



Examining the “Messiness” of Transitions Between Related Artifacts

Nicole Panorkou¹ · Toni York¹ · Erell Germia²

Accepted: 4 September 2022

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Abstract

Instructional designs that include two or more artifacts (digital manipulatives, tables, graphs) have shown to support students’ development of reasoning about covarying quantities. However, research often neglects how this development occurs from the student point of view during the interactions with these artifacts. An analysis from this lens could significantly justify claims about what designs really support students’ covariational reasoning. Our study makes this contribution by examining the “messiness” of students’ transitions as they interact with various artifacts that represent the same covariational situation. We present data from a design experiment with a pair of sixth-grade students who engaged with the set of artifacts we designed (simulation, table, and graph) to explore quantities that covary. An instrumental genesis perspective is followed to analyze students’ transitions from one artifact to the next. We utilize the distinction between static and emergent shape thinking to make inferences about their reorganizations of reasoning as they (re-)form a system of instruments that integrates previously developed instruments. Our findings provide an insight into the nature of the synergy of artifacts that offers a constructive space for students to shape and reorganize their meanings about covarying quantities. Specifically, we propose different subcategories of complementarities and antagonisms between artifacts that have the potential to make this synergy productive.

Keywords Transitions · Artifacts · Synergy of artifacts · Instructional design · Covariational reasoning · STEM reasoning · Static and emergent shape thinking · Design experiments · Instrumental genesis · Systems of instruments · Complementarities, redundancies, and antagonisms

✉ Nicole Panorkou
panorkoun@montclair.edu

¹ Montclair State University, Montclair, NJ, USA

² Kean University, Union, NJ, USA

Introduction: Artifacts and Transitions in Math Education

Throughout their math development, children interact with various artifacts, such as physical manipulatives, tables, and graphs, that contribute to how they construct a mental image of their mathematical world. One example of this is research about students' construction and interpretation of graphs, which has been shown to support the learning of mathematical and physical concepts such as covariational reasoning (e.g., Johnson et al., 2016) and modeling motion (e.g., Doorman, 2005), respectively. With the inclusion of technology as a tool for math teaching and learning in the early 1980s, the mathematics education community has developed innovative digital artifacts for teaching and learning that were not accessible before, such as dynamic geometry environments, computer algebra systems, and dynamic simulations. By having students engage with these technological artifacts, researchers explored new ways of students' reasoning about various mathematical concepts. Examples of these studies include the design of digital artifacts to support students' reasoning about functions (e.g., Roschelle et al., 2012), pattern generalization (Noss et al., 2009), area and volume measurement (Panorkou, 2020a, b), and probability (Pratt, 2000).

Transitioning between these artifacts has been considered important in mathematics education, but also a challenge. Students rarely use concepts they constructed by interacting with a digital medium, because they fail to understand the connection to the formal mathematics they examine with paper-and-pencil tools (e.g., Gurtner, 1992). In response to this, Geraniou and Mavrikis (2015) suggested the design of bridging activities to engineer students' connection-making as they transition from an exploration of algebra using a digital artifact to typical paper-and-pencil tasks. Other studies, such as the one by Soury-Lavergne (2021), examined students' transitions between physical and digital forms of the same manipulative and found that the most effective approach for constructing knowledge involves combining the two types in learning situations.

In a series of studies (e.g., Basu & Panorkou, 2019; Panorkou & Germia, 2021; Panorkou & York, 2020), our research group examined students' reasoning as they explored covarying quantities of various scientific phenomena by first engaging with a dynamic simulation, and then using the simulation to collect values of those quantities in a table, and finally using the pairs of values collected in the table to construct a graph. We found that these transitions between the three artifacts (simulation, table, graph) supported students in developing more sophisticated forms of covariational reasoning and also a deeper understanding of the scientific phenomenon being modeled. We also noticed that through this process, students were able to conceptualize a graph as a dynamic relationship of two quantities and not just as a static object as students often do (e.g., Moore & Thompson, 2015).

Our prior research is one of the multiple studies that found that students can develop their covariational reasoning as they explore phenomena with quantities that covary in a combination of artifacts. Indeed, studies exploring *what* forms of covariational reasoning or shape thinking students exhibit as they interact with

various artifacts, or a combination of artifacts, have been conducted extensively (e.g., Ellis et al., 2015; Johnson et al., 2016). One could also say that there are multiple studies exploring *why* this development of reasoning happens, pointing to specific artifacts or a combination of artifacts during the learning process (e.g., Ellis et al., 2018). However, these studies rarely examine questions about *how* this process happens. While both the *what* and *why* questions give us a reason to wonder, the *how* question can point us to specific conditions, methods, and degrees in which students' reasoning and artifact interactions influence each other in the learning process. This lens can be important for informing the design of artifacts, tasks, and questioning around these ideas, as well as the analysis of students' reasoning during those transitions.

To make this contribution, the current study aims to give an insight into *how* students' covariational reasoning may be shaped and reorganized as students transition between artifacts, and *how* the artifacts' synergy may provide a constructive space for students to shape and reorganize their reasoning. In this article, we analyze data from a design experiment with sixth-grade students studying quantities that covary using a digital simulation, a table, and a graph. We focus on Moore and Thompson's (2015) notions of *emergent* and *static* shape thinking to describe the development of students' reasoning during transitions and use an instrumental genesis approach, and Mariotti and Montone's (2020) concept of *synergy* to discuss how a collection of artifacts and students' reasoning work together. The outcome of our analysis is a set of specific subcategories of *complementarities*, *redundancies*, and *antagonisms* (Soury-Lavergne, 2021) between artifacts that make this synergy productive.

Earlier Research on Covariational Reasoning

In this section, we provide a definition of covariational reasoning and a brief summary of the research that has been conducted to characterize students' covariational reasoning (the *what*). We then discuss the role that artifacts played in these studies for engaging students with particular forms of reasoning, pointing to *why* this development of reasoning happens. This research is the context for our exploration of *how* transitions between different artifacts can support students' covariational reasoning.

Defining Covariational Reasoning

Covariational reasoning involves mentally imagining two quantities' values (magnitudes) changing simultaneously (Carlson et al., 2002; Confrey & Smith, 1994). For instance, in explaining the Earth's climate, a person may co-ordinate the simultaneous change in two quantities: as the distance from the equator increases, the air temperature tends to decrease. In the past three decades, mathematics educators have tried to characterize students' covariational reasoning in different ways (e.g., Confrey & Smith, 1994, 1995; Saldanha & Thompson, 1998) and proposed frameworks to characterize students' progression of covariational reasoning over time (Carlson et al., 2002; Thompson & Carlson, 2017). Multiple studies used these two

frameworks to characterize students' covariational reasoning about various mathematical ideas such as rate of change (e.g., Johnson, 2012) and functional relationships (Ellis et al., 2015, 2018). More recent studies utilized those frameworks to also support students' learning of scientific phenomena, such as gravity (Panorkou & Germia, 2021) and the greenhouse effect (Basu & Panorkou, 2019).

According to Moore and Thompson (2015), one of the difficulties that students have in relation to covariation is that, while graphing, students often focus on the shape of a graph, failing to reason about the covariational relationship between the quantities represented. While characterizing students' ways of thinking about graphs, they distinguished between *static* and *emergent* shape thinking. Students who think statically operate on a graph "as an object in and of itself, essentially treating the graph as a piece of wire" (p. 784). In contrast, students with emergent shape thinking show an understanding of a graph as a trace in progress, or as a snapshot of an emergent trace, that illustrates the relationship between two covarying quantities. Students are also able to identify the same functional relationship as they move between different graphs that represent it. For example, Moore and Thompson discuss a student who was able to identify the same relationship in both a conventional and an unconventional graph (e.g., axes switched) of the arcsine function.

The Role of Artifacts in Students' Covariational Reasoning

What is common in the studies mentioned above is a focus on supporting students to reason covariationally in different levels or ways as they explore mathematical concepts and scientific phenomena. To achieve this, most of these studies mention the role of technology and multiple artifacts (simulations, tables, graphs) in their designs (e.g., Ellis et al., 2015; Johnson et al., 2016). For instance, Johnson and colleagues used a Ferris wheel animation designed on *The Geometer's Sketchpad* (Jackiw, 2001), where students were able to control the motion and change the animation's speed. Students were asked to use the simulation to graph the relationship between the car's height from the ground and its distance traveled within one revolution. Although these studies used different technologies, the common element is that, by engaging with digital artifacts, students were provided with opportunities for direct and dynamic manipulation of quantities and a prompt to explore their continuous change.

Often, the goal of these studies is for students to connect the dynamic representations of relationships with the graphing of those relationships. This has been shown to advance students' conceptions of graphs of functions as a representation of co-ordinated change (e.g., Ellis et al., 2018; Panorkou & Germia, 2021). For example, in our prior research, we asked students to examine covarying quantities by manipulating digital simulations of scientific phenomena (e.g., gravity or the rock cycle), collecting data in a table, and then generating a graph. We found that having the students generate graphs by gathering data from a simulation was productive for supporting those who have just started learning graphs to understand that each point on the graph is a co-ordinated pair of two quantities' values (e.g., Panorkou & York, 2020).

While the transitions that students make from one artifact to the next were found in our study and the studies above to be important for shaping students' reasoning, none of these studies examined *how* these transitions are made. The following section describes how the current study takes this different lens to examine the process by which students make connections within and across different types of artifacts.

Theoretical Framework: Studying the Transitions Between Artifacts

To study students' connection-making within and across artifacts, we considered theories that focus on students' interactions with tools and artifacts. We first acknowledge that students' development of mathematics is dependent on the material and symbolic tools that they have available in a learning situation (Artigue, 2002). In other words, tools shape the learning environment as well as part of the actions and web of ideas embedded in that environment (Noss & Hoyles, 1996). While *tools* could be any object that is available for human activity, in this article we use the term *artifact* to refer to a tool made by humans that is used for performing a specific task (Trouche, 2004; Verillon & Rabardel, 1995). As the student interacts with an artifact, the artifact imposes some affordances (Gibson, 1977) and constraints which shape both the actions and the ideas emerging from the activity. For instance, students can dynamically manipulate variables in a digital simulation in a way not possible with a static table or graph. Conversely, tables and graphs may offer the opportunity to study patterns in how the quantities are changing in more detail.

Through their interaction with an artifact, students build a mental construction of the tool or what research refers to as an *instrument* (Lagrange et al., 2001; Verillon & Rabardel, 1995). This mental construction consists partly of the artifact and partly of cognitive schemes (Artigue, 2002). After distinguishing between an artifact and an instrument, our focus led to the theory of *instrumental genesis* (Verillon & Rabardel, 1995) to describe the process by which an artifact becomes an instrument. Instrumental genesis is linked both to the artifact's characteristics (affordances and constraints) and to the user's prior knowledge and experience (Rabardel, 2000).

In this study, we use Trouche's (2003) notion of *instrumental orchestration* to discuss "the various devices that a teacher organizes with an aim of assisting the instrumental genesis of students" (p. 792). Instrumental orchestration is a plan of action that consists of a set of individuals, a set of objectives, a didactic configuration, and a set of exploitation of this configuration (Guin & Trouche, 2002). The teacher can construct this plan of action at different levels (Trouche, 2005): first, at the level of the artifact itself, by forming some conjectures based on the affordances and constraints of the artifact; second, at the level of instruments or sets of instruments, by acting as an orchestra conductor engineering the instrumental genesis of the whole class or a group of students; third, at the level of the relationship, the student maintains with the instrument, by engineering opportunities for students to reflect on their own activity and their observations of others.

When the user interacts with new artifacts, the new instruments formed from the instrumental genesis do not develop in isolation. As Rabardel and Bourmaud (2003)

explain, “the new functions form an overall system with the functions of instruments developed earlier” (p. 679). As a result, this system of interlinked instruments is constantly reorganized from interactions with new related artifacts and the development of new related instruments. We use the term *reorganization* (Piaget, 1977/2001) to refer to humble inferences about their reflections and projections of particular forms of reasoning and their connection-making to a higher conceptual level where these initial forms of reasoning become part of a more coherent whole. For instance, students’ meanings of covariational reasoning are part of a system that links instruments that they formed related to covariation as they interacted with various artifacts, and these meanings are reorganized to a higher conceptual level when they interact with another related artifact.

The “web of connections” (Noss & Hoyles, 1996, p. 105; *italics in original*) that students (re)construct is influenced by the students’ prior knowledge and experience, and this led our theoretical exploration to the notion of transfer. Marton (2006) referred to “transfer” to describe “how what is learned in one situation affects or influences what the learner is capable of doing in another situation” (p. 501). There have been many definitions of transfer in the literature, and what is common is that transfer is determined from the expert’s point of view (Lobato, 2003). For instance, current studies on students’ covariational reasoning involve the design of artifacts and sequences of artifacts for supporting students’ reorganizations of reasoning from the expert’s view, neglecting to examine the similarities and differences that students themselves view as relevant within and across these situations. Building on the work of Hoffding (1892) and Lave (1988), Lobato (2003, 2006) presented a reconceptualization of transfer from an actor-oriented (learner) view as an active process that involves the learner’s personal construction of relations of similarity and difference between situations. This includes identifying the connections that the students make within and across different types of instructional experiences and examining how and why those connections could be constructive for students’ understanding.

To study students’ connection-making between situations, the works of Soury-Lavergne (2021) and Mariotti and Montone (2020) provided an entry point. Soury-Lavergne claimed that a system of instruments can be supported by having students interact with a *duo of artifacts* and their combined instrumental geneeses. For the combined instrumental geneeses to work, the two artifacts can differ with respect to their functional characteristics, but contribute to the same specific mathematical concept, even though the development of meanings about the concept is different in each one. The goal of the duo is for students to relate these different meanings emerging from the different uses of these artifacts: in other words, intertwining the instrumental genesis related to the interaction of each artifact. Mariotti and Montone refer to this phenomenon as a *synergy* between two artifacts where “an implicit or explicit reference to both artifacts creates a relationship between meanings emerging from their use” (p. 113). By taking a semiotic perspective, they argue that this results in “deepening and weaving the semiotic web” of mathematical meaning.

In designing experiences for students to engage with the synergy between artifacts, we followed Soury-Lavergne’s (2021) three characteristics of design that the duo of artifacts needs to have in order to lead to a system of instruments. First, a *complementarity* between the two artifacts needs to make the use of each

artifact necessary. Second, there must be some *redundancy* of some characteristics of one artifact in the other. This may result in some continuity between them and make their connection visible. Third, there must be an *antagonism* between artifacts, where in each one some functionalities are constrained, and users need to challenge and reorganize their initial instruments to adapt to new conditions.

The duos used both by Soury-Lavergne (2021) and by Mariotti and Montone (2020) consisted of a physical and a digital artifact. As mentioned earlier in this article, studies examining students' covariational reasoning usually combine two or more artifacts in their designs. They often use either an instructional sequence of a dynamic simulation followed by a graph or a sequence combining a dynamic simulation, a table, and a graph. While we may design a sequential instructional process, our experience observing students' transfer shows that the distinction between these different characterizations in the child's actions and reasoning is rather blurred. Instead, students shift back and forth in unrestrained ways between different artifacts. In other words, each artifact becomes a transitional instrument for the continuous (re-)forming of a system of instruments. Accordingly, we use the term *transitions* to refer to the dynamic, continuous, transitional, and sometimes "messy" shifts (physical and cognitive) that the individual makes between artifacts as they (re-)form their system of instruments.

By situating our work within the above theories, we formulated the following research questions:

- (a) *How* is students' covariational reasoning shaped and reorganized through transitions between artifacts?
- (b) *How* does the artifacts' synergy provide a constructive space for students to shape and reorganize their reasoning?

The following section describes the methods we employed to study these questions.

Methods

We followed a design experiment methodology (Cobb et al., 2003) to develop and test a theory about both the process of learning and the nature of the synergy of artifacts that supported that learning. These experiments have a cyclic and iterative nature. The research team formed some preliminary conjectures about a projected process of learning, as students transition from one artifact to the next, as well as the characteristics of the synergy of artifacts which may support this process. Then, as the experiment unfolds, these conjectures evolve and are open to modifications. In the first part of this section, we describe the design and initial conjectures of the study. In the second part, we provide details on the methods of data collection and analysis.

Design and Conjectures

Our prior studies showed that students as young as sixth grade are able to construct sophisticated forms of reasoning about covarying quantities when these were presented in a context meaningful to them, such as examining various scientific phenomena (e.g., Panorkou & Germia, 2021). Consequently, our study's instrumental orchestration included the design of artifacts based on the scientific phenomenon of climate, which involved the covarying quantities of temperature and latitude.

Our review of the literature showed that, by dynamically manipulating quantities using digital artifacts, students were able to reason covariationally about these quantities (e.g., Ellis et al., 2018). Accordingly, for our design, we developed the climatic zones simulation (Fig. 1) which allows the student to examine how temperature and latitude covary dynamically. Specifically, the simulation presents the three major climatic zones of polar, temperate, and tropical. To explore the simulation, the student uses their mouse to move the arrow on the right side of the screen up and down, in order to observe the changes in the two quantities.

We selected four cities that would represent the different climatic zones. Temperatures in these zones are determined in this simulation mainly by the distance from the equator, which is referred to as *latitude*. The equator is at 0° latitude, and latitude always increases as the distance from the equator increases to the north or south. Although the relationship between latitude and temperature can be quite complex in reality, due to the effects of many other variables, this simulation models a

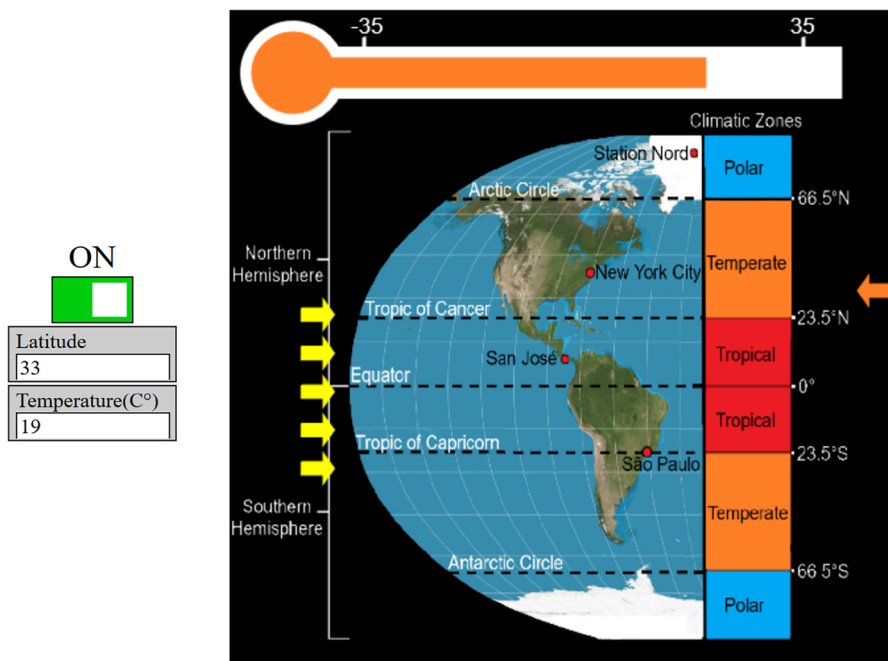


Fig. 1 The climatic zones simulation

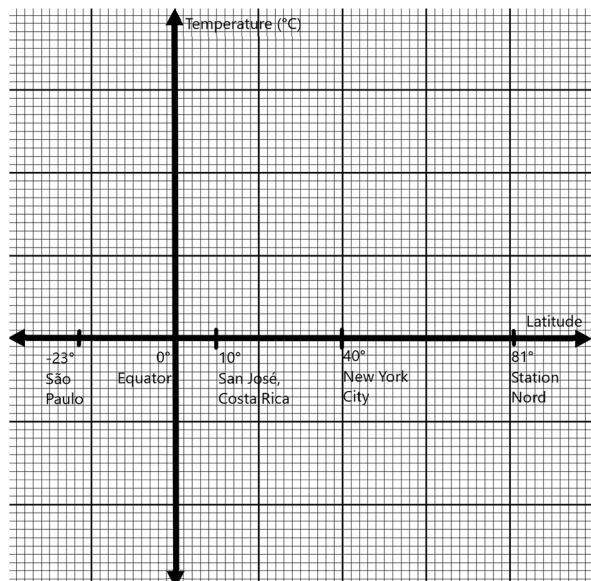
simplified version of this relationship in which the temperature always decreases as the latitude increases. While this is not perfectly accurate to true scientific realism, it offers an accessible data set for middle-school students to explore and is therefore a simplified model of a real scientific process designed to have useful pedagogical features (Weintrop et al., 2016).

Similar to earlier studies (e.g., Ellis et al., 2018), our conjecture was that, by interacting with the dynamic simulation first, students would be supported in constructing an understanding of graphs as a representation of co-ordinated change. Accordingly, this would help them demonstrate emergent shape thinking (Moore & Thompson, 2015). We chose to have students construct the graphs themselves, so that they understand that each point on the graph is a co-ordinated pair of two quantities' values. While some studies asked students to transition straight from the digital tool to the graph (e.g., Johnson et al., 2016), other studies found that including a table before the graph was found to advance students' conceptions of graphs as a representation of co-ordinated change (e.g., Ellis et al., 2015; Panorkou & Germia, 2021). We decided to design two days of activities to examine both scenarios of instrumental orchestration.

Day 1: Transitioning Between a Digital Simulation and a Graph

On the first day of the design experiment, the students were prompted to identify latitudes and distinguish the climatic zones based on them. Specifically, we asked the students to explore the simulation and then graph the air temperature in the four cities and at the equator (Fig. 2).

Fig. 2 Graph students were asked to construct representing the latitudes in four cities and the equator and their temperatures



Our goal was to examine how students would reason as they transition between the simulation and the graph. Our conjecture was that, by examining the quantities dynamically with the simulation, students would reason covariationally about the two quantities in the context of the digital artifact, constructing an instrument. Subsequently, we hoped that, by using the simulation to graph the relationship, students would be able to reason emergently about the graph and reorganize their system of previous instruments to include this new instrument.

Day 2: Transitioning Between a Digital Simulation, a Table, and a Graph

On the second day of the design experiment, students were prompted to further explore how the atmospheric temperature varies in different climatic zones. Contrary to day 1, on day 2, students were also asked to interact with a table. To examine this type of transition as well, we asked students to use the simulation to complete the table in Fig. 3.

Our goal was to examine how students would transition between the digital simulation and the table, as well as to explore how students would reorganize their system of instruments to incorporate the new instrument they constructed by interacting with this new artifact. Our conjecture was that the table could support their emergent shape thinking because it represents different values of the quantities as ordered pairs.

After creating the table, students were asked to use it to graph the values of the two quantities as shown in Fig. 4. In contrast to the day 1 graph, this graph did not include any scales and students had to create their own intervals on each axis prior to plotting the values.

Fig. 3 The table that students were asked to complete on day 2

Latitude	Air Temperature
-67	
- 47	
-23	
-10	
0	
10	
23	
47	
67	

Our goal was two-fold. First, we were interested in examining how students would transition between the three representations to organize the grid and plot the graph. Second, we again aimed to explore how students would reorganize their system of instruments by transitioning between the different artifacts. Special attention was given to their construction of emergent shape thinking, and how the transitions supported or impeded this type of thinking. Here, it is important to clarify that, although our design may seem to promote a sequential learning process, we expected that the student's actions and reasoning would illustrate a variety of "messy" transitions between artifacts, both within each day and across the two days.

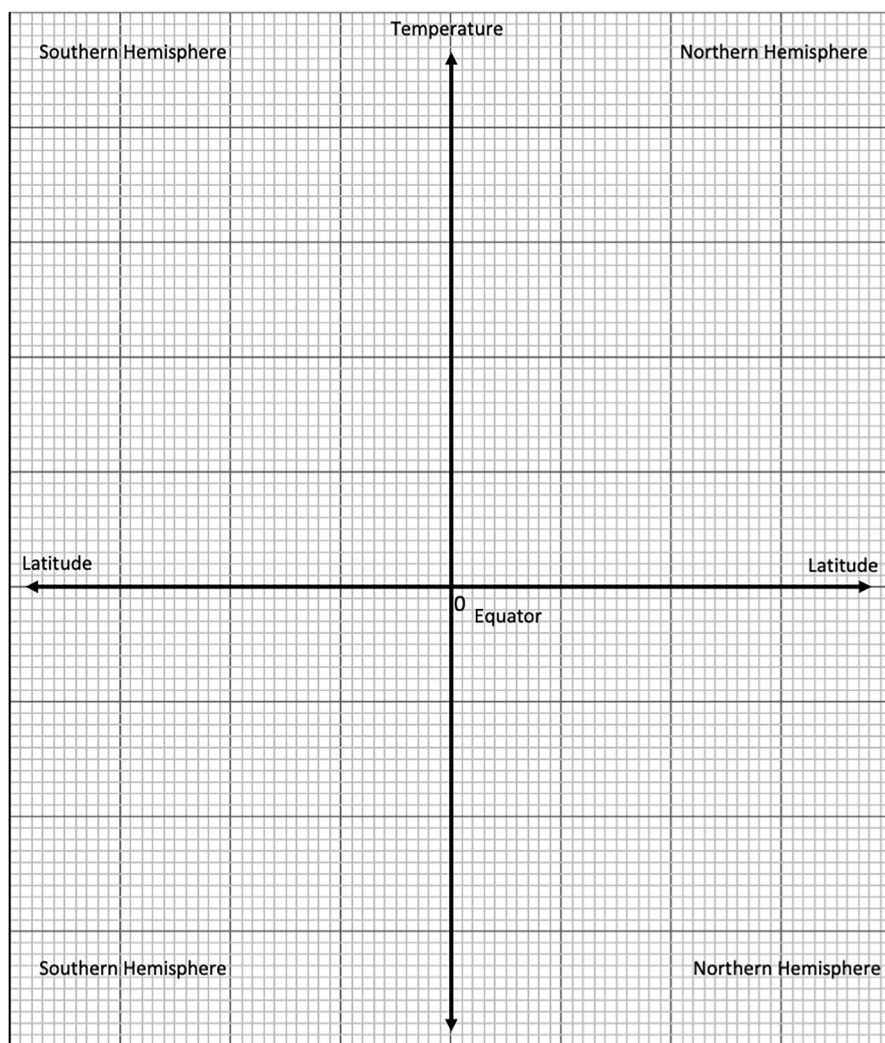


Fig. 4 The graph that students were asked to complete on day 2

Data Collection and Analysis

We collected data from a whole-class design experiment (Cobb et al., 2003) in a sixth-grade classroom from the northeast of the USA. The design experiment consisted of two 20- to 25-min sessions of virtual classes. Because of COVID-19 restrictions, the class was held on Google Meet (Google LLC, 2022a). The simulation is hosted on our ACMES (Assimilating Computational and Mathematical Thinking into Earth and Environmental Science) project website (<https://acmes.online/>). Students were asked to use digital handouts and the image creation and editing tools of Google Slides (Google LLC, 2022b) to create their tables and graphs. The students were divided in multiple breakout rooms where they engaged with the tasks individually and also in pairs. Students were assigned to a breakout room by the classroom teacher, who was asked by the researchers to pair together students who had all consent/assent forms signed and were known to be active participants in the class's online sessions. A researcher joined each pair to record their activity and conduct interviews as the students engaged with the task design. The interview focused on the students' creation and interpretation of the artifacts, including questions such as "What does your table show?" and "How did you create your graph?".

To discuss in depth the transitions that students made between artifacts, in this article, we present the analysis of one pair of students, Mikhail and Tasif, who were placed in the same breakout room together for both sessions. This pair was chosen for analysis because they were present in the class throughout the data collection period and both showed a high level of engagement with the online format of the lessons. Screen-casting software was used to record the Google Meet breakout room sessions, including each student's video, voice, and shared on-screen activity. All the recordings were transcribed for the analysis.

This article focuses on the retrospective analysis at the end of the design experiment. During this analysis, the data was analyzed in three stages. In the first stage of the analysis, we identified excerpts that showed evidence of students' covariational reasoning and shape thinking throughout the design experiment based on the first column in Table 1.

Table 1 Framework of data analysis

Characterizing students' reasoning	Characterizing students' transitions
<i>Covariational reasoning</i> Reasoning about two quantities' values changing simultaneously	<i>Complementarity</i> between the two artifacts makes the use of each artifact necessary
<i>Static shape thinking</i> Making inferences about the behavior of a simulation, table, or graph as an object in and of itself	<i>Redundancy</i> of some characteristics of one artifact in the other. This may result in some continuity between them and make their connection visible
<i>Emergent shape thinking</i> Interpreting a simulation, table, or graph as an emergent trace of covarying quantities and identifying the same relationship by transitioning between representations	<i>Antagonism</i> between artifacts, where in each one some functionalities are constrained, and users need to challenge and reorganize their initial instruments to adapt to new conditions

In particular, we used the Moore and Thompson (2015) framework for shape thinking to identify students' forms of reasoning, and characterize them either as static shape thinking or as emergent shape thinking. For example, reasoning about the steepness of a graph as going downhill exhibits static shape thinking, while reasoning about the covariational relationship between the temperature and the distance from the equator illustrates emergent shape thinking. Even though Moore and Thompson use this distinction to talk about students' behavior in different representations of graphs, we considered students' shape thinking to be a process that could possibly develop throughout their interactions with the previous artifacts. In other words, a student who is thinking emergently may be able to build a structure of covarying quantities that is not inherently tied to one representational context, but is operative with other representational contexts, such as simulations and tables, that illustrate that same abstracted structure.

After identifying and characterizing students' reasoning, in the second stage of analysis we reviewed the excerpts from each student chronologically to observe the progression of their reasoning as they transitioned from one artifact to the next. In particular, we examined how their forms of reasoning emerged as a reorganization of prior ways of reasoning, and identified the transitions that students made during those reorganizations. The outcome of this second stage is a set of reorganizations of reasoning students exhibited in specific transitions between artifacts.

In the third stage of analysis, we tracked the features of the artifacts that were necessary and contingent in supporting those reorganizations during transitions. As shown in the second column in Table 1, we analyzed how the artifacts' complementarities, redundancies, and antagonisms (Soury-Lavergne, 2021) provided a constructive space for students to shape and reorganize their reasoning. We used Lobato's (2003) actor-oriented transfer framework to study students' dynamic (re-)construction of relations of similarity and difference as they transition between artifacts. We identified these relations by examining students' reasoning and actions, including their gestures. Our goal was to analyze the affordances and constraints of the design functions of each artifact and observe how these artifacts are interpreted and used as instruments in a system. Our analysis in this stage was based on questions suggested by Lobato such as, *What relations of similarity or difference are created by the students?* and *How are these similarities or differences supported by the instructional environment?*

To illustrate the dynamic process of transitions, in the following "Results" section, we present a chronological set of "situated accounts of learning" (Cobb et al., 2003) that relate students' reorganizations to the synergy of artifacts (Mariotti & Montone, 2020) by which they were supported, structured, and ordered. Afterwards, in the "Discussion" section, we use these accounts to present a profile of the types of relations between artifacts that students utilized to develop their reasoning.

Results

In this section, we present the excerpts we analyzed from the data to describe situated accounts of learning that related students' reorganizations as they transitioned between a simulation and a graph in day 1, and between a simulation, a table, and a graph in day 2.

Day 1: Transitions Between a Simulation and a Graph

On the first day of the design experiment, students were asked to explore the simulation and then graph the air temperature in the four cities and at the equator. First, we asked the students freely to explore the simulation and to describe what they noticed. Mikhail and Tasif observed the simulation's controls and outputs, noting that moving the arrow highlights different climatic zones with varying temperatures. Mikhail then discussed the observed changes in temperature in different climate zones.

Mikhail: The closer you go to the tropical, the hotter it is [moves the cursor slowly from the northern polar zone to the tropical zone]. The closer you go to the polar, the colder it is [moved the cursor from the tropical zone to the southern polar zone]. And the temperature is more in the middle. And, and could go each way [moved the cursor within the southern temperate zone].

While exploring the simulation, Tasif made similar observations (see Fig. 5) and also used the term *latitude*, noting how the temperature could be seen to change in different zones: “in the northern hemisphere, the more south you go, [it] gets hotter. But if in the southern hemisphere the more north you go, it’ll be a little hotter until we reach zero latitude.” During this exploration, both students identified the changing quantities of latitude and temperature in the simulation. Moreover, although no numerical values were mentioned for the quantities, both students discussed how the more the location changes in one direction or the other, the more the temperature also changes. This conceptual connection of the changes they observed in the two quantities shows that both students were reasoning covariationally as they manipulated the simulation.

After the free exploration session, the students were further prompted to discuss how they saw the quantities of temperature and latitude changing in different regions in the simulation. In response, they began connecting positive and negative values to physical locations on the globe.

Mikhail: It’s [the temperature] also positive around here [pointing to the southern Temperate zone]. It depends where you go [moves cursor around the other northern and southern zones].

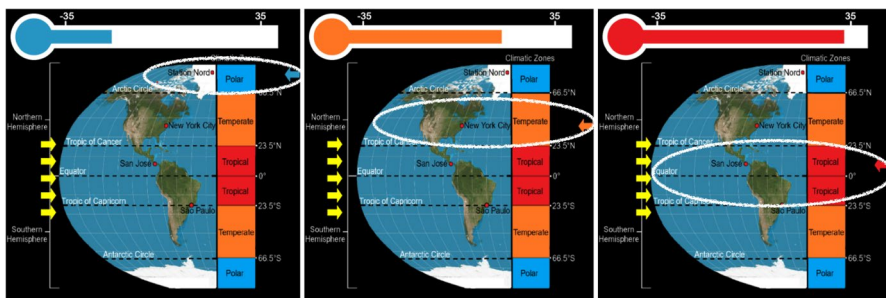


Fig. 5 Screenshots of Tasif’s simulation exploration

Tasif: Yeah.

Interviewer: Is the latitude positive if you go down?

Mikhail: No, it goes negative [moves cursor over the southern hemisphere].

Similarly, Mikhail also described that, “The equator lies at zero because over here [is] the equator line, and zero degrees [latitude] is over here [moving cursor along the equator line].”

Next, the students were asked to read temperature data about different locations shown in the simulation and then graph this information on the provided set of axes. The students worked together but each produced their own graph. When asked what information they needed to complete their graphs, Mikhail described that, “it gives us the locations [points to some of the location names on the x -axis], but it doesn’t give us the temperature [moves the cursor along the y -axis]. The temperature is what we need to find.” His reasoning highlights the complementarity between the artifacts, as his identification of “what we need to find” shows that he recognized that he would need to call back to information from the simulation in order to complete the graphing task. At the same time, a redundancy between these two artifacts and their ability to represent the same information is also implied.

The students then began discussing their first data point, and in negotiating how to represent it on the graphs, they immediately realized that they needed to create an appropriate scale and range on the y -axis. As Mikhail stated, “We need to type out the temperatures [pointing to the y -axis].” They then discussed what spacing to use to best fit the data they expected to plot.

Tasif: What should each big line [darker lines on the y -axis] represent? Should it be twenty?

Mikhail: I think it should be with fives, right? zero, five, ten and then negatives and there. Wait, no, it needs to start from minus thirty-five.

The students’ discussion shows that the graphing task required them to reason about how temperature information could be represented in this new artifact. This is an antagonism between how the two artifacts encode information, as the simulation by itself did not require this kind of reasoning from students when they worked with it. Furthermore, as their process of labeling the axis continued, the students reasoned about which locations on their graphs represented different locations on the simulation, showing again the redundancy between the two artifacts. For example, in Fig. 6, Mikhail noted that, “this [pointing at 0° latitude on the y -axis] would be zero on the equator” and Tasif agreed.

Once their axis labeling was complete, the students were able to then plot their first data point for Station Nord. Tasif estimated that it should be “Around minus twenty degrees, right?” While Mikhail verbally agreed, he instead placed the marker for the data point closer to -30° . The students’ use of estimation in placing their data points shows an antagonism between the encoding of information

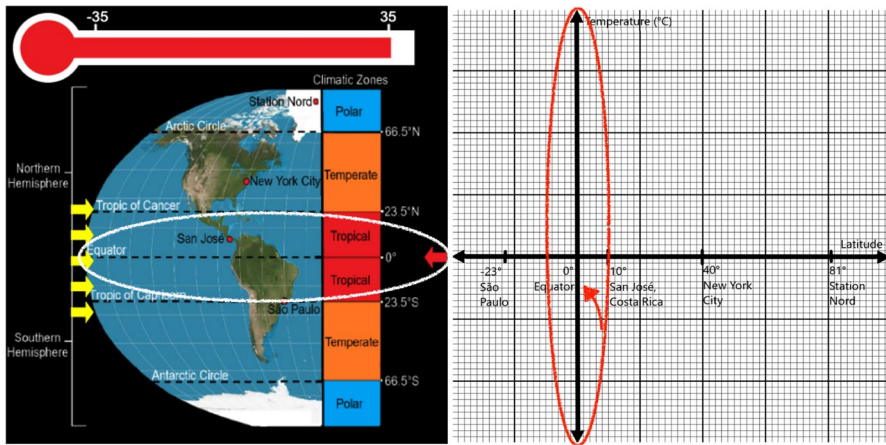


Fig. 6 Students identified zero on the graph as representing the equator from the simulation

in the simulation and graph. The graph seemed to require the students to actively reason about how to place and interpret data points where the simulation only required them to passively read this information. However, after being prompted by the interviewer to “check it again,” Mikhail referred back to the simulation and used this to refine his estimation of where the data point should be located (Fig. 7).

Mikhail’s use of the simulation to revise his reasoning about a data point when questioned about its placement is another kind of complementarity, in which one artifact was used to check and revise the students’ reasoning about another. The students then continued to graph the rest of the required data points, each time referring to the simulation and then estimating where to place the marker on their graphs. The students’ process of calling back to the simulation for each data point seems to be

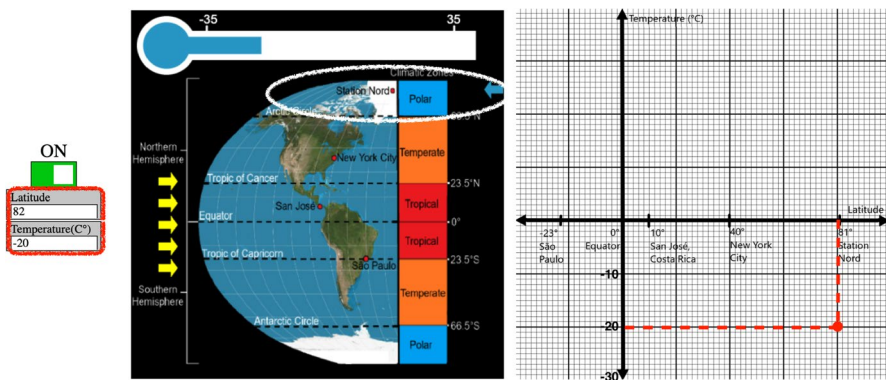


Fig. 7 Mikhail used the simulation to estimate data for Station Nord while creating a graph

another result of the complementarity between the two artifacts, as one was needed in order to create the other.

As they progressed through their graphing process, the students also continued to make connections between locations in the simulation and how they were represented on their graphs, illustrating evidence of the redundancy between the two artifacts. For example, Tasif described that “for the equator, I put a latitude zero. So I got thirty-five Celsius.” Similarly, Mikhail reasoned that “The equator would be, it would be on this line [y-axis], and it would be over thirty [places a point on the y-axis between 30 °C and 40 °C]” (Fig. 8). This also again shows the antagonism of the graphing task requiring the students to actively reason about how this information they had read from the simulation should be represented in their graph.

The interviewer asked them to describe what their completed graph looked like, and both students reasoned about the shapes of their graphs.

Mikhail: My graph looks like it's very low in the beginning [uses cursor to trace a path between the points starting at the right side (Station Nord) to the left]. But then the numbers get closer together [uses hand gesture, moving thumb and pointer finger closer together] and it slowly goes higher and then dips down a bit [traces this path with a hand]. And add that little dip.

Interviewer: How about you, Tasif?

Tasif: Mine looks like from Station Nord, it goes up and up. But once you reach to Sao Paulo, it goes a little bit down [makes a hand gesture not fully visible on the video]. So, like a hill, sort of.

Fig. 8 Recreation of Mikhail's completed day 1 graph

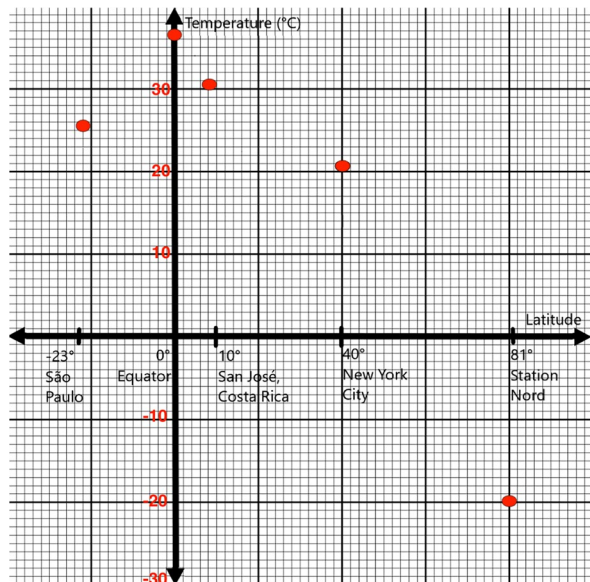


Table 2 Summary of day 1

Category	Forms of evidence in our data
Complementarity	Students call back to the simulation to gather data to create the graph Student changes the location of a graphed data point after using the simulation to check his estimation
Redundancy	Students identify the equator in both artifacts
Antagonism	Students need to reason about how data is encoded in the graph but can simply read it from the simulation Students tend to reason statically about the graph and emergently about the simulation

Whereas their prior discussion using the simulation had resulted in statements of covariational reasoning, in discussing their graphs the students relied on reasoning about visual features such as “dips down a bit” and “like a hill” without describing what these might mean in terms of the quantities of the graph. This may imply another important antagonism between the simulation and graph, in which the former seems to support covariational reasoning, an example of emergent shape thinking, while the latter seems to encourage static shape thinking instead. We also acknowledge the possibility that students’ static shape thinking might have been influenced by the nature of the question we asked pointing to what the graph “looks like.”

Table 2 summarizes the evidence we found in the students’ reasoning on the first day of the experiment that illustrates the complementarities, redundancies, and antagonisms between the two artifacts. In terms of complementarity, the students’ transitions between the simulation and graph showed how they called back to the simulation in order to create the graph as well as how the simulation could be used to revise their thinking about the data in their graph. Being able to identify the location of the equator in both artifacts shows evidence of redundancy. In terms of antagonism, there appeared to be differences in how the students worked with the two artifacts based on how information is encoded differently in each and how each artifact seems to support particular forms of shape thinking (static or emergent).

Day 2: Transitions Between a Simulation, a Table and a Graph

During the second day of the experiment, students further explored how the atmospheric temperature varies in different latitudes by using the simulation to collect data in a table and then graph the relationship. To begin with, we first asked students to use the simulation to complete a table by recording the air temperature for every given latitude. The two students agreed that Tasif would share his screen and record the temperatures on the table (Fig. 9), while Mikhail read the values out loud from the simulation.

Mikhail: How about this Tasif, you stay on the thing [table]. And I’ll go on my ACMES [simulation]. And I’ll just find the air temperatures. I’ll tell you, then you type them in. For minus twenty-three, it’s twenty-three. For minus ten, it’s twenty-four [...]

Fig. 9 Students' completed table with values taken from the simulation

Latitude	Air Temperature
-67	- 19
- 47	12
-23	23
-10	24
0	35
10	31
23	26
47	8
67	- 9

Mikhail and Tasif worked together to transition between the simulation and the table as they looked for the values of air temperature corresponding to the given values of latitude. The students' actions show both the redundancy and the complementarity between the simulation and the table as the information they found in the simulation provided the values they needed to complete the table.

Next, we asked the students to describe any patterns they found in their table. Mikhail stated, "there is some type of pattern repeating. [...] when the latitude is minus twenty-three, the air temperature is twenty-three. And when the latitude was twenty-three, it was twenty-six. It is kind of similar." We infer that Mikhail was exhibiting a form of static shape thinking by reasoning about a visual pattern in the temperature and latitude where "twenty-three" was a repeated value. His reasoning also shows an antagonism between the simulation and the table, where the table seems to encourage comparison of numerical values without constructing a relationship between quantities as the students had done with the simulation. Mikhail also exhibited static shape thinking when we asked them to describe how the temperature changed in their tables.

Mikhail: The temperature, I say it would rise up from negative, I mean, it would become very low for negative [latitude] since minus nineteen [°C]. [...] But then when it reaches minus forty-seven latitude, the temperatures start rising. And for the next, it's like a slowly curved-rising [tracing a hill-like motion with his hand]. And then it slopes down again [moves from the peak to the original starting height].

We interpret that Mikhail's reasoning illustrates static shape thinking because, although he discussed temperatures at different latitudes in his table, he focused on a visual description of the shape of the slope rather than a relationship between the quantities. On the other hand, Tasif reasoned covariationally about the distance from the equator and the temperatures based on what he observed from his table, stating,

“the farther you go from the equator, the colder it will be.” In contrast to Mikhail, Tasif’s reasoning illustrates emergent shape thinking.

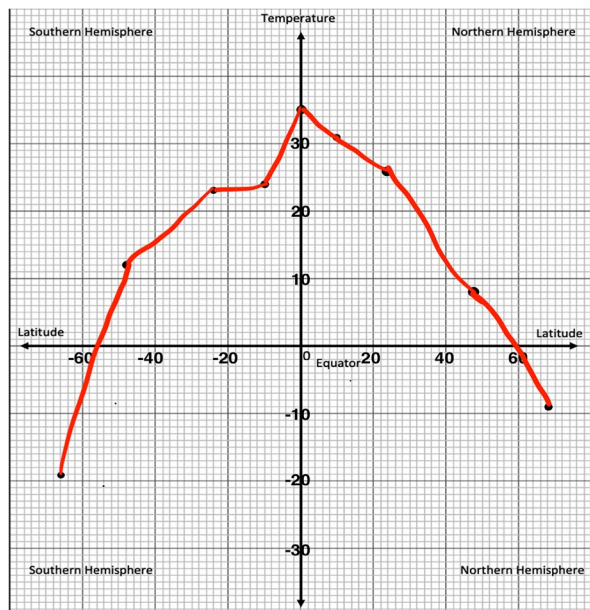
After listening to Tasif, Mikhail added, “because there’s less direct sunlight. So, I’m guessing that the graph would be some type of curve hill.” Mikhail built from Tasif’s covariational reasoning about the temperature and the distance from the equator to explain why it was colder “because there’s less direct sunlight” when you go farther from the equator. Mikhail’s reasoning shows an interesting complementarity calling forward between the table and the graph he would create later, as he imagined the future shape of the graph while discussing the changes in latitudes and temperatures shown on the table. We also interpret Mikhail’s statements about the relationship between the directness of sunlight and the temperature as an example of reorganizing his static shape thinking about the possible shape of the graph into emergent shape thinking about the relationship it represents. Mikhail later finished his graph and shared it on his screen (Fig. 10).

Mikhail: It wasn’t as I predicted. It was a curve, a very steep curve from the beginning. But then the curve smallens, and then it steepens again [moved hand at about a 45° angle upwards] and goes to the highest point. It kind of slowly curves down until it comes to its peaks.

We interpret Mikhail’s reasoning about the steepness of the curve in the graph to be illustrating static shape thinking. This reasoning also shows a kind of complementarity between the table and graph artifacts. To elaborate, the information of the values taken from the table complemented the shape of the actual graph, which prompted Mikhail to revise his description of his predicted graph.

When we asked Tasif to talk about his graph, he also reasoned about the change in steepness in different sections of the graph.

Fig. 10 Recreation of Mikhail’s completed day 2 graph



Tasif: I've seen that at first it was going really steep but then, after it reached ten Celsius, it started, it got a little less steeper, as you can see right here [moving his mouse along the portion of his graph that corresponds with -55 to 0 on the x -axis (latitude)]. But when it hits the equator [pointing at the peak of his graph where $x = 0$], it looks all the way down like this [moving his mouse along his graph from where $x = 0$ to $x = 70$].

Similar to Mikhail, Tasif reasoned about the visual features of the graph which illustrates his static shape thinking. His response shows the antagonism between the graph that encouraged static shape thinking and the simulation which did not.

When we asked Mikhail about the highest temperature that he found in his graph, he reasoned that it was 35°C and “that’s the equator latitude.” When we asked him why that was the case, Mikhail reasoned:

Mikhail: I think that’s the case because if you look at the variables inside the ACMES simulation, and you put it [latitude] exactly at zero, it’ll rise exactly and stop at thirty-five degrees Celsius. And I think this is because the equator is where it’s [the sunlight] more direct, it’s the most direct that it can be. And when it’s more direct, that means it’ll be more. [...] And that’s why the equator’s super-hot.

Mikhail transitioned from the graph to the simulation and reasoned about the features of the graph that were related to the simulation, using both artifacts to support his argument about the highest temperature (Fig. 11). His reasoning shows how the two artifacts share a redundancy of information.

When we asked the students to find where the temperature was at its lowest in their graphs, Mikhail reasoned about the shape of the graph while correctly identifying the lowest and highest temperatures.

Mikhail: When it gets far from the equator, like Tasif said. Like, the farther it goes the more, that’s how both of ours became steep. [Tasif points to the

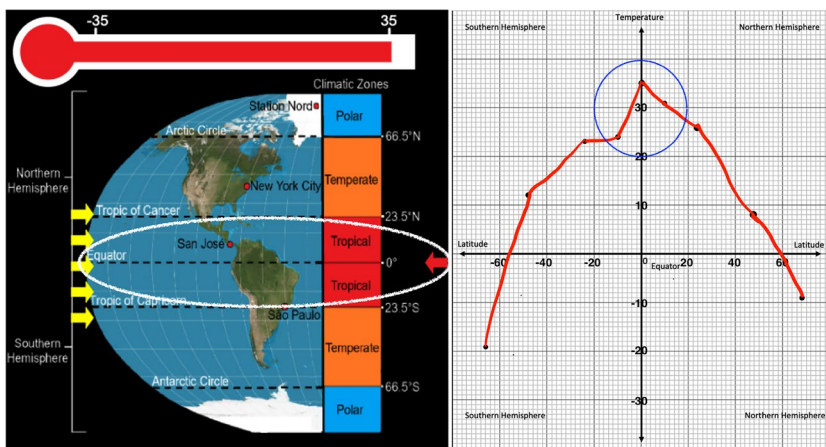


Fig. 11 Mikhail referred to the latitude of the equator from the simulation to explain the peak of the temperature at the equator on the graph

far left side of the graph with his mouse.] Because it raised the latitude, it got closer to the equator from the negative latitude. And since it was rising up, as soon as the equator the temperature raised as well. But also, when it, when you go higher past the equator, it [temperature] also goes down.

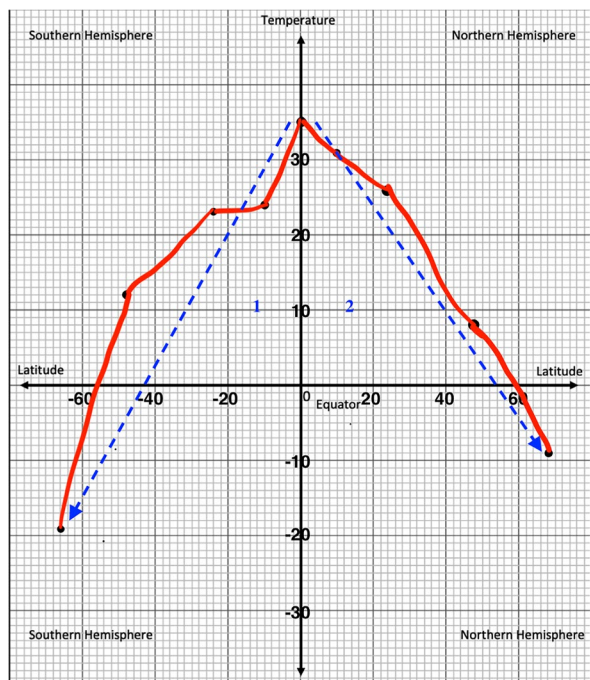
Although he is again discussing the visual shape of the graph, in this excerpt Mikhail also displays emergent shape thinking in his connection of the steepness and the distance from the equator. He had earlier described this while working with the simulation, so this reasoning also again shows the redundancy between the graph and the simulation representing the same information. This exemplifies a tendency we noticed for students' thinking to shift back and forth between static and emergent shape thinking while discussing their graphs.

We found another example of this tendency when we probed the students to talk about the pattern in the northern and southern hemispheres (see Fig. 12).

Mikhail: Except for when you get to the northern hemisphere area. That's where it's just steep. But in the southern hemisphere, it's steep at the beginning, but then it starts clustering when it reached the equator.

Tasif: The more south you go [points his mouse to the left side of the graph], the more colder it is than compared to the northern hemisphere [points to the right side of the graph]. As you can see, it's a way down here [points to approximately $(-70, -19)$]. But then, at the northern hemisphere, it's up here [points to approximately $(70, -8)$], which isn't as low as this [points back to approximately $(-70, -19)$]. And they're both sixty-seven latitude.

Fig. 12 Recreation of Tasif's illustration of the change in temperatures in the southern and northern hemispheres



Mikhail's reasoning shows static shape thinking about the different amounts of steepness he saw in the graph, although he had previously reasoned emergently about his graph. Tasif's reasoning shows emergent shape thinking that connects these visual features to a meaningful relationship between the latitude and the temperature. However, in other instances he reasoned about the shape of the graph, such as stating that "from the equator is pretty much the same [tracing from the y-intercept to the end of the graph], which would mean that they have the same latitude. [...] it's sort of symmetrical [pointing at the y-axis]." In this instance, Tasif did not explain this symmetry in terms of a relationship between the latitude and temperature, illustrating static shape thinking.

When we further prompted Tasif to reason about the changes in temperature as the distance from the equator changed, he used the simulation to respond emergently.

Tasif: When you are at the Temperate zone, it [temperature] slowly decreases as you can see in simulation [points the mouse within the Temperate zones in both hemispheres], but once you reach the Polar zones [points the mouse within the Polar zones], it [temperature] starts decreasing rapidly.

Tasif transitioned to the simulation to reason about the rate of change in the temperature while he co-ordinated this with the changes in climatic zones. His use of the simulation seemed to directly support his reasoning about the relationship between temperature and distance from the equator. This also again illustrates the same antagonism between the simulation that encouraged emergent shape thinking and the graph that seemed more likely to encourage static shape thinking.

Towards the end of the interview, when we asked the students to use their graphs to define the relationship between the latitude and the air temperature just in the northern hemisphere, Tasif shared his graph and reasoned, "from our graph, you can see the farther you go from the equator the colder it will be." When they were asked a similar question about the relationship between the latitude and the air temperature for the southern hemisphere, Mikhail responded, "as the distance from the equator increases, the air temperature decreases. It goes for both hemispheres." Then Mikhail added a generalization about the relationships for the two hemispheres, saying, "that is why it is the rule of global climate: the farther you get [from the equator], the colder you are." Here, both students showed emergent shape thinking as they reasoned about the covariational relationship in their graphs. This shows the redundancy between the graph and the simulation, since after moving back and forth between the different artifacts, the students reasoned about how the same information was also represented in their graphs.

Table 3 summarizes the evidence we found in the students' reasoning on day 2 that illustrates the complementarities, redundancies, and antagonisms between the three artifacts. Much of the evidence found on day 2 reflected the evidence on day 1. To elaborate, students were able to identify the equator as the location with the highest temperature in both artifacts, showing similar evidence of

Table 3 Summary of day 2

Category	Forms of evidence in our data
Complementarity	Students use the simulation to gather data to create the table and then use the simulation and table to create the graph Student uses the table to make a prediction about the graph before creating it Student compares his prediction about the graph with the actual result and notes the differences
Redundancy	Students identify the equator as the point of highest temperature in both artifacts
Antagonism	Students need to reason about how data is encoded in the graph but can simply read it from the table and the simulation Students tend to reason statically about the table and the graph and emergently about the simulation

redundancy as in day 1. Also, similar to day 1, we have further evidence of antagonisms created by the different encoding and different support offered by the three artifacts. However, on day 2, Mikhail also used his table to make a conjecture about what he believed his graph would look like and then revised this thinking after actually completing his graph. Using one artifact to make a conjecture about another shows evidence of a form of complementarity that was not noted in day 1.

Discussion

In this section, we discuss the situated accounts of students' learning from two different but interrelated perspectives. To respond to the research question (a), in the subsection below titled "Reorganizations of Students' Systems of Instruments," we first discuss *how* students' covariational reasoning was shaped and reorganized through transitions from one artifact to the next. Through this discussion, we make inferences about their reorganizations of their system of instruments. To respond to the research question (b), in the subsection below entitled "A Profile of Types of Relations Between Artifacts," we then present a profile of the types of relations between artifacts that students constructed to discuss *how* the artifacts' synergy provided a constructive space for students to shape and reorganize their reasoning.

Reorganizations of Students' Systems of Instruments

By taking an actor-oriented perspective (Lobato, 2003), our analysis was able to give an insight into *how* students may reorganize (Piaget, 1977/2001) their

reasoning as they interact with multiple artifacts representing the same relationship with two covarying quantities. The students' moves between the artifacts were not sequential, even though the new artifacts were introduced sequentially. Instead, students moved back and forth between the different artifacts in a "messy" process of on-going transitions, giving us the opportunity to examine their reorganizations of their reasoning. Similar to their "messy" transitions between the artifacts, we observed that the progression of their reasoning was also a "messy" process of generating and reorganizing their system of instruments (Rabardel & Bourmaud, 2003) with which to reason about how temperature changes with latitude.

To illustrate, on the first day of the interview, Mikhail and Tasif exhibited similar reasoning while engaging with both the simulation and the graph. Both students reasoned covariationally as they manipulated the simulation by connecting the changes observed in temperature and latitude in terms of the climatic zones or distance from the north or south poles. This covariational reasoning is a form of emergent shape thinking (Moore & Thompson, 2015). However, when we asked them to describe their graphs, both students exhibited static shape thinking as they discussed the visual features of their graphs without connecting these to any covariational meaning. While the simulation had become part of an instrument with which the students could reason covariationally, the graph did not yet serve this purpose for them.

During the second day of the interview, the two students continued generating and reorganizing their systems of instruments to include new table and graph artifacts. The two students exhibited different forms of reasoning as they worked on their tables and graphs, indicating that their processes of instrumental genesis and reorganization were proceeding in different ways. Mikhail's reasoning tended to show static shape thinking as he identified visual patterns in his table and described the shape of the curves in his graph without connecting these to any covariational meaning. Tasif also exhibited static shape thinking when he described similar visual features of his graph. However, Tasif also displayed emergent shape thinking when he described the temperature changes in his table by stating that, "the farther you go from the equator, the colder it will be." Mikhail engaged with Tasif's emergent reasoning when he agreed that this was "because there's less direct sunlight" before then returning to his static reasoning about the possible shape of a graph created from the data in his table. As they responded to further questions about the patterns and relationships shown in their graphs, both students displayed a mix of static and emergent shape thinking, with Mikhail's reasoning tending to be more static while Tasif showed more emergent reasoning.

After moving back and forth between the different artifacts, both students had shown evidence of reorganizing their reasoning in a higher conceptual level that included reasoning covariationally about the quantities in the graph, which is considered to be a typical difficulty that students face when interpreting graphs (e.g., Moore & Thompson, 2015). The transition process led students to

observe a structure of covarying quantities that was not tied to one artifact but rather was operative with all other artifacts. In other words, they were able to reason emergently about the quantities across multiple representations. Therefore, to characterize students' development of shape thinking as they transition between artifacts, in our analysis we extended the Moore and Thompson distinction between static and emergent shape thinking from focusing on graphs only to include student inferences about the behavior of a simulation, table, or graph. Table 4 presents this expanded framework and provides examples of student reasoning from our analysis. This framework can be useful for other studies examining students' development of reasoning during their transitions between artifacts.

Table 4 Examples of static and emergent shape thinking in different artifacts

	Simulation	Table	Graph
<i>Static Shape Thinking:</i> Making inferences about the behavior of an artifact as an object in and of itself	Reasoning about simulation features without connecting them to an underlying covariational meaning e.g., discussing the simulation: "the yellow arrows are the sun rays hitting this side of the earth. [...] it shows you the polar, the temperate, or the tropical, tropical zones and the things that separate them."	Reasoning about a visual pattern of numbers without connecting this to an underlying covariational meaning e.g., discussing the table: "it's a close repeated pattern, but it really isn't completely repeated. [...] when the latitude is minus twenty-three, the air temperature is twenty-three. And when the latitude was twenty-three, it was twenty-six. It is kind of similar."	Reasoning about the visual features of a graph's shape without connecting this to an underlying covariational meaning e.g., discussing the graph: "Mine looks like from Station Nord, it goes up and up. But once you reach to Sao Paolo, it goes a little bit down [makes a downward hand gesture]. So, like a hill, sort of."
<i>Emergent Shape Thinking:</i> Interpreting an artifact as representing an emergent trace of covarying quantities and identifying the same relationship by transitioning between artifacts	Reasoning about how simultaneously varying quantities in the simulation are related to each other e.g., discussing the simulation: "in the northern hemisphere, the more south you go [it] get[s] hotter."	Reasoning about numerical patterns that describe a covariational relationship e.g., discussing the table: "the farther you go from the equator, the colder it will be."	Reasoning about the graph as a trace that records the relationship between covarying quantities e.g., discussing the graph: "the farther you go from the equator, the colder it will be [moves the pointer from $x=0$ to $x=-65$]."

A Profile of Types of Relations Between Artifacts

Our findings show how students' reorganizations of their systems of instruments were supported by the synergy (Mariotti & Montone, 2020) of complementarities, redundancies, and antagonisms (Soury-Lavergne, 2021) between these different artifacts, as summarized in Tables 2 and 3. Our analysis expanded the Soury-Lavergne framework to identify subcategories of complementarities and antagonisms that are possible when students engage with these transitions. A profile of these subcategories is presented with descriptions and examples from our data in Table 5.

To elaborate, we found that the complementarity between the artifacts took three kinds of forms in the students' work: *callbacks* to previous artifacts, *conjectures* about subsequent artifacts, and *revisions* of reasoning based on considering other artifacts. *Callbacks* to previous artifacts were common, for example, as the students first discussed and then completed the process of finding data in the simulation in order to create their table or graph. The students also referenced back to the simulation while reasoning about their graph, making connections about features common to both artifacts. *Conjectures* about subsequent artifacts were rarer, though one student did make comments reasoning about what he expected the shape of his graph to look like based on what he saw in his table before he had actually begun graphing. *Revision* of reasoning then occurred later, when this same student considered his completed graph and revised his earlier conjecture about its shape. Similarly, the students used their further consideration of the simulation to revise their reasoning about finding maximum points on their graph.

In terms of redundancies, the simulation, table, and graph artifacts all served to represent the same information, namely the covariational relationship between the two quantities of latitude and temperature. This redundancy supported students in reasoning about these quantities and their relationship across each of the different representations. For example, the students identified features of the simulation such as the equator as also being found in their tables and graphs and also used the simulation to check or justify their reasoning about their graphs.

In contrast, the multiple antagonisms we noted between the artifacts were highlighted by the different ways in which the students reasoned about them. There were two notable subcategories of these differences. First, we found that the graphs required active placement and interpretation of data points as opposed to the passive reading and recording of data in the simulation and table. This difference in how information is *encoded* in the different artifacts resulted in students actively reasoning about axis labels and point estimation while graphing and interpreting their graphs. The students thus interacted with the data in their graphs in a way that they did not need to in order to use the simulation or table. Second, we found that the table and graph artifacts seemed to *support* students in engaging in static shape thinking where the simulation did not. While the students had reasoned covariationally while working with the simulation, their discussions of their tables and graphs showed many examples of static reasoning about the visual features of these artifacts without connecting those to a covariational meaning. It was only after moving back and forth between the different artifacts that both students reasoned emergently about their graphs.

Table 5 A profile of complementarities, redundancies, and antagonisms

	Description	Example from our data
Complementarity		
<i>Callback</i>	Student references a previous artifact while creating the next	“How about this Tasif, you stay on the thing [table]. And I’ll go on my ACMES [simulation]. And I’ll just find the air temperatures. I’ll tell you, then you type them in.”
<i>Conjecture</i>	Student uses an artifact to make a prediction about a new artifact	After creating a table: “So I’m guessing that the graph would be some type of curve hill.”
<i>Revision</i>	Student changes their thinking after considering other artifacts	After creating a graph: “It [the graph] wasn’t as I predicted... it was a curve, a very steep curve from the beginning. But then it goes, but then the curve smallens [gets smaller].”
Redundancy		
<i>Same information</i>	Student identifies how different artifacts represent the same information	“The equator [a feature found in the simulation] would be, it would be on this line [the y-axis of the graph].”
Antagonism		
<i>Different encoding</i>	Student operates differently on different artifacts based on how the information is encoded	Student <i>reasons</i> about how data is encoded in a graph: “So, what should each big line [the darker lines on the temperature axis] represent? Should it be twenty?” Student <i>reads</i> data from a table or simulation without reasoning about how it is encoded: “It’s [the temperature] also positive around here [pointing to the southern temperate zone]. It depends where you go [moves cursor around the other northern and southern zones].”
<i>Different support</i>	Student is supported in reasoning at different levels of sophistication by different artifacts	Student is supported in reasoning <i>statically</i> about the visual features of a graph’s shape or the numbers in a table: “Mine [graph] looks like from Station Nord, it goes up and up. But once you reach to Sao Paolo, it goes a little bit down [makes a hand gesture not fully visible on the video]. So like a hill sort of.” “There is some type of pattern repeating [in my table]. [...] when the latitude is minus twenty-three, the air temperature is twenty-three. And when the latitude was twenty-three, it was twenty-six. It is kind of similar.” Student is supported in reasoning <i>emergently</i> about the covariation in a simulation: “So, the closer you go to the tropical, the hotter it is [moves the cursor slowly from the northern polar zone to the tropical zone in the simulation].”

The development of an expanded profile of the types of relations between artifacts that students utilize to develop their reasoning will inform future studies aiming to design and analyze instructional learning experiences to support connection-making. Recall that previous studies examining students' covariational reasoning emphasized in their designs the role of dynamic simulations in combination with tables and graphs (e.g., Ellis et al., 2015; Johnson et al., 2016). Studies supported that this instructional design seems to advance students' conceptions of graphs of functions as a representation of co-ordinated change (e.g., Ellis et al., 2018; Panorkou & Germia, 2021). Our work provides information to explain *how* this instructional design is effective for student learning. To elaborate, both students tended to transition back to the simulation to support their covariational reasoning during the discussion of their tables and graphs. We thus found that the dynamic simulation helped students to reason emergently about graphs because the simulation's inclusion set up a synergy between artifacts that supported them in reorganizing their reasoning.

Concluding Remarks

This was an exploratory study examining students' transitions between related artifacts designed to illustrate the same relationship between two covarying quantities. Our findings provide an insight into *how* students' transitions between artifacts may shape their constructions and reorganizations of reasoning. We use the term "insight" here to emphasize our study's small sample limited to one pair of students. We focused on only one pair in order to present a detailed analysis of their transition processes; however, more studies are necessary before we can make claims that are generalizable to a wider population.

Our results show the significance of design for engineering a potential synergy between artifacts and providing a constructive space for such constructions and reorganizations. We observed that although our design led students to a specific sequence in exploring the different artifacts, students' transitions involved dynamic, continuous, and "messy" shifts between these artifacts that were not restricted by our sequence. Our study provided evidence that the extension of the Moore and Thompson (2015) distinction of static versus emergent shape thinking in graphs to include a description of similar student behaviors in simulations and tables is productive for describing the "messy" reorganizations of students' reasoning. However, other studies would need to validate whether this extended version is applicable for describing the behavior of other students and different populations of students. Our contribution is thus to characterize this extension to Moore and Thompson with examples that may or may not be applicable elsewhere, but that provide insight into avenues for further research.

Our analysis assembled a profile of the types of relations between artifacts that students utilized to construct and reorganize their system of instruments. The profile expands Soury-Lavergne's (2021) work to include specific types of complementarities and antagonisms as subcategories. This expanded profile can be useful for orchestrating future synergies of artifacts to support students' reorganizations

of systems of instruments. At the same time, it can be used as a tool for teachers and researchers to self-evaluate whether such orchestrations have engineered constructive transitions for students. Specifically, from a teacher standpoint, this study provides an insight into how teachers may design appropriate activities with a synergy of artifacts in mind that could support students' transitions and instrumental genesis, and engineer opportunities for them to collectively construct knowledge through peer interaction and targeted questioning. This design insight touches on all three levels of instrumental orchestration (Trouche, 2005). At the first level, a teacher might choose or design individual artifacts around a single topic which offer different affordances and constraints. At the second level, a particular group of these artifacts might be presented to students to highlight certain redundancies, complementarities, and antagonisms which support their instrumental genesis. Finally, at the third level, the teacher might design and pose certain questions to the students to prompt them to reflect on their activity and the relationship they developed with the set of instruments.

While our work sheds light on how instrumental orchestration may be used to engineer and evaluate these transitions, the profile was developed based only on the particular synergy of our study. Other studies are needed to put this profile in harm's way to validate, compare, and add new forms of complementarities, redundancies, and antagonisms between artifacts. In conjunction with examining whether this profile is representative of other designs (artifacts, tasks, questioning), future studies also need to explore the "messiness" of students' transitions between artifacts that may support the learning about other quantities that covary in different ways (e.g., linear, piecewise linear, non-linear) or even other mathematical concepts aside from covariation.

To conclude, our study showed the importance of studying the *how* questions in the learning process. In contrast to other studies examining the *what* and *why* of the development of students' reasoning through interactions with specific artifacts, our study explains *how* this development happens as students transition between these artifacts. This actor-oriented lens (Lobato, 2003) proved to be important for examining the "messy" development of mathematical thinking that is constantly organized and reorganized through corresponding "messy" transitions.

Acknowledgements We would like to thank our earth science colleague Jay Singh and our mathematics education colleague Debasmita Basu for their help in the design of the set of activities on climate, and our computer science colleagues Sowmith Etikyala and Anthony Cuvillo for their help in the design of the climate simulation.

Funding This research was supported by a grant from the National Science Foundation (#1742125). The data presented, statements made, and views expressed are solely the responsibility of the authors.

Data Availability The datasets analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of Interest The authors declare no competing interests.

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