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Dynamics for a hybrid non-autonomous prey–predator system with generalist predator and impulsive conditions on time scales

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In this paper, we investigate the dynamical behavior for a hybrid non-autonomous predator–prey system with Holling Type II functional response, impulsive effects and generalist predator on time scales, where our proposed model commutes between a continuous-time dynamical system and discrete-time dynamical system. By using comparison theorems, we first study the permanence results of the proposed model. Also, we established the uniformly asymptotic stability for the almost periodic solution of the proposed model. Finally, in the last section, we provide some examples with numerical simulation.

Keywords: Hybrid non-autonomous prey–predator system; Lyapunov functional; impulsive effect; time scales.

Mathematical Subject Classification 2020: 37B55, 93A30, 37B25, 34A37, 34N05

1. Introduction

In ecological system [1], dynamics of prey and predator models have consistently been one of the focal subjects in the modern days. Since the first prey and predator model was presented by Lotka and Volterra [2, 3], after introducing this, the researchers started working on it and developed different mathematical models of prey–predator interactions to get valuable biological insights, e.g. [4–8] and reference therein.

The dynamics of many evolutionary processes are characterized by the fact that at a specific moment, they experience a sudden change in their state, such as

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harvesting, natural disasters and shocks. These processes are subject to short-term perturbations, whose time period is minimal in analogy with the whole evolution. In dynamical systems associated with such sudden changes, we assume these changes in the form of impulses. Therefore, impulsive differential equations have been developed in modeling impulsive problems in industrial robotics, population dynamics, optimal control, biological systems, physics, biotechnology, ecology, pharmacokinetics, etc. [9]. The concept of impulsive differential equations provided beneficial results in the field of science and engineering [10]. There is some abrupt perturbation in the ecological system, such as drought, fire, hunting, harvesting, flood, and breeding, which are not suitable to be considered continually. For example, several paths beat the pest outbreak, such as chemical control and biological control. Some pesticides and other beneficial species are used to control the crops and other things from pests. These procedures consistently lead to a quick diminishing or increment of the number of species on fixed occasions. Therefore, to model these kinds of rapid changes in the ecological system, we use impulsive differential equations instead of initial valued simple differential equations. We refer [11–13] and references therein for more information about the impulsive population system.

It is notable that environmental and biological parameters are naturally subject to fluctuation in time, so almost periodically varying parameters are considered as significant. Also, if the different constituent parts of the temporally non-uniform environment are non-integral multiples periods, then one needs to consider the environment to be almost periodic. Likewise, as of now, few outcomes are available for the existence of positive almost periodic solutions to population dynamical system with impulses [14, 15]. In [15], authors established the local existence of positive periodic solutions of the predator–prey system with time delays on time scales. Since permanence is one of the most important topic in the study of the population dynamics. Therefore, in this paper, we will study the permanence results of prey–predator system on time scales with impulsive effects. In the past few years, permanence of different kind of continuous or discrete ecosystem has been studied widely; we refer the readers to [16–21] and the reference therein. Also, in [22], authors considered the following prey–predator model:

$$
\begin{aligned}
\frac{dy_1}{d\delta} &= y_1(\delta)\left[r_1(\delta)\left(1-\frac{y_1(\delta)}{K_1(\delta)}\right)-\frac{a(\delta)y_2(\delta)}{1+h(\delta)a(\delta)y_1(\delta)}\right],\\
\frac{dy_2}{d\delta} &= y_2(\delta)\left[r_2(\delta)\left(1-\frac{y_2(\delta)}{K_2(\delta)}\right)-d(\delta)+\frac{e(\delta)a(\delta)y_1(\delta)}{1+h(\delta)a(\delta)y_1(\delta)}\right],
\end{aligned}
$$

where y_1 and y_2 denotes the densities of prey species and predator species at time δ, respectively. In this model, predator species y_2 is generalist. In nature, numerous predators are generalist and their preys comprise of various species [23–26]. The investigation on predator–prey models including generalist predators has pulled in wide consideration and gave extra biological insights, for more about generalist predator, one can see [27]. In [28], authors studied the existence and global asymptotic stability of positive periodic solutions of the periodic single-species model with periodic impulsive perturbations and in [29], authors investigated the permanence of the impulsive system with seasonal effects. Also, in [30], author considered the following impulsive prey and predator model:

$$\begin{aligned}
z_1'(\delta) &= z_1(\delta)[r_1(\delta) - b_1(\delta)z_1(\delta)] - \frac{c_1(\delta)z_1(\delta)z_2^m(\delta)}{f(\delta)+n(\delta)z_1(\delta)+m(\delta)z_2(\delta)}, \\
&\qquad\qquad\qquad\qquad\qquad\qquad \delta \neq \delta_k, \quad k \in \mathbb{N}, \\
z_2'(\delta) &= z_2(\delta)[-r_2(\delta) - b_2(\delta)z_2(\delta)] + \frac{c_2(\delta)z_1(\delta)z_2^m(\delta)}{f(\delta)+n(\delta)z_1(\delta)+m(\delta)z_2(\delta)}, \quad \delta \neq \delta_k, \\
z_1(\delta_k^+) &- z_1(\delta_k) = \theta_{1k}z_1(\delta_k), \quad \delta = \delta_k, \\
z_2(\delta_k^+) &- z_2(\delta_k) = \theta_{2k}z_2(\delta_k), \quad \delta = \delta_k,
\end{aligned}$$

where z_1 and z_2 denote the prey and predator densities at time δ, respectively, and all other coefficients are continuous almost periodic functions, θ_{1k} and θ_{1k} are positive constants and $0 = \delta_0 < \delta_1 < \delta_2 < \cdots < \delta_k < \delta_{k+1} < \cdots$ are fixed impulsive points with $\lim_{k\to+\infty} \delta_k = +\infty$, for more one can see [30].

In general, one investigates the continuous system and discrete dynamical systems separately and the majority of the outcomes are demonstrated independently by using the discrete analysis or continuous analysis. In numerous realistic models, we frequently need to think about continuous and discrete processes at the same time or on some different time scales. For example, to demonstrate the development cycle of certain species, such as *Pharaoh cicada*, *Magicicada septendecim* and *Magicicada cassini* [31], we need a particular time scale of type $\mathbb{T} = \cup_{r=1}^{\infty}[r(\delta_1+\delta_2), r(\delta_1+\delta_2)+\delta_2]$ with $\delta_1, \delta_2 \in \mathbb{R}^+$, since it depends on both continuous and discrete times. By choosing either a difference equation or a differential equation, we cannot precisely describe the dynamic behavior of such kinds of models. Subsequently, we need an equation that works simultaneously for continuous and discrete analysis. As a consequence, Hilger [32], presented the idea of time scales theory in 1988, in his Ph.D. thesis. It leads to a new understanding and analysis of dynamical system on any non-uniform time domains that are closed subset of $\mathbb{R}$. As expected, once a results has been established for dynamic equations on an arbitrary time scale, this results holds for standard continuous differential equations (i.e. $\mathbb{R}$) and standard continuous difference equations (i.e. $h\mathbb{Z}, h$ is a real number). Besides these two time scales, there are many interesting time scales with non-uniform step size. Extension and unification are two main features of the time scale calculus [33]. Also, the motivation to use hybrid system is that populations may or may not interact and grow continuously on specific time domain.

Over the most recent couple of years, numerous authors have researched and discovered numerous applications on dynamic system on time scales such as population dynamics, control theory, and thermal physics [34–39]. Particularly, in [40], authors consider the Leslie–Gower system on time scales and studied the permanence results by using some dynamical inequality on time scales and comparison theorems. In [41], authors studied the permanence, existence and uniform asymptotic stability of unique positive almost periodic solution of a multi-species Lotka–Volterra-type competitive system with delays and feedback controls on time scales. In [42], authors concerned with a generalized almost periodic predator–prey model with impulsive effects and time delays and obtained sufficient conditions to guarantee the permanence and global asymptotic stability of the system. Also, in [43], authors discussed the dynamics of a prey–predator model with impulsive state feedback control. As per our knowledge there is not a single published paper on non-autonomous prey–predator model with a Holling Type II functional response and impulsive effect on time scale. Therefore, motivated by this reason, the above discussion and by paper [44], in this work, we shall explore the dynamics for the following hybrid non-autonomous prey–predator model with a Holling Type II functional response with impulsive conditions on time scales:

$$\gamma_1^{\Delta}(\delta) = s_1(\delta) - \frac{s_1(\delta)}{k_1(\delta)}e^{\gamma_1(\delta)} - \frac{\theta(\delta)e^{\gamma_2(\delta)}}{1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}}, \quad \delta \neq \delta_k,\ \ \delta \in [\delta_0,\infty)_{\mathbb{T}},\ \ k \in \mathbb{N}, \tag{1.1}$$

$$\gamma_2^{\Delta}(\delta) = s_2(\delta) - \frac{s_2(\delta)}{k_2(\delta)}e^{\gamma_2(\delta)} - d(\delta) + \frac{e(\delta)\theta(\delta)e^{\gamma_1(\delta)}}{1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}}, \quad \delta \neq \delta_k,\ \ \delta \in [\delta_0,\infty)_{\mathbb{T}}, \tag{1.2}$$

$$\gamma_1(\delta_k^+) = \gamma_1(\delta_k) + \ln(1+\eta_{k_1}), \quad \delta = \delta_k, \tag{1.3}$$

$$\gamma_2(\delta_k^+) = \gamma_2(\delta_k) + \ln(1+\eta_{k_2}), \quad \delta = \delta_k, \tag{1.4}$$

where γ_1 and γ_2 denote prey and predator densities at time δ, respectively. Furthermore, the coefficients $k_1, k_2, s_1, s_2, \theta, d, h, e$ are almost periodic functions for $\delta \geq 0$ and the parameters have the following meanings:

(1) k_1 is the carrying capacity of species γ_1.
(2) s_1 is the intrinsic growth rate of species γ_1.
(3) θ is the capturing efficiency of a predator.
(4) k_2 is the carrying capacity of species γ_2.
(5) d denotes the death rate of predator due to attacking or hunting.
(6) s_2 is the intrinsic growth rate of species γ_2.
(7) h is the predator handling time.

Also, $\gamma_1(0) > 0$ and $\gamma_2(0) > 0$, δ_k is an impulsive points for every k and $0 \leq \delta_0 < \delta_1 < \delta_2 < \cdots < \delta_k < \cdots$. $\gamma_i(\delta_k^+) = \lim_{a\to 0^+}\gamma_i(\delta_k + a), \gamma_i(\delta_k^-) = \lim_{a\to 0^+}\gamma_i(\delta_k - a)$ denote the right and left limit of $\gamma_i(\delta)$ at $\delta = \delta_k, i = 1, 2$. We assume that the $\{\eta_{k_i}\},\ i = 1, 2$ are the real sequence and there exists an integer $p > 0$ such that $\eta_{k_i+p} = \eta_{k_i},\ \delta_{k+p} = \delta_k$ also, $\eta_{k_1} > -1$. $\gamma_1(\delta_k^+),\ \gamma_1(\delta_k^-)$ represent the right and left limit of $\gamma_1(\delta)$ in the sense of time scales, respectively, and $\gamma_1(\delta_k^-) = \gamma_1(\delta_k),\ \gamma_2(\delta_k^-) = \gamma_2(\delta_k)$.

Clearly, the model (1.1)–(1.4) is non-autonomous since all biological or environmental parameters have been assumed to be dependent in time. Also, the autonomous version of this model is given as follows:

$$\gamma_1^{\Delta}(\delta) = s_1 - \frac{s_1}{k_1} e^{\gamma_1(\delta)} - \frac{\theta e^{\gamma_2(\delta)}}{1 + h\theta e^{\gamma_1(\delta)}}, \quad \delta \neq \delta_k, \ \ \delta \in [\delta_0, \infty)_{\mathbb{T}}, \ \ k \in \mathbb{N}, \tag{1.5}$$

$$\gamma_2^{\Delta}(\delta) = s_2 - \frac{s_2}{k_2} e^{\gamma_2(\delta)} - d + \frac{e\theta e^{\gamma_1(\delta)}}{1 + h\theta e^{\gamma_1(\delta)}}, \quad \delta \neq \delta_k, \quad \delta \in [\delta_0, \infty)_{\mathbb{T}}, \tag{1.6}$$

$$\gamma_1(\delta_k^+) = \gamma_1(\delta_k) + \ln(1 + \eta_{k_1}), \quad \delta = \delta_k, \tag{1.7}$$

$$\gamma_2(\delta_k^+) = \gamma_2(\delta_k) + \ln(1 + \eta_{k_2}), \quad \delta = \delta_k, \tag{1.8}$$

where all the biological or environmental parameters have been assumed to be constants in time. In case of real life modeling, this is rarely to consider autonomous system, because most natural environments are physically highly variable and accordingly, birth rates, death rates and other vital rates of populations vary greatly in time. Once the environment fluctuation is taken into account, a model must be non-autonomous. Therefore, in this work we are interested to investigate the hybrid non-autonomous prey–predator system on time scales.

Remark 1. Let $y_1(\delta) = e^{\gamma_1(\delta)}$ and $y_2(\delta) = e^{\gamma_2(\delta)}$, $\mathbb{T} = \mathbb{R}$, then considered system (1.1)–(1.4) becomes

$$\frac{dy_1}{d\delta} = y_1(\delta)\left[s_1(\delta)\left(1 - \frac{y_1(\delta)}{k_1(\delta)}\right) - \frac{\theta(\delta)y_2(\delta)}{1 + h(\delta)\theta(\delta)y_1(\delta)}\right], \quad \delta \neq \delta_k, \quad \delta \in [\delta_0, \infty)_{\mathbb{T}},$$

$$\frac{dy_2}{d\delta} = y_2(\delta)\left[s_2(\delta)\left(1 - \frac{y_2(\delta)}{k_2(\delta)}\right) - d(\delta) + \frac{e(\delta)\theta(\delta)y_1(\delta)}{1 + h(\delta)\theta(\delta)y_1(\delta)}\right],$$
$$\delta \neq \delta_k, \quad \delta \in [\delta_0, \infty)_{\mathbb{T}},$$

$$y_1(\delta_k^+) = y_1(\delta_k)(1 + \eta_{k_1}), \quad \delta = \delta_k,$$

$$y_2(\delta_k^+) = y_2(\delta_k)(1 + \eta_{k_2}), \quad \delta = \delta_k,$$

and if $\mathbb{T} = \mathbb{Z}$, then the system (1.1)–(1.4) becomes

$$y_1(n+1) = y_1(n)\exp\left\{s_1(\delta)\left(1 - \frac{y_1(\delta)}{k_1(\delta)}\right) - \frac{\theta(\delta)y_2(\delta)}{1 + h(\delta)\theta(\delta)y_1(\delta)}\right\},$$
$$n \neq n_k, \quad n \in [\delta_0, \infty)_{\mathbb{Z}},$$

$$y_2(n+1) = y_2(n)\exp\left\{s_2(\delta)\left(1 - \frac{y_2(\delta)}{k_2(\delta)}\right) - d(\delta) + \frac{e(\delta)\theta(\delta)y_1(\delta)}{1 + h(\delta)\theta(\delta)\ y_1(\delta)}\right\},$$
$$n \neq n_k, \quad n \in [\delta_0, \infty)_{\mathbb{Z}},$$

$$y_1(n_k^+) = y_1(n_k)(1 + \eta_{k_1}), \quad n = n_k,$$

$$y_2(n_k^+) = y_2(n_k)(1 + \eta_{k_2}), \quad n = n_k.$$

Thus, from Remark 1, we can conclude that our model will serve for both the discrete as well as continuous cases. This paper is organized as follows. In Sec. 2, we introduce some notations, definitions and state some preliminary results. In Sec. 3, we establish the permanence of the proposed model (1.1)–(1.4). In Sec. 4, we establish the asymptotic stability of the unique almost periodic solution of the proposed model (1.1)–(1.4). Section 5 gives some simulated examples to show the feasibility of obtained analytical outcomes in different–different time domains.

2. Preliminaries

We briefly describe some fundamental definitions, important lemmas and useful assumptions that we will use to prove the main results.

Definition 2.1 ([33]). A time scale (which is special case of a measure chain [33]) is an arbitrary nonempty closed subset of the real number. For example: $\mathbb{R}$, $\mathbb{Z}$, $[0,1]\cup\mathbb{N}$ and Cantor set.

Definition 2.2 ([33]). A time scales interval is defined by $[\delta_1,\delta_2]_{\mathbb{T}} = \{\delta \in \mathbb{T} : \delta_1 \leq \delta \leq \delta_2\}$.

Definition 2.3 ([33]). The forward jump operator (σ) and backward jump operator (ρ) is defined as follows:

$$\sigma : \mathbb{T} \to \mathbb{T}, \quad \text{with } \sigma(\delta) = \inf\{\theta \in \mathbb{T} : \theta > \delta\} \quad \text{and} \quad \inf\emptyset = \sup\mathbb{T},$$
$$\rho : \mathbb{T} \to \mathbb{T}, \quad \text{with } \rho(\delta) = \sup\{\theta \in \mathbb{T} : \theta < \delta\} \quad \text{and} \quad \sup\emptyset = \inf\mathbb{T}.$$

Definition 2.4 ([33]). The graininess function $\mu : \mathbb{T} \to [0,\infty)$ is defined by $\mu(\delta) = \sigma(\delta) - \delta, \forall\, \delta \in \mathbb{T}$.

Definition 2.5 ([33]). The set $\mathbb{T}^k$ is derived from time scale $\mathbb{T}$ which is defined as follows:

If $\mathbb{T}$ has a left-scattered maximum δ_1, then $\mathbb{T}^k = \mathbb{T} - \{\delta_1\}$, otherwise $\mathbb{T}^k = \mathbb{T}$.

Definition 2.6 ([33]). Let $\psi : \mathbb{T} \to \mathbb{R}$ is a function and let $\delta \in \mathbb{T}^k$. Then, the Δ-derivative of ψ at the point δ is defined by for each $\epsilon > 0$, $\exists$ neighborhood $\mathcal{U}$ of δ such that

$$\big|[\psi(\sigma(\delta)) - \psi(\theta)] - \psi^{\Delta}(\delta)[\sigma(\delta) - \theta]\big| \leq \epsilon|\sigma(\delta) - \theta|, \quad \forall\, \theta \in \mathcal{U}.$$

If $\mathbb{T} = \mathbb{R}$, then $\psi^{\Delta}(\delta) = \psi'(\delta)$.
If $\mathbb{T} = \mathbb{Z}$, then $\psi^{\Delta}(\delta) = \Delta\psi(\delta) = \psi(\delta+1) - \psi(\delta)$.

Definition 2.7 ([33]). A function $q : \mathbb{T} \to \mathbb{R}$ is said to be regressive if $1 + \mu(\delta)q(\delta) \neq 0$, $\forall\, \delta \in \mathbb{T}$. The set of all regressive functions is denoted by $\mathcal{R}$. Moreover,

q is said to be positive regressive if $1+\mu(\delta)q(\delta)>0,\ \forall\ \delta\in\mathbb{T}$ and the set of all positive regressive functions is denoted by $\mathcal{R}^+$.

Definition 2.8 ([33]). If $q\in\mathcal{R}$, then we define the exponential function by

$$e_q(\delta,r)=\exp\left(\int_r^t \zeta_{\mu(\varphi)}(q(\varphi))\Delta\varphi\right),\quad \text{for } \delta,r\in\mathbb{T},$$

where

$$\zeta_{\mu(r)}(q(r))=\begin{cases}\dfrac{1}{\mu(r)}\mathrm{Log}(1+q(r)\mu(r)), & \text{if } \mu(r)\neq 0,\\[2ex] q(r), & \text{if } \mu(r)=0.\end{cases}$$

Lemma 2.1 ([33]). *If $\delta_0,\delta_1,a\in\mathbb{T}$ and $q\in\mathcal{R}$, then*

$$\int_{\delta_0}^{\delta_1} q(\varphi)e_q(a,\sigma(\varphi))\Delta\varphi=e_q(a,\delta_0)-e_q(a,\delta_1).$$

Lemma 2.2 ([30]). *Let $\psi:\mathbb{T}\to\mathbb{R}$ be a continuously increasing function and $\psi(\delta)>0$ also $\psi^\Delta(\delta)\geq 0\ \forall\ \delta\in\mathbb{T}$. Then*

$$\frac{\psi^\Delta(\delta)}{\psi^\sigma(\delta)}\leq[\ln(\psi(\delta))]^\Delta\leq\frac{\psi^\Delta(\delta)}{\psi(\delta)}.$$

If $\psi(\delta)>0$ and $\psi^\Delta(\delta)<0$ for $\delta\in\mathbb{T}$, then

$$\frac{\psi^\Delta(\delta)}{\psi(\delta)}\leq[\ln(\psi(\delta))]^\Delta\leq\frac{\psi^\Delta(\delta)}{\psi^\sigma(\delta)}.$$

Lemma 2.3 ([33]). *Assume that $q\in\mathcal{R}$ and $\delta_0\in\mathbb{T}$, if $q\in\mathcal{R}^+$ on $\mathbb{T}^k$, then $e_q(\delta,\delta_0)>0$ for all $\delta\in\mathbb{T}$.*

Lemma 2.4 ([30]). *Let us suppose $\gamma\in PC^1[\mathbb{T},\mathbb{R}]$, where PC^1 denotes the space of all rd-continuous functions, for more, one can see [30] and*

$$\gamma^\Delta(\delta)\leq(\geq)p(\delta)\gamma(\delta)+q(\delta),\quad \delta\neq\delta_k,\ \ \delta\in[\delta_0,\infty)_{\mathbb{T}},$$
$$\gamma(\delta_k^+)\leq(\geq)d_k\gamma(\delta_k)+b_k,\quad \delta=\delta_k,\ \ k\in\mathbb{N}.$$

Then, for $\delta\geq\delta_0\geq 0$

$$\gamma(\delta)\leq(\geq)\gamma(\delta_0)\prod_{\delta_0<\delta_k<\delta}d_k e_p(\delta,\delta_0)+\sum_{\delta_0<\delta_k<\delta}\left(\prod_{\delta_0<\delta_k<\delta}d_k e_p(\delta,\delta_k)\right)b_k$$
$$+\int_{\delta_0}^{\delta}\prod_{s<\delta_k<\delta}d_k e_p(\delta,\sigma(s))q(s)\Delta s.$$

Lemma 2.5 ([30]). *Suppose that* $\gamma \in PC^1[\mathbb{T},\mathbb{R}]$, $-g \in \mathcal{R}^+$, $\nu \leq \prod_{\delta_0<\delta_k<\delta} d_k \leq \varphi$ *for* $\delta \geq \delta_0, \mathcal{R}^+$ *denotes the set of all positive regressive functions* [33].

(i) *If*

$$\gamma^{\Delta}(\delta) \leq m - g\gamma(\delta), \quad \delta \neq \delta_k, \;\; \delta \in [\delta_0,\infty)_{\mathbb{T}},$$
$$\gamma(\delta_k^+) \leq d_k\gamma(\delta_k) + b_k, \quad \delta = \delta_k, \;\; k \in \mathbb{N},$$

then for $\delta \geq \delta_0$

$$\gamma(\delta) \leq \gamma(\delta_0)\varphi e_{(-g)}(\delta,\delta_0) + \sum_{\delta_0<\delta_k<\delta} \varphi e_{(-g)}(\delta,\delta_k)b_k + \frac{m\varphi}{g}[1 - e_{(-g)}(\delta,\delta_0)].$$

Moreover, if $g > 0$, $m > 0$, *we have* $\limsup_{\delta\to\infty}\gamma(\delta) \leq \frac{m\varphi}{g}$.

(ii) *If*

$$\gamma^{\Delta}(\delta) \geq m - g\gamma(\delta), \quad \delta \neq \delta_k, \;\; \delta \in [\delta_0,\infty)_{\mathbb{T}},$$
$$\gamma(\delta_k^+) \geq d_k\gamma(\delta_k) + b_k, \quad \delta = \delta_k, \;\; k \in \mathbb{N},$$

then for $\delta \geq \delta_0$

$$\gamma(\delta) \geq \gamma(\delta_0)\nu e_{(-g)}(\delta,\delta_0) + \sum_{\delta_0<\delta_k<\delta} \nu e_{(-g)}(\delta,\delta_k)b_k + \frac{m\nu}{g}[1 - e_{(-g)}(\delta,\delta_0)].$$

Moreover, if $g > 0$, $m > 0$, *then* $\liminf_{\delta\to\infty}\gamma(\delta) \geq \frac{m\nu}{g}$.

Lemma 2.6 ([30]). *Assume* $-m \in \mathcal{R}^+$, $\gamma \in PC^1[\mathbb{T},\mathbb{R}]$ *and* $\gamma(\delta) > 0$ *for* $\delta \in \mathbb{T}$ *and* $\nu \leq \prod_{\delta_0<\delta_k<\delta} d_k \leq \varphi$ *for* $\delta \geq \delta_0$.

(i) *If*

$$\gamma^{\Delta}(\delta) \leq \gamma^{\sigma}(\delta)(m - g\gamma(\delta)), \quad \delta \neq \delta_k, \;\; \delta \in [\delta_0,\infty)_{\mathbb{T}},$$
$$\gamma(\delta_k^+) \leq d_k\gamma(\delta_k), \quad \delta = \delta_k, \;\; k \in \mathbb{N},$$

then for $\delta \geq \delta_0$

$$\gamma(\delta) \leq \frac{m\varphi}{g}\left[1 + \left(\frac{m}{g\gamma(\delta_0)} - 1\right)e_{(-m)}(\delta,\delta_0)\right]^{-1}.$$

Moreover, if $g > 0$, $m > 0$, *we have* $\limsup_{\delta\to\infty}\gamma(\delta) \leq \frac{m\varphi}{g}$.

(ii) *If*

$$\gamma^{\Delta}(\delta) \geq \gamma^{\sigma}(\delta)(m - g\gamma(\delta)), \quad \delta \neq \delta_k, \;\; \delta \in [\delta_0,\infty)_{\mathbb{T}},$$
$$\gamma(\delta_k^+) \geq d_k\gamma(\delta_k), \quad \delta = \delta_k, \;\; k \in \mathbb{N},$$

then for $\delta \geq \delta_0$

$$\gamma(\delta) \geq \frac{m\nu}{g}\left[1 + \left(\frac{m}{g\gamma(\delta_0)} - 1\right)e_{(-m)}(\delta, \delta_0)\right]^{-1}.$$

Moreover, if $g > 0,\ m > 0,$ *then* $\liminf_{\delta\to\infty} \gamma(\delta) \geq \frac{m\nu}{g}$.

Lemma 2.7 ([30]). *Assume* $-m \in \mathcal{R}^+,\ \gamma \in PC^1[\mathbb{T}, \mathbb{R}], \gamma(\delta) > 0$ *for* $\delta \in \mathbb{T}$ *and* $\nu \leq \prod_{\delta_0<\delta_k<\delta} d_k \leq \varphi$ *for* $\delta \geq \delta_0,\ \bar{\mu} = \sup_{\delta\in\mathbb{T}} \mu(\delta)$. *If*

$$\gamma^{\Delta}(\delta) \geq \gamma(\delta)(m - g\gamma(\delta)), \quad \delta \neq \delta_k,\ \ \delta \in [\delta_0, \infty)_{\mathbb{T}},$$

$$\gamma(\delta_k^+) \geq d_k\gamma(\delta_k), \quad \delta = \delta_k,\ \ k \in \mathbb{N},$$

then for $\delta \geq \delta_0$

$$\gamma(\delta) \geq \frac{m\nu}{g}\left[1 + \left(\frac{m}{g\gamma(\delta_0)} - 1\right)e_{(-\frac{m}{1+\bar{\mu}m})}(\delta, \delta_0)\right]^{-1}.$$

Moreover, if $g > 0,\ m > 0,$ *then* $\liminf_{\delta\to\infty} \gamma(\delta) \leq \frac{m\nu}{g}$.

For convenience, the following notations are introduced:

$$\psi^+ = \sup_{\delta\in\mathbb{T}} \psi(\delta), \quad \psi^- = \inf_{\delta\in\mathbb{T}} \psi(\delta).$$

Throughout this paper, we take the following assumptions:

(H1) We assume $\{\eta_{k_i}\},\ i = 1, 2,$ are the almost periodic sequence and $0 < \eta_{k_i} \leq e - 1$ for $k \in \mathbb{N}$.

(H2) $\theta(\delta)$, $d(\delta)$, $e(\delta)$, $s_1(\delta)$, $k_1(\delta)$, $h(\delta)$, $s_2(\delta)$ and $k_2(\delta)$ all are bounded nonnegative almost periodic functions on $\mathbb{T}$ such that $s_1^- > 0$, $k_1^- > 0$, $s_2^- > 0$ and $k_2^- > 0$.

3. Permanence

Here, we will prove the permanence of proposed model (1.1)–(1.4).

Definition 3.1. Model (1.1)–(1.4) is permanent, if there are the positive constants m, n, M, N such that following inequality hold:

$$\lim_{\delta\to\infty} \inf \gamma_1(\delta) \geq m, \quad \lim_{\delta\to\infty} \inf \gamma_2(\delta) \geq n,$$

$$\lim_{\delta\to\infty} \sup \gamma_1(\delta) \leq M, \quad \lim_{\delta\to\infty} \sup \gamma_2(\delta) \leq N.$$

Theorem 3.2. *If the assumptions* (H1) *and* (H2) *are hold true, then the system* (1.1)–(1.4) *is permanent.*

Proof. From Eq. (1.1), we obtain

$$\gamma_1^{\Delta}(\delta) \leq s_1^+ - \frac{s_1^-}{k_1^+}e^{\gamma_1(\delta)} \leq \left(s_1^+ - \frac{s_1^-}{k_1^+}\right) - \frac{s_1^-}{k_1^+}\gamma_1(\delta),$$

and so

$$\gamma_1^{\Delta}(\delta) \leq \left(s_1^+ - \frac{s_1^-}{k_1^+}\right) - \frac{s_1^-}{k_1^+}\gamma_1(\delta), \quad \delta \neq \delta_k, \ \delta \in [\delta_0, \infty)_{\mathbb{T}},$$

$$\gamma_1(\delta_k^+) \leq \gamma_1(\delta_k) + \ln(1 + \eta_{k_1}), \quad \delta = \delta_k, \ k \in \mathbb{N}.$$

Therefore, by Lemma 2.5 and for $k_1^+(s_1^+ - \frac{s_1^-}{k_1^+}) > s_1^-$, we have

$$\lim_{\delta\to\infty} \sup \gamma_1(\delta) \leq \frac{(s_1^+ - \frac{s_1^-}{k_1^+})}{\frac{s_1^-}{k_1^+}} = M.$$

Hence, for arbitrary $\epsilon > 0$, $\exists\ k_0 > 0$, such that $\gamma_1(\delta) \leq M + \epsilon$ for all $\delta > k_0$.

Now, from Eq. (1.2), we have

$$\gamma_2^{\Delta}(\delta) \leq s_2^+ - \frac{s_2^-}{k_2^+}e^{\gamma_2(\delta)} + \frac{e^+\theta^+e^{M+\epsilon}}{1 + h^-\theta^-},$$

$$\gamma_2^{\Delta}(\delta) \leq \left[s_2^+ + \frac{e^+\theta^+e^{M+\epsilon}}{1 + h^-\theta^-} - \frac{s_2^-}{k_2^+}\right] - \frac{s_2^-}{k_2^+}\gamma_2(\delta),$$

and so

$$\gamma_2^{\Delta}(\delta) \leq \left[s_2^+ + \frac{e^+\theta^+e^{M+\epsilon}}{1 + h^-\theta^-} - \frac{s_2^-}{k_2^+}\right] - \frac{s_2^-}{k_2^+}\gamma_2(\delta), \quad \delta \neq \delta_k, \ \delta \in [\delta_0, \infty)_{\mathbb{T}},$$

$$\gamma_2(\delta_k^+) \leq \gamma_2(\delta_k) + \ln(1 + \eta_{k_2}), \quad \delta = \delta_k, \ k \in \mathbb{N}.$$

Therefore, by Lemma 2.5 and for $k_2^+\left[s_2^+ + \frac{e^+\theta^+e^M}{1+h^-\theta^-} - \frac{s_2^-}{k_2^+}\right] > s_2^-$, we have

$$\lim_{\delta\to\infty} \sup \gamma_2(\delta) \leq \frac{\left[s_2^+ + \frac{e^+\theta^+e^{M+\epsilon}}{1+h^-\theta^-} - \frac{s_2^-}{k_2^+}\right]}{\frac{s_2^-}{k_2^+}}.$$

Now, letting $\epsilon \to 0$, we get

$$\lim_{\delta\to\infty} \sup \gamma_2(\delta) \leq \frac{\left[s_2^+ + \frac{e^+\theta^+e^{M}}{1+h^-\theta^-} - \frac{s_2^-}{k_2^+}\right]}{\frac{s_2^-}{k_2^+}} = N.$$

Hence, for arbitrary $\epsilon > 0$, $\exists\ k_1 > 0$, such that

$$\gamma_2(\delta) \leq N + \epsilon, \quad \forall\ \delta > k_1.$$

Moreover, from Eq. (1.1), we get

$$\gamma_1^{\Delta}(\delta) \geq s_1^- - \frac{s_1^+}{k_1^-}e^{\gamma_1(\delta)} - \frac{\theta^+e^{N+\epsilon}}{1 + h^-\theta^-},$$

$$\gamma_1^{\Delta}(\delta) \geq \left[s_1^- - \frac{\theta^+e^{N+\epsilon}}{1 + h^-\theta^-}\right] - \frac{s_1^+}{k_1^-}e^{\gamma_1(\delta)}.$$

Let $\psi(\delta) = e^{\gamma_1(\delta)}$. Then, we see $\psi(\delta) > 0$ and we can write above inequality as

$$[\ln(\psi(\delta))]^{\Delta} \geq \left[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}\right] - \frac{s_1^+}{k_1^-}\psi(\delta).$$

If $\psi^{\Delta}(\delta) > 0$, then from Lemma 2.2, we have

$$\frac{\psi^{\Delta}(\delta)}{\psi(\delta)} \geq \left[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}\right] - \frac{s_1^+}{k_1^-}\psi(\delta),$$
$$\psi^{\Delta}(\delta) \geq \psi(\delta)\left[\left[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}\right] - \frac{s_1^+}{k_1^-}\psi(\delta)\right].$$

Thus

$$\psi^{\Delta}(\delta) \geq \psi(\delta)\left[\left[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}\right] - \frac{s_1^+}{k_1^-}\psi(\delta)\right], \quad \delta \neq \delta_k, \ \ \delta \in [\delta_0, \infty)_{\mathbb{T}},$$
$$\psi(\delta_k^+) \geq \psi(\delta_k)\ln(1+\eta_{k_1}), \quad \delta = \delta_k, \ \ k \in \mathbb{N}.$$

By using Lemma 2.7, we have

$$\lim_{\delta\to\infty}\inf \psi(\delta) \geq \frac{\left[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}\right]}{\frac{s_1^+}{k_1^-}}.$$

If $\psi^{\Delta}(\delta) < 0$, then by Lemma 2.2, we have

$$\frac{\psi^{\Delta}(\delta)}{\psi^{\sigma}(\delta)} \geq \left[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}\right] - \frac{s_1^+}{k_1^-}\psi(\delta),$$
$$\psi^{\Delta}(\delta) \geq \psi^{\sigma}(\delta)\left[\left[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}\right] - \frac{s_1^+}{k_1^-}\psi(\delta)\right].$$

Thus

$$\psi^{\Delta}(\delta) \geq \psi^{\sigma}(\delta)\left[\left[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}\right] - \frac{s_1^+}{k_1^-}\psi(\delta)\right], \quad \delta \neq \delta_k, \ \ \delta \in [\delta_0, \infty)_{\mathbb{T}},$$
$$\psi(\delta_k^+) \geq \psi(\delta_k)\ln(1+\eta_{k_1}), \quad \delta = \delta_k, \ \ k \in \mathbb{N}.$$

By applying Lemma 2.6 and for $k_1^-[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}] > s_1^+$, we have

$$\lim_{\delta\to\infty}\inf \psi(\delta) \geq \frac{\left[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}\right]}{\frac{s_1^+}{k_1^-}}.$$

That is

$$\lim_{\delta\to\infty}\inf \gamma_1(\delta) \geq \ln\left(\frac{\left[s_1^- - \frac{\theta^+ e^{N+\epsilon}}{1+h^-\theta^-}\right]}{\frac{s_1^+}{k_1^-}}\right) = m.$$

Now, letting $\epsilon \to 0$, we get

$$\lim_{\delta\to\infty}\inf \gamma_1(\delta) \geq \ln\left(\frac{\left[s_1^- - \frac{\theta^+ e^N}{1+h^-\theta^-}\right]}{\frac{s_1^+}{k_1^-}}\right) = m.$$

Now, from Eq. (1.2)

$$\gamma_2^\Delta(\delta) \geq s_2^- - \frac{s_2^+}{k_2^-}e^{\gamma_2(\delta)} - d^+,$$

$$\gamma_2^\Delta(\delta) \geq s_2^- - d^+ - \frac{s_2^+}{k_2^-}e^{\gamma_2(\delta)}.$$

Let $\psi_1(\delta) = e^{\gamma_2(\delta)}$. Then, it is obvious that $\psi_1(\delta) > 0$. Then, from the above inequality, we get

$$[\ln(\psi_1(\delta))]^\Delta \geq s_2^- - d^+ - \frac{s_2^+}{k_2^-}\psi_1(\delta).$$

If $\psi_1^\Delta(\delta) \geq 0$, then

$$\psi_1^\Delta(\delta) \geq \psi_1(\delta)\left[s_2^- - d^+ - \frac{s_2^+}{k_2^-}\psi_1(\delta)\right].$$

Thus

$$\psi_1^\Delta(\delta) \geq \psi_1(\delta)\left[s_2^- - d^+ - \frac{s_2^+}{k_2^-}\psi_1(\delta)\right], \quad \delta \neq \delta_k,\ \ \delta \in [\delta_0,\infty)_{\mathbb{T}},$$

$$\psi_1(\delta_k^+) \geq \psi_1(\delta_k)\ln(1+\eta_{k_2}), \quad \delta = \delta_k,\ \ k \in \mathbb{N}.$$

Now, from Lemma 2.6, we get

$$\lim_{\delta\to\infty}\inf \psi_1(\delta) \geq \frac{s_2^- - d^+}{\frac{s_2^+}{k_2^-}}.$$

If $\psi_1^\Delta(\delta) < 0$, then from Lemma 2.5, we have

$$\frac{\psi_1^\Delta(\delta)}{\psi_1^\sigma(\delta)} \geq s_2^- - d^+ - \frac{s_2^+}{k_2^-}\psi_1(\delta),$$

$$\psi_1^\Delta(\delta) \geq \psi_1^\sigma(\delta)\left[s_2^- - d^+ - \frac{s_2^+}{k_2^-}\psi_1(\delta)\right].$$

Thus

$$\psi_1^\Delta(\delta) \geq \psi_1^\sigma(\delta)\left[s_2^- - d^+ - \frac{s_2^+}{k_2^-}\psi_1(\delta)\right], \quad \delta \neq \delta_k,\ \ \delta \in [\delta_0,\infty)_{\mathbb{T}},$$

$$\psi_1(\delta_k^+) \geq \psi_1(\delta_k)\ln(1+\eta_{k_2}), \quad \delta = \delta_k,\ \ k \in \mathbb{N}.$$

Now, from Lemma 2.6 and for $k_2^-(s_2^- - d^+) > s_2^+$, we get

$$\lim_{\delta\to\infty} \inf \psi_1(\delta) \geq \frac{s_2^- - d^+}{\frac{s_2^+}{k_2^-}}.$$

That is

$$\lim_{\delta\to\infty} \inf \gamma_2(\delta) \geq \ln\left(\frac{s_2^- - d^+}{\frac{s_2^+}{k_2^-}}\right) = n.$$

Therefore, from the above, we conclude that

$$m + \epsilon \leq \gamma_1(\delta) \leq M + \epsilon, \quad \text{and} \quad n + \epsilon \leq \gamma_2(\delta) \leq N + \epsilon.$$

Thus, from Definition 3.1, our model (1.1)–(1.4) is permanent. □

4. Almost Periodic Solution

Here, we will work on the stability of the considered system (1.1)–(1.4). For this, we need the following definitions and lemmas given below.

Definition 4.1 ([45]). A time scale $\mathbb{T}$ is called an almost periodic time scale if

$$\Pi = \{\omega \in \mathbb{R} : \delta \pm \omega \in \mathbb{T},\ \forall\, \delta \in \mathbb{T}\} \neq \{0\}.$$

Definition 4.2 ([45]). The ϵ-translation set of ψ is defined by

$$E\{\epsilon, \psi\} = \{\delta \in \Pi : |\psi(\delta + \omega) - \psi(\delta)| < \epsilon,\ \forall\ \delta \in \mathbb{T}\},$$

is dense $\forall\ \epsilon > 0$ and $\mathbb{T}$ be an almost periodic time scale.

Definition 4.3 ([45]). A function $\psi \in C(\mathbb{T}, \mathbb{R}^n)$ is said to be an almost periodic function if for preassign $\epsilon > 0$, $\exists$ a constant $l(\epsilon) > 0$ such that

$$|\psi(\delta + \omega) - \psi(\delta)| < \epsilon, \quad \forall\, \delta \in \mathbb{T}.$$

ω is known as ϵ-translation number of ψ.

Suppose that the set $\bar{\omega} = \{\gamma_1, \gamma_2\ : m \leq \gamma_1 \leq M, n \leq \gamma_2 \leq N\}$ is the solution of our considered model (1.1)–(1.4). Then, the following lemma:

Lemma 4.1 ([30]). *Let the assumptions* (H1) *and* (H2) *are hold. Then,* $\bar{\omega} \neq \emptyset$.

Theorem 4.4. *Let the assumptions* (H1) *and* (H2) *are hold. Then, model* (1.1)–(1.4) *has a unique almost periodic solution denoted by* $X(\delta) = (\gamma_1, \gamma_2)$. *Also, the solution is asymptotically stable.*

Proof. Let us define a norm on $\mathbb{R}^2$

$$\|(\gamma_1, \gamma_2)\| = \sup_{\delta \in \mathbb{T}} |\gamma_1| + \sup_{\delta \in \mathbb{T}} |\gamma_2|.$$

Let $X = (\gamma_1, \gamma_2)$ and $Y = (z_1, z_2)$ be two solution of system (1.1)–(1.4). Then, the Lyapunov function on $\mathbb{T}^+ \times \bar{\omega} \times \bar{\omega}$ is defined as follows:

$$V(\delta, X, Y) = \sum_{i=1}^{2} (\gamma_i - z_i)^2,$$

which satisfies property (1) and (2) of [30, Lemma 4.1]. Now, for condition (3) of [30, Lemma 4.1], we have

$$\begin{aligned}
&V(\delta_k^+, X(\delta_k^+), Y(\delta_k^+)) \\
&\quad = (\gamma_1(\delta_k^+) - z_1(\delta_k^+))^2 + (\gamma_2(\delta_k^+) - z_2(\delta_k^+))^2 \\
&\quad = [\ln(1+\lambda_k)\gamma_1(\delta_k) - \ln(1+\lambda_k) z_1(\delta_k)]^2 + [\ln(1+\eta_k)\gamma_2(\delta_k) \\
&\qquad - \ln(1+\eta_k) z_2(\delta_k)]^2 \\
&\quad = [\ln(1+\lambda_k)]^2 (\gamma_1(\delta_k) - z_1(\delta_k))^2 + [\ln(1+\eta_k)]^2 (\gamma_2(\delta_k) - z_2(\delta_k))^2 \\
&\quad \le (\gamma_1(\delta_k) - z_1(\delta_k))^2 + (\gamma_2(\delta_k) - z_2(\delta_k))^2 \\
&\quad \le V(\delta, X(\delta_k), Y(\delta_k)).
\end{aligned}$$

Hence, condition (3) also satisfied. Now, for condition (4) of [30, Lemma 4.1], we have

$$\begin{aligned}
D^+ V^{\Delta}(\delta, X, Y) &= \sum_{i=1}^{2} [2(\gamma_i(\delta) - z_i(\delta)) + \mu(\delta)(\gamma_i(\delta) - z_i(\delta))^{\Delta}](\gamma_i(\delta) - z_i(\delta))^{\Delta} \\
&= \sum_{i=1}^{2} (2w_i + \mu(\delta) w_i^{\Delta}(\delta)) w_i^{\Delta}(\delta) \\
&= V_1(\delta) + V_2(\delta),
\end{aligned}$$

where $w_i(\delta) = \gamma_i(\delta) - z_i(\delta), V_1(\delta) = (2w_1 + \mu(\delta) w_1^{\Delta}(\delta)) w_1^{\Delta}(\delta)$ and $V_2(\delta) = (2w_2 + \mu(\delta) w_2^{\Delta}(\delta)) w_2^{\Delta}(\delta)$. Now, computing delta derivative of $w_i(\delta)$, we get

$$\begin{aligned}
w_1^{\Delta}(\delta) = s_1(\delta) - \frac{s_1(\delta)}{k_1(\delta)} e^{\gamma_1(\delta)} - \frac{\theta(\delta) e^{\gamma_2(\delta)}}{1 + h(\delta)\theta(\delta) e^{\gamma_1(\delta)}} - s_1(\delta) + \frac{s_1(\delta)}{k_1(\delta)} e^{z_1(\delta)} \\
+ \frac{\theta(\delta) e^{z_2(\delta)}}{1 + h(\delta)\theta(\delta) e^{z_1(\delta)}}
\end{aligned}$$

$$= -\frac{s_1(\delta)}{k_1(\delta)}[e^{\gamma_1(\delta)} - e^{z_1(\delta)}] - \frac{\theta(\delta)e^{\gamma_2(\delta)}}{1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}} + \frac{\theta(\delta)e^{z_2(\delta)}}{1+h(\delta)\theta(\delta)e^{z_1(\delta)}}$$

$$= -\frac{s_1(\delta)}{k_1(\delta)}e^{\xi(\delta)}w_1(\delta)$$

$$- \frac{\theta(\delta)e^{\eta(\delta)}w_2(\delta) + h(\delta)\theta^2(\delta)[e^{\gamma_1(\delta)}e^{\eta(\delta)}w_2(\delta) - e^{z_1(\delta)}e^{\xi(\delta)}w_1(\delta)]}{[1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}][1+h(\delta)\theta(\delta)e^{z_1(\delta)}]}.$$

Now, for delta derivative of $w_2(\delta) = \gamma_2(\delta) - z_2(\delta)$, we obtain

$$w_2^{\Delta}(\delta) = s_2(\delta) - \frac{s_2(\delta)}{k_2(\delta)}e^{\gamma_2(\delta)} - d(\delta) + \frac{e(\delta)\theta(\delta)e^{\gamma_1(\delta)}}{1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}} - s_2(\delta)$$

$$+ \frac{s_2(\delta)}{k_2(\delta)}e^{z_2(\delta)} + d(\delta) - \frac{e(\delta)\theta(\delta)e^{z_1(\delta)}}{1+h(\delta)\theta(\delta)e^{z_1(\delta)}}$$

$$= -\frac{s_2(\delta)}{k_2(\delta)}e^{\eta(\delta)}w_2(\delta) + \frac{e(\delta)\theta(\delta)e^{\xi(\delta)}}{[1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}][1+h(\delta)\theta(\delta)e^{z_1(\delta)}]}w_1(\delta).$$

Now, from the above inequality we obtain

$$w_1^{\Delta}(\delta) = -\frac{s_1(\delta)}{k_1(\delta)}e^{\xi(\delta)}w_1(\delta)$$

$$- \frac{\theta(\delta)e^{\eta(\delta)}w_2(\delta) + h(\delta)\theta^2(\delta)[e^{\gamma_1(\delta)}e^{\eta(\delta)}w_2(\delta) - e^{z_1(\delta)}e^{\xi(\delta)}w_1(\delta)]}{[1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}][1+h(\delta)\theta(\delta)e^{z_1(\delta)}]},$$

$$\delta \neq \delta_k, \quad \delta \in [\delta_0, \infty)_{\mathbb{T}},$$

$$w_2^{\Delta}(\delta) = -\frac{s_2(\delta)}{k_2(\delta)}e^{\eta(\delta)}w_2(\delta) + \frac{e(\delta)\theta(\delta)e^{\xi(\delta)}}{[1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}][1+h(\delta)\theta(\delta)e^{z_1(\delta)}]}w_1(\delta),$$

$$\delta \neq \delta_k, \quad \delta \in [\delta_0, \infty)_{\mathbb{T}},$$

$$w_1(\delta_k^+) = w_1(\delta_k), \quad \delta = \delta_k,$$

$$w_2(\delta_k^+) = w_2(\delta_k), \quad \delta = \delta_k.$$

Hence

$$V_1(\delta) = (2w_1(\delta) + \mu(\delta)w_1^{\Delta}(\delta))w_1^{\Delta}(\delta)$$

$$= \Bigg(2w_1(\delta) + \mu(\delta)\Bigg[-\frac{s_1(\delta)}{k_1(\delta)}e^{\xi(\delta)}w_1(\delta)$$

$$- \frac{\theta(\delta)e^{\eta(\delta)}w_2(\delta) + h(\delta)\theta^2(\delta)[e^{\gamma_1(\delta)}e^{\eta(\delta)}w_2(\delta) - e^{z_1(\delta)}e^{\xi(\delta)}w_1(\delta)]}{[1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}][1+h(\delta)\theta(\delta)e^{z_1(\delta)}]}\Bigg]\Bigg)$$

$$
\begin{aligned}
&\times\Bigg(-\frac{s_1(\delta)}{k_1(\delta)}e^{\xi(\delta)}w_1(\delta)\\
&-\frac{\theta(\delta)e^{\eta(\delta)}w_2(\delta)+h(\delta)\theta^2(\delta)[e^{\gamma_1(\delta)}e^{\eta(\delta)}w_2(\delta)-e^{z_1(\delta)}e^{\xi(\delta)}w_1(\delta)]}{[1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}][1+h(\delta)\theta(\delta)e^{z_1(\delta)}]}\Bigg)\\
\le{}&-\Bigg[2\frac{s_1^-}{k_1^-}e^m-\frac{2h^+(\theta^+)^2e^{2M}}{(1+h^-\theta^-e^m)^2}-\mu\left(\frac{s_1^+}{k_1^-}e^M\right)^2+2\mu\frac{s_1^-h^-(\theta^-)^2e^{3m}}{k_1^+(1+h^+\theta^+e^M)^2}\\
&-\left(\mu\frac{h^+\theta^+e^{2M}}{(1+h^-\theta^-e^m)^2}\right)^2+\frac{e^n}{(1+h^+\theta^+e^M)^2}+\frac{1}{2}\frac{h^-(\theta^-)^2e^ne^m}{(1+h^+\theta^+e^M)^2}\\
&-\mu\frac{s_1^+}{k_1^-}\frac{\theta e^Ne^M}{(1+h^-\theta^-e^m)^2}-\mu\frac{s_1^+}{k_1^-}\frac{h^+(\theta^+)^2e^{2M}e^N}{(1+h^-\theta^-e^m)^2}+\mu\frac{(\theta^-)^3h^-e^{2m}e^n}{(1+h^+\theta^+e^M)^4}\\
&+\mu\frac{(h^-)^2(\theta^-)^4e^{3m}e^n}{(1+h^+\theta^+e^M)^4}\Bigg]\times w_1^2(\delta)-\Bigg[\frac{e^n}{(1+h^+\theta^+e^M)^2}+\frac{1}{2}\frac{h^-(\theta^-)^2e^ne^m}{(1+h^+\theta^+e^M)^2}\\
&-\mu\left(\frac{\theta e^N}{(1+h^-\theta^-e^m)^2}\right)^2-2\mu\frac{h^+(\theta^+)^3e^{2N}e^M}{(1+h^-\theta^-e^m)^4}-\mu\left(\frac{h^+(\theta^+)^2e^Me^N}{(1+h^-\theta^-e^m)^2}\right)^2\\
&-\mu\frac{s_1^+}{k_1^-}\frac{h^+(\theta^+)^2e^{2M}e^N}{(1+h^-\theta^-e^m)^2}-\frac{\mu}{2}\frac{s_1^+}{k_1^-}\frac{\theta e^Me^M}{(1+h^-\theta^-e^m)^2}+\mu\frac{(h^-)^2(\theta^-)^4e^{3m}e^n}{(1+h^+\theta^+e^M)^4}\\
&+\mu\frac{h^-(\theta^-)^3e^{2m}e^n}{(1+h^+\theta^+e^M)^4}\Bigg]\times w_2^2(\delta)\le -P_1w_1^2(\delta)-P_2w_2^2(\delta),
\end{aligned}
$$

where P_1 and P_2 are

$$
\begin{aligned}
P_1={}&\Bigg[2\frac{s_1^-}{k_1^-}e^m-\frac{2h^+(\theta^+)^2e^{2M}}{(1+h^-\theta^-e^m)^2}-\mu\left(\frac{s_1^+}{k_1^-}e^M\right)^2+2\mu\frac{s_1^-h^-(\theta^-)^2e^{3m}}{k_1^+(1+h^+\theta^+e^M)^2}\\
&-\left(\mu\frac{h^+\theta^+e^{2M}}{(1+h^-\theta^-e^m)^2}\right)^2+\frac{e^n}{(1+h^+\theta^+e^M)^2}+\frac{1}{2}\frac{h^-(\theta^-)^2e^ne^m}{(1+h^+\theta^+e^M)^2}\\
&-\mu\frac{s_1^+}{k_1^-}\frac{\theta e^Ne^M}{(1+h^-\theta^-e^m)^2}-\mu\frac{s_1^+}{k_1^-}\frac{h^+(\theta^+)^2e^{2M}e^N}{(1+h^-\theta^-e^m)^2}+\mu\frac{(\theta^-)^3h^-e^{2m}e^n}{(1+h^+\theta^+e^M)^4}\\
&+\mu\frac{(h^-)^2(\theta^-)^4e^{3m}e^n}{(1+h^+\theta^+e^M)^4}\Bigg],\\
P_2={}&\Bigg[\frac{e^n}{(1+h^+\theta^+e^M)^2}+\frac{1}{2}\frac{h^-(\theta^-)^2e^ne^m}{(1+h^+\theta^+e^M)^2}-\mu\left(\frac{\theta e^N}{(1+h^-\theta^-e^m)^2}\right)^2\\
&-2\mu\frac{h^+(\theta^+)^3e^{2N}e^M}{(1+h^-\theta^-e^m)^4}-\mu\left(\frac{h^+(\theta^+)^2e^Me^N}{(1+h^-\theta^-e^m)^2}\right)^2-\mu\frac{s_1^+}{k_1^-}\frac{h^+(\theta^+)^2e^{2M}e^N}{(1+h^-\theta^-e^m)^2}\\
&-\frac{\mu}{2}\frac{s_1^+}{k_1^-}\frac{\theta e^Me^M}{(1+h^-\theta^-e^m)^2}+\mu\frac{(h^-)^2(\theta^-)^4e^{3m}e^n}{(1+h^+\theta^+e^M)^4}+\mu\frac{h^-(\theta^-)^3e^{2m}e^n}{(1+h^+\theta^+e^M)^4}\Bigg].
\end{aligned}
$$

Furthermore, we have

$$
\begin{aligned}
V_2(\delta) &= (2w_2(\delta) + \mu(\delta)w_2^{\Delta}(\delta))w_2^{\Delta}(\delta) \\
&= \Bigg(2w_2(\delta) + \mu(\delta)\Bigg[-\frac{s_2(\delta)}{k_2(\delta)}e^{\eta(\delta)}w_2(\delta) \\
&\quad + \frac{e(\delta)\theta(\delta)e^{\xi(\delta)}}{[1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}][1+h(\delta)\theta(\delta)e^{z_1(\delta)}]}w_1(\delta)]\Bigg) \\
&\quad \times \left(-\frac{s_2(\delta)}{k_2(\delta)}e^{\eta(\delta)}w_2(\delta) + \frac{e(\delta)\theta(\delta)e^{\xi(\delta)}}{[1+h(\delta)\theta(\delta)e^{\gamma_1(\delta)}][1+h(\delta)\theta(\delta)e^{z_1(\delta)}]}w_1(\delta)\right) \\
&\quad \leq \left(\frac{\theta^+e^+e^M}{(1+h^-\theta^-e^m)^2} - \mu^-\frac{s_2^-}{k_2^+}\frac{\theta^-e^-e^ne^m}{(1+h^+\theta^+e^M)^2} + \mu^+\left(\frac{\theta^+e^+e^M}{1+h^-\theta^-e^m}\right)^2\right) \\
&\quad \times w_1^2(\delta) + \Bigg(-2\frac{s_2^-}{k_2^+}e^n + \frac{\theta^+e^+e^M}{(1+h^-\theta^-e^m)^2} + \mu^+\left(\frac{s_2^+}{k_2^-}e^N\right)^2 \\
&\quad - \mu^-\frac{s_2^-}{k_2^+}\frac{\theta^-e^-e^ne^m}{(1+h^+\theta^+e^M)^2}\Bigg) \times w_2^2(\delta) \\
&\quad \leq -P_3w_1^2(\delta) - P_4w_2^2(\delta),
\end{aligned}
$$

where P_3 and P_4 are

$$
P_3 = \left(\frac{\theta^+e^+e^M}{(1+h^-\theta^-e^m)^2} - \mu^-\frac{s_2^-}{k_2^+}\frac{\theta^-e^-e^ne^m}{(1+h^+\theta^+e^M)^2} + \mu^+\left(\frac{\theta^+e^+e^M}{1+h^-\theta^-e^m}\right)^2\right),
$$

$$
P_4 = \left(-2\frac{s_2^-}{k_2^+}e^n + \frac{\theta^+e^+e^M}{(1+h^-\theta^-e^m)^2} + \mu^+\left(\frac{s_2^+}{k_2^-}e^N\right)^2 - \mu^-\frac{s_2^-}{k_2^+}\frac{\theta^-e^-e^ne^m}{(1+h^+\theta^+e^M)^2}\right).
$$

Combining the above results, we get

$$
\begin{aligned}
D^+V^{\Delta}(\delta, X, Y) &\leq -(P_1+P_3)w_1^2(\delta) - (P_2+P_4)w_2^2(\delta) \\
&\leq -\kappa V(\delta, X, Y),
\end{aligned}
$$

where

$$
\kappa = \min\{(P_1+P_3), (P_2+P_4)\}.
$$

For, $\kappa > 0$, we see that condition (4) of [30, Lemma 4.1] satisfies. So, all the conditions of [30, Lemma 4.1] are hold true. Therefore, by [30, Lemma 4.1], there exists a unique almost periodic solution $(\gamma_1(\delta), \gamma_2(\delta)) \in \bar{\omega}$, which is uniformly asymptotically stable. $\square$

5. Example

Example 5.1. In this example, we consider $\mathbb{T} = [0, 10]_{\mathbb{R}}$, i.e. $\mu = 0$, $\delta_1 = 5$ is an impulsive point and consider the following coefficients:

$$
\begin{aligned}
&s_1(\delta) = 20 + \cos(6\pi\delta); \quad s_2(\delta) = 3.198; \quad k_1(\delta) = 2.3 + \cos(6\pi\delta);\\
&k_2(\delta) = 1.6 - 0.5\cos(6\pi\delta); \quad \theta(\delta) = 0.01 + 0.01\cos(6\pi\delta);\\
&h(\delta) = 0.1 + 0.1\sin(6\pi\delta); \quad e(\delta) = 6.05 + 6.05\cos(6\pi\delta);\\
&d(\delta) = 0.1 + 0.1\sin(6\pi\delta); \quad \eta_{k_1} = 0.301; \quad \eta_{k_2} = 0.301.
\end{aligned}
$$

We can easily see that

$$
\begin{aligned}
&s_1^+ = 21, \quad s_1^- = 19, \quad k_1^+ = 3.30, \quad k_1- = 1.30, \quad s_2^+ = 3.198,\\
&s_2^- = 3.198, \quad k_2^+ = 2.10, \quad k_2^- = 1.10, \quad \theta^+ = 0.02, \quad \theta^- = 0, \quad h^+ = 0.20,\\
&h^- = 0, \quad e^+ = 12.10, \quad e^- = 0, \quad d^+ = 0.20, \quad d^- = 0.
\end{aligned}
$$

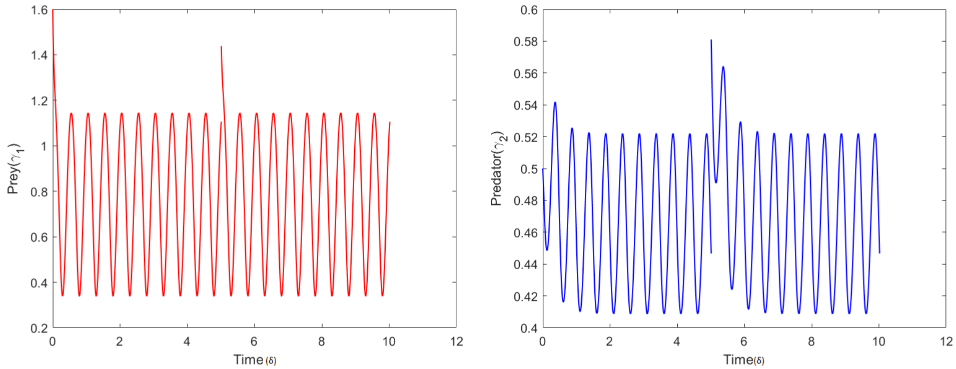
Thus, the system (1.1)–(1.4) becomes

$$
\begin{aligned}
\gamma_1^{\Delta}(\delta) &= 20 + \cos(6\pi\delta) - \frac{20 + \cos(6\pi\delta)}{2.3 + \cos(6\pi\delta)} e^{\gamma_1(\delta)}\\
&\quad - \frac{(0.01 + 0.01\cos(6\pi\delta))e^{\gamma_2(\delta)}}{1 + (0.1 + 0.1\sin(6\pi\delta))(0.01 + 0.01\cos(6\pi\delta))e^{\gamma_1(\delta)}},\\
\gamma_2^{\Delta}(\delta) &= 3.198 - \frac{3.198}{1.6 - 0.5\cos(6\pi\delta)} - (0.1 + 0.1\sin(6\pi\delta))\\
&\quad + \frac{(6.05 + 6.05\cos(6\pi\delta))(0.01 + 0.01\cos(6\pi\delta))e^{\gamma_1(\delta)}}{1 + (0.1 + 0.1\sin(6\pi\delta))(0.01 + 0.01\cos(6\pi\delta))e^{\gamma_1(\delta)}},\\
\gamma_1(\delta_k^+) &= \gamma_1(\delta_k) + \ln(1 + 0.301),\\
\gamma_2(\delta_k^+) &= \gamma_2(\delta_k) + \ln(1 + 0.301).
\end{aligned}
$$

After doing some simple calculation, we get

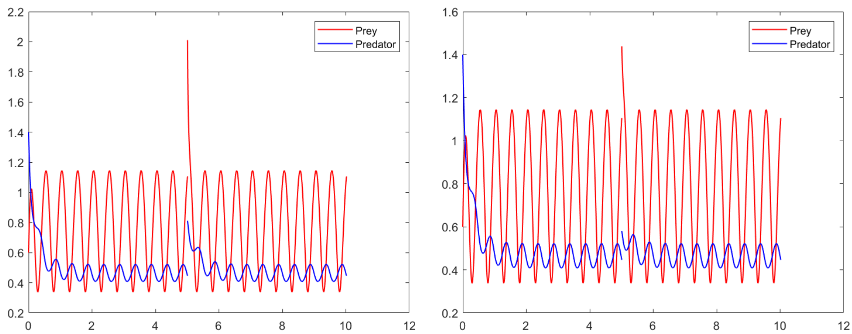
$$
\begin{aligned}
&m = 0.1320, \quad M = 2.6474, \quad n = 0.0307, \quad N = 3.3433 \quad \text{and}\\
&P_1 = 14.0324, \quad P_2 = 0.9728, \quad P_3 = 3.4163 \quad \text{and} \quad P_4 = 0.2755.
\end{aligned}
$$

Therefore, $\kappa = \min\{(P_1 + P_3), (P_2 + P_4)\} = 1.2484 > 0$. Hence, all the assumptions of Theorem 4.4 are satisfied. Thus, our model has a unique almost periodic solution, which is asymptotically stable. Also, from Fig. 1, we conclude that the solution is permanent.

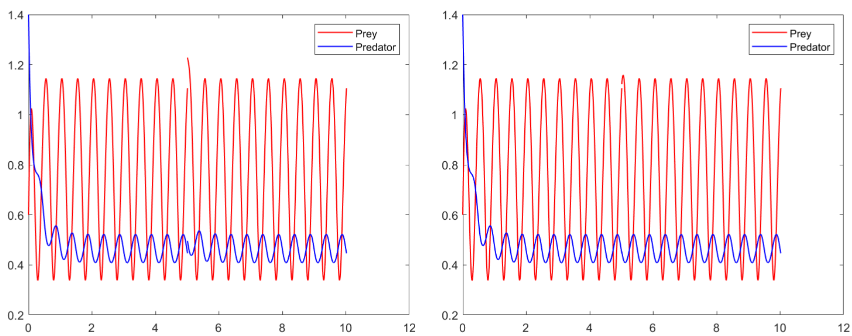


(a) (b)

Fig. 1. Almost periodic solution with time domain $\mathbb{T} = [0, 10]_{\mathbb{R}}$ and initial conditions ($\gamma_1(0) = 1.6,\ \gamma_2(0) = 0.5$).



(a) $\eta_{k_1} = 0.8187$ (b) $\eta_{k_1} = 0.3012$



(c) $\eta_{k_1} = 0.1108$ (d) $\eta_{k_1} = 0.0149$

Fig. 2. Impulsive effect at $\delta_1 = 5$ with time domain $\mathbb{T} = [0, 10]_{\mathbb{R}}$ and initial conditions ($\gamma_1(0) = 1.6,\ \gamma_2(0) = 0.5$).

Example 5.2. In this example, we consider the time domain $\mathbb{T} = [0, 20]_{\mathbb{Z}}$, i.e. $\mu = 1$, $\delta_1 = 10$ is an impulsive point and consider the following coefficients:

$$s_1(\delta) = 1 + \cos(4\pi\delta); \quad s_2(\delta) = 2; \quad k_1(\delta) = 7 + \cos(4\pi\delta);$$

$$k_2(\delta) = 2.6 - 0.5\cos(4\pi\delta); \quad \theta(\delta) = 0.01 + 0.01\cos(6\pi\delta);$$

$$h(\delta) = 0.1 + 0.1\sin(6\pi\delta); \quad e(\delta) = 0.05 + 0.05\cos(6\pi\delta);$$

$$d(\delta) = 0.1 + 0.1\sin(6\pi\delta); \quad \eta_{k_1} = 0.2466; \quad \eta_{k_2} = 0.6065.$$

Thus, the system (1.1)–(1.4) becomes

$$\begin{aligned}
\gamma_1^{\Delta}(\delta) &= 1 + \cos(4\pi\delta) - \frac{1+\cos(4\pi\delta)}{7+\cos(4\pi\delta)} e^{\gamma_1(\delta)} \\
&\quad - \frac{(0.01 + 0.01\cos(4\pi\delta))e^{\gamma_2(\delta)}}{1 + (0.1 + 0.1\sin(4\pi\delta))(0.01 + 0.01\cos(4\pi\delta))e^{\gamma_1(\delta)}}, \\
\gamma_2^{\Delta}(\delta) &= 2 - \frac{2}{2.6 - 0.5\cos(4\pi\delta)} - (0.1 + 0.1\sin(4\pi\delta)) \\
&\quad + \frac{(0.05 + 0.05\cos(4\pi\delta))(0.01 + 0.01\cos(4\pi\delta))e^{\gamma_1(\delta)}}{1 + (0.1 + 0.1\sin(4\pi\delta))(0.01 + 0.01\cos(4\pi\delta))e^{\gamma_1(\delta)}}, \\
\gamma_1(\delta_k^+) &= \gamma_1(\delta_k) + \ln(1 + 0.2466), \\
\gamma_2(\delta_k^+) &= \gamma_2(\delta_k) + \ln(1 + 0.6065).
\end{aligned}$$

After doing some simple calculation, we get $m = 1.7248$, $M = 7$, $n = 0.6906$, $N = 3.3983$ and $\kappa = 1240.6 > 0$. Hence, all the assumptions of Theorem 4.4 are hold true. Thus, our model has a unique almost periodic solution, which is also asymptotically stable. Also, from Fig. 3, we conclude that the solution is permanent.

Example 5.3. In this example, we consider the time domain $\mathbb{T} = [1/2, 2] \cup [3, 9/2]$, i.e. $\mu = 0$, $\delta_1 = 1.2$, $\delta_2 = 3.9$ are the impulsive point and consider the following coefficients:

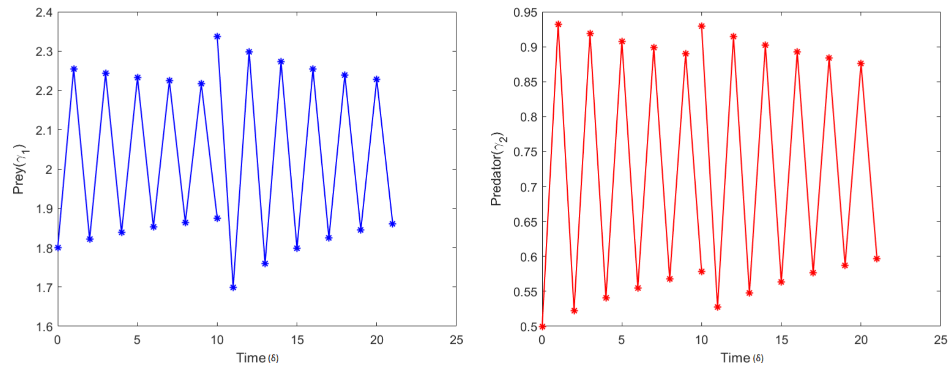
$$s_1(\delta) = 20 + \cos(6\pi\delta); \quad s_2(\delta) = 3.198;$$

$$k_1(\delta) = 2.3 + \cos(6\pi\delta); \quad k_2(\delta) = 1.6 - 0.5\cos(6\pi\delta);$$

$$\theta(\delta) = 0.01 + 0.01\cos(6\pi\delta); \quad h(\delta) = 0.1 + 0.1\sin(6\pi\delta);$$

$$e(\delta) = 6.05 + 6.05\cos(6\pi\delta); \quad d(\delta) = 0.1 + 0.1\sin(6\pi\delta);$$

$$\eta_{k_1} = 0.301; \quad \eta_{k_2} = 0.301.$$



(a) (b)

Fig. 3. Almost periodic solution with time domain $\mathbb{T} = [0, 20]_{\mathbb{Z}}$ and initial conditions ($\gamma_1(0) = 1.8$, $\gamma_2(0) = 0.5$).

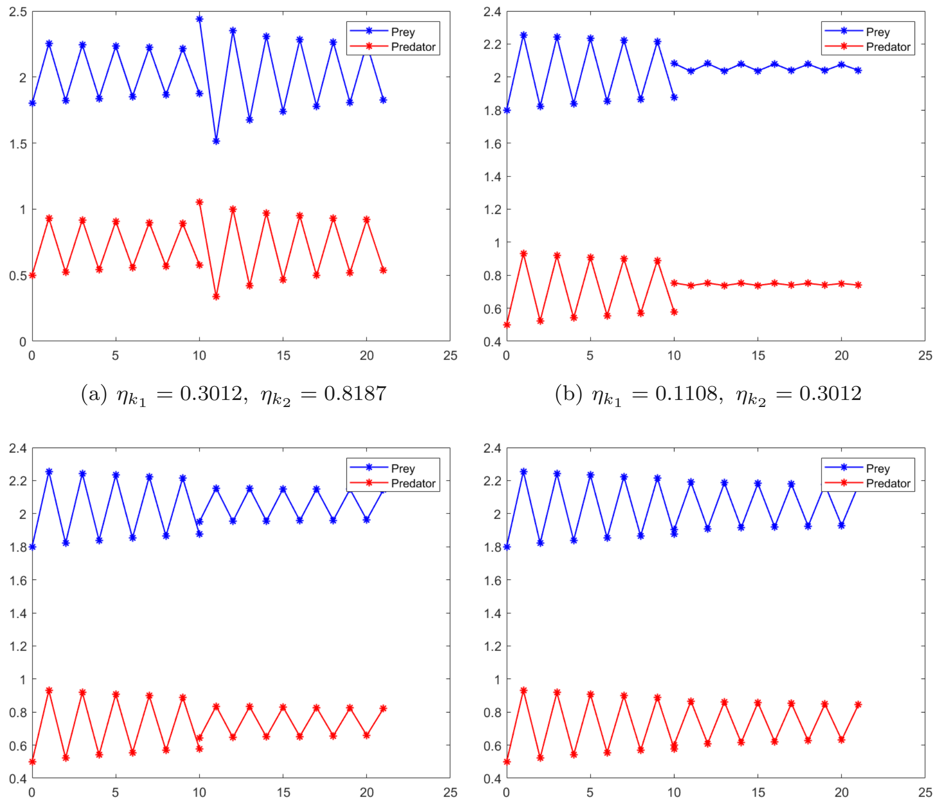
We can easily see that

$$s_1^+ = 21, \quad s_1^- = 19, \quad k_1^+ = 3.30, \quad k_1- = 1.30,$$
$$s_2^+ = 3.198, \quad s_2^- = 3.198, \quad k_2^+ = 2.10, \quad k_2^- = 1.10,$$
$$\theta^+ = 0.02, \quad \theta^- = 0, \quad h^+ = 0.20, \quad h^- = 0,$$
$$e^+ = 12.10, \quad e^- = 0, \quad d^+ = 0.20, \quad d^- = 0.$$

With the help of Figs. 5(a), 5(b) and 6, we conclude that the solution of model (1.1)–(1.4) is permanent. Also, all the assumptions of Theorem 4.4 are hold true. Therefore, our model (1.1)–(1.4) has a unique almost periodic solution, which is also asymptotic stable.

Remark 2. In Fig. 2, we have taken the time domain interval $\mathbb{T} = [0, 10]_{\mathbb{R}}$ and impulsive point $\delta_1 = 5$. We can conclude from Fig. 2, when we increase or decrease the values of the impulsive functions $\{\eta_{k_i}\}$, $i = 1, 2$, the corresponding population densities of the predator and prey are increasing and decreasing, respectively, i.e. some abrupt changes in nature like fire, hunting, harvesting, breeding, etc., lead the abrupt changes in population densities of prey and predator species. Figure 7(a) represents the phase portrait when $\mathbb{T} = \mathbb{R}$ in Example 5.1.

Remark 3. In Fig. 4, we have taken the time domain interval $\mathbb{T} = [0, 20]_{\mathbb{Z}}$ and impulsive point $\delta_1 = 10$. In Fig. 4, when we increase or decrease the values of the impulsive function $\{\eta_{k_i}\}$, $i = 1, 2$, the corresponding population densities of the predator and prey also increase and decrease, respectively. Also, from these figures, we observe that after the abrupt changes in prey and predator populations the periodicity of the system also changes. Figure 7(b) represents the phase portrait when $\mathbb{T} = \mathbb{Z}$ in Example 5.2.



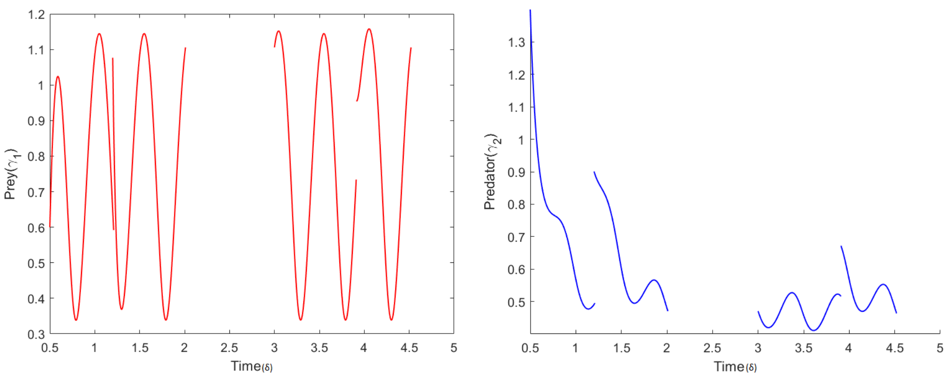
(a) $\eta_{k_1} = 0.3012,\ \eta_{k_2} = 0.8187$

(b) $\eta_{k_1} = 0.1108,\ \eta_{k_2} = 0.3012$

(c) $\eta_{k_1} = 0.0407,\ \eta_{k_2} = 0.1108$

(d) $\eta_{k_1} = 0.0149,\ \eta_{k_2} = 0.0407$

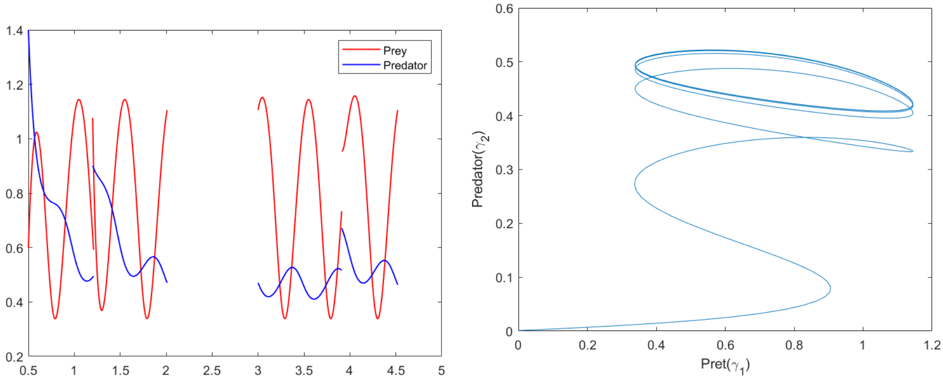
Fig. 4. Impulsive effect at $\delta_1 = 10$, with time domain $\mathbb{T} = [0, 20]_{\mathbb{Z}}$ and initial conditions ($\gamma_1(0) = 1.8,\ \gamma_2(0) = 0.5$).



(a)

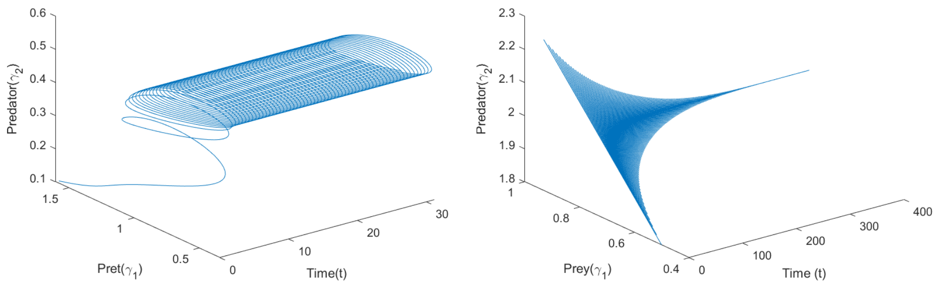
(b)

Fig. 5. Almost periodic solution with time domain $\mathbb{T} = [1/2, 2]_{\mathbb{R}} \cup [3, 9/2]_{\mathbb{R}}$ and initial conditions ($\gamma_1(0) = 1.6,\ \gamma_2(0) = 0.5$).



(a) (b)

Fig. 6. (a) Time domain $\mathbb{T} = [1/2, 2]_{\mathbb{R}} \cup [3, 9/2]_{\mathbb{R}}$ and initial conditions $(\gamma_1(0) = 1.6,\ \gamma_2(0) = 0.5)$. (b) Phase diagram of prey and predator with initial condition $(\gamma_1(0) = 0.001,\ \gamma_2(0) = 0.001)$ and time domain $\mathbb{T} = \mathbb{R}$.



(a) Time domain $\mathbb{T} = \mathbb{R}$ (b) Time domain $\mathbb{T} = \mathbb{Z}$

Fig. 7. In (a) $(\gamma_1(0) = 1.6,\ \gamma_2(0) = 0.5)$ and (b) $(\gamma_1(0) = 1.8,\ \gamma_2(0) = 0.5)$.

6. Conclusion

In this work, we have proposed a hybrid non-autonomous prey–predator system with impulsive conditions on time scales where the predator being generalist means predators have alternative food option.

First, we have discussed the permanence of considered system (1.1)–(1.4) which ensures the long-term survival of species. In Theorem 3.2, we have established the sufficient conditions for permanence by comparison theorem and time scale calculus.

Notice that the considered system (1.1)–(1.4) is non-autonomous meaning that all the biological and environmental parameters appearing in the considered model are time-dependent, whereas in autonomous cases all the parameters are considered as constant. Therefore, our model is more general as compared to autonomous prey–predator system. In Examples (5.1)–(5.3), we are considering the non-autonomous prey–predator system which means all the parameters are time-dependent, i.e. the values of the parameters are changed due to environmental disturbance and with respect to time. From these considered examples, we can see that our system will satisfy the conditions of permanence.

Also, we have considered the impulsive effects in system (1.1)–(1.4). Impulsive effects also play an important role in modeling a certain kind of rapid disturbance (like fire, drought, deforestation, and landslides). These kinds of disturbances will lead to a rapid change in densities of prey and predator species. Thus, we consider the impulsive effects. In our simulated results, we can easily see the effect of impulses. From Figs. 3–5, we can conclude that at a certain point of time if we apply impulsive conditions, then corresponding to that the densities of prey and predator are also affected.

Moreover in this work, we have discussed the stability results. In Theorem 4.4, we have established some sufficient conditions for stability of considered system (1.1)–(1.4) by using the permanence results, time scale calculus and Lyapunov function. For this, in Sec. 5, to show the effectiveness of the obtained theoretical results related to stability, we give some simulated examples on different–different time scales, e.g. $\mathbb{T} = \mathbb{R}$, $\mathbb{Z}$ and $[1/2, 2]_{\mathbb{R}} \cup [3, 9/2]_{\mathbb{R}}$. Also, from these simulated examples, we clearly see that our solution is permanent (see Figs. 1, 3, 5 and 6). Figures 2 and 4 show the importance of the impulsive effects.

In this work, our main focus is to study the dynamics of prey–predator system on time scale. But in future, we can extend these results in prey–predator system with Allee effect on time scales, prey–predator system with feedback control strategy on time scales and prey–predator system with cooperative hunting on time scales, etc.

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