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# Data-Driven Modeling and Control of Cyclic Variability of an Engine Operating in Low Temperature Combustion Modes

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**Abstract:** Combustion cyclic variability in an internal combustion engine leads to cyclic variations in the engine torque output and emissions. Combustion cyclic variability is often characterized by coefficient of variation of indicated mean effective pressure ($COV_{IMEP}$) that is used as an indicator of combustion stability. These cyclic variations are inevitable and cannot be completely eliminated but can be controlled to allow stable engine operation. This work focuses on control oriented modeling of $COV_{IMEP}$ to limit engine cyclic variations in low temperature combustion (LTC) modes. $COV_{IMEP}$ is generally stochastic in nature; thus, a data-driven approach is used to develop a predictive model of $COV_{IMEP}$ for Homogeneous Charge Compression Ignition (HCCI) and Reactivity Controlled Compression Ignition (RRCI) modes. This work presents the development of a cycle-by-cycle model predictive controller for a 2.0 liter multi-mode LTC engine. Physics-based control-oriented models for combustion phasing (CA50) and IMEP are augmented with the new data-driven $COV_{IMEP}$ model to limit the cyclic variations below 3% for HCCI and RCCI modes. These models are then used to design closed-loop non-linear model predictive controllers to control CA50 and IMEP while constraining $COV_{IMEP}$ to ensure stable engine operation for varying load conditions.



*Keywords:* Combustion Stability; Machine Learning; Non-linear Model Predictive Control; Homogeneous Charge Compression Ignition (HCCI); Reactivity Controlled Compression Ignition (RCCI)

## 1. INTRODUCTION

Advanced combustion modes including low temperature combustion offers high thermal efficiency and low engine-out emissions. Premixed mixtures in LTC modes reduce the local fuel rich zones which prevents very high peak in-cylinder gas temperatures that in turn, helps in restricting oxides of nitrogen (NOx) formation. The LTC engines can offer thermal efficiency comparable to the conventional diesel engines and produce NOx, and PM emissions substantially less than conventional diesel engines. This work focuses on the two common LTC modes i.e.; homogeneous charge compression ignition (HCCI) and reactivity controlled compression ignition (RCCI).

Combustion control in the LTC modes is important to avoid knock and too high maximum pressure rise rate, partial burns and misfires. Combustion phasing ($CA50$) and indicated mean effective pressure ($IMEP$) are the common controlled parameters in the LTC modes. In addition, combustion in LTC modes is mainly restrained by high cyclic variations and maximum pressure rise rate. Large cyclic variations are usually observed at low loads in LTC modes Yao et al. (2016). That is why, peak pressure rise rate Bengtsson et al. (2006) and cyclic variability Hellstrom et al. (2014) are important control parameters that need to be monitored for safe and stable engine operation.

Combustion variations on cycle-to-cycle basis are indicated by the coefficient of variation of indicated mean effective pressure ($COV_{IMEP}$). High cyclic variations in IMEP result in engine speed fluctuations and affect the noise, vibration and harshness (NVH) performance of a vehicle Qilun et al. (2016). Moreover, cyclic variability increases engine-out emissions Di Mauro et al. (2019). In addition, desirable ultra-lean engine operations with high thermal efficiency are prone to high cyclic variations. Therefore, it is important to minimize combustion cyclic variations to achieve stable combustion in order to allow the maximum thermal efficiency, and lower engine-out emissions. This study focuses on model based control of HCCI and RCCI modes by constraining $COV_{IMEP}$ to limit the cyclic variations in the combustion to ensure stable engine operation.

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Several studies have been conducted to evaluate the parameters responsible for these cyclic variations in LTC engines. Jia et al. (2015) studied the cyclic variations in the LTC modes with main focus on dual fuel RCCI combustion. The study suggested that cyclic variations in RCCI combustion can be reduced by retarding the injection timing, increasing the injection pressure and using boosted intake pressure. Low exhaust gas recirculation (EGR) rate, high equivalence ratio and high intake temperature provide lower cyclic variations in HCCI mode Shahbakhti and Koch (2008). Kalghatgi et. al. showed a linear regression model using $CA50$ and equivalence ratio ($\phi$) to differentiate low and high cyclic variability for HCCI combustion Kalghatgi and Head (2006). Hellstrom et. al. developed proportional integral (PI) and linear quadratic Gaussian (LQG) controllers to regulate $CA50$ in order to reduce cyclic variability in HCCI engine operation Hellstrom et al. (2014). In another study, $CA50$ was adjusted to ensure stable RCCI combustion and limiting $COV_{IMEP}$ for varying load and speed conditions Raut et al. (2018).

To the best of the authors' knowledge, this is the first study undertaken to develop a learning based engine cyclic variability classifier to create a predictive model and nonlinear MPC for LTC engines to control load and CA50 while stabilizing the engine combustion. This paper focuses on developing an approach to categorize stable and unstable combustion into different classes on the basis of the $COV_{IMEP}$ using supervised learning algorithm for HCCI and RCCI modes. The contributions of this work include:

(1) Development of a control oriented predictive model for $COV_{IMEP}$ using supervised learning classification algorithm for HCCI and RCCI combustion modes;
(2) Development of a nonlinear MPC for an HCCI engine regulating $CA50$ to ensure stable combustion while delivering requested $IMEP$;
(3) Development of a nonlinear MPC for an RCCI engine to control $CA50$, $IMEP$ and constraining $COV_{IMEP}$.

## 2. EXPERIMENTAL SETUP AND DATA

A GM 2.0L, 4-cylinder gasoline direct injection EcoTec engine coupled with a 460hp AC dynamometer is used in this study. The original engine is modified to include two port-fuel injection (PFI) systems and one direct injection (DI) system. The engine is run at wide open throttle under naturally aspirated conditions without external exhaust gas recirculation. An air heater is used to pre-heat the intake air to the desired temperature. In-cylinder gas pressure is measured with a resolution of 1 CAD using PCB piezoelectric pressure transducers. A dSPACE MicroAutoBox (MABX) is used as the engine control unit. A Xilinx Spartan-6 field propgrammable gate array (FPGA) is used for the real time feedback of combustion parameters. More details about the engine instrumentation can be found in reference Kannan (2016).

In HCCI mode, both iso-octane and n-heptane are injected during the exhaust stroke of the previous cycle via two PFI systems. While in RCCI mode, iso-octane is injected during intake stroke via PFI and n-heptane is directly injected during the compression stroke. The premixed ratio of the two fuels is calculated using (1):

$$PR = \frac{\mathrm{m_{iso}LHV_{iso}}}{\mathrm{m_{iso}LHV_{iso} + m_{nhep}LHV_{nhep}}} \tag{1}$$

where, $m_{iso}$ and $m_{nhep}$ are the mass of injected iso-octane and n-heptane, respectively. $\mathrm{LHV_{iso}}$ and $\mathrm{LHV_{nhep}}$ are the lower heating values of iso-octane and n-heptane, respectively.

Operating conditions of the engine data used in this study are shown in Table 1. These include 210 steady state HCCI operating points and 300 steady state RCCI operating points.

Table 1. Range of experimental data used in this paper for developing control models for HCCI and RCCI modes

| **Parameters** | **HCCI** | **RCCI** |
|---|---|---|
| IAT ($^{\circ}C$) | 40-100 | 40-80 |
| $\mathrm{P_{man}}$ (kPa) | 96 | 96 |
| Engine Speed (RPM) | 800-1600 | 800-2200 |
| PR (-) | 0-40 | 10-40 |
| SOI (CAD bTDC) | 450 | 20-60 |
| Equivalence ratio, $\phi$(-) | 0.32-0.67 | 0.32-1.00 |

The engine data is recorded for 100 consecutive cycles at different steady state operating conditions. The $COV_{IMEP}$ is a measure of variability in the $IMEP$ and is defined as the ratio of standard deviation of $IMEP$ to the mean of $IMEP$. It is calculated by:

$$COV_{IMEP}(\%) = \frac{\sigma_{IMEP}}{\mu_{IMEP}} \times 100. \tag{2}$$

where, $\mu_{IMEP}$ and $\sigma_{IMEP}$ are the mean and standard deviation of $IMEP$, respectively.

The in-cylinder pressure data from all operating conditions are analyzed and combustion parameters are determined. Figure 1 shows in-cylinder pressure traces for 100 consecutive engine cycles running in HCCI mode. It shows unstable combustion with partial burns and misfire in a number of the engine cycles for HCCI mode. This unstable combustion results in very high $COV_{IMEP}$ of 10.8%. High cyclic variations may occur due to slower burning cycles which result in reduced work output. In this work, a predictive model is developed to confine the engine cyclic variations below 3%.

## 3. DATA-DRIVEN MODELING OF $COV_{IMEP}$

In this work, machine learning classification is used to develop a model to predict engine combustion stability level based on $COV_{IMEP}$. The data can be classified into three distinct classes based on $COV_{IMEP}$ to indicate stable, semi-stable and unstable combustion. Each class is provided with combustion stability index (CSI) based on $COV_{IMEP}$. The data points with $COV_{IMEP} \leq 3\%$ are grouped into class-I having CSI of 1. These data points showed stable and complete combustion.Class-II consists of data points with $3 < COV_{IMEP} \leq 5\%$ with CSI of 2. These include engine operation with some of the cycles have abnormal combustion (e.g., high MPRR or light knocking). Depending on engine load, an engine can tolerate $COV_{IMEP}$ by 5% without any major oscillations in torque delivery or vehicle driveability concerns. Therefore,

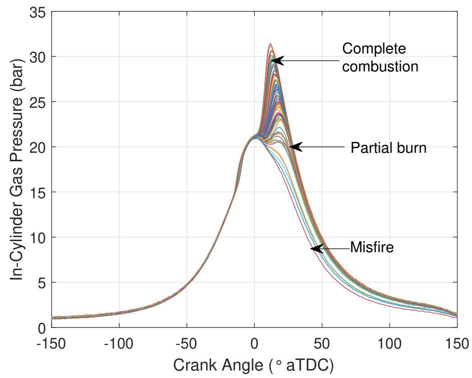


Fig. 1. In-cylinder pressure traces for 100 consecutive steady-state cycles of HCCI engine operating at FQ = 11.2 (mg/cycle), PR = 20 (-), Tman = 353 (K); $COV_{IMEP}$ = 10.8%, CSI = 3

class-II is considered semi-stable. The data points having $COV_{IMEP} > 5\%$ are grouped into class-III with CSI of 3 to represent unstable combustion, see Fig. 1. Class-III contains some data points with misfire and/or partial burns. Partial burn usually occurs when the burning rate is sufficiently slow and combustion is not completed by the time exhaust valves open Heywood (1988).

Classification is a supervised machine learning algorithm

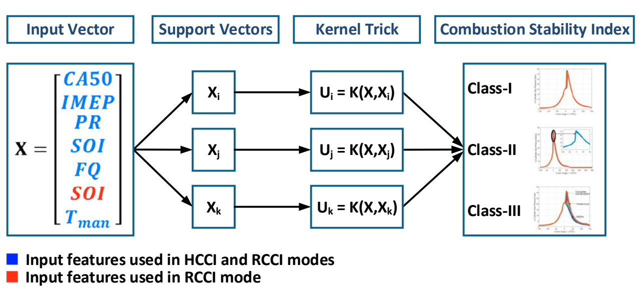


Fig. 2. Classification of engine combustion stability using support vector machines for $COV_{IMEP}$ data

which categorizes the data into different classes. There are various algorithms which can be used for classification such as logistic regression, support vector machines (SVMs), neural networks (NN), Naive Bayes, K nearest neighbors (KNN), boosted decision trees and random decision forests. We investigated SVM, KNN, Naive Bayes and neural networks classifiers and their accuracy were compared. In this study, support vector machine (SVM) is used for multi-class classification based on providing the best prediction accuracy. SVM works well with small data sets Pasupa and Sunhem (2016). SVM is a vector space based machine learning approach that maximizes the margin between two classes Cortes and Vapnik (1995). The $COV_{IMEP}$ predictive modeling is formulated as a non-linear multi-class classification problem as the classes are linearly inseparable. SVM maps the data ($x_i \in R^p$) in the input space (X) to a feature space F:

$$F = \{\phi(x) : x_i \in X\} \tag{3}$$

$$f(x) = \sum_{i=1}^{N} w_i \phi(x) + \beta_0 \tag{4}$$

The linearly inseparable data in the input space (X) can be linearly separated in the feature space (F) Cristianini (2000). These linear boundaries provide better separation and translates into nonlinear boundaries in the input space (X) Hastie T. (2009). The classifier is given by:

$$G(x) = sign(f(x)) \tag{5}$$

Lagrange dual objective function is of the form:

$$L_D = max \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j (\phi(x_i), \phi(x_j)) \tag{6}$$

Subject to the constraints

$$\sum_{j} y_j \alpha_j = 0; \quad 0 \leq \alpha_j \leq C \tag{7}$$

The dual optimization problem is solved by maximizing $L_D$ using quadratic programming. The solution of the optimization problem is given by:

$$f(x) = \sum_{i=1}^{N} \alpha_i y_i (\phi(x), \phi(x_i)) + \beta_0 \tag{8}$$

where, $\alpha_i$, $\beta_0$ and $y_i$ are the Lagrange multipliers, bias and class labels, respectively. C is box constraint to limit the values of Lagrange multiplier.

$$y_i f(x_i) = 1 \tag{9}$$

The transformation to feature space (F) and computation of their corresponding inner product can be computationally expensive. Therefore, kernel trick is used to compute the inner products in the feature space without any transformation using kernel function. The commonly used nonlinear kernel functions in the SVM are polynomial (10), radial basis (Gaussian) (11) and sigmoid.

$$K(x_i, x_j) = (1 + (x_i, x_j))^d \tag{10}$$

$$K(x_i, x_j) = exp(-\gamma ||x_i - x_j||^2 \tag{11}$$

### 3.1 $COV_{IMEP}$ Modeling for HCCI and RCCI Modes

For HCCI and RCCI modes, SVM is used to train the model for $COV_{IMEP}$ classification using linear, polynomial and radial basis kernel functions. Combustion parameters including $CA50$, burn duration ($BD$), peak pressure ($Pmax$), location of peak pressure ($\theta_{Pmax}$), $IMEP$, maximum pressure rise rate ($MPRR$), location of maximum pressure rise rate ($\theta_{MPRR}$), premixed ratio (PR), fuel quantity, intake air temperature and engine speed were initially used in developing $COV_{IMEP}$ classifier for both modes. $CA50$ has a linear relationship with ($\theta_{Pmax}$) and ($\theta_{MPRR}$). $Pmax$, $MPRR$ and engine speed didn't improve the prediction accuracy of the model. Therefore, this study uses $CA50$, $IMEP$, premixed ratio, fuel quantity and intake air temperature as features to develop a control oriented model for the $COV_{IMEP}$ for the HCCI mode. However, SOI is also used as a feature for classification in RCCI mode. HCCI engine data for 210 different operating conditions are used for training and testing. Engine data at 300 different steady state operating conditions are used for RCCI mode. 72% of the data is used for training the model and the remaining 28% is used for testing. The data is standardized before training because of different scales of the predictor variables. The classification models for each mode are tuned by optimizing the kernel scale and box constraint (C). The choice of C is a trade-off between variance and bias. The models are trained using 10 fold cross validation approach to prevent over-fitting. Sequential minimal optimization is used as a solver for the optimization problem. The radial basis (Gaussian) kernel function (11) showed better results for HCCI mode as compared to linear and cubic polynomial kernel functions. The trained classification model of HCCI consists of three classifiers, one for each class using one-vs-all classification technique. Due to imbalanced data set, the classes with small data size are oversampled. The confusion matrix for the test data set of HCCI is shown in Fig. 3a. Class-I and II show one misclassified data point for each class while class-III shows 100% accuracy. Overall, the developed model shows 96.5% accuracy in predicting engine CSI (combustion stability index).

For RCCI mode, the cubic kernel function (10) showed better results as compared to linear and Gaussian kernel functions. The trained model is validated for 80 different test conditions, as shown in Fig. 3b. The test data for class-I and III show 100% prediction accuracy. However class-II shows two misclassifications with prediction accuracy of 92.8%.

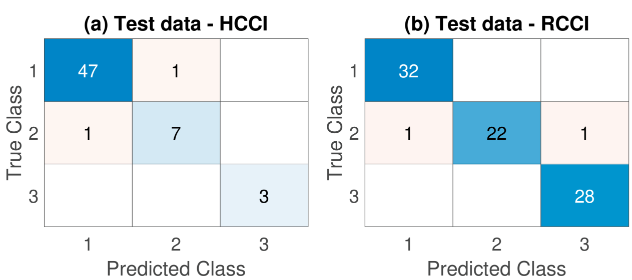


Fig. 3. Confusion matrix for $COV_{IMEP}$ classification for HCCI and RCCI modes

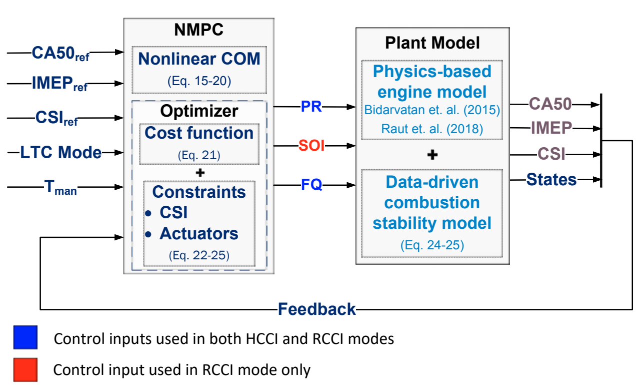


Fig. 4. Schematic of nonlinear MPC and plant model

## 4. NON-LINEAR MODEL PREDICTIVE CONTROL DEVELOPMENT

To control cycle-by-cycle $CA50$ and $IMEP$, experimentally validated control oriented models (COMs) are used from our prior works Raut et al. (2018); Bidarvatan et al. (2015). The objective of this work is to control $CA50$ and $IMEP$ while ensuring combustion stability by limiting the $COV_{IMEP}$ below 3%. For both HCCI and RCCI modes, COMs are augmented with data-driven models to constrain $COV_{IMEP}$. Crank angle for 50% fuel mass fraction burned ($CA50$), temperature at start of combustion ($T_{soc}$), pressure at start of combustion ($P_{soc}$) and indicated mean effective pressure ($IMEP$) are chosen as the states of the MIMO nonlinear HCCI and RCCI COMs. The outputs for both HCCI and RCCI modes are $CA50$, $IMEP$ and $COV_{IMEP}$. The control structure is shown in Fig. 4.

The nonlinear COM of HCCI and RCCI modes can be represented as follows:

$$x(k+1) = f(x(k), u(k)) \tag{12}$$

$$y(k) = f(x(k)) \tag{13}$$

where:

$$x = [CA50 \quad T_{soc} \quad P_{soc} \quad IMEP]^T \tag{14}$$

$$u_{HCCI} = [PR \quad FQ]^T \tag{15}$$

$$u_{RCCI} = [SOI \quad FQ \quad PR]^T \tag{16}$$

$$y = [CA50 \quad IMEP \quad COV_{IMEP}]^T \tag{17}$$

The developed COMs are used in a nonlinear model predictive control (NMPC) platform to control the LTC engine in HCCI and RCCI modes. The NMPC uses real-time iterative optimization of the plant model over a finite number of time steps and yields an optimized control strategy for a provided reference input. The first element of the optimal control sequence is provided as a feedback control for the next sampling interval. The prediction and control horizons for both HCCI and RCCI modes are chosen to be 5 and 3 engine cycles, respectively. The cost function is designed by penalizing the control efforts and least square error of reference tracking as shown in (18)

$$J = \sum_{k=1}^{N} \frac{1}{2}(Y_k - Rs)^T Q (Y_k - Rs) + U_k^T R U_k \tag{18}$$

subject to the constraints on $COV_{IMEP}$ and actuators limits

$$h(x(k), u(k)) : CSI - 1 = 0 \tag{19}$$

$$A_{cons} U \leq B_{cons} \tag{20}$$

where Q and R are the tuning weights. Combustion stability index (CSI) is used to specify the classes that is based on $COV_{IMEP}$. Constraint h(x(k),u(k)) is a nonlinear equality constraint on $COV_{IMEP}$ which is given by (21) and (22) for HCCI and RCCI modes, respectively.

$$CSI_{HCCI} = h_{HCCI}(x(k), u_{HCCI}(k)) \tag{21}$$

$$CSI_{RCCI} = h_{RCCI}(x(k), u_{RCCI}(k)) \tag{22}$$

$Rs$ is the reference signal for $CA50$ and $IMEP$ available for next 5 time steps.

$$Rs = [CA50_{ref} \quad IMEP_{ref}]^T \tag{23}$$

This nonlinear programming problem (NLP) is solved in Matlab by using "fmincon" command and specifying sequential quadratic programming (SQP) algorithm. SQP solves this minimization problem with an active nonlinear constraint. SQP is an iterative approach to search for a local optimal solution Meadows (1997). The QP subproblem is formulated to account for the local properties of the NLP for each iteration step using quasi-Newton method. The subproblem is of the following form:

$$min \; \nabla J^T_{(x_k, u_0)} \Delta U + \frac{1}{2} \Delta U^T H \Delta U \tag{24}$$

over $\Delta U \in R^n$
subject to

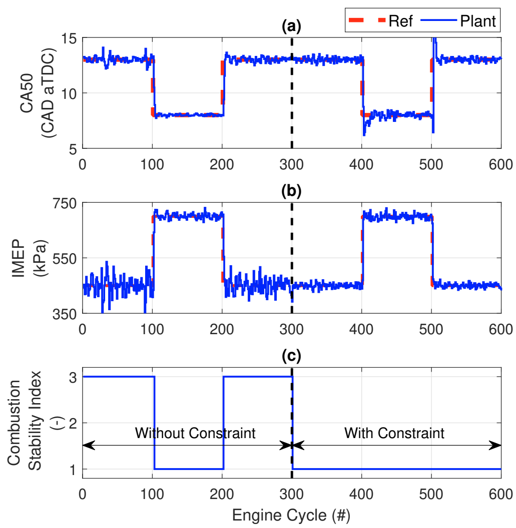


Fig. 5. Controller response for reference tracking of CA50 and IMEP with and without constraint on $COV_{IMEP}$ for RCCI mode.

$$h(x_k, u_0) + \nabla h(x_k, u_0)^T \Delta U = 0 \tag{25}$$

$$g(x_k, u_0) + \nabla g(x_k, u_0)^T \Delta U \leq 0 \tag{26}$$

The solution of this subproblem is used to form a new iteration.

$$U(j+1) = U(j) + \alpha \Delta U(k) \tag{27}$$

where, $\alpha$ is a scale factor that determines the length of the search step in direction of $\Delta U$. $\alpha$ is determined by line search approach subject to the decrease in original NLP merit function. $H$ is the positive definite Hessian matrix, computed by taking the second derivative of J(x(k),u(k)) w.r.t $U$.

## 5. RESULTS AND DISCUSSIONS

### 5.1 Results for RCCI Mode

In RCCI, there are three control knobs available to control combustion. $CA50$ can be controlled by adjusting either SOI or PR. Fuel quantity is used to control $IMEP$. Constraint on $COV_{IMEP}$ is added along with the actuator constraints. Figures 5 and 6 show the controller performance with and without constraint on $COV_{IMEP}$. During first 300 cycles, the controller is capable of tracking $CA50$ and $IMEP$ but the optimal solution lies in the region of unstable combustion as the constraint on $COV_{IMEP}$ is inactive. However, when the constraint on $COV_{IMEP}$ is activated, the controller adjusts the premixed ratio while tracking $CA50$ and $IMEP$ to provide an optimal solution in the region of stable combustion, as shown in Fig. 5 during 300-600 cycles. Moreover, the controller is capable of reference tracking $CA50$ and $IMEP$ with settling time of one engine cycle.

### 5.2 Results for HCCI Mode

Nonlinear MPC controller is implemented in Matlab for the reference tracking of $CA50$ and $IMEP$ while constraining $COV_{IMEP}$ for HCCI mode. The outputs and states are computed using engine plant model and provided as feedback to the controller. In HCCI mode,

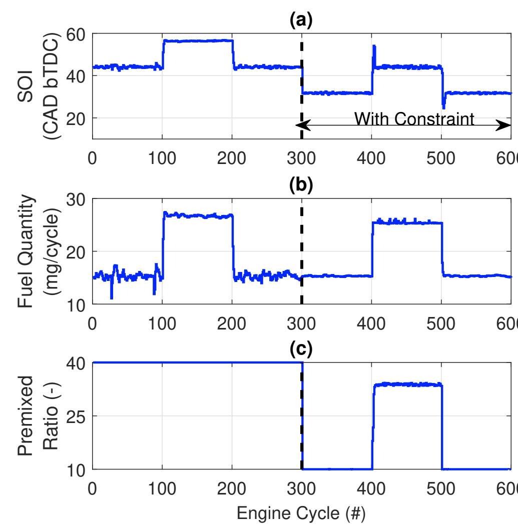


Fig. 6. Control inputs of the NMPC for the results in Fig. 5

premixed ratio, fuel quantity and manifold temperature are the available control actuators. Manifold temperature shows very slow response; thus, it cannot be used to control the parameters on cycle-to-cycle basis. PR is used as manipulated variable to control $CA50$, and $IMEP$ is controlled by adjusting the fuel quantity. Figures 7 and 8 show the controller response for tracking $CA50$ and $IMEP$ with and without constraint on $COV_{IMEP}$. For the first 100 cycles, when the $IMEP$ is 250 kPa and $CA50$ is 13 CAD (aTDC), the model predicts unstable combustion as the combustion stability index is 3. This means that the $COV_{IMEP}$ for these 100 cycles is greater than 5%. For the next 100 cycles, the combustion is stable as CSI is 1 ($COV_{IMEP}$ <3%). However, the controller shows good performance for reference tracking of $CA50$ and $IMEP$. It takes one engine cycle to reach the targeted $IMEP$ while $CA50$ takes four engine cycles to reach the reference value. For cycles 301-600, the constraint on $COV_{IMEP}$ is activated. The controller tracks $IMEP$ well but it adjusts $CA50$ such that the combustion stability index remains 1. As shown in Fig. 7, when the constraint on $COV_{IMEP}$ becomes active, premixed ratio regulates $CA50$ to avoid constraint violation. Hence, $COV_{IMEP}$ stays below 3% ensuring stable combustion.

## 6. CONCLUSIONS

In this work, a data-driven predictive model of combustion stability classification is developed for the low temperature combustion (LTC) modes in a 2-liter 4-cylinder engine. The engine data at 510 operating conditions were used to develop and assess the model. Support vector machine (SVM) is used to classify $COV_{IMEP}$ and create a predictive model of the engine combustion stability. $CA50$, $IMEP$, premixed ratio, fuel quantity, intake manifold temperature and start of injection (for RCCI only) are used as predictors to classify $COV_{IMEP}$ into three different classes including class-I consisting of $COV_{IMEP} \leq$ 3%, class-II with $3 < COV_{IMEP} \leq 5\%$ and class-III with $COV_{IMEP} > 5\%$. Experimental data with $COV_{IMEP} >$ 5% mostly showed either partial burn or misfire. The de-

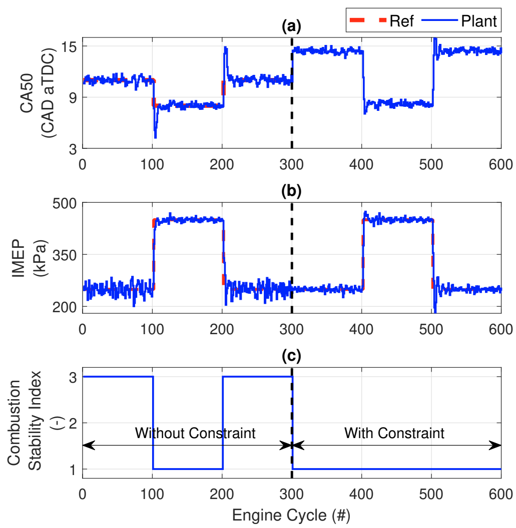


Fig. 7. Simulation results for reference tracking of CA50 and IMEP with and without constraint on $COV_{IMEP}$ for HCCI mode.

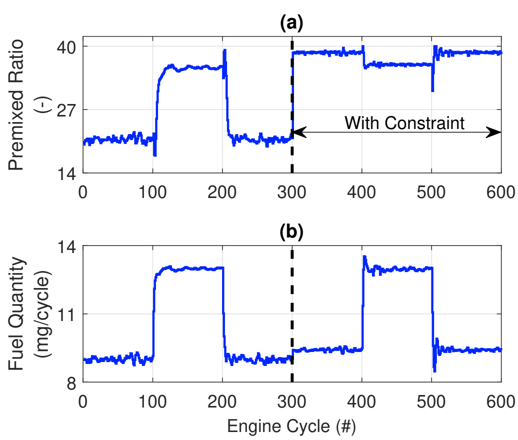


Fig. 8. Control inputs of the NMPC for the results in Fig. 7

veloped combustion stability models for both LTC modes show more than 97% prediction accuracy.

Using the combustion stability models, closed-loop nonlinear model predictive controllers are designed for HCCI and RCCI combustion modes. The optimal control action is determined using SQP algorithm. The nonlinear MPC controller for HCCI is capable of tracking $CA50$ and $IMEP$ in the absence of constraint on $COV_{IMEP}$ and regulates $CA50$ whenever needed for keeping the combustion stable. For the RCCI mode, SOI and fuel quantity are used to control $CA50$ and $IMEP$. In the presence of active constraint on $COV_{IMEP}$, the designed controller adjusts premixed ratio to ensure stable combustion during engine load changes. These allow the multi-mode LTC engine to run stably in both HCCI and RCCI modes.

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