HOW TRANSITIONS BETWEEN RELATED ARTIFACTS SUPPORT STUDENTS' COVARIATIONAL REASONING

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Many studies use instructional designs that include two or more artifacts (digital manipulatives, tables, graphs) to support students' development of reasoning about covarying quantities. While students' forms of covariational reasoning and the designs are often the focus of these studies, the way students' interactions and transitions between artifacts shape their actions and thinking is often neglected. By examining the transitions that students make between artifacts as they construct and reorganize their reasoning, our study aimed to justify claims made by various studies about the nature of the synergy of artifacts. In this paper, we present data from a design experiment with a pair of sixth-grade students to discuss how their transitions between artifacts provided a constructive space for them to reason about covarying quantities in graphs.

Keywords: Instructional Activities and Practices; Mathematical Representations; Design Experiments.

Over the past three decades, mathematics educators have characterized students' reasoning about the simultaneous change of two quantities in various ways (e.g., Confrey & Smith, 1994; 1995; Saldanha & Thompson, 1998). Some proposed frameworks to describe students' progression of covariational reasoning over time (Carlson et al., 2002; Thompson & Carlson, 2017) and a handful of studies used these to examine students' reasoning of various mathematical ideas, such as the rate of change (e.g., Johnson, 2012), and scientific phenomena, such as gravity (e.g., Panorkou & Germia, 2021). To examine *what* forms of covariational reasoning students exhibit, these studies often engage students with various artifacts such as dynamic manipulatives, tables, or graphs (e.g., Castillo-Garsow, 2012; Ellis, 2011).

Multiple studies point to the role of particular artifacts or combinations of artifacts used during the learning process to explain *why* this development of students' reasoning happens or does not happen (e.g., Ellis et al., 2018). The role of technology has been to advance students' reasoning because it can be utilized to illustrate change in progress (Castillo-Garsow, 2012) as well as the ability to reverse change. For example, Johnson et al. (2016) used a Ferris wheel animation on Geometer's Sketchpad (GSP) and asked the students to click and drag the car to control the motion and graph the relationship between the car's height from the ground and its distance traveled within one revolution. Examining relationships between quantities using dynamic manipulatives before graphing has shown to advance students' conceptions of graphs of functions as a representation of coordinated change (e.g., Ellis et al., 2018). While some asked students to transition straight from the digital tool to the graph (e.g., Ellis et al., 2018; Stevens & Moore, 2016), others asked students to create a table as an intermediate artifact before transitioning to the graph (e.g., Ellis et al., 2015).

Although each study uses different technologies (GSP, GeoGebra, Desmos), often their goal is for students to connect these dynamic representations of relationships with the graphing of those relationships. One of the prevailing difficulties that students have with graphing is that they focus on the shape of the graph and ignore the relationship between the two covarying quantities that it illustrates. To explain this, Moore and Thompson (2015) distinguished between *static* or *emergent* shape thinking. Students who think statically operate on a graph as an object, such as a piece of wire. In contrast, students with emergent shape thinking illustrate an understanding of a graph as a trace (or a snapshot of a trace) in progress that depicts the relationship between two covarying quantities. To examine students' reasoning across multiple representations, we expanded the ideas of static and emergent thinking in this study to also include applying them to reasoning with simulations and tables as well as graphs. To reason emergently in this way, a student must construct a more abstracted structure of covarying quantities that is operative in each of the representational contexts, rather than being inherently tied to any one of them.

While research studying *what* forms of reasoning students develop during the transition between artifacts and *why* this might happen prevails, an examination of *how* this process happens is rarely the focus of the investigation. In this paper we argue that the *how* lens can show us specific methods and conditions in which students' actions and reasoning are influenced during the interactions with the artifacts. This can make a contribution for informing both the design of artifacts, tasks, and questioning around these ideas and also the analysis of students' reasoning during those transitions. Considering the above, this study aims to examine *how* students make connections between different representations of the same covarying quantities as they transition between artifacts, and to explore the conditions that made those transitions productive for their learning. Specifically, we aim to examine the following research question: *How may students' transitions between artifacts provide a constructive space for them to reason emergently with a graph?*

Theoretical Framework: Examining the Transitions Between Artifacts

In this paper we use the term *artifact* to refer to a tool made by humans that is used for performing a specific task (Verillon & Rabardel, 1995; Trouche, 2004). As the student interacts with an artifact, the artifact imposes on the user some affordances and constraints, which shape both the actions and the ideas emerging from the activity (Artigue, 2002; Noss & Hoyles 1996). Through their interaction with an artifact, students build an *instrument* (Verillon & Rabardel, 1995; Lagrange et al., 2001) which is a mental construction that consists in part of the artifact and in part of cognitive schemes. *Instrumental genesis,* or the process by which an artifact becomes an instrument, is therefore influenced by both the artifact's characteristics and the user's prior knowledge and experience (Rabardel, 2000).

The new instruments formed from the instrumental genesis do not develop in isolation but instead become part of a system that consists of instruments that students developed earlier (Rabardel & Bourmaud, 2003). Mariotti and Montone (2020) discussed the nature of a *synergy* between two artifacts as "an implicit or explicit reference to both artifacts [that] creates a relationship between meanings emerging from their use" (p. 113). To support students in relating these different meanings emerging from the different uses of artifacts, Soury-Lavergne (2021) proposed three characteristics of design that the group of artifacts needs to have. Specifically, she talked about making the use of each artifact necessary, what she refers to as *complementarity* between the two artifacts. To make their relation visible, she argued that there must be some form of *redundancy* of some characteristics of one artifact in the other. To her "redundancy in the system of instruments produces robustness and adaptability of the system" (p. 6). Finally, she stated that each artifact should have different constraints that would lead to students to adapt, by challenging and reorganizing their initial system of instruments. She referred to this characteristic as an *antagonism* between artifacts.

We may view each artifact as becoming a transitional instrument for the continuous (re)organization of a system of instruments. We refer to *reorganizations* (Piaget, 2001) of reasoning as the inferences we make about students' projections and reflections of particular forms of reasoning and their connections as these are (re)structured into a more coherent whole. While designs may imply a sequential instructional process from one artifact to the next, our experience shows that students' transitions do not follow a linear process but rather shift back and forth in unrestrained ways between different artifacts. Consequently, in this paper we use the term *transitions* to refer to the dynamic, continuous, and "messy" shifts (physical and cognitive) that the individual makes between artifacts as they (re)structure their system of instruments.

Methods

The findings we report here are part of a design experiment (Cobb et al., 2003) in which we iteratively developed and tested theories about both the process of learning and the nature of the synergy of artifacts that supported that learning. In this section we discuss the design and initial conjectures explored in this study as well as our methods of data collection and analysis.

Design and Initial Conjectures

Since our previous work (e.g., Panorkou & Germia, 2021) showed that students as young as sixth grade can engage in sophisticated reasoning about covarying quantities when these are presented in meaningful contexts such as scientific phenomena, we focused on the scientific phenomenon of climate in this study. We thus designed a set of artifacts based on this phenomenon, specifically involving the exploration of the covarying quantities of temperature and latitude in the earth's climate.

The first of these artifacts is the Climatic Zones simulation (Figure 1), with which students can explore how temperature and latitude covary in the earth's polar, temperate, and tropical zones. The temperature in these zones is largely determined by the distance away from the earth's equator, or latitude. To explore the simulation, students control the location of the arrow on the right side of the screen by moving their mouse into the different zones. The readouts above and to the left allow the student to observe resulting changes in the quantities. Similar to other studies (e.g., Ellis et al., 2018), our conjecture was that students' interactions with this simulation would support them in constructing their understanding of graphs as records of covariation, thereby engaging in emergent shape thinking (Moore & Thompson, 2015).

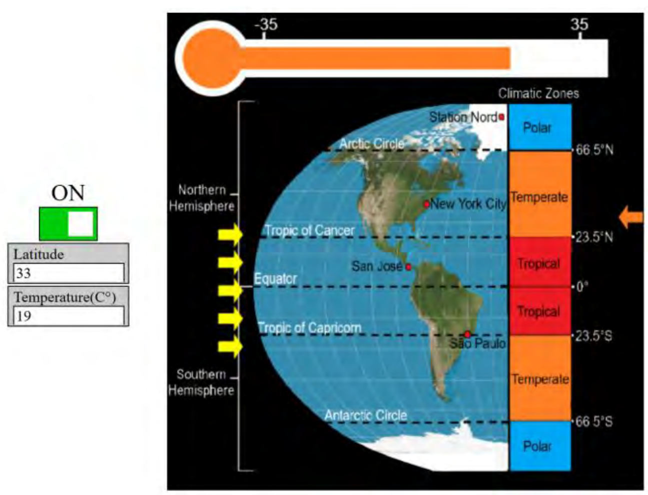

Figure 1: The Climatic Zones Simulation

We also chose to include both a table and a graph in order to explore both students' transitions between these different representations as well as how they would reorganize their systems of instruments to include each new artifact. Therefore, the next artifact we designed is a table which students were asked to use the simulation to complete a list of temperatures for given latitudes. We conjectured that this could support their emergent shape thinking since the table represents different values of the two quantities as ordered pairs. Students were then asked to complete the corresponding graph. This third artifact was designed without any scales, requiring students to create their own intervals on each axis before they could plot the values. We conjectured that this would allow us to examine how students would transition between the three artifacts to organize the data in the table as well as how they might reorganize their systems of instruments as they did so.

Finally, although this design may seem to be a strictly sequential learning activity, we set no restrictions on when or how often students could move back and forth between the artifacts as they worked. We thus expected that the students' actions and reasoning would illustrate the "messiness" of their transitions back and forth between the artifacts.

Data Collection and Analysis

The data presented in this paper was collected during a whole-class design experiment (Cobb et al., 2003) that took place in a sixth-grade classroom in the Northeast of the U.S. We conducted two 25-minute virtual sessions via Google Classroom due to COVID-19 restrictions. The students were paired off into breakout rooms where they engaged in the tasks and were interviewed by the researchers. The students' video and shared screens were recorded during the sessions. These recordings were then transcribed for the analysis.

This paper focuses on the retrospective analysis of the activity of one pair, Jami and Gaelyn. We analyzed the data in three stages. In the first stage, we identified episodes of the students' covariational reasoning and static or emergent shape thinking (Moore & Thompson, 2015). In the second stage, we reviewed each students' data chronologically in order to observe the progressions and reorganizations of their reasoning as they transitioned between the artifacts. In the third stage, we analyzed the features of the artifacts that seemed to support those progressions and reorganizations. We used the Soury-Lavergne (2021) framework of complementarities, redundancies, and antagonisms to characterize how this group of artifacts served as instruments in a system that provided a productive space for the students to reorganize their reasoning.

Findings

In this section we present examples of complementarity, redundancy, and antagonism in the order they emerged from the data.

Complementarity - Callback

We asked the students to explore the simulation and identify the quantities that change. Jami reasoned about the latitude, saying that "as you go up [using the cursor] the latitude is positive and then as you go down, the latitude is negative." She also reasoned about how the temperature changed in each climatic zone, explaining,

> Jami: The polar zone is actually very cold because it is up north [latitude], and the temperate zone since it is right between the equator where it is most hottest and the polar zone, it is pretty warm, like the average temperature, and the tropical zones are very hot because they are more close to the equator.

Her reasoning illustrated emergent shape thinking about how the closer from the equator a zone is, the hotter the temperature.

Next, we asked Jami and Gaelyn to use the simulation to complete the table (Figure 2, left). Jami shared her screen showing the simulation on the left side of the monitor and the table on the right side. The students took turns identifying the temperature using the simulation. For example, Jami stated “[negative] sixty-seven is in the latitude, the air temperature is negative 19.” Then, Gaelyn added “negative 23 is 23 degrees.” Both students transitioned between the table and simulation as they looked for the values of temperature that correspond to the given latitude. This reasoning exhibited the complementarity between these two artifacts as they referenced back to the information from the simulation to complete the table.

Latitude	Air Temperature
-67	- 19
- 47	12
-23	23
-10	24
0	35
10	31
23	26
47	8
67	- 9

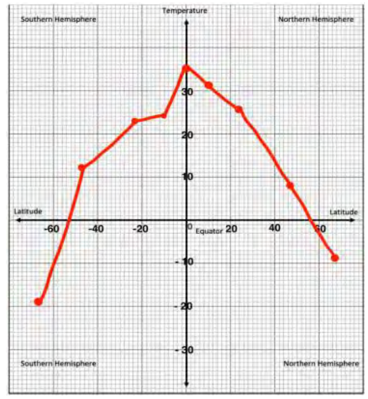

Figure 2: Gaelyn’s Completed Table and Graph

Then we asked the students to use the values in the table to create a graph showing the relationship between latitude and temperature (Figure 2, right). Gaelyn transitioned back and forth between the table and the graph as she plotted the values, reasoning,

> Gaelyn: The first thing was negative 67 to negative 19 [toggling back and forth between the table and graph]. … So that would be about here to [plotting in the first quadrant]. No wait, this is negative, so that would be about here [plotting in the third quadrant] to 19 right above the 20.

Gaelyn’s reorganization of plotting (-67, -19) from the first quadrant to the third quadrant also showed her understanding of graphing negative values. Next, Jami helped Gaelyn by guiding her with the succeeding values from the table as Gaelyn continued plotting them on the graph. Their actions of toggling between the two artifacts showed the complementarity of the table and the graph, as the former provided the information needed to create the latter.

Students’ calling back to the previous artifacts was a necessary action to collect relevant information and create the subsequent artifacts. The students needed to look for the data from the simulation to complete the table and then use the values from table to create the graph.

Antagonism

The students’ reasoning about the simulation, table, and graph also showed an antagonism in how the information in the three artifacts was encoded differently by each. Their reading of data points from the simulation and the table illustrated a straightforward encoding of information about the corresponding values of latitude and temperature. However, the graph required them to coordinate the two quantities and locate them along the x- and y-axes in order to represent the same information. As the students were plotting the points on the graph, Gaelyn moved her cursor along the y-axis and stated, “this is zero [latitude]” (Figure 3, middle). Jami also reasoned about the y-axis by stating that the “equator is right in the middle. It is the origin on the graph” (Figure 3, right).

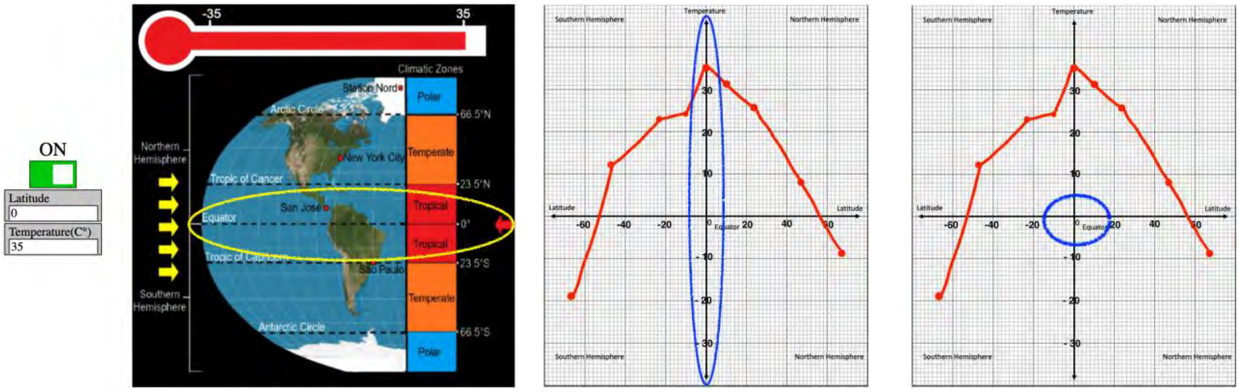

Figure 3: Relation of the Latitude of the Equator in the Simulation (left) to the Y-Axis (Gaelyn, middle) and the Origin (Jami, right) in the Graph

The students' reasoning about the location of the equator being at 0 degrees latitude and in the middle of the graph illustrated their understanding of the y-axis and origin of the graph. This also shows how the students needed to make meaning of the values on the graph, whereas they only needed to read the values from the simulation and the table.

Complementarity - Revision

Next, we asked the students to use the graph to describe the relationship between the latitude and temperature in the northern hemisphere. Gaelyn stated, "the higher the latitude, the lower the temperature," describing the relationship between the two quantities from the equator to the right side of the graph. This statement showed that she was able to identify how the latitude and the temperature in the northern hemisphere covaried in the graph. However, when we asked the students to use the graph to explain the same relationship in the southern hemisphere, Gaelyn wondered, "Would it be the opposite? So, like the lower the latitude, wait, no, never mind." Since Gaelyn was not sure about her response, Jami used the simulation to reason about the values of latitude. She stated that "if you go to the southern, the lower the latitude, because it's a negative, so when you go more negative numbers, on the number line, it gets more like lower, the numbers [moving the arrow downwards]" as shown in Figure 4. Jami probably imagined a "number line" across the different climatic regions in the simulation similar to what can be found on the graph to describe the changes in the values of the latitude. She referred to the values of the latitude as "negative numbers" to reason about "the lower the latitude" as she moved the arrow down in the simulation from temperate to polar climatic zones. In that way, she connected the change in the latitude in the simulation with the x-axis in the graph.

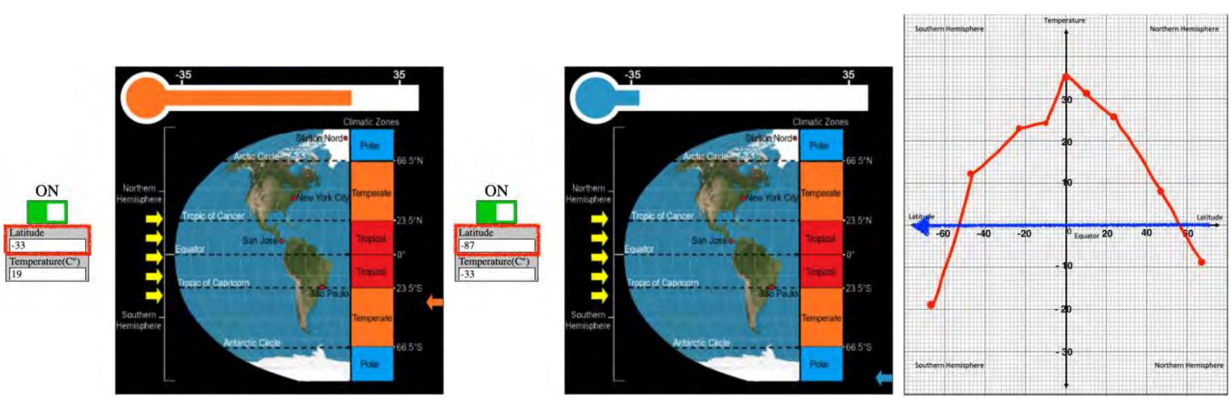

Figure 4: Jami Using the Simulation to Explain Latitude in the Graph

Associating the change in values with the direction on a number line did not seem to make sense for the students because the latitude was decreasing as Jami moved the arrow downwards in the simulation. Instead of reasoning about the negative values of latitude in the south, Gaelyn revised their discussion of latitude in terms of the distance from the equator:

> Gaelyn: … it says the same from the equator, as the distance from the equator increases, the air temperature decreases. Because we can't really say, because they're both going down [referring to the values of the latitude and temperature at the southern hemisphere], so we just have to do the distance.

Jami agreed and restated the relationship between the quantities in the southern hemisphere: "as the distance from the equator increases, the air temperature decreases."

The students' use of the two artifacts to discuss the relationship between latitude and temperature in the two hemispheres led them to revise their understanding of the latitude quantity from a positive or negative number into a measure of the distance from the equator. Since the latitudes on the right side of the graph were positive, it may have been easier for the students to express that covariational relationship than when working with the negative latitudes on the left side of the graph. The simulation allowed them to dynamically explore both regions of this relationship and became a necessary support for their reasoning about the relationship in the graph. Additionally, the students' reasoning exhibited a form of emergent shape thinking in both the graph and simulation. This shows a complementarity between the simulation and the graph as the students moved back and forth between these artifacts in order to revise their reasoning.

Redundancy

The students' activity with the artifacts also showed evidence of the redundancy of the information represented by each. For example, when we asked students to use their graph to describe the location of the highest temperature, Gaelyn reasoned that "the highest temperature, it would be right here [putting the cursor on the y-intercept of the graph] at the equator" (Figure 5). Gaelyn located the highest temperature at the y-intercept on the graph and then related the same point with the equator in the simulation where the highest temperature was also found. Jami added, "because the line is labeled temperature [making a vertical motion with her index finger]." This kind of reasoning showed the redundancy of information offered by the graph and simulation by connecting the information about the y-axis on the graph with both the location of the equator (Gaelyn) and the temperature data (Jami) from the simulation.

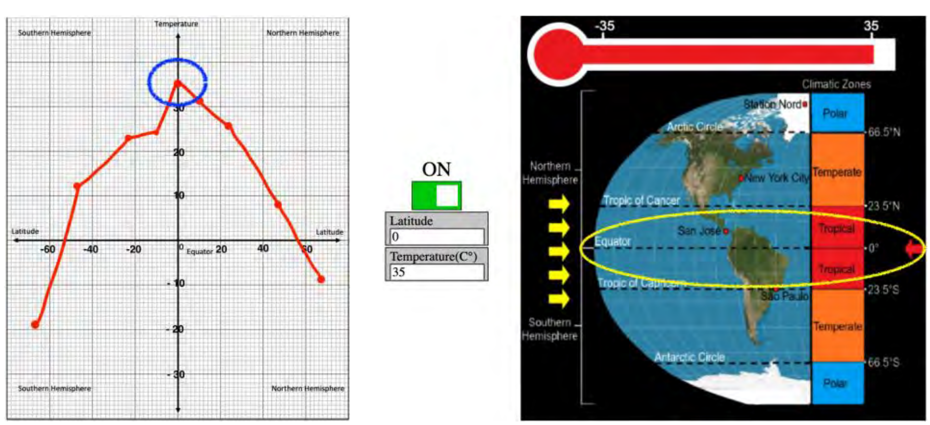

Figure 5: Gaelyn's Relation of the Highest Temperature in the Graph and the Simulation

The students' transitioning between the graph and simulation offered them opportunities to explore the similarities in the features of the two artifacts. Finding such information gave them opportunities to reason about how the quantities and their relationship are represented in both the simulation and the graph.

Conclusions

By examining *how* students transition between artifacts, our work informs the design of artifacts, tasks, and questioning that support them in making connections between different representations as they (re)organize their *systems* of instruments (Rabardel & Bourmaud, 2003). Jami and Gaelyn's transitions between the three related artifacts we designed influenced how they reasoned about the quantities and covariational relationships represented by those artifacts. Specifically, their transitions between the artifacts offered a constructive space for them to reason emergently with the graph instead of thinking about its static shape as described in the literature. Furthermore, they reasoned emergently across all of the artifacts, showing that they constructed a structure of covarying quantities not inherently tied to a particular representation.

Our findings highlight the specific forms of complementarities, antagonisms, and redundancies (Soury-Lavergne, 2021) that supported the synergy (Mariotti & Montone, 2020) between these related artifacts. The need for the students to use each artifact to create the next prompted them to reference back to previous artifacts as they continued to work. These callbacks show one kind of complementarity between the artifacts that supported the students' reasoning. The students' use of one artifact to revise their reasoning about another shows another kind of complementarity. Both of these complementarities worked hand-in-hand with the redundancy of information found between the artifacts, as the students explored and reasoned about the same features, quantities, and relationships across the different artifacts. At the same time, the antagonism between the different ways each artifact encoded this information offered a constructive space for the students to reason emergently (Moore & Thompson, 2015) across these different contexts and especially to make covariational meaning of their graphs.

Our analysis contributes to an expansion of Soury-Lavergne's (2021) work to include two possible subcategories of complementarity: callbacks to previous artifacts to support the creation of subsequent artifacts and revisions of reasoning based on the exploration of other related artifacts. More studies are needed to further investigate these identified subcategories and to look for other possible forms of complementarities, redundancies, and antagonisms in different sets of data. There is also a need to examine the similarities and differences in the progressions of reasoning among other students. For instance, it would be interesting to examine the progressions of other pairs who did not immediately reason emergently as they transitioned to the table and graph, and how the synergy between the artifacts may have influenced such progress.

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