarantee faster convergence.
- The adaptive update of the penalty parameters has been implemented based on the change in primal and dual residuals which as result make the convergence faster.
- Most importantly, the proposed distributed approach is developed in a convex optimization model.

The rest of the paper is organized as follows. Section II discusses the mathematical preliminaries of ADMM formulation and various features including acceleration and residual balancing based auto-tuning. Section III discusses the formulation of the proposed OPF viz., D-SDP ADMM. In section IV, the numerical result and discussions are presented on two IEEE test systems and the conclusion and future work are discussed in section V.

II. Mathematical Preliminaries

In this section, the fundamental theory behind distributed and decentralized optimization based on ADMM is discussed first. Then, the theoretician framework on algorithm acceleration, and auto-tuning is discussed. Lastly, the linear algebraic characteristic of semidefinite programmin is briefly discussed.

ADMM is an algorithm that leverages the better convergence properties of method of multipliers to solve constrained optimization problems. Assume a problem in following form,

$$\begin{aligned} &Min\ f(x) + g(y) \\ &s.t. Ax + By = c \end{aligned} \tag{1}$$

where $x \in \mathbb{R}$ and $y \in \mathbb{R}$ are the variables and $A, B \in \mathbb{R}$ are parameter matrices. Based on the field of the application, these variables may refer to various entities. In this case of optimal power flow formulation, the variables which are considered to be in consensus are bus voltages, V, branch current, I, branch power P, Q.

The augmented Lagrangian of this problem can be written as:

$$L_\rho(x,z,\beta) = f(x) + g(y) + \beta^T(Ax + By - c) \quad (2)$$
$$+ \frac{\rho}{2}||Ax + By - c||_2^2$$

ADMM solves the problem in three updation steps. First, x is updated with fixed y, then y is solved with updated x from previous step and in the final step β is updated from fixed values of x and y. These steps are as follows.

$$x^{k+1} := \arg\min_x\{f(x) + (\beta^k)^T(Ax + By^k - c) \quad (3)$$
$$+ \frac{\rho}{2}||Ax + By^k - c||_2^2\}$$

$$y^{k+1} := \arg\min_y\{g(y) + (\beta^k)^T(Ax^{k+1} + By - c) \quad (4)$$
$$+ \frac{\rho}{2}||Ax^{k+1} + By - c||_2^2\}$$

$$\beta^{k+1} := \beta^k + \rho(Ax^{k+1} + By^{k+1} - c) \quad (5)$$

where $\rho > 0$ is the penalty factor and β is the vector of lagrangian multipliers. The convergence of the ADMM depends on the following criterion,

$$\lim_{k\to\infty}(Ax^{k+1} + By^{k+1} - c) = 0$$

A. Consensus Optimization via ADMM

If the objective function of the ADMM problem consists of N terms, then the problem takes new form which is known as consensus ADMM. This form of objective function may represent to minimize the loss function of an individual area of the distribution system, or to minimize the line losses of a region of a large distribution network written as

$$Min \sum_{i=1}^{N} f(x) \quad (6)$$
$$s.t. x_i - y = 0$$

where x_i is the local variable and y is the global variable.

The objective is to have all the local variables converge to a global value. For this work, the objective is to minimize the line power loss in the network. The variables of the branch flow model formulation are bus voltage magnitude, line current, and active and reactive line power flow. Thus in the consensus formulation, the constraint would be to have the bus voltage and line power flow of certain buses and lines between the regions converge as observed from each region. Definition of these local and global variables are discussed in section III where the ADMM-based OPF problem is formulated.

The difference between conventional ADMM and consensus based ADMM is in the update process of the global variable, which is done as shown below:

$$y^{k+1} := \frac{1}{n}\sum_{i=1}^{N}(x_i^{k+1}) \quad (7)$$

In the consensus ADMM approach all the regions solve their OPF problem for a constraint set and a global variable y. The iterative updating process continues till the error reduces below the threshold value. After each iteration the primal and dual residuals of all the subproblems are calculated using the following equations:

$$r_a^{k+1} = ||\beta_a^{k+1} - \beta_a^k|| \quad (8)$$

$$d_a^{k+1} = \rho * ||y_a^{k+1} - y_a^k|| \quad (9)$$

where r_a^{k+1} and d_a^{k+1} are the primal and dual residuals for area a after k-th iteration.

Once the maximum value of the residuals become less than the threshold ϵ then the convergence is considered to be achieved.

$$max(r_a^{k+1}, d_a^{k+1}) \le \epsilon \quad (10)$$

B. Decentralized ADMM by Substituting Lagrange Multiplier

The consensus ADMM as well as the original formulation of ADMM does not ensure the fully decentralized formation. The local variable and lagrange multiplier update using (3) and (6) can be performed locally but the update of the global variables using (7) for overlapping regions requires a central controller to execute. By replacing the global variable y and lagrange multiplier β it is possible to formulate a fully decentralized model. For that purpose, a new local variable vector w is introduced for area a corresponding to the lagrange multiplier β_i,

$$w_a^k := y_a^k - \beta_a^k/\rho \quad (11)$$

Further, leveraging the features of radial distribution networks, ADMM can be reformulated for any local problem as

$$x_a^{k+1} := \arg\min_x\{f(x_a) + \frac{\rho}{2}||x_a - w^k||_2^2\} \quad (12)$$

$$w_a^k := w_a^k + x_a^{k+1} - \frac{x_a^{k+1} + x_b^{k+1}}{2} \quad (13)$$

C. Auto Tuning of Penalty Parameter by Residual Balancing

The convergence of ADMM based OPF problem is mathematically proven although, the speed to convergence depends significantly on the choice of penalty parameter. One way to accelerate the ADMM convergence is to vary the penalty parameter depending on the residual values from each iteration. Various approaches have been proposed by the researchers to implement a self-tuning penalty parameter model. Most of those approaches requires a central controller to look at the residual values and update the penalty parameter. In the decentralized approach, the penalty parameter for each area can be updated based on the local primal and dual residual values.So the central co-ordination is not required anymore. The penalty parameter tuning can be performed as

$$\rho_i^{k+1} = \begin{cases} \frac{\rho_i^k}{1+\tau}, & \text{if } ||r_i^k||_2 \le \xi||d_i^k||_2, \\ (1+\tau)\rho_i^k, & \text{if } ||d_i^k||_2 \le \xi||r_i^k||_2, \\ \rho_i^k, & \text{otherwise.} \end{cases} \quad (14)$$

where ξ and τ are parameters whose values are usually selected as $= 0.1$ and $\tau = 1.0$.

This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2022.3217023

Algorithm 1: Residual Balanced ADMM algorithm

Initialize the Lagrange multiplier β and global variable y and penalty parameter ρ for each subsystem.
Initialize variables and τ
Initialize the error value to a large number.
Solve local OPF for each subsystems using objective function as (4).
All the adjacent subsystems share the solution for consensus variables y.
Update the global variables using (7).
Update the Lagrange multiplier β using (5).
error = $\max(r_a^{k+1}, d_a^{k+1})$
if $(error \geq \epsilon)$ **then**
  if $||r_a^{k+1}||_2 < ||d_a^{k+1}||_2$ **then**
    $\rho_a^{k+1} = \frac{\rho_a^k}{1+\tau}$
  end
  else if $||d_a^{k+1}||_2 < ||r_a^{k+1}||_2$ **then**
    $\rho_a^{k+1} = (1+\tau)\rho_a$
  end
  else
    $\rho_a^{k+1} = \rho_a^k$
  end
end

Algorithm 2: Accelerated ADMM algorithm

1 Initialize the Lagrange multiplier β and global variable y for each subsystem.
2 Each subsystem solves the local optimal power flow problem.
3 Copy variables corresponding to y are sent to the leading subsystems of the respective variables.
4 Leading subsystem receives the copy variables and update the global variable y^{k+1} using (7) .
5 Leading subsystems transmit the updated y^{k+1} to the adjacent subsystems.
6 **if** *(Stopping criteria (10) satisfied)* **then**
7   Algorithm terminates
8 **end**
9 **else**
10   Each subsystem sends r_a^{k+1} and d_a^{k+1} to global controller to calculate α using (17)
11   The global controller transmits α to all the subsystems.
12   The subsystems update y and β using (15) and (16).
13   Go to step 2.
14 **end**

D. Accelerated ADMM Method

In accelerated ADMM approach, additional steps included to update the global variable y^{k+1} and lagrange multiplier β^{k+1} as follows

$$\hat{y_i}^{k+1} = \alpha^k.y^{k+1} + (1-\alpha^k).y^k \tag{15}$$

$$\hat{\beta^{k+1}} = \alpha^k.\beta^{k+1} + (1-\alpha^k).\beta^k \tag{16}$$

$$\alpha^k = \begin{cases} 1 + \frac{\gamma^k - 1}{\gamma^{k+1}}, & if \frac{max(||r^k||_2, ||s^k||_2)}{max(||r^{k-1}||_2, ||s^{k-1}||_2)} \leq 1 \\ 1, & otherwise \end{cases} \tag{17}$$

where $\gamma = [1 + \sqrt{1 + 4(\gamma^{k-1})^2)}]/2$ for $k > 1$. Here r and s stands for the primal and dual residuals.

E. Semidefinite Programming

Let us consider a simple linear programming (LP) example,

$$\begin{aligned} &\text{minimize } c.x \\ &\text{subject to, } A.x = b \\ &x \geq 0 \end{aligned}$$

Here, x is the control variable, c and A are the parameter matrices. All the equations in objective function and constraints are linear or piecewise linear. Thus, the whole problem is convex. Semidefinite programming is generalization of linear programming where the inequality constraints are represented by general inequalities which corresponds to the cone of positive semidefinite matrices [36], [37]. This is a pure primal form of a semidefinite programming based optimization problem,

$$\begin{aligned} &\text{Minimize} \quad trace(CX) \\ &\text{Subject to,} \quad trace(A_i X) = b_i, \ for \ i = 1, ..n \\ &X \succcurlyeq 0 \end{aligned}$$

Here, $X \in S^n$ is the decision variable, it is also a positive semidefinite matrix. Others, b, C and A are symmetric matrices which values are already known to the model. The feasible set defined by the set of constraints are always convex. The objective function is linear by nature. Thus the whole problem is linear and convex.

III. Proposed Decentralized Distributed Optimization Approach with Semi-Definite Programming (SDP) Based ADMM

The proposed OPF approach is based on Branch Flow Model (BFM) of the power distribution system. The model is then convexified based on SDP. Further the model is made distributed ad fully decentralized using the ADMM including features such as auto-tuning and acceleration.

A. BFM Model of Power Distribution System

Let us assume a graph $G = (N, E)$ represents a radial distribution network where, N is the set of all vertices and E is the set of all branches. Branch flow model comprises of the branch variables such as branch current, branch active and reactive power flow. Let, V_i is the voltage of node i, S_{ij} and I_{ij} is the complex power and current flown through branch $i-j$, then branch flow model can be stated as follows

$$V_i - V_j = z_{ij} I_{ij}, \forall (i,j) \in E \tag{18}$$

$$S_{ij} = V_i I_{ij}^*, \forall (i,j) \in E \tag{19}$$

$$\sum_{k:j \to k} S_{jk} - \sum_{i:i \to j} (S_{ij} - z_{ij}|I_{ij}|^2) + y_j^*|V_j|^2 = s_j \tag{20}$$

where z_{ij} is the branch impedance and s_j is the injected complex power at node j.

This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2022.3217023

B. Convexification of BFM using SDP Framework

The relaxed BFM can be formulated from (18)-(20) by ignoring the angles of the variables. For this, first, substituting the expression of current I_{ij} from (19) into (18) yields $V_i - V_j = z_{ij} S_{ij}^* / V_i^*$. Then taking the square of the magnitudes of this expression derives (22) as

$$s_j = \sum_{k:j\to k} S_{jk} - \sum_{i:i\to j} (S_{ij} - z_{ij} l_{ij}) + y_j v_j, \forall j \in E \quad (21)$$

$$v_j = v_i - 2(z_{ij}^* S_{ij} + z_{ij} S_{ij}^*) + z_{ij} l_{ij} z_{ij}^*, \forall (i,j) \in E \quad (22)$$

$$l_{ij} = \frac{|S_{ij}|^2}{v_i}, \forall (i,j) \in E \quad (23)$$

It is worth noting that, in the relaxed model the squared terms of the node voltage and branch current replaces the previous variables as $v_i = |V_i|^2$ and $l_{ij} = |I_{ij}|^2$.

The non-linear equation (23) can be expressed in terms of a positive semidefinte matrix as follows:

$$\begin{bmatrix} v_i & S_{ij} \\ S_{ij}^* & \lambda_{ij} \end{bmatrix} \succcurlyeq 0$$

$$rank \begin{bmatrix} v_i & S_{ij} \\ S_{ij}^* & \lambda_{ij} \end{bmatrix} = 1$$

The aforementioned model still hold the non-convexity due to the rank-1 constraint of the PSD matrix. Relaxing the model by adopting the semidefinite relaxation (SDR), the BFM-SDP OPF problem is formulated:

$$Min \sum_{i:i\to j} z_{ij} I_{ij} \quad (24)$$

subject to

$$s_j = \sum_{k:j\to k} S_{jk} - \sum_{i:i\to j} (S_{ij} - z_{ij} |l_{ij}|^2) + y_j v_j \quad (25)$$

$$v_j = v_i - (S_{ij} z_{ij}^* + z_{ij} S_{ij}^*) + z_{ij} \lambda_{ij} z_{ij}^* \quad (26)$$

$$\begin{bmatrix} v_i & S_{ij} \\ S_{ij}^* & \lambda_{ij} \end{bmatrix} \geq 0 \quad (27)$$

$$v_{ref} = V_{ref} V_{ref}^* \quad (28)$$

$$v^{min} \leq v_i \leq v^{max} \quad (29)$$

$$S^{min} \leq S_i \leq S^{max} \quad (30)$$

In this paper, the reactive power dispatch of the DERs are considered as the control variable. Assuming the apparent power capacity of the DER as 120% of the active power generation, the reactive power upper and lower bounds are calculated as

$$Q_{G,i} = \sqrt{(1.2 * P_{G,i})^2 - (P_{G,i})^2} \quad (31)$$

C. Distributed BFM-SDP OPF in ADMM framework

Based on the consensus ADMM and the BFM-SDP OPF formulation, the distributed problem can be formulated for each region. Before that, the global variable can be defined as, $y' = [P_{mn} Q_{mn} P_{lt} Q_{lt} V_m V_l]$. Now, the augmented OPF problem for each region can be formulated as follows. For the master network all the nodes as shown in Fig. 1 along with the consensus region nodes are considered to formulate the augmented OPF problem for master network.

$$Min \sum_{i:i\to j} z_{ij} I_{ij} + (\beta_1^k)^T (x_1 - y_1^k) + \frac{\rho}{2} ||x_1 - y_1^k||_2^2 \quad (32)$$

subject to

(25) - (30)

where $y_1 = y$.

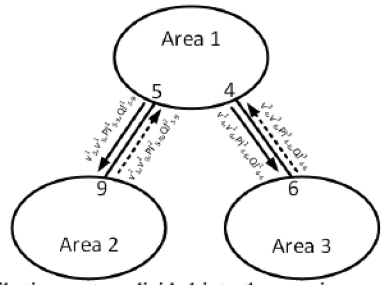

Fig. 1. A distribution system divided into three regions.

Algorithm 3: Proposed D-SDP ADMM

Initialize the Lagrange multiplier β and global variable y and penalty parameter ρ for each subsystem.
Initialize variables and τ
Initialize the error value to a large number.
Solve local OPF for each subsystems using objective function as (12).
All the adjacent subsystems share the solution for consensus variables.
Update the local auxiliary variables using (13).
Broadcast the updated local auxiliary variables to the adjacent subsystems.
Calculate the local primal and dual residuals, r_a, d_a in all subsystems. error = $\max(r_a^{k+1}, d_a^{k+1})$
if $(error \geq \epsilon)$ **then**
    Update the penalty parameter in each subsystems using (14)
end

Similarly for sub-network 1 the augmented OPF problem can be formulated with updated z as follows

$$y_2 = [P_{mn}, Q_{mn}, V_m]^T$$

The augmented Lagrangian objective function for sub-network 1 is as follows:

$$Min \sum_{i:i\to j} z_{ij} I_{ij} + (\beta_2^k)^T (x_2 - y_2^k) + \frac{\rho}{2} ||x_2 - y_2^k||_2^2 \quad (33)$$

Further for sub-network 2 the augmented OPF problem can be formulated with updated z as follows,

$$y_3 = [P_{lt}, Q_{lt}, V_l]^T$$

With the objective function as

$$Min \sum_{i:i\to j} z_{ij} I_{ij} + (\beta_3^k)^T (x_3 - y_3^k) + \frac{\rho}{2} ||x_3 - y_3^k||_2^2 \quad (34)$$

Once all the regions done solving for the variable **x** then, the global variable z is updated using (30) as,

$$y(1,3,5) = 0.5 * [y_1(1,3,5) + y_2] \quad (35)$$

$$y(2,4,6) = 0.5 * [y_1(2,4,6) + y_3]$$

This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2022.3217023

The primal and dual residual of the formulation are denoted as follows,

$$r^{k+1} = ||\beta^{k+1} - \beta^k||_2 \tag{36}$$
$$d^{k+1} = \rho||y^{k+1} - y^k||_2$$

The dual variable are updated using (32). Then the error is being calculated as,

$$error^k = \left\| \begin{matrix} r^{k+1} \\ s^{k+1} \end{matrix} \right\|^2 \tag{37}$$

The threshold cut-off value for error is considered as $10e-4$. If the error value becomes less than the threshold then a global consensus is achieved.

D. Proposed D-SDP ADMM

In the fully decentralized proposed ADMM approach, the main contributions when compared to the state-of-the-art are a) relaxing the global variable and introducing an auxiliary local variable b) auto tune the penalty parameters to improve the convergence, and c) introduce the convex model in the ADMM framework, The combined formulation takes the form as

$$x_i^{k+1} := \arg\min_x \{f(x_i) + \frac{\rho}{2}||x_i - w^k||_2^2\} \tag{38}$$

$$w_i^{k+1} := w_i^k + x_i^{k+1} - \frac{x_i^k + x_j^k}{2} \tag{39}$$

$$\rho_i^{k+1} = \begin{cases} \frac{\rho_i^k}{1+\tau}, & \text{if } ||r_i^k||_2 \leq ||d_i^k||_2, \\ (1+\tau)\rho_i^k, & \text{if } ||d_i^k||_2 \leq ||r_i^k||_2, \\ \rho_i^k, & \text{otherwise.} \end{cases} \tag{40}$$

For a generic network as shown in Fig. 1 the detailed implementation of the proposed approach is explained below. Assume the network is consist of three sub-networks as $N_a = \{1-5, 6, 9\}, N_b = \{4, 6-8\}, N_c = \{5, 9-12\}$. Now the set of adjoining buses are $N_a \cap N_b = \{4, 6\}$, $N_a \cap N_c = \{5, 9\}$. The boundary bus i which is shared by adjoining areas a and b will have it's variables denoted as $x_{a,i}$ and $x_{b,i}$. For the network in example, $x_{1,4} = x_{2,4} = y_4$, $x_{1,6} = x_{2,6} = y_6$, $x_{1,5} = x_{3,5} = y_5$ and $x_{1,9} = x_{3,9} = y_9$. Now, to implement the decentralized approach, a local auxiliary variable is introduced to replace the global variable as

$$w_{1,4} = y_4 - \frac{\beta_{1,4}}{\rho} \qquad w_{1,6} = y_6 - \frac{\beta_{1,6}}{\rho}$$
$$w_{2,4} = y_4 - \frac{\beta_{2,4}}{\rho} \qquad w_{2,6} = y_6 - \frac{\beta_{2,6}}{\rho}$$
$$w_{1,5} = y_5 - \frac{\beta_{1,5}}{\rho} \qquad w_{1,9} = y_9 - \frac{\beta_{1,9}}{\rho}$$
$$w_{3,5} = y_5 - \frac{\beta_{3,5}}{\rho} \qquad w_{3,9} = y_9 - \frac{\beta_{3,9}}{\rho}$$

With the help of these local auxiliary variables, the update equation for area 1 can be written as,

$$x_{1,i}^{k+1} := \arg\min_x \{f(x_{1,i}) + \frac{\rho}{2} \sum_{j \in N_1 \cap N_2 \cap N_3} ||x_{1,j} - w_{1,j}^k||_2^2\} \tag{41}$$

$$w_{1,j}^{k+1} := w_{1,j}^k + x_{2,j}^{k+1} - \frac{x_{1,j}^k + x_{2,j}^k}{2}; j \in N_1 \cap N_2 \cap N_3 \tag{42}$$

Here, the variable vectors can be expressed elementwise as, $x_1 = \{v_4, v_6, P_{4,6}, Q_{4,6}, v_5, v_9, P_{5,9}, Q_{5,9}\}$ and $w_1^k = \{\hat{v}_4^k, \hat{v}_6^k, \hat{P}_{4,6}^k, \hat{Q}_{4,6}^k, \hat{v}_5^k, \hat{v}_9^k, \hat{P}_{5,9}^k, \hat{Q}_{5,9}^k\}$. Then the local OPF problem for area 1 will take the form as shown below:

$$Min \sum_{i:i \to j} z_{1,ij} I_{1,ij} + \frac{\rho_1}{2}[(x_1 - w_1^k)^2] \tag{43}$$

subject to

(25) - (30)

Once the OPF is solved, the local auxiliary variable w_1^{k+1} is updated using (36) and then the residuals are calculated for area 1 using following equations:

$$r_1^k = ||(x_{1,j}^k - x_{2,j}^k)/2|| \tag{44}$$
$$d_1^k = ||((x_{1,j}^k + x_{2,j}^k) - (x_{1,j}^{k-1} + x_{2,j}^{k-1}))/2|| \tag{45}$$

The convergence is assumed to be achieved once all the residuals are calculated and $max(r^k, d^k) \leq \epsilon$. If the convergence is not reached for the subproblem, then the penalty parameter is updated using (34) based on the ratio of primal and dual residual. And since the penalty parameter update depends only on the local residual, the decentralized approach stays operational without the requirement of a central coordinator.

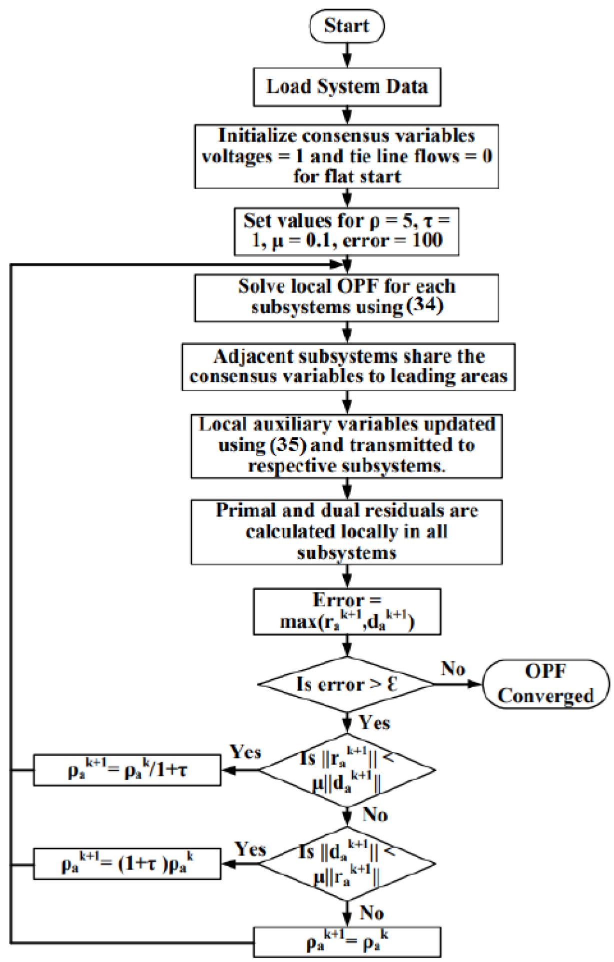

Fig. 2. Flowchart for the proposed decentralized distributed convex OPF based on ADMM (D-SDP ADMM).

This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2022.3217023

IV. Simulation Results and Discussions

The proposed methodology is implemented on the following two IEEE test systems that are real-life feeders of power distribution systems. They are a) modified IEEE 123 bus system as shown in Fig 3 and, b) modified IEEE 8500 bus system as shown in Fig 3

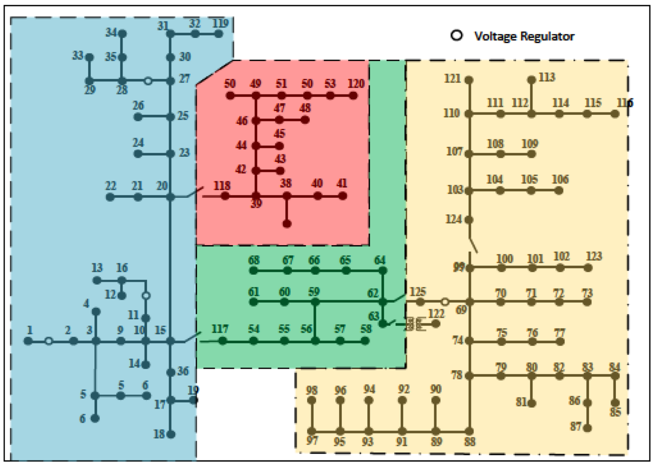

Fig. 3. Modified IEEE 123 bus system with IBR Based DERs.

The IEEE-123 bus system is a heavily loaded feeder with one three-phase and 3 single-phase voltage regulator and four shunt capacitors. This power grid model has been used to prove the applicability of the proposed OPF algorithm on a system with more number of regulators. For this purpose, the converted single-phase network is considered for OPF modeling, using the OpenDSS software. First, a single-phase representation of the $Ybus$ is performed using a positive sequence representation of the three-phase $Ybus$. Then from the Y bus matrix, the line impedance values are extracted. The connected loads are also converted similarly. Table. I represents the power grid loading. The 8500-node test feeder consists of multiple feeder regulators, capacitor banks, split-phase service transformers, and feeder secondaries. The circuit has a $115kV$ source, $12.47kV$ medium voltage feeder sections, and a $120V$ low voltage feeder section. There are 4876 three-phase, two-phase, and single-phase medium-voltage nodes. The single-phase nodes are connected to 1177 split phase transformers. The two secondaries of these transformers are connected to load nodes using triplex lines. In total, there are 3041 A phase nodes, $2830B$ phase nodes, and $2660C$ phase nodes. Table. I represents the power grid loading.

TABLE I
Test systems description

Sl No	Test System	Volt. Reg.	Trans.	Shunt Caps	Avg R/X	Total Load
1	IEEE 123	4	1	4	0.2645	1.1633 MW 0.64 MVAR
2	IEEE 8500	4	1177	4	0.2145	3.3252 MW 0.8335 MVAR

For evaluating the performance of the proposed approach on the power grid with DER, a 10%, 30%, and 50% DER penetration is considered by placing DERs randomly at different locations on the feeder. The capacity of the DERs is considered to be equal to the loads connected to that bus. The reference bus voltage is considered as 1.05 pu. The upper and lower bound of voltage magnitude are set as 1.05 pu and 0.95 pu. For IEEE 123 bus system, the base MVA is 5MVA and for the IEEE 8500 bus, the base MVA is set as 1MVA. The details of the DERs including the location (bus number), size, and total number are illustrated in Table. Table. II and Table. III for IEEE 123 and IEEE 8500 node system. The active power generation of the DERs is considered to be equal to the active power demand of the bus. The solution of the proposed decentralized method is compared with other

TABLE II
DER location and rating for different penetration levels in IEEE 123 bus system

DER %	DER Location	DER Power Capacity (KW)	DER Power Capacity (KVA)
10%	11,30,89,102	13.33	16
	50	70	84
	60	6.667	8
	67	46.667	56
	78	81.66	98
30%	8,11,18,2130,32,37,45, 64,77,89,101,102,106,109	13.33	16
	53,57,60,86,98,113,116	6.667	8
	50	70	84
	67	46.667	56
	78	81.66	98
50%	8,11,18,21,24,26,30,32,35, 37,45,55,64,71,77,81,84,89, 92,96,101,102,106,109,111	13.33	16
	4,14,19,34,40,43,47,53,57,60, 86,98,113,116	6.667	8
	50	70	84
	67	46.667	56
	68	25	30
	78	81.66	98

TABLE III
DER location and active power rating for 10% DER penetration in 8500 bus system

DER %	DER Location	DER Active Power Rating (KW)	DER Capacity (KVAR)
10%	1102,1183,1274,1368,1408, 1502,1642,1669,1674,1691, 1740,1816,1868,1883,2018, 1928,1969,1992,2043,2054, 2081,2092,2112,2139,2149, 2167,2180,2209,2299,2340, 2355,2364,2404,2420,2456, 2462,2516	2.9570	3.5484
	34,43,59,62,66,69,102,120, 149,151,162,194,200,213,224, 230,239,247,252,267,284,329, 363,372,384,402,409,432,486, 502,607,612,621,690,761,794, 800,823,833,850,885,899,928, 951,995,1038,1138,1146,1239, 1304,1313,1321,1416,1466,1473, 1647,1713,1723,1809,1860,1907, 1911,1938,2066,2097,2188,2194, 2311,2321,2326,2429,2468	3.3900	4.068
	23,40	5.0870	6.1044
	2520	5.9130	7.0956
	2485	29.560	35.472

This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2022.3217023

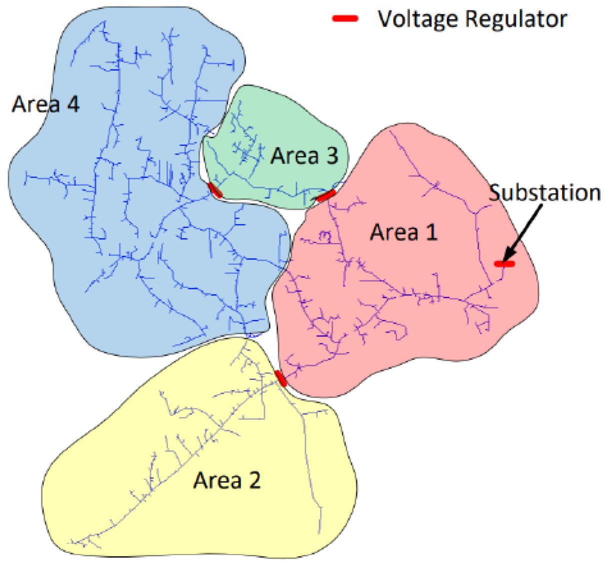

Fig. 4. Modified IEEE 8500 bus one line diagram IBR Based DERs location.

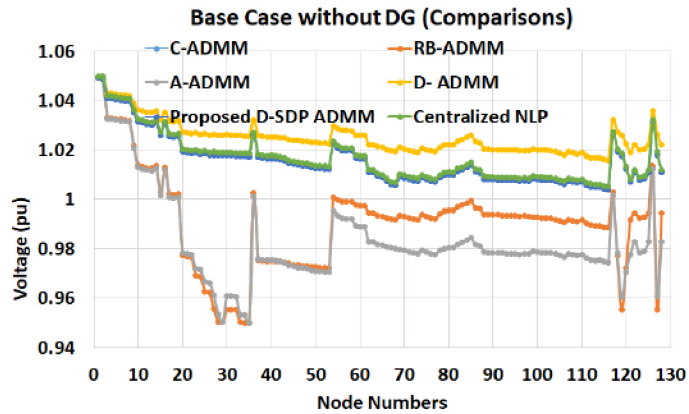

Fig. 5. Voltage profile comparison of modified IEEE 123 bus system with no DG penetration.

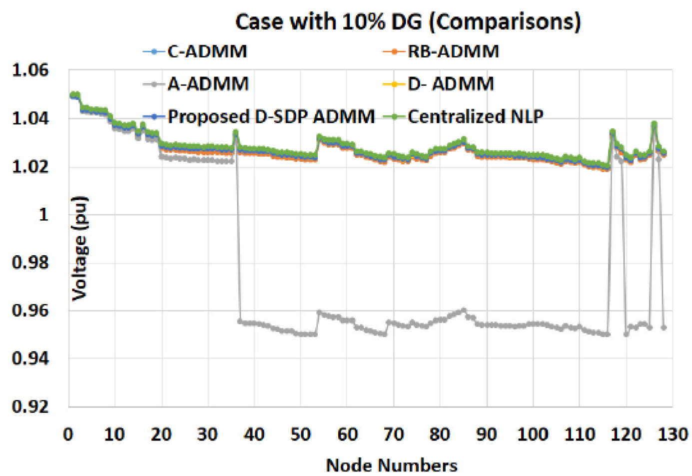

Fig. 6. Voltage profile comparison of modified IEEE 123 bus system with 10% DER penetration.

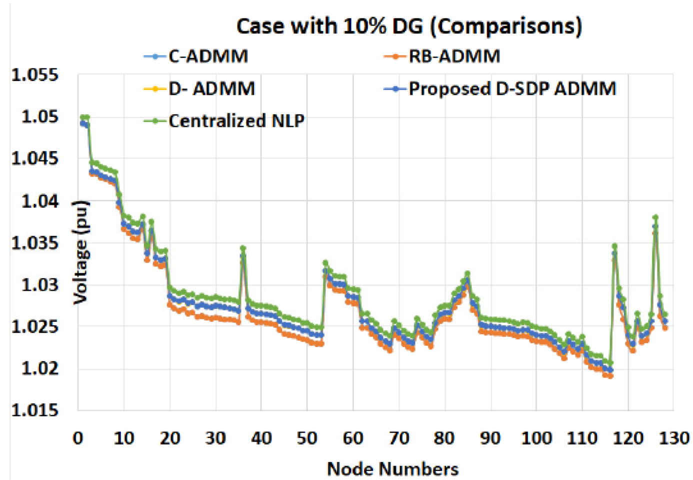

Fig. 7. Voltage profile comparison of modified IEEE 123 bus system with 10% DER penetration.

state-of-the-art including centralized OPF, consensus ADMM based OPF, residual balanced ADMM based OPF, Accelerated ADMM OPF, Decentralized ADMM based SDP-OPF. All the simulations were performed on a windows computer with a 2.5GHz Intel Core i5 processor and 16GB RAM. All the coding was done in the MATLAB platform using the YALMIP optimization toolbox and MOSEK solver.

A. IEEE 123 node system:

First, accuracy of the proposed method is analyzed on the base case (which is without any DERs). The analysis is when compared to other centralized and distributed method. As the Nonlinear Programming (NLP) formulation provide global optimal solution (for near equilibrium conditions) the proposed approach is compared with the NLP. The comparisons are with four distributed optimal power flow algorithms viz. consensus ADMM (C-ADMM), residual balanced ADMM (RB-ADMM), Accelerated ADMM (A-ADMM), and decentralized ADMM (D-ADMM), one centralized approach based on NLP and the proposed D-SDP ADMM. Fig. 5 shows the comparisons, It can be seen that the proposed approach provides close optimal solution when compared with NLP. It can also be seen that the solutions from other approaches are not accurate when compared to NLP.

In Fig 5, the comparison of voltage profiles from different approaches is shown. It can be observed that the distributed approaches such as A-ADMM, RB-ADMM, D-ADMM, and C-ADMM are deviating from the global optimal solution. The proposed approach can provide the closest solution to the globally optimal values in the NLP as illustrated. Further comparisons for higher levels of DER penetrations are performed. From Fig. 6 it can see that the solution from A-ADMM is deviating most from the NLP solution and the voltage profile is very close to the lower bound for most of the buses. To get a better comparison Fig. 7 is provided where all other profiles except A-ADMM are compared. It can be seen that the profiles are close but there are gaps among NLP solutions and other approaches while the proposed approach was able to be the most accurate method.

Fig. 8 and Fig. 9 shows the similar trend of the solution from A-ADMM approach. In Fig. 9, it can be seen that as the level of penetration increased the gap among the distributed ADMM profiles when compared to the central solutions while

This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2022.3217023

TABLE IV
COMPARISON OF CONVERGENCE PROPERTIES OF DIFFERENT DISTRIBUTED OPTIMIZATION METHODS

	Centralized NLP		Consensus ADMM (C-ADMM)		Residual Balanced ADMM (RB-ADMM)		Accelerated ADMM (A-ADMM)		D-ADMM		Proposed D-SDP ADMM	
	Iteration	Time (s)	Iteration	Time (s)	Iteration	Time (s)	Iteration	Time (s)	Iteration	Time (s)	Iteration	Time (s)
123 Bus 4 Partitions	N/A	48.10	108	25.57	97	23.45	88	20.76	173	41.29	107	25.41
8500 Bus 4 Partitions	N/A	192.38	154	136.65	N/A	N/A	N/A	N/A	N/A	N/A	189	178.71

TABLE V
COMPARISON OF SUBSTATION POWER OF DIFFERENT DISTRIBUTED OPTIMIZATION METHODS

	Centralized NLP		Consensus ADMM (C-ADMM)		Residual Balanced ADMM (RB-ADMM)		Accelerated ADMM (A-ADMM)		D-ADMM		Proposed D-SDP ADMM	
	Psub (KW)	Qsub (KVAR)	Psub (KW)	Qsub (KVAR)	Psub (KW)	Qsub (KVAR)	Psub (KW)	Qsub (KVAR)	Psub (KW)	Qsub (KVAR)	Psub (KW)	Qsub (KVAR)
123 Bus 4 Partitions	921.07	251.081	921.5	287.1	921.4	287.1	1.3931	1.3319	921.2	251.3	921.2	251.14
8500 Bus 4 Partitions	3150.79	576.734	3255.8	1051.6	N/A	N/A	N/A	N/A	N/A	N/A	3150.8	576.8

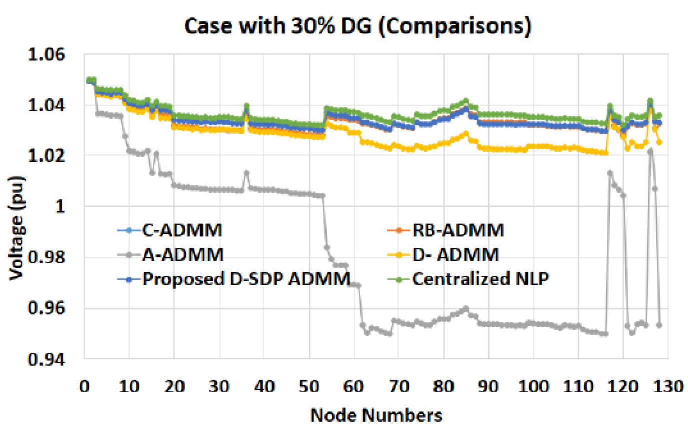

Fig. 8. Voltage profile comparison of modified IEEE 123 bus system with 30% DER penetration.

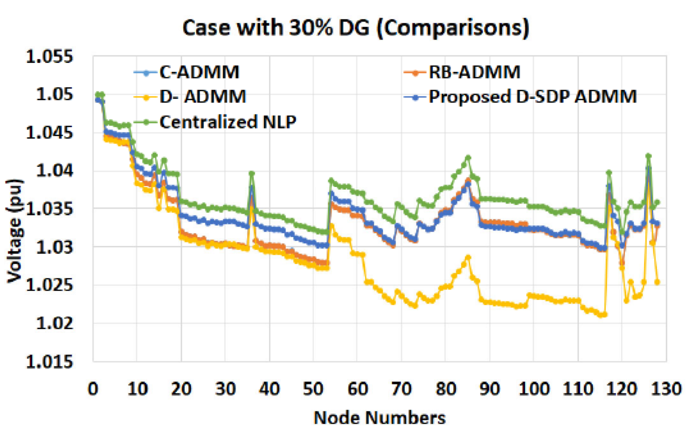

Fig. 9. Voltage profile comparison of modified IEEE 123 bus system with 30% DER penetration.

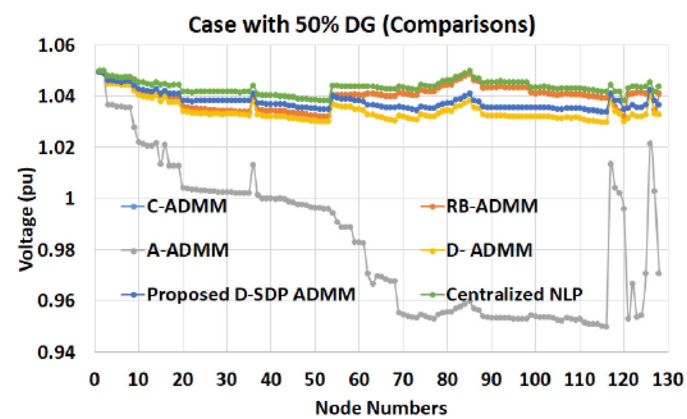

Fig. 10. Voltage profile comparison of modified IEEE 123 bus system with 50% DER penetration.

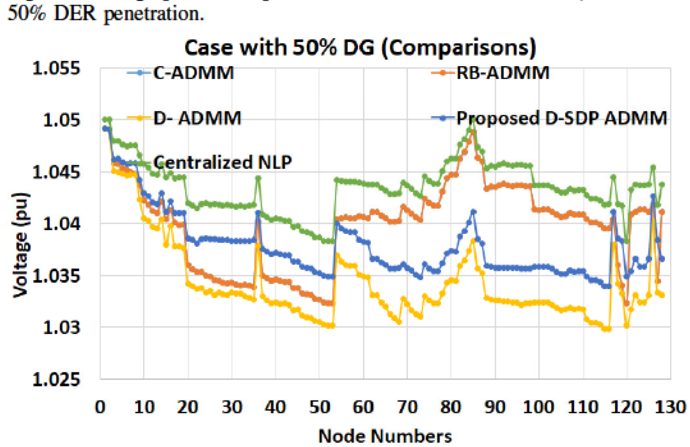

Fig. 11. Voltage profile comparison of modified IEEE 123 bus system with 50% DER penetration.

the proposed method still gives the most accurate solution. Results showed Fig. 10 and Fig. 11 validate the claim that the A-ADMM fails to provide the global optimal solution while the proposed D-SDP ADMM approach still guaranty the exact solution for any level of DER penetration.

The consensus ADMM-based OPF converged to the optimal solution with a minor gap compared with the global

This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2022.3217023

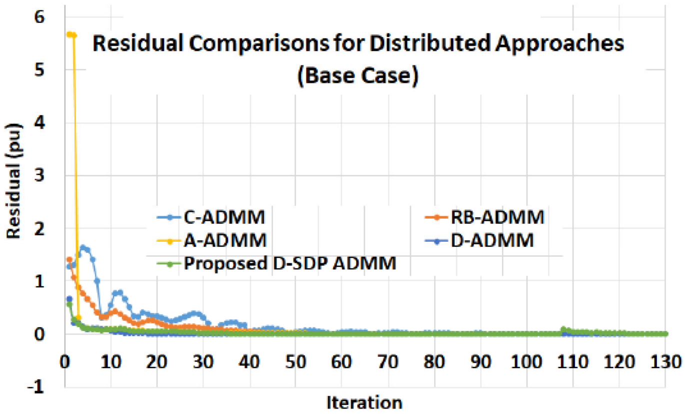

Fig. 12. Residual comparison of modified IEEE 123 bus system base case.

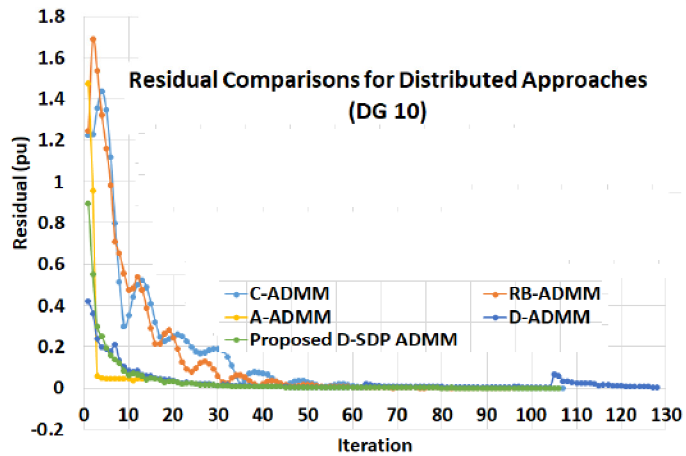

Fig. 13. Residual comparison of modified IEEE 123 bus system with 10% DER penetration.

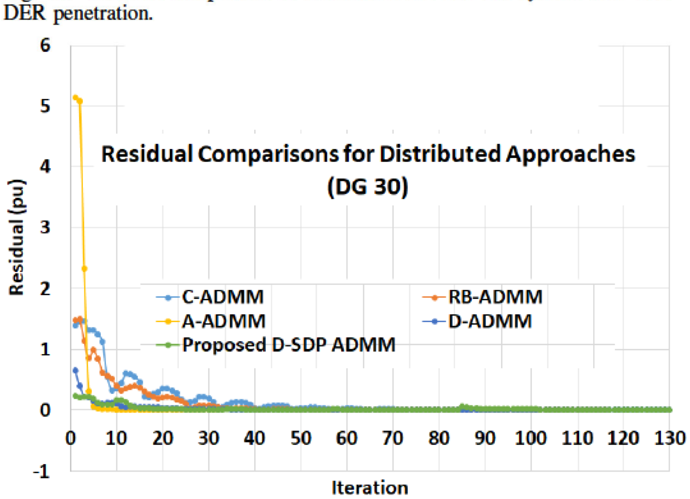

Fig. 14. Residual comparison of modified IEEE 123 bus system with 30% DER penetration.

optimal solution from centralized OPF. Similar convergence was achieved using the residual balanced ADMM-based OPF. Since, in the residual balanced approach, the penalty parameter is updated after each iteration, it shows a faster convergence speed. This is evident from data provided in Table IV. In the accelerated ADMM-based distributed OPF the global variable and Lagrangian multiplier are updated in additional steps. It is noted that this approach does not guarantee faster convergence and globally optimal solutions all the time. The solution from the decentralized ADMM was closest to the global optimal solution although it takes more iterations to achieve convergence. In the proposed approach, the auto-tuning of the penalty parameter helps to speed up the convergence.

The exactness of the solution from the proposed D-SDP ADMM method is further illustrated from the information provided in Table. VI. In the Table. VI substation active and reactive power dispatch, along with the number of iterations and total computational time to converge from different distributed methods i.e., centralized NLP, C-ADMM, RB-ADMM, A-ADMM, D-ADMM, and proposed D-SDP ADMM for base system and 10%, 30% and 50% DER penetration cases are compiled. The speed of the convergence for different methods can also be visualized from the plotting of residuals. Fig 12-Fig 14, shows the residual profiles from different approaches for different test system cases. In chronological order, the figures represent the base system, 10%, and 30% DER penetration cases. It can be seen that the profiles from C-ADMM and RB-ADMM have similar slop while A-ADMM has the steepest slope among the profiles. However, in the earlier discussion, it has been shown that A-ADMM fails to provide the global optimal solution. The proposed D-SDP ADMM method doesn't have the fastest convergence property among all but it is faster than the C-ADMM and RB-ADMM approaches and it ensures the global optimal point. Please note that residue for 50% DER penetration case is similar to that of 20% DER penetration case thus been omitted.

B. Scalability Analysis (IEEE 8500 node system):

Once the proposed model was able to provide satisfactory results for the modified IEEE 123 bus system, it was tested on another real-world test network, the modified IEEE 8500 node system. In this case, 10% DER penetration was considered. The total network was partitioned into four interconnected subsystems. The numerical comparison of the solution for different approaches is showcased in Table. VI. Although, few of the methods i.e., residual balanced ADMM, accelerated ADMM and decentralized SDP ADMM were not able to converge to an optimal solution for the given threshold value. As shown before, the proposed method shows similar convergence properties compared with the consensus ADMM method. The number of iterations and time of convergence is higher in the proposed method but the solution is the closest to the global optimal point. From Fig. 15 it can be seen that there is a significant gap in the solution from consensus ADMM while the profile from the proposed approach is almost similar to the centralized solution. Fig. 16 shows the maximum residual profile in each iteration while solving the 8500 bus system using the proposed method. It is evident from the slop of the plot that, the auto-tuning of the penalty parameter significantly improved the speed of convergence.

C. Validation Through Real-time Simulation

The numerical solution comparison from Table VI shows that the proposed distributed approach is able to converge at the global optimal solution with conclusive tightness in

This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2022.3217023

TABLE VI
COMPARISON OF SUBSTATION POWER AND NUMBER OF ITERATIONS OF DISTRIBUTED OPTIMIZATION METHODS

IEEE 123 Bus System with base case and different DG Penetration. All proposed methods are compared with 4 partitions						
Comparisons	Sub. Power	Base	10% DG	30% DG	50% DG
Centralized NLP	Psub (KW)	1192.136	921.07	728.471	518.527
	Qsub (KVAR)	447.85	251.081	146.675	91.66
	Iteration	N/A	N/A	N/A	N/A
	Time (s)	0.4781	0.4772	0.4851	0.4869
Consensus ADMM (C-ADMM)	Psub (KW)	1192.38	921.5	729.017	519.819
	Qsub (KVAR)	448.146	287.1	222.618	166.464
	Iteration	166	108	130	179
	Time (s)	38.3792	25.5744	31.213	42.1903
Residual Balanced ADMM (RB-ADMM)	Psub (KW)	1390.77	921.4	729.122	519.795
	Qsub (KVAR)	1327.48	287.1	222.662	166.502
	Iteration	94	97	117	134
	Time (s)	22.5885	23.4449	28.3374	32.264
Accelerated ADMM (A-ADMM)	Psub (KW)	1152.40	922.847	607.636	581.744
	Qsub (KVAR)	1499.73	1266.08	1110.347	490.63
	Iteration	52	14	32	54
	Time (s)	12.2044	3.3418	7.5776	12.6954
D-ADMM	Psub (KW)	921.536	921.2	729.112	518.746
	Qsub (KVAR)	309.92	251.3	274.48	24.256
	Iteration	200+	173	129	200+
	Time (s)	47.84+	41.2951	31.1019	47.76+
Proposed Approach	Psub (KW)	1192.362	921.2	725.548	518.507
	Qsub (KVAR)	448.024	251.144	153.271	114.81
	Iteration	187	107	200+	180
	Time (s)	44.2442	25.4125	48.36+	42.876

TABLE VII
SUMMARY OF OBSERVATIONS

Objective Function- Loss Minimization				
Methods	**Feasibility**	**Optimality**	**Accuracy**	**Scalability**
Centralized NLP	Feasible	Global Optimal	Most Accurate	No
C-ADMM	Feasible	Global/Local optimal	Accurate	Scalable
RB-ADMM	Feasible	Global/Local optimal	less accurate	Scalable
A-ADMM	Feasible	Local optimal	Inaccurate	Scalable
D-ADMM	Feasible	Local optimal	less accurate	Scalable
Proposed D-SDP ADMM	Feasible	Global optimal	Very accurate	Scalable

solution when compared to the centralized and non-linear approaches. For further validation and real-time applicability of the proposed method, the solution was validated using a real time power system simulator Opal-RT. The setup for the real time simulation and validation is shown in Fig 17. The DER setpoints, such as active and reactive power dispatches for each individual DER inverter are transmitted to a similar

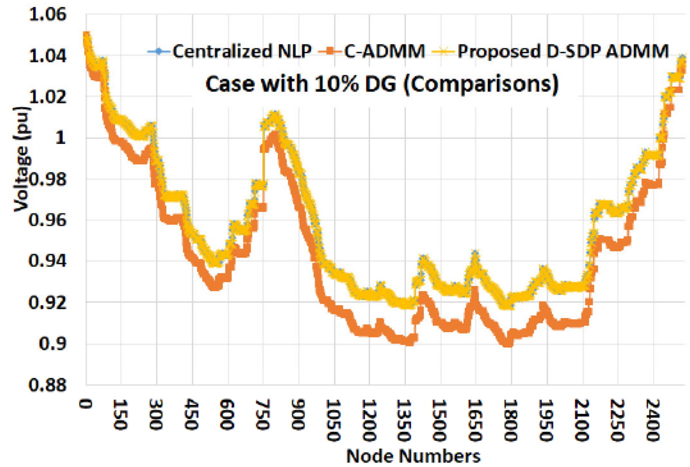

Fig. 15. Voltage profile vomparison of modified IEEE 8500 bus system with 10% DG penetration.

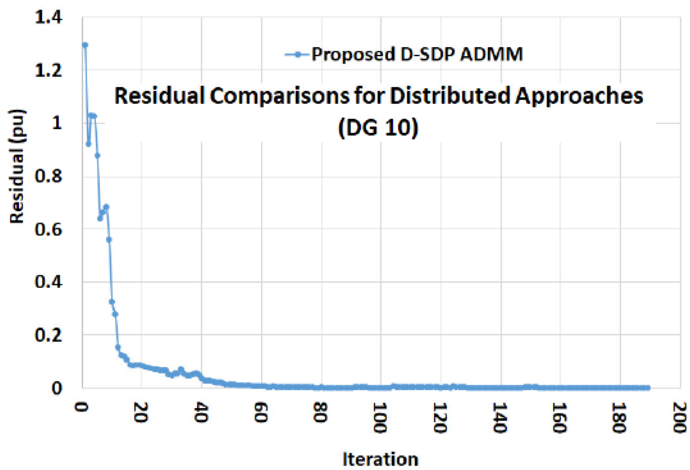

Fig. 16. Residual comparison of modified IEEE 8500 bus system with 10% DG penetration.

model built inside the Opar-RT simulator and power flow was solved. Once done, the active and reactive power dispatches from the substation from Opal-RT simulation and the similar from the proposed method are very conclusive. The % error of the voltage profile from these two approaches are shown in Fig 18. We can see that the maximum error is around 1% which indicates that the solutions are very similar. Also the substation active and reactive power dispatches from Oplar-RT simulations are 924.844KW and 182.885KVAR while the same from the proposed approach were 921.2KW and 251.144KVAR. Since the objective function selected was to minimize the line active power losses, thus by comparing the substation active power dispatch we can confirm that the proposed approach's solution is conclusive. Also, in Opal-RT platform, each simulation takes around 60ms while the proposed approach consumes around 25.41s. In real world DSO, the operators usually perform the real time dispatch on a 5 min time resolution. Since, the computational time of our proposed approach is well below the 300s mark, it can be confirmed that the proposed approach can also be implemented in real world operation.

This article has been accepted for publication in IEEE Transactions on Industry Applications. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TIA.2022.3217023

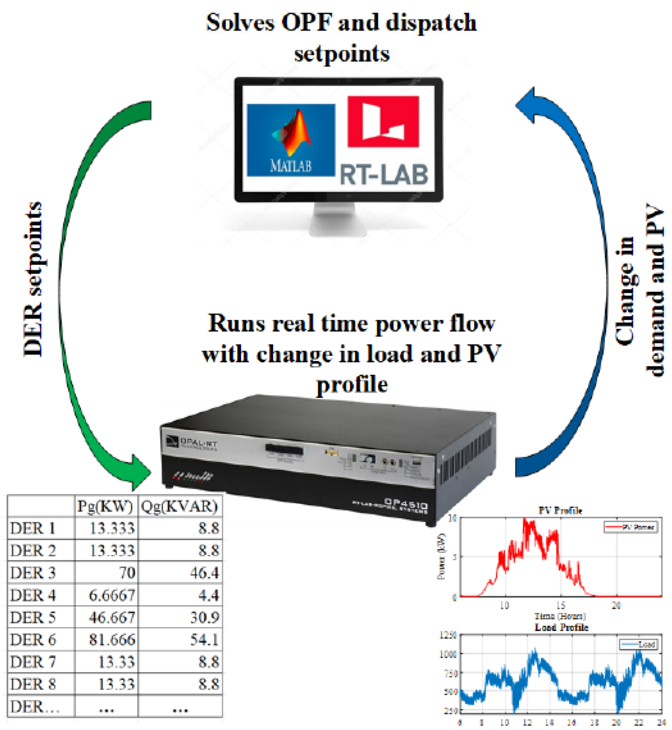

Fig. 17. Setup used for real time simulation and validation using Opal-RT.

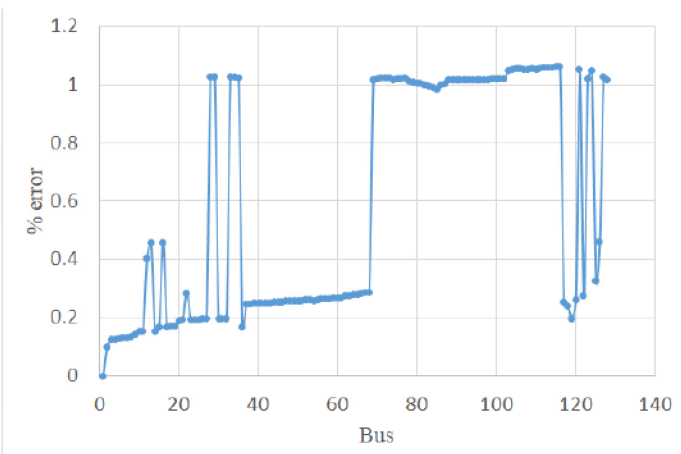

Fig. 18. % Error of bus voltage magnitudes from proposed approach and OpalRT simulation.

D. Summary of Observations:

The proposed method provides advantages of distributed approaches such as C-ADMM, D-ADMM, and RB-ADMM. Also, the optimality gap is minimal in the proposed architecture due to the convexification of the model and the solution is globally optimal. To analyze this criteria such as tightness of the solution, speed of convergence, and scalability of the method are compared. Based on the performances on these criteria for different methods, a Table. VII is presented, which summarises the feasibility, optimality, accuracy, scalability, and usability of different distribution methods. In the analysis, the solution from the centralized NLP is considered the benchmark for all other approaches. The NLP method ensures the global optimal solution if the size and type of the problem are not beyond its capacity. Thus it may not be able to provide the global optimal solution for any size of the problem. The C-ADMM approach is found to be feasible for different types of networks and able to provide a solution that may not be the global optimal but is very close to it. Then, the RB-ADMM method speeds up the convergence but it cost the accuracy of the solution. Next, the A-ADMM method is the fastest among all the methods discussed here but it lacks accuracy by a very high margin, thus losing its usability for implementation. Next, the D-ADMM method is an improved method that enables to implementation of the decentralized approach for distributed optimization, albeit its solution also contains some gaps compared with the benchmark solution, This method is further improved in our proposed D-SDP ADMM using the auto-tuning of the penalty parameter. This modification improves the speed of convergence and the solution is validated to be the global optimal point. Also, the proposed D-SDP ADMM method holds its performance for a larger IEEE 8500 node system and a higher level of DER penetration.

V. Conclusions and Future Work

In this paper, an adaptive ADMM-based fully decentralized formulation of OPF based on the BFM-SDP architecture is proposed. The method of adaptive tuning of penalty parameter has been discussed which improves the speed of convergence while achieving the global optimal solution. The proposed method has been validated by implementing on two standard distribution test networks, IEEE 123 bus system, and IEEE 8500 bus system. The solution of the proposed method is validated by comparing that with the centralized solution. Although the proposed method takes additional convergence time when compared to certain distributed approaches, it always reaches the global optimal point thus being more accurate. Since the underlying OPF formulation is based on the BFM-SDP model, it provides exact relaxation when compared to the approximated linearized and SOCP, relaxed model. The convergence properties and solution profiles for the IEEE 8500 bus system prove the scalability of the proposed method for larger systems. The plan as an extension of this work is to combine acceleration in the local auxiliary variable update to further speed up the convergence while maintaining the tightness of the solution. Also extending the formulation for unbalanced multiphase network is a work in progress.

References

[1] J. Carpentier, "Contribution a l'etude du dispatching economique," *Bulletin de la Societe Francaise des Electriciens*, vol. 3, no. 1, pp. 431–447, 1962.

[2] M. Huneault and F. D. Galiana, "A survey of the optimal power flow literature," *IEEE Transactions on Power Systems*, vol. 6, no. 2, pp. 762–770, May 1991.

[3] J. A. Momoh, R. Adapa, and M. E. El-Hawary, "A review of selected optimal power flow literature to 1993. i. nonlinear and quadratic programming approaches," *IEEE Transactions on Power Systems*, vol. 14, no. 1, pp. 96–104, Feb 1999.

[4] J. A. Momoh, M. E. El-Hawary, and R. Adapa, "A review of selected optimal power flow literature to 1993. ii. newton, linear programming and interior point methods," *IEEE Transactions on Power Systems*, vol. 14, no. 1, pp. 105–111, Feb 1999.

[5] K. Pandya and S. Joshi, "A survey of optimal power flow methods," *Journal of Theoretical and Applied Information Technology*, vol. 4, p. 450–458, 01 2008.

[6] S. Frank, I. Steponavičė, and S. Rebennack, "Optimal power flow: a bibliographic survey i," *Energy Systems*, vol. 3, 09 2012.
[7] ——, "Optimal power flow: a bibliographic survey ii," *Energy Systems*, vol. 3, 09 2012.
[8] C. Coffrin and P. Van Hentenryck, "A linear-programming approximation of ac power flows," *INFORMS Journal on Computing*, vol. 26, 06 2012.
[9] A. Mary, B. Cain, and R. O'Neill, "History of optimal power flow and formulations," *Fed. Energy Regul. Comm.*, vol. 1, pp. 1–36, 01 2012.
[10] C. D'Adamo, S. Jupe, and C. Abbey, "Global survey on planning and operation of active distribution networks-update of cigre c6. 11 working group activities," in *CIRED 2009-20th International Conference and Exhibition on Electricity Distribution-Part 1*. IET, 2009, pp. 1–4.
[11] T. T. Hashim, A. Mohamed, and H. Shareef, "A review on voltage control methods for active distribution networks," *Prz. Elektrotech*, vol. 88, no. 6, pp. 304–312, 2012.
[12] S. Chowdhury, S. Chowdhury, and P. Crossley, *Microgrids and active distribution networks*, 2022.
[13] M. Khanabadi and S. Kamalasadan, "Day ahead scheduling of distribution system with distributed energy resources considering demand response and energy storage," in *2013 North American Power Symposium (NAPS)*. IEEE, 2013, pp. 1–6.
[14] M. Farivar, C. R. Clarke, S. H. Low, and K. M. Chandy, "Inverter var control for distribution systems with renewables," in *2011 IEEE international conference on smart grid communications (SmartGridComm)*. IEEE, 2011, pp. 457–462.
[15] M. Farivar and S. H. Low, "Branch flow model: Relaxations and convexification—part i," *IEEE Transactions on Power Systems*, vol. 28, no. 3, pp. 2554–2564, 2013.
[16] S. Moghadasi and S. Kamalasadan, "An architecture for voltage stability constrained optimal power flow using convex semi-definite programming," in *2015 North American Power Symposium (NAPS)*, 2015, pp. 1–6.
[17] B. D. Biswas, S. Moghadasi, S. Kamalasadan, and S. Paudyal, "Integrated transmission systems convex optimal power flow considering security constraints," in *2019 North American Power Symposium (NAPS)*, 2019, pp. 1–6.
[18] P. Wang, Q. Wu, S. Huang, C. Li, and B. Zhou, "Admm-based distributed active and reactive power control for regional ac power grid with wind farms," *Journal of Modern Power Systems and Clean Energy*, vol. 10, no. 3, pp. 588–596, 2021.
[19] B. Kocuk, S. S. Dey, and X. A. Sun, "Strong socp relaxations for the optimal power flow problem," *Operations Research*, vol. 64, no. 6, pp. 1177–1196, 2016.
[20] M. Wang, Y. Su, L. Chen, Z. Li, and S. Mei, "Distributed optimal power flow of dc microgrids: A penalty based admm approach," *CSEE Journal of Power and Energy Systems*, vol. 7, no. 2, pp. 339–347, 2019.
[21] L. Gan and S. H. Low, "Chordal relaxation of opf for multiphase radial networks," in *2014 IEEE International Symposium on Circuits and Systems (ISCAS)*. IEEE, 2014, pp. 1812–1815.
[22] R. Baldick, B. H. Kim, C. Chase, and Y. Luo, "A fast distributed implementation of optimal power flow," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 858–864, 1999.
[23] M. Ma, L. Fan, and Z. Miao, "Consensus admm and proximal admm for economic dispatch and ac opf with socp relaxation," in *2016 North American power symposium (NAPS)*. IEEE, 2016, pp. 1–6.
[24] M. Kraning, E. Chu, J. Lavaei, S. Boyd *et al.*, "Dynamic network energy management via proximal message passing," *Foundations and Trends® in Optimization*, vol. 1, no. 2, pp. 73–126, 2013.
[25] G. Chen and J. Li, "A fully distributed admm-based dispatch approach for virtual power plant problems," *Applied Mathematical Modelling*, vol. 58, pp. 300–312, 2018.
[26] T. Erseghe, "Distributed optimal power flow using admm," *IEEE transactions on power systems*, vol. 29, no. 5, pp. 2370–2380, 2014.
[27] S. Boyd, N. Parikh, and E. Chu, *Distributed optimization and statistical learning via the alternating direction method of multipliers*. Now Publishers Inc, 2011.
[28] S. Boyd, S. P. Boyd, and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.
[29] R. Gupta, V. Sovljanski, F. Sossan, and M. Paolone, "Performance comparison of alternating direction optimization methods for linear-opf based real-time predictive control," in *2021 IEEE Madrid PowerTech*. IEEE, 2021, pp. 1–6.
[30] Y. Wang, L. Wu, and S. Wang, "A fully-decentralized consensus-based admm approach for dc-opf with demand response," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 2637–2647, 2016.
[31] T. Goldstein, B. O'Donoghue, S. Setzer, and R. Baraniuk, "Fast alternating direction optimization methods," *SIAM Journal on Imaging Sciences*, vol. 7, no. 3, pp. 1588–1623, 2014.
[32] C. Mavromatis, M. Foti, and M. Vavalis, "Auto-tuned weighted-penalty parameter admm for distributed optimal power flow," *IEEE Transactions on Power Systems*, vol. 36, no. 2, pp. 970–978, 2020.
[33] W. Zheng, W. Wu, B. Zhang, H. Sun, and Y. Liu, "A fully distributed reactive power optimization and control method for active distribution networks," *IEEE Transactions on Smart Grid*, vol. 7, no. 2, pp. 1021–1033, 2015.
[34] S. Mhanna, G. Verbič, and A. C. Chapman, "Adaptive admm for distributed ac optimal power flow," *IEEE Transactions on Power Systems*, vol. 34, no. 3, pp. 2025–2035, 2018.
[35] B. D. Biswas and S. Kamalasadan, "Distributed convex optimal power flow model based on alternating direction method of multipliers for power distribution system," in *2021 IEEE Industry Applications Society Annual Meeting (IAS)*. IEEE, 2021, pp. 1–6.
[36] J. Nie, K. Ranestad, and B. Sturmfels, "The algebraic degree of semidefinite programming," *Mathematical Programming*, vol. 122, no. 2, pp. 379–405, 2010.
[37] P. A. Parrilo and S. Lall, "Semidefinite programming relaxations and algebraic optimization in control," *European Journal of Control*, vol. 9, no. 2-3, pp. 307–321, 2003.

Biswajit Dipan Biswas (S'19) received his B.Eng. degree in Electrical and Electronics Engineering from Bangladesh University of Engineering and Technology, Bangladesh, in 2013. He worked as an Operation Engineer at KEPCO-KPS in Bangladesh from 2013 to 2017. He is currently working as a PhD candidate in Electrical Engineering at UNC Charlotte. His current research interests include convex optimization in power distribution systems with high penetration with distributed energy resources.

Md Shamim Hasan (S'21) received his B.Eng. degree in Electrical and Electronics Engineering from Bangladesh University of Engineering and Technology, Bangladesh, in 2016. He worked as an Assistant Engineer at Gas Transmission Company and Power Grid Company of Bangladesh Limited (PGCB) in Bangladesh from 2016 to 2021. He is currently working as a PhD candidate in Electrical Engineering at UNC Charlotte. His current research interests include optimization and control of power system with high penetration with distributed energy resources.

Sukumar Kamalasadan (SM'17) received the B.Tech. degree in electrical and electronics engineering from the University of Calicut, India, in 1991, the M.Eng. degree in electrical power systems management from the Asian Institute of Technology, Bangkok, Thailand, in 1999, and the Ph.D. degree in electrical engineering from The University of Toledo, Toledo, OH, USA, in 2004. He is a Professor in the Department of Electrical and Computer Engineering at the University of North Carolina at Charlotte, Charlotte, NC, USA. His research interests include intelligent and autonomous control, power systems dynamics, stability and control, smart grid, microgrid, and real-time optimization and control of power system. He is a recipient of the National Science Foundation (NSF) CAREER Award and IEEE Best Paper Awards. He is the chair of IEEE Power and Energy Society (PES) Power and Energy Education Committee, member of 2022 IEEE PES Leadership Team and Council member of IEEE Industry Application Society (IAS).