Distributed Convex Optimal Power Flow Model Based on Alternating Direction Method of Multipliers For Power Distribution System

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***Abstract*—The objective of OPF is to find an operating point for a network minimizing certain cost functions such as line losses or generation costs. In recent years, a significant rise in distributed generation (DG) penetration in the distribution network made the OPF problem a greater computational burden. As a result, the centralized OPF formulation is facing more challenges. In this paper a fully distributed approach has been proposed that utilizes the convergence proper of alternating direction method of multipliers (ADMM) and split the central OPF problem into small problems of regions. All the regional OPF problems are parallelizable and computationally cheaper than the centralized approach. The non-linear, non-convex AC-OPF problem in this approach uses SDP relaxation to convexify. The proposed approach is tested on the modified IEEE 123 bus system to prove it's scalability.**

***Index Terms*—Optimal Power Flow(OPF), Distribution System, Convex Optimization, Alternating Direction Method of Multipliers (ADMM), Semi-Definite Programming (SDP).**

I. Introduction

THE objective of OPF is to minimize or maximize a cost function such as minimizing the generation cost, line losses, or maximizing voltage stability, DG generation. Numerous economic operations of power systems such as economic dispatch, unit commitment, demand response, volt-var control are designed around OPF. Since the very first approach to solve the OPF problem, proposed by J. Carpentier in 1962 [1], a lot of approaches have been proposed by the researchers to solve the problem. Detailed survey literature on different formulations of OPF and evolution of the problem formulation can be found in [2]–[11]. The original alternating current optimal power flow (AC-OPF) problem is a non-linear, non-convex optimization problem. Different relaxation methods have been explored to handle the non-convexity of the problem. Among them, semi-definite programming (SDP), second-order cone programming (SOCP), and chordal relaxation are the most popular ones. Initially bus injection models (BIM) of transmission network utilized both SDP relaxation [12]–[14] and SOCP relaxation [15], [16] for OPF formulation. Since these are the relaxed model of the original problem, the formulation is said to be exact if the solution of the original problem can be recovered from the relaxed model. However, radial network modeling requires additional considerations for exact modeling.

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Another aspect of the conventional OPF formulations is that they are mostly centralized operations. This means, the original network is formulated as one single problem and solved as one model. However, as the distributed generation is becoming more and more popular in today's power system, it increases the total number of variables in the formulation, thus increases the difficulty level of the problem [17]. So, OPF formulation of real-world distribution networks with thousands of nodes and high DG penetration is extremely difficult to solve with a centralized approach. Thus there is a real need to solve distributed formulation of the OPF problem for the future distribution grid. In that regard, various distributed approaches have been proposed by different researchers. The generalized approach is to break down the OPF problem into subproblems that can be solved simultaneously. There are distributed formulations based on the AC non-convex OPF problem as in [18], [19] which used the method of multipliers. In [20] the formulation leveraged ADMM for distributed optimization but the main disadvantage of such formulation is that it does not guarantee convergence. On the other hand, the distributed formulation of the convexified OPF problem ensures convergence especially ADMM based convex methods combines the benefits of the dual decomposition [21].

In this paper, an approach has been proposed where a radial system is divided into multiple regions. This division can be based on different criteria such as geographical location, the position of the SVR, placement of the transformer, or switches. In this approach, two main aspects are significant which are intra-regional optimization and inter-regional coordination. The intra-regional optimization model has been formulated by utilizing the SDP relaxed branch flow model and the inter-regional coordination is implemented with the help of ADMM. The main contributions of this paper are as follows. This approach provides a simplified architecture to implement the distributed formulation of the OPF problem for the radial distribution network. It identifies the consensus region for the split network and implements ADMM to solve the OPF problem in a fully distributed approach. All the regional OPF problems are parallelizable and computationally cheaper when compared to other distributed OPF counterparts.

The rest of the paper is organized in the following order. Section II describes the mathematical preliminaries regarding the ADMM and OPF problem formulation. Section III describes the proposed distributed OPF formulation based on ADMM, system description, and numerical case studies are discussed in section IV. Finally, section V concludes the paper and briefly discusses the future extension of this work.

II. Mathematical Preliminaries

ADMM is an algorithm that leverages the better convergence properties of method of multipliers to solve constrained optimization problems. Assume a problem in following form,

$$Min\ f(x) + g(y) \tag{1}$$
$$s.t. Ax + By = c$$

Here, $x \in$ and $y \in$ are the variables and A and B are parameter matrices. The augmented Lagrangian equation of this problem can be written as:

$$L_\rho(x, z, \beta) = f(x) + g(y) + \beta^T(Ax + By - c) \tag{2}$$
$$+ \frac{\rho}{2}||Ax + By - c||_2^2$$

ADMM solves the problem in three updation steps. First, x is updated with fixed y, then y is solved with updated x from previous step and in the final step β is updated from fixed values of x and y. These steps are as follows.

$$x^{k+1} := \arg\min_x \{f(x) + (\beta^k)^T(Ax + By^k - c) \tag{3}$$
$$+ \frac{\rho}{2}||Ax + By^k - c||_2^2\}$$
$$y^{k+1} := \arg\min_y \{g(y) + (\beta^k)^T(Ax^{k+1} + By - c) \tag{4}$$
$$+ \frac{\rho}{2}||Ax^{k+1} + By - c||_2^2\}$$
$$\beta^{k+1} := \beta^k + \rho(Ax^{k+1} + By^{k+1} - c) \tag{5}$$

Here $\rho > 0$ is the penalty factor and β is the vector of lagrangian multipliers. The convergence of the ADMM depends on the following criterion,

$$\lim_{k \to \infty} (Ax^{k+1} + By^{k+1} - c) = 0$$

A. Consensus Optimization via ADMM

If the objective function of the ADMM problem consists of N terms, then the problem takes new form which is known as consensus ADMM. This form of objective function may represent to minimize the loss function of an individual area of the distribution system, or to minimize the line losses of a region of a large distribution network. The problem can be written as

$$Min \sum_{i=1}^{N} f(x) \tag{6}$$
$$s.t. x_i - y = 0$$

Here, x_i are the local variable and y is the global variable, where the objective is to converge all the local variables to the global value. In our application, the objective is to minimize the line power loss in the network. The variables of the branch flow model formulation are bus voltage magnitude, line current, active and reactive line power flow. Thus in the consensus formulation, the constraint would be to converge the bus voltage and line power flow of certain buses and lines between the regions observing from each region. Definition of these local and global variables are discussed in section III where the ADMM based OPF problem is formulated. The augmented Lagrangian function for this scenario can be written as,

$$L_\rho(\mathbf{x}, y, \beta) = \sum_{i=1}^{N}(f(x_i) + \beta^T(x_i - y) + \frac{\rho}{2}||x_i - y||_2^2)$$

The local variables x_i and the global variable y are updated using the following steps,

$$x_i^{k+1} := \arg\min_x \{f(x_i) + (\beta^k)^T(x_i - y^k) + \frac{\rho}{2}||x_i - y^k||_2^2\} \tag{7}$$

$$y^{k+1} := \frac{1}{n}\sum_{i=1}^{N}(x_i^{k+1}) \tag{8}$$

$$\beta^{k+1} := \beta^k + \rho(x_i^{k+1} - y^{k+1}) \tag{9}$$

We propose a consensus ADMM approach to solve the OPF problem of a large distributed network where all the regions solve their OPF problem for a constraint set and a global variable z. This iterative updating process continues till the error reduces below the threshold value.

III. ADMM Based OPF Formulation

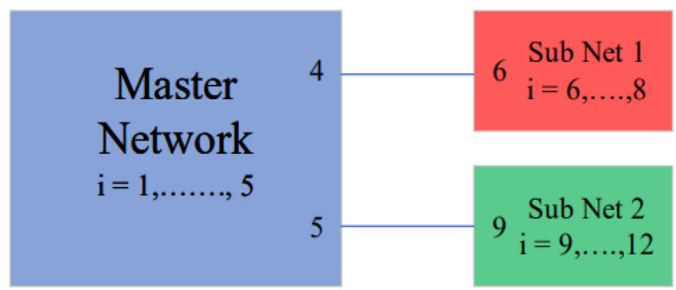

Fig. 1: A distribution system divided into three regions

In the distributed approach to solving the OPF of a power network, consider the network is divided into multiple areas. Among them, one is considered as the master network and others are as the sub-networks. There are communication links established between master and sub-networks to exchange information. As shown in Figure 1, let us assume the whole network is divided into 3 regions, where nodes $1, ...5$ belongs to the master network, nodes $6, ..., 8$ belongs to sub-network 1, and nodes $9, ...12$ belongs to sub-network 2. Also, the branches between $4-6$ shared by both master network and sub-network 1 and $5-9$ shared by master network and sub-network 2. Basically the area covered by branches $4-6$ and $5-9$ is the consensus area and the variables $P_{4-6}, P_{5-9}, Q_{4-6}, Q_{5-9}, V_4, V_5, V_6, V_9$ represents the global variable Z. Each area will solve the OPF problem of its region in parallel and assign the values. Next, the global variable will be updated based on values calculated by each local iteration. Then consensus will be achieved considering the preset threshold value.

A. BFM-SDP OPF

In this paper we are mostly focusing on the formulation of OPF problem for the distribution systems. Hence the Branch Flow Model of the system is adopted to formulate the OPF problem. Let us assume a graph $G = (N, E)$ represents a radial distribution network where, N is the set of all vertices and E is the set of all branches. Branch flow model comprises of the branch variables such as branch current, branch active and reactive power flow. Let, V_i is the voltage of node i, S_{ij} and I_{ij} is the complex power and current flown through branch $i-j$, then branch flow model can be stated as follows

$$V_i - V_j = z_{ij} I_{ij}, \forall(i,j) \in E \tag{10}$$

$$S_{ij} = V_i I_{ij}^*, \forall(i,j) \in E \tag{11}$$

$$\sum_{k:j\rightarrow k} S_{jk} - \sum_{i:i\rightarrow j} (S_{ij} - z_{ij}|I_{ij}|^2) + y_j^*|V_j|^2 = s_j \tag{12}$$

Here, z_{ij} is the branch impedance and s_j is the injected complex power at node j. The relaxed branch flow model is adopted from this equations by ignoring the angles of the variables. By substituting the expression of current I_{ij} from (11) into (10) yields $V_i - V_j = z_{ij} S_{ij}^*/V_i^*$. Then taking the square of the magnitudes of this expression derives the equation (14) as shown below. In the relaxed model the squared terms of the node voltage and branch current replaces the previous variables as $v_i = |V_i|^2$ and $l_{ij} = |I_{ij}|^2$. The relaxed BFM model is

$$s_j = \sum_{k:j\rightarrow k} S_{jk} - \sum_{i:i\rightarrow j} (S_{ij} - z_{ij} l_{ij}) + y_j v_j, \forall j \in E \tag{13}$$

$$v_j = v_i - 2(z_{ij}^* S_{ij} + z_{ij} S_{ij}^*) + z_{ij} l_{ij} z_{ij}^*, \forall(i,j) \in E \tag{14}$$

$$l_{ij} = \frac{|S_{ij}|^2}{v_i}, \forall(i,j) \in E \tag{15}$$

The non-linear equation (15) can be expressed in terms of a positive semidefinte matrix as follows:

$$\begin{bmatrix} v_i & S_{ij} \\ S_{ij}^* & \lambda_{ij} \end{bmatrix} \succcurlyeq 0$$

$$rank \begin{bmatrix} v_i & S_{ij} \\ S_{ij}^* & \lambda_{ij} \end{bmatrix} = 1$$

The aforementioned equations still hold the non-convexity due to the rank-1 constraint of the PSD matrix. Relaxing the equation by adopting the semidefinite relaxation (SDR), the BFM-SDP OPF problem is formulated:

$$Min \sum_{i:i\rightarrow j} z_{ij} I_{ij} \tag{16}$$

$$s.t. \begin{cases} s_j = \sum_{k:j\rightarrow k} S_{jk} - \sum_{i:i\rightarrow j} (S_{ij} - z_{ij}|l_{ij}|^2) + y_j v_j \\ v_j = v_i - (S_{ij} z_{ij}^* + z_{ij} S_{ij}^*) + z_{ij} \lambda_{ij} z_{ij}^* \\ \begin{bmatrix} v_i & S_{ij} \\ S_{ij}^* & \lambda_{ij} \end{bmatrix} \geq 0 \\ v_{ref} = V_{ref} V_{ref}^* \\ v^{min} \leq v_i \leq v^{max} \\ S^{min} \leq S_i \leq S^{max} \end{cases}$$

B. Implementing Consensus ADMM Based BFM-SDP-OPF

Based on the consensus ADMM and the BFM-SDP OPF formulation, the distributed problem can be formulated for each region. Before that, the global variable z can be defined as, $y^{'=[P_{mn} Q_{mn} P_{lt} Q_{lt} V_m V_l]}$. Now, the augmented OPF problem for each region can be formulated as follows. For the master network all the nodes as shown in Fig. 1 along with the consensus region nodes are considered to formulate the augmented OPF problem for master network.

$$Min \sum_{i:i\rightarrow j} z_{ij} I_{ij} + (\beta_1^k)^T (x_1 - y_1^k) + \frac{\rho}{2} ||x_1 - y_1^k||_2^2 \tag{17}$$

$$s.t. \begin{cases} s_j = \sum_{k:j\rightarrow k} S_{jk} - \sum_{i:i\rightarrow j} (S_{ij} - z_{ij}|l_{ij}|^2) + y_j v_j \\ v_j = v_i - (S_{ij} z_{ij}^* + z_{ij} S_{ij}^*) + z_{ij} \lambda_{ij} z_{ij}^* \\ \begin{bmatrix} v_i & S_{ij} \\ S_{ij}^* & \lambda_{ij} \end{bmatrix} \geq 0 \\ v_{ref} = V_{ref} V_{ref}^* \\ v^{min} \leq v_i \leq v^{max} \\ S^{min} \leq S_i \leq S^{max} \end{cases}$$

where $y_1 = y$

Similarly for sub-network 1 the augmented OPF problem can be formulated with updated z as follows

$$y_2 = [P_{mn}, Q_{mn}, V_m]^T$$

The augmented Lagrangian objective function for sub-network 1 is as follows:

$$Min \sum_{i:i\rightarrow j} z_{ij} I_{ij} + (\beta_2^k)^T (x_2 - y_2^k) + \frac{\rho}{2} ||x_2 - y_2^k||_2^2 \tag{18}$$

Further for sub-network 2 the augmented OPF problem can be formulated with updated z as follows,

$$y_3 = [P_{lt}, Q_{lt}, V_l]^T$$

With the objective function as

$$Min \sum_{i:i\rightarrow j} z_{ij} I_{ij} + (\beta_3^k)^T (x_3 - y_3^k) + \frac{\rho}{2} ||x_3 - y_3^k||_2^2 \tag{19}$$

Once all the regions done solving for the variable **x** then, the global variable z is updated using Eq. (8) as,

$$y(1,3,5) = 0.5 * [y_1(1,3,5) + y_2] \tag{20}$$
$$y(2,4,6) = 0.5 * [y_1(2,4,6) + y_3]$$

The primal and dual residual of the formulation are denoted as follows,

$$r^k = ||x^k - y^k||_2 \tag{21}$$
$$s^k = \rho||y^k - y^{k-1}||_2$$

After that, the dual variable is updated using Eq. (9). Finally the error is being calculated as,

$$error^k = \left\| \begin{matrix} r^k \\ s^{k-1} \end{matrix} \right\|^2 \tag{22}$$

The threshold cut-off value for error is considered as $10e-4$. If the error value becomes less than the threshold then a global consensus is achieved.

IV. Real-life Implementation

To test the scalability of the proposed approach a large distribution network such as a modified IEEE 123 bus system is utilized. The actual network is three-phase and unbalanced. For this approach, the single-phase version is used, which is the positive sequence equivalent of the actual network. The system operates at 4.16KV. The total load connected to the system is 1163.3KW and 640KVAR. Some further modifications were also done. Some distributed generation (DG) plants are introduced in the system. The capacity of

Algorithm 1 Proposed Distributed OPF

Step: 1 Initialize network data and boundary values for each region.
Step: 2 Initialize ρ and y for ADMM formulation.
Step: 3 Initialize threshold values for primal and dual residual.

while $error^k \leq 10^{-4}$ **do**

Update x_1 for region 1 using (17)
Update x_2 for region 2 using (18)
Update x_3 for region 3 using (19)
Update y using (20)
Update β for each region using (9)
Calculate primal and dual residual to get the error Calculate primal and dual residual to get the error

the DG generation is 10% of the total connected load. The maximum active power generation capacity of the DG plants is considered to be equal to the active power demand of the respective bus. And, the KVA rating of the DG plants is considered to be 120% of the active power rating. That's how the upper and lower bound for the reactive power generation capacity is calculated for the plants.

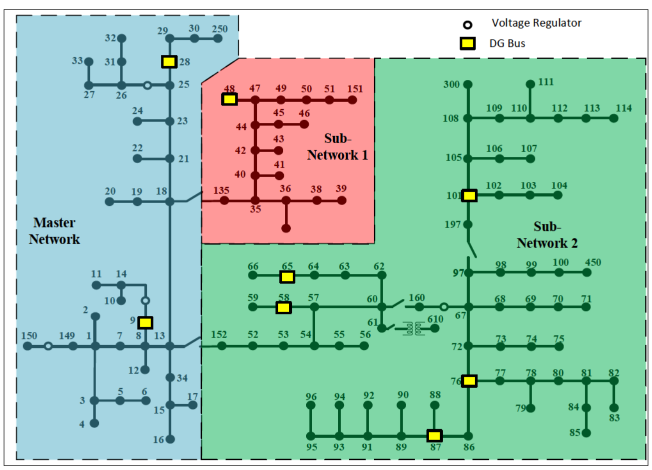

Fig. 2: Modified IEEE 123 bus system with 10% DG penetration

Then the whole network is divided into three regions. There are switches between nodes 20-118 and 15-117. The partitions are made on the location of those two switches. The area containing substation node 1 is considered the master network. This area is marked with a blue line in the figure. Next, the area enclosed by the red line is considered sub-network 1. This is connected to the master network through the switch between 20-118. Finally, the rest of the network is considered as sub-network 2 which is connected to the master network through the switch at 15-117 and marked by a green line in the figure. A single line diagram of the network along with the DG plant's location in all three regions is shown in Fig. 2.

In this case study, different scenarios were run for different values of penalty factor ρ. It is known that primal and dual residual values, as well as the convergence speed, depend greatly on the value of the penalty factor. A higher-valued penalty factor increases dual variables, on the other hand, primal residual increases for smaller penalty factors. Here we ran the simulation for different values of ρ such as, $\rho = 0.7, 1.0, 10, 50, 100$. The change in the number of iterations for convergence with the change of penalty factor is observed. We can see in Fig. 6 that, as the penalty factor value increases the value of dual residual increases. Though with a higher value of penalty parameter, the gap between primal and dual residual decreases faster yet the solution for the lower value of ρ is more optimal. That statement can be proved by the numerical results showcased in Table I. The performance of the proposed approach is also compared with another distributed OPF method proposed in [22] which is noted as "Distributed (Non-ADMM) in the table and figures. To compare the solution of the proposed approach with the centralized OPF solution, the active and reactive power generation from the substation, the total active power loss in the system, and the node voltage profile are compared in Fig. 3. It can be seen from Table I that, with the decrease of the value of ρ, the number of iterations increases, albeit the resultant voltage profile is closer to the centralized OPF solution's profile. The comparison of the voltage profiles is shown in Fig. 5. It is also evident the significance of choosing an appropriate penalty parameter. Since, for a lower value as $\rho = 0.1$, the formulation fails to converge. The percentage optimality of the solution from different values of the penalty factor can also be realized using the % error with respect to the solution from the centralized approach. The % error in substation active and reactive power is shown in Fig. 7.

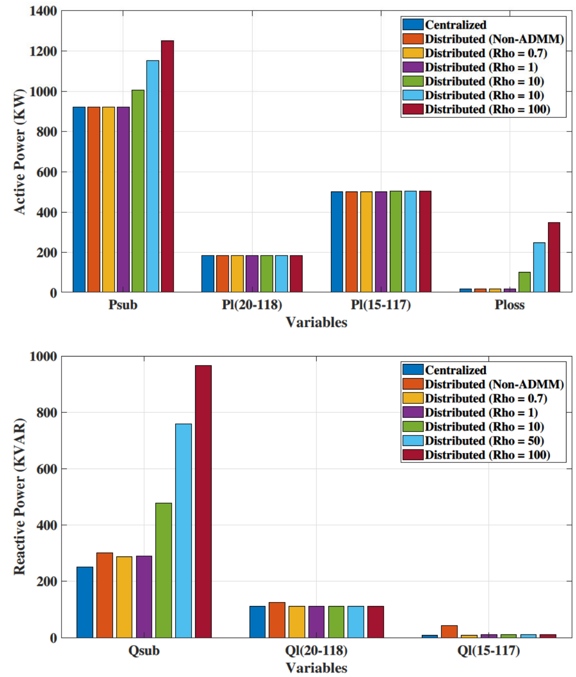

Fig. 3: Comparison of substation active and reactive power, line power through connecting lines and active power loss for different scenarios.

TABLE I: OPF solution comparison

	Centralized BFM-SDP	Distributed Non-ADMM	Distributed ($\rho =$)			
			0.7	1	10	50
P_sub (KW)	921.0474	921.5347	921.4553	921.5349	1006.326	1152.632
Q_sub (KVAR)	251.0378	299.4837	287.1654	288.0038	477.5972	760.4315
P_loss (KW)	16.0574	16.5447	16.4653	16.5449	101.3359	247.6421
$P_{20-118}(KW)$	182.0654	182.1052	182.0667	182.0667	182.0837	182.0843
$Q_{20-118}(KVAR)$	111.0626	126.2216	111.1589	111.149	111.6502	111.1006
$P_{15-117}(KW)$	502.3250	502.3033	502.3359	502.4029	502.6494	502.6225
$Q_{15-117}(KVAR)$	10.61	42.9596	16.4653	11.4644	11.3007	11.1947
Time (s)	0.31	0.35	0.32	0.34	0.31	0.30

A. *Update in formulation approach*

In the current formulation, the consensus regions are considered overlapped. Which means, the boundary buses belongs to both adjacent regions and all the power flow constraints are satisfied for all the buses of the subsystems. The issue it raises is that, in some cases the active or reactive power flow through the tie-line struggle to converge. Similar case seen for IEEE 123 bus system if subsystem 2 is splitted in another subsystem the across the line between node 60 and 160. To resolve the issue, few changes are considered in formulation. Now, the subsystems are considered to be fully isolated and no overlapping region. Albeit, the information of the boundary bus of a leading subsystem will be know to the adjacent subsystem and that node will not be considered while solving the bus power balance constraint. In this way, the mismatch in the global variables was being able to rectified.

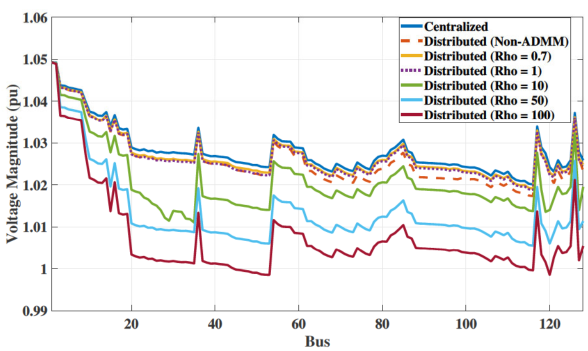

Fig. 4: Voltage profile comparison among centralized OPF solution and proposed distributed approach for different penalty factor values.

V. Conclusions and Future Work

In this paper, a fully distributed approach has been formulated to solve the convexified OPF problem for a radial power system. The scalability of the formulation has been tested on a modified IEEE 123 bus system with 10% DG penetration. This formulation can also apply to larger networks. The significance of choosing a proper penalty factor is shown by simulating different case scenarios. This formulation can improve the time of convergence for realistic large networks by splitting the system into small regions and solve the problem in parallel while ensuring inter-regional coordination. The future extension of this work includes the finding of the optimal value

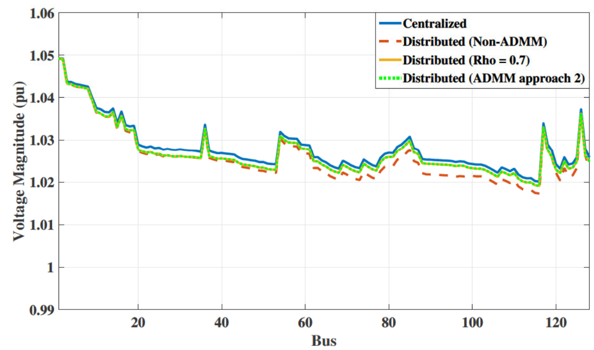

Fig. 5: Voltage profile comparison among centralized OPF solution and proposed distributed approach for different penalty factor values.

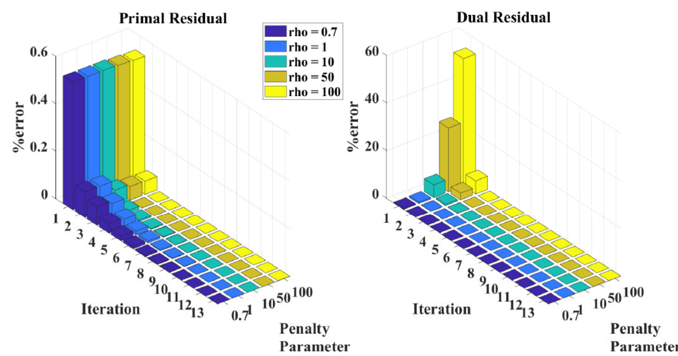

Fig. 6: Primal and Dual residual values for different magnitude of penalty parameter.

for the penalty parameter since this has a significant impact on the final solution for the problem.

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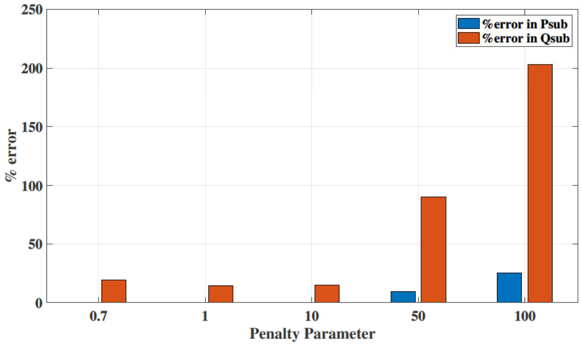

Fig. 7: Percentage error in substation active and reactive power for different value of ρ with respect to centralized OPF dispatch.

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