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Context Matters: Understanding the Relationship Between Instructor's Beliefs and the Amount of Time Spent Lecturing

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Abstract

Prior studies have identified the impact beliefs have on mathematics instructors' instructional practice, such as their choice to (or not to) lecture. However, the role of instructional context role in influencing beliefs and instruction has not been thoroughly researched. This paper explores how course context and beliefs could impact mathematics instructors' propensity to lecture by investigating two very different instructional contexts in undergraduate mathematics in the United States: Calculus and Abstract Algebra. The results of our regression analyses were significant in both data sets and, we did find beliefs in each context that predicted the amount of time spent lecturing. For instance, the more calculus instructors believed in the effectiveness of teacher-centered instructional practices, the more likely they were to lecture. Whereas the more abstract algebra instructors believed in their student's capacity to learn the less likely they were to lecture. However, while the regression model for the abstract algebra instructors accounted for 37.8% of the variability in the reported amount of time spent lecturing, the model for Calculus instructors only accounted for 2.7% of the variability. Thus our analyses indicate that there are contextual differences, such as course coordination, student demographics, and the job security of the instructors, that may be mitigating the extent to which beliefs impact instructional practice.

Keywords Mathematics instructors · Beliefs · Institutional constraints · Quantitative analysis · Calculus · Abstract algebra

Introduction

Instructional practices across undergraduate science, technology, engineering, and mathematics (STEM) courses in the United States (US) have, again and again, been identified as a reason why students discontinue STEM degrees (e.g., Seymour &

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Hewitt, 1997; Seymour & Hunter, 2019; PCAST, 2012). While there is a growing body of research indicating that the use of active learning, as opposed to a more traditional lecture mode of instruction, improves learning and retention of students in undergraduate STEM courses (see Freeman et al., 2014) for a meta-analysis of this research), it has also been well documented that instructional practice is slow to change (Johnson, 2019) and many of the dissemination efforts currently in use are minimally effective (Henderson et al., 2011). Instructors' decision-making around instructional practice, in general, and instructional change, in particular, is complex and involves various individual and contextual factors (e.g., Henderson & Dancy, 2007, Johnson et al., 2019).

In this study, we focus on a particular set of individual factors — beliefs — and their impact on a particular indicator of instructional practice — the self-reported amount of class time spent lecturing. In general, researchers have found that descriptive self-reports of instructional practice generally align with what is recorded with descriptive instructional measures and observations (Burstein et al., 1995; Mayer, 1999; Hayward et al., 2018; Ross et al., 2003). Thus, while researchers have found inaccuracies in self-reports of the quality of instructors' teaching, self-reports of the frequency of certain instructional practices, such as lecturing, is fairly accurate (Kaufman et al., 2016).

Here "lecturing" is used to characterize an instructional activity colloquially referred to as *chalk talk,* in which the instructor presents pre-prepared material by "writing out a mathematical narrative on the board while talking aloud" (Artemeva & Fox, 2011, p. 345). This pervasive, and largely passive, form of instruction is typically presented as antithetical to more "active" instructional practices, such as small group work and whole-class discussion. For instance, in a 2015 call for instructional change, the Common Vision Committee — a collaboration between the American Mathematics Association for Two-Year Colleges (AMATYC), the American Mathematical Society (AMS), the American Statistical Association (ASA), the Mathematical Association of America (MAA), and the Society for Industrial and Applied Mathematics (SIAM) — stated:

> Across the guides we see a general call to move away from the use of traditional lecture as the sole instructional delivery method in undergraduate mathematics courses [...] Even within the traditional lecture setting, we should seek to more actively engage students than we have in the past [...] Oft-cited examples are active learning models where students engage in activities such as reading, writing, discussion, or problem solving that promote analysis, synthesis, and evaluation of class content. Cooperative learning, problem-based learning, and the use of case studies and simulations are also approaches that actively engage students in the learning process. (Saxe & Braddy, 2015, p 19)

As lecturing has been practically defined as the converse of active learning, investigating the factors that encourage or inhibit the use of lecturing simultaneously (to some extent) investigates the factors that encourage and inhibit the use of active learning.

While educational researchers consistently argue that beliefs are undoubtedly influential factors on instructional practice (e.g., Speer, 2008), the research literature is rife with examples of inconsistencies between beliefs and practice (e.g., Cooney, 1985; Johnson et al., 2017; Smith et al., 2014). One avenue for reconciling these discrepancies is by looking at reward structures and structural constraints, such as course coordination, administrative and departmental support, and promotion and tenure considerations (Allman et al., 2018; Hayward et al., 2016; Hagman et al., 2017; Henderson & Dancy, 2007; Hora 2012; Johnson et al., 2013; McDuffie & Graeber, 2003). Researchers have hypothesized that such contextual factors could influence instruction by contributing to content coverage concerns and loss of autonomy (e.g., Hagman et al., 2017).

To better understand how contextual factors may influence the relationship between beliefs and practice, we look at instruction in two very different undergraduate mathematics courses: Calculus I and Abstract Algebra. In the U.S. these two very different courses allow us to consider how elements such as course coordination (e.g., common textbooks, common exams, set syllabi), pressures from client disciplines, and required follow-up courses may mediate the relation between beliefs and practice. By better understanding the relationships between beliefs and practice, and the extent to which this relationship may be mediated by instructional context, researchers and change agents may be better positioned to consider how contextual and individual factors can be leveraged for instructional change in undergraduate mathematics.

Literature Review

Our goal here is to better understand how beliefs impact instructional practice and how course context may be mitigating that impact. To frame our work, we posit that instructors make *choices within constraints* (Ingram & Clay, 2000). We assume that instructors are rational beings and, without constraints, there would be a strong relationship between an instructor's beliefs about instruction and their actual instructional practice. However, what the literature consistently finds is a lack of agreement between these. Instead of taking such inconsistencies as an indication of irrationality on the instructor's part, we follow the call of Hoyles (1992) and Leatham (2006) and, instead, assume both espoused and enacted beliefs vary sensibly with context.

Beliefs

In his chapter in the *Second Handbook of Research on Mathematics Teaching and Learning,* Philipp (2007) acknowledges some of the disparities in the research literature around the use of the terms beliefs, knowledge, and attitudes and offers descriptions and definitions that can be used to parse these terms. Beliefs are described as "psychologically held understandings, premises, or propositions about the world that are thought to be true" and the difference between beliefs and knowledge as "whether one holds the conception as beyond question" (p. 259). For the practical purposes of this paper however, arguing if a teacher "knows" that their students are academically prepared or simply "believes" that to be true is beyond what we could reasonably justify. Thus, we follow Pajares (1992) conceptions of beliefs that do away with distinctions between attitudes, beliefs, knowledge, and values and adopt Leatham (2006) conception of beliefs as what we "just believe":

> Of all the things we believe, there are some things that we ''just believe'' and other things that we ''more than believe – we know.'' Those things we ''more than believe'' we refer to as knowledge and those things we ''just believe'' we refer to as beliefs. (p. 92)

As previously stated, although educational researchers consistently argue that beliefs are influential factors on instructional practice (e.g., Speer, 2008), there are extensive examples of inconsistencies between beliefs and practice within the literature (e.g., Cooney, 1985; Johnson et al., 2017; Smith et al., 2014). Here we present an overview of both, and argue for, the importance of considering contextual factors that may be mitigating the strength of the relationship between beliefs and instructional practice.

Beliefs and Instructional Practice

It has been documented that how instructors teach is influenced by what they believe their role is as a teacher (e.g., Burn & Mesa, 2015; Mesa et al., 2014; Speer, 2008; Weber, 2004). For example, in a fine-grain investigation of one instructor's beliefs, Speer (2008) identified that this instructor "saw his role as 'guide' where his goal was to help students learn to use their resources (cognitive and others) well, to support their problem solving, and to ensure that their understanding of the ideas was strong" (p. 234). Speer connected this set of beliefs to how this instructor sought to provide support and guidance during instruction. Additionally, Mesa et al. (2014) found connections between instructors' beliefs about their role and their instructional practice: instructors who incorporated an active learning approach believed that students should build self-confidence and motivation instead of mastering the content; and, instructors who implemented a lecture-based mode of instruction viewed their role as an authority meant to disseminate knowledge to students.

Authoritative views of one's role are not the only beliefs that could contribute to a traditional lecture-based mode of instruction. For instance, Weber (2004) found that, although an instructor in his study wanted to focus on higher-order skills, they were concerned that students might become frustrated and give up. One interpretation of this concern is that the instructor believed it was their role to reduce frustration and provide support and step-by-step explanations for students, and therefore may have felt a conflict between their instructional goals and how they envisioned their own role in the classroom. As a result of this conflict, instructional practice may favor a "procedural lecture style" (p. 128).

Instructors also choose instructional approaches depending on their view of learning (Burn & Mesa, 2015; Mesa et al., 2014; Speer, 2008; Weber, 2004). For instance, Johnson et al. (2017) found instructors' views of learning, particularly their views of lecture as an instructional practice, to be a significant predictor of their instructional practice. Some instructors may define learning as doing mathematics, while others define it as knowing and understanding (Speer, 2008). For example, Burn and Mesa (2015) found that faculty believe students should focus on higher-order thinking skills but also believe that students most often focus on memorization. Instructors also agreed that procedural fluency was necessary before understanding ideas in Calculus. The number and type of assignments given in class then varied based on this view of learning accordingly. Some instructors used quizzes throughout the semester as formative assessments, while other instructors used mostly exams as summative assessments. These included basic procedural tasks as well as complex, rich tasks depending on how the instructor viewed assessments of learning.

Similarly, Weber (2004) found that one instructor believed that students learned best when they were provided a foundation to build their knowledge upon. This began with providing students with experiences to work with the content, understand procedures, and then replicate the procedures. In another study, Mesa et al. (2014) found that some instructors who incorporated an active learning approach believed that students should build self-confidence and motivation instead of mastering the content. However, some of the instructors who adopted a lecture style expressed a belief that indicated students could not learn mathematics independently. This set of beliefs was summarized by Mesa et al, as:

> Students remain at a different level, a level in which mathematics is inaccessible, whereas mathematics instructors represent an elite with access to knowledge that the majority does not have… This perception ensures the importance of the instructor in a traditional model; the instructor must bring mathematics down to the students' level. (p. 134)

With these beliefs, students cannot have direct access to new content on their own, not even with their textbooks. Thus, the teacher must serve as a ‘‘translator’’ of mathematics, lecturing the material to their students in a way that they can grasp.

Instructors' beliefs about mathematical content have also been found to impact instructional practice. Remillard (2005) found that the type of curriculum instructors implemented in the classroom was centered on their teaching beliefs. In other instances, some instructors reported pressure to cover content, and as a result, they felt they did not have enough time to teach difficult concepts (Johnson et al., 2017). For example, Johnson et al. (2017) found that both lecturers and non-lecturers who taught Abstract Algebra agreed "that there wasn't enough time for all the content and that they felt pressured to cover topics quickly without enough time to help students understand difficult ideas" (p. 273).

Especially in a "gatekeeper" course such as Calculus, instructors may feel it is particularly important for students to master content. Mesa et al. (2014) support this finding by indicating that many undergraduate instructors who choose to adopt a lecture style of teaching prefer to "cover predefined content in a set time" (p. 132). These instructors believe that it is more important to cover all the content "even if students cannot keep up with the pace of the lecture" (p. 132). However, in contrast, it has been shown that most "Calculus I faculty believe they have enough time to cover required material and do not feel undue pressure to move too quickly through material" (Burn & Mesa, 2015, p. 47). Johnson et al. (2016) support this finding by noting that the majority of Calculus I instructors indicated they had enough time to cover material. However, they found that most of those instructors taught in a variety of lecture and active learning classes, whereas instructors in more lecture-based classes felt they did not have enough time to cover the content.

Despite the body of research exemplifying how instructors' beliefs about their own roles, students, learning, and content influence instructional practice, the literature also contains many examples of "inconsistencies" between beliefs and instructional practice. For instance, in a study conducted by Johnson et al. (2017) "Of the 45 participants who disagreed with the statement "*I think lecture is the best way to teach*" only 16 did not lecture" (p. 24) Findings such as these may give the impression that instructors' instructional practices are inconsistent with what instructors reported they do in the classroom. This framing of inconsistency is more broadly documented in Hoyles (1992) meta-analysis of beliefs literature. Hoyles highlights one trend in beliefs research: the focus on identifying inconsistencies between instructors' classroom actions and professed beliefs. Hoyles notes that such work often takes the deficit stance that teachers and their beliefs can be accounted for as obstacles to curricula reform.

The Importance of Context

Hoyles (1992), Leatham (2006), and Speer (2005) critique the prior deficit framing of instructors' beliefs on methodological and theoretical grounds. Their critiques range from the need to account for context to researchers and participating instructors lacking a shared understanding of terms. As an alternative, Hoyles (1992) advocates a situated beliefs framework where beliefs are products of activity, context, and culture and that "any individual can hold multiple (even contradictory) beliefs and "mismatch", "transfer" and "inconsistency" are irrelevant considerations and replaced by notions of constraint and scaffolding within settings" (p. 40). Leatham (2006) holds similar views. In this paper, following Hoyles (1992) and Leatham (2006), we also take the anti-deficit perspective that beliefs vary sensibly with context. Instead of concluding that instructors are inconsistent, we hold that there are a host of other factors that shift their beliefs and resulting instructional decision-making.

As a way to discuss the factors that constrain instructional choices, we consider the *external framing* of a course (Hoadley, 2003). External framing refers to the influence of agents outside of the classroom—such as administrators, professional societies, policy documents, and client disciplines—over various aspects of teaching, including coverage and pacing. Consider the external framing of these two very different courses: Calculus I and Abstract Algebra. A survey conducted in 2010 concluded that, in the US, 98% of Calculus I students did not indicate math as their career goal, with 31% indicating a career in engineering (Bressoud, 2015). As a result, Calculus I is primarily a service course, where client disciplines (e.g., engineering, physics, chemistry) and the university more broadly (in enforcing quantitative course requirements) have a strong influence over what topics are taught. Additionally, because of the large number of sections of the same course taught at a university in any given semester, Calculus I courses are often coordinated (Rasmussen & Ellis, 2015). This coordination frequently includes a common syllabus, schedule, textbook, exams, and homework. In these ways, Calculus I instructors may be experiencing this course as having strong external framing, in that these instructors may not feel like they have control or agency over how they teach (Hoadley, 2003; Hagman et al., 2017).

In comparison, Abstract Algebra in the U.S. is normally taught in small classes (typically fewer than 40 students per section of the same course) consisting of junior and senior mathematics majors, with only a small proportion that will subsequently enroll in a follow-up course (Johnson et al., 2017). It is common for at most a handful of sections to run per year, and thus, issues of coordination across multiple sections and instructors are few. More often than not, these courses are taught by tenured or tenure-track professors who perhaps enjoy greater job security and more autonomy over course material than non-tenure track or itinerant faculty. In these ways, Abstract Algebra instructors may be more likely to experience this course as having weak external framing, in that they feel a greater sense of control and agency over this course. For instance, in a study by Johnson et al. (2017), only 4 out of 129 abstract algebra instructors answered "no" when asked if they believed they had the freedom to make changes to the content in their course.

If our assumptions are correct, we would expect to see a stronger relationship between instructors' beliefs and their instruction in less constrained contexts. To test this hypothesis, we investigated the following research questions:

1. For Calculus I instructors, what is the relationship between beliefs and instructional practice?
2. For Abstract Algebra instructors, what is the relationship between beliefs and instructional practice?

By running parallel factor analyses and regression models to answer these two questions, we aim to understand the extent to which external framing may be mitigating the relationship between beliefs and practice. Our discussion focuses on comparing our parallel analyses with an eye towards the ways in which these courses differ in U.S. universities. As such, our discussion addresses a third research question:

3. How might contextual factors mitigate the relationship between instructors' beliefs and instructional practice?

Methods

This paper draws on two sets of pre-existing data. One is from the Mathematical Association of America's (MAA) 2010-2012 National Science Foundation (NSF) supported survey study on the *Characteristics of Successful Programs in College Calculus* (CSPCC). The CSPCC data includes Calculus instructor pre- and post-survey responses from over 200 institutions across the U.S. (community college up to Ph.D. granting institutions). Instructors surveyed were those teaching the first course in the calculus sequence. In the US, calculus content is typically divided up into a sequence of undergraduate courses. While there is variation in content, the first course typically covers functions, limits, derivatives, derivative rules and applications, and initial integration techniques (Johnson, 2016). Previous analyses of the CSPCC data set indicated that students typically take calculus to meet general college mathematics requirements or requirements for specific majors, such as science, technology, engineering, or mathematics (Selinski & Milbourne, 2015). Additionally, we know that calculus courses are taught by tenured or tenure-track faculty, non-tenure-track faculty, part-time faculty, and/or graduate students (Selinski & Milbourne, 2015).

The CSPCC data was part of the initial phase of understanding calculus instruction nationwide. Two surveys were sent out to instructors, once before the fall 2010 term and then at the end of the term. Instructors were surveyed on demographic information, expectations of student success, teaching practices, departmental influences, and beliefs on instruction. In looking at the descriptive statistics for the instructors in our analysis, 47% were tenure-stream, suggesting that more than half the instructors may not have the flexibility in their teaching that a tenure-stream faculty might have as prior research has posited (e.g., Hagman et al., 2017). Additionally, 68% of these instructors reported having a common final exam or a combination of a common final and their own exam (62% and 6%, respectively). This suggests a high degree of external framing, which may impede instructors' autonomy in making instructional decisions. Further details of the CSPCC study can be found in Bressoud et al. (2015).

The other data set comes from a 2015/2016 survey of Abstract Algebra (AA) instructors. Specifically, the AA data came from surveys of undergraduate abstract algebra instructors across approximately 200 Bachelor's, Master's, and Ph.D. granting institutions nationwide. Undergraduate AA in the US is taught as a single course or as a pair of courses. In the introductory or single course, instructors emphasize proofs and cover various aspects of groups (e.g., homomorphisms, isomorphisms) and (occasionally) rings and fields (Johnson et al., 2017). Students taking AA are usually students majoring in mathematics and are nearing the end of their undergraduate careers (Johnson et al., 2017). Typically, tenure or tenure-track faculty with greater job security and decision-making autonomy over course content are in charge of teaching AA (Johnson et al., 2017). Previous research on the data found 92% were tenure-stream professors (Johnson et al., 2017). Compared to the instructors represented in the CSPCC data set, the instructors in the AA data set are likely to experience greater autonomy and fewer (if any) course coordination obligations.

In the AA survey, the survey developers adapted survey questions from Henderson and Dancy (2009) physics education survey and the CSPCC survey (Johnson et al., 2017). Questions asked instructors about demographic information, factors that influenced their teaching, their teaching practices and motives behind them, beliefs about teaching and learning, and their willingness to change teaching practices (and why). Details on the AA study can be found in Fukawa-Connelly et al. (2016).

Given the influence of the CSPCC survey on the creation of the AA survey, and the overlap of survey items and sections (e.g., self-reported teaching practice, beliefs, and influences), we felt these two data sets were similar enough to be comparable. However, there were also some significant differences in the wording of some of the items – most notably in the questions focused on instructional practice. As such, we chose not to combine these two data sets. Rather, we focused on running two parallel models from the two surveys. This study design reflects the post-hoc, exploratory, nature of our investigation. The survey instruments, and the sampling techniques, used to construct the two data were different enough that we could not directly measure the impact of course context. Instead, each model will, independently, assess the relationships between the beliefs (as determined by factor analyses) and instructional practice (as determined by regression analyses). Then, by comparing the two models, we hope to gain insights into the extent to which course context may be influencing this relationship.

Survey Item Identification

For our study, we sought to identify groups of instructors' beliefs that might relate to instructional practices. The two researchers in this study independently coded the data, looking for survey items related to beliefs, and then met to discuss their coding and reconcile differences.

The first author decided to forefront survey items that asked respondents to express the value of, or belief about, certain instructional practices. While many of the survey items asked instructors questions about the frequency of certain practices (e.g., "*Approximately what percentage of class time do you spend having students work in* groups?"), other survey items were Likert scale responses on a scale from disagree to agree regarding statements such as "*Calculus students learn best from lectures, provided they are clear and well-organized*." Such statements, which sanction specific instructional practices, were taken as indicators of instructors' beliefs in favor of such practices. Other items were selected in accordance with the research literature on beliefs. For instance, we selected instructors' estimates of students who would pass or fail because of the literature on how instructors' beliefs about students impact instruction (Burn & Mesa, 2015; Mesa et al., 2014; Speer, 2008; Weber, 2004). Another example was the inclusion of questions asking instructors how influential they believed their experiences as students and instructors were on their teaching because prior literature identified such experiences as impacting instruction (Oleson & Hora, 2014).

The second author identified belief items by looking through the items and noting the ones that focused on how instructors believe or think about teaching practices and learning. Examples of these items included asking instructors questions that began with the statement "*I think*." These items asked instructors to agree or disagree with such statements, thus providing insight into their personal beliefs concerning teaching practices and learning. For example, items including statements such as "*I think lecture is the best way to teach*" or "*I think that all students can learn advanced mathematics*" provide insight into instructors' beliefs about the best instructional practices for helping students learn. Other items identified as belief items related to the order in which instructors thought the material should be covered and how their personal experience as a student and teacher impact their instructional practices. For example, as Oleson & Hora (2014) noted, an instructor's instructional practice is impacted by their personal experiences in a mathematics classroom and as a learner. Furthermore, it has been documented that instructors' beliefs about content impact how they teach, whether they feel pressured to cover content (Johnson et al., 2017; Mesa et al., 20140) or take their time and focus on helping students master the content (Burn & Mesa, 2015; Johnson et al., 2016).

We retained items that we could reach a consensus on as indications of beliefs. Once this was done, we had numerous survey items in each data set, and we wanted to see their effect on instruction. We took the reported percentage of class time spent lecturing to be our sole dependent variable capturing instructional practice for both the CSPCC and AA data sets. As lecturing has been practically defined as the converse of active learning (e.g., Johnson et al., 2017), this variable serves as a proxie (albeit imperfect) for the amount of time spent doing "active learning" instructional approaches.

Survey Items and Factor Analyses

Based on our literature review, we identified numerous survey questions related to instructors' beliefs which we suspected would influence the amount of time spent lecturing; 16 were initially identified in the CSPCC survey and 23 in the AA survey. However, some of these items appeared to be related and we suspected they might be different dimensions of an underlying latent variable. For instance, in the CSPCC survey, we identified a handful of questions all related to beliefs about student preparation and ability (e.g., "*Approximately what percentage of students currently enrolled in your Calculus I course do you expect are academically prepared for the course*" and "*Estimate the percentage of students currently enrolled in your Calculus I course that will receive a grade of D or F*"). In the AA survey, we had multiple questions related to beliefs about lecture as an instructional approach (e.g., "*I think lecture is the best way to teach*" and "*I think lecture is the only way to teach that allows me to cover the necessary content*").

Even though there appeared to be some relationships between these survey items, we decided to begin our analysis with an exploratory factor analysis, as we were working with existing instruments which did not necessarily have the validity and reliability testing we would have liked in order to make a priori constructs representing such groups of beliefs. The exploratory factor analysis was then used to created composite independent variables based on the factors. These composites were then put into a regression analysis to predict time spent lecturing for each data set; however, only factors that could be theoretically supported based on the available research literature were used to create composite variables for the regression analysis.

Iterative factor analyses were run with different numbers of items while eliminating cross-loaded items or items with loadings less than 0.4. Cross-loaded items were removed because they carried multiple interpretations, each of which related to factors involving different meanings. Items with loadings less than 0.4 on all factors were considered too weak to impact interpretation. Deletions in both cases were thus undertaken to avoid confounding our interpretation of the factors our factor analyses would produce. Ultimately, we included 13 and 20 items in our final CSPCC and AA factor analyses, respectively. The CSPCC data resulted in a four-factor solution

(PROMAX rotated), explaining 54.72% of the variance in teacher responses on each factor. The AA data resulted in a five-factor solution (PROMAX rotated), explaining 68.22% of the variance in teacher responses on each factor.

As our intention in running the factor analyses was to create constructs that would predict time spent lecturing, we then examined each factor to see if the items that grouped together represented an actual construct that made theoretical sense. For factors that did seem to carry an underlying representative belief or set of beliefs, items under that factor were standardized, with items that loaded negatively being reverse coded. These items were then averaged together to create a composite variable representing that factor and underlying belief. This was not done for factors that could not be made sense of theoretically, and thus these factors were not carried forward into the regression analyses.

Regression Analysis

For this study, we were interested in looking for composites with significant effects on the amount of time spent lecturing (as a measure of teaching practice). The dependent variable for our CSPCC analyses had instructors rate on a scale from 0 (*Not at all*) to 5 (*Very often*), the statement "*During class time, how frequently did you lecture"* (mean= 4.20, median=5, mode=5). For the AA analyses, teachers answered on a scale from 0 (*Never*) to 4 (*75-100%*) the question "*While teaching, what is the approximate amount of time per class that you are lecturing"* (mean= 2.64, median=3, mode=3). These are ordered categorical dependent variables (with at least five categories); thus, multiple regression was an appropriate statistical test. For each data set, the dependent variable of time spent lecturing was regressed on the centered composite independent variables specific to that data set. Descriptive statistics for the variables of interest for each data set are presented in Tables 1 and 8 of the Appendix.

In terms of diagnostic tests and model assumptions, VIF values for both data sets were close to 1 (as indicated in Tables 4 and 7), indicating that multicollinearity was not an issue. We tested linearity by fitting a Loess line on the plots of standardized predicted values against standardized residuals and entering centered power terms sequentially into separate regression models. We checked homoscedasticity by examining the spread of the plots for irregularities. For the CSPCC data, the spread

Table 1 Predictors of Time Spent Lecturing

CSPCC Lecture Variable (*N*=493)	Frequency	AA Lecture Variable (*N*=219)	Frequency
0 (not at all)	11	Never	5
1	11	0-25% of the time	33
2	17	25-50% of the time	54
3	62	50-75% of the time	71
4	118	75-100% of the time	56
5 (very often)	274		

of the data suggested homoscedasticity was a reasonable assumption while the Loess line and statistically significant quadratic model ($F[4, 424] = 2.61$, $p < .05$) suggested linearity may not hold (refer to Tables 9 and 10 and Fig. 1). For the AA data, the spread of the data suggested homoscedasticity was not met. However, this is a typical occurrence when dependent variables are ordinal, as was the case with our data. Curvilinear tests for the AA data suggested linearity was met (Tables 9 and 11 and Fig. 1). Histograms of residuals and P-P plots indicated normality of residuals was satisfied for the AA data but not for the CSPCC data (Figs. 2 and 3). We checked for outliers by plotting centered Leverage values against an instructor ID variable, which indicated the CSPCC data had outliers that could skew the regression analysis results (Fig. 3). This issue is further compounded by the fact that the dependent variable in the CSPCC data is highly skewed, with over half the participants indicating the highest value. This violation of the regression model assumptions may lead to incorrectly estimated p-values (Fig. 4).

Taken together, these tests suggest that the results of our regression analyses may be inflated for both data sets, particularly for the CSPCC data. This means statistically significant relationships we find may not actually be significant (i.e., a false positive), or non-significant results may actually be significant (i.e., false negative). Given that some assumptions were met, violated assumptions were reasonable, and the robustness of regression analysis to violations of assumptions, we determined it was still appropriate to continue with our analysis.

Results

In what follows, we present the results according to each data set separately. We first present the factor analysis then the regression analysis. With regard to the factor analysis, factors and detailed descriptions of included variables are presented below. More concise tables with factor loadings and Cronbach's alpha for each factor's items are provided in tables following the detailed descriptions.

CSPCC Factor Analysis

Table 2 displays the factor loadings for the CSPCC data. The CSPCC survey was designed to gain insight into faculty beliefs concerning "Student Success, Their Primary Role as Instructors, and Students' Approach to Studying Calculus I" (Burn & Mesa, 2015, p. 49). As such, we named our factors with an eye towards the constructions Burn and Mesa put forth. The variables loading onto the first factor asked instructors to estimate what percentage of their students were prepared for the course, and would pass, fail, or withdraw. For instance, reporting higher values for the number of students who will withdraw or fail (the two highest loading items) suggests instructors believe more of their students will achieve a grade lower than a C. As such, we interpreted the factor as representing *Expectations of student success*. Higher values for the composite variable corresponded to believing *less* in their students' academic abilities and success.

Table 2 Results of Factor Analysis of CSPCC Variables with PROMAX Rotation

Variables	Factor Loading
Factor 1: Expectations of student success (α = .81)	
Approximately what percentage of students currently enrolled in your Calculus I course do you expect are academically prepared for the course	.61
Estimate the percentage of students currently enrolled in your Calculus I course that will withdraw	.80
Estimate the percentage of students currently enrolled in your Calculus I course that will receive a grade of D or F	.79
Estimate the percentage of students currently enrolled in your Calculus I course that will receive a grade of C or better	-.99
Factor 2: Transmissive Beliefs about student learning (α = .49)	
When students make unsuccessful attempts when solving Calculus I problems	.63
Calculus students learn best from lectures, provided they are clear and well-organized	.78
Understanding ideas in calculus typically comes after achieving procedural fluency	.55
Factor 3: Conceptions of mathematics as connected (α = .46)	
Students' success in Calculus I PRIMARILY relies on their ability	.75
My primary role as a Calculus instructor is to	.71
I intend to show how mathematics is relevant	.59
Factor 4 (α = .34)	
In solving Calculus I problems, graphing calculators or computers help students	-.46
I would continue to teach calculus	.68
Familiarity with the research literature on how students think about ideas in calculus would be useful for teaching	.76

The second factor consisted of the following questions:

1. *From your perspective, when students make unsuccessful attempts when solving a Calculus I problem, it is: 0 (a natural part of solving the problem) to 5 (an indication of their weakness in mathematics)*,
2. Rate on a scale of 0 (*Strongly Disagree*) to 5 (*Strongly Agree*) the statement *Calculus students learn best from lectures, provided they are clear and well-organized*,
3. Rate on a scale of 0 (*Strongly Disagree*) to 5 (*Strongly Agree*) the statement *Understanding ideas in calculus typically comes after achieving procedural fluency*.

Burn and Mesa (2015) had considered these items as instructor's cognitive goals for students and beliefs about studying. We saw the second two items as representing views that students learn by first digesting clearly presented material (2nd question, a teacher-centric view) and then reproducing presented procedures to really learn (3rd question), a possible characteristic of teacher-centered instruction (Fukawa-Connelly et al., 2012). For instructors taking such a perspective, making a mistake in reproducing a procedure could then indicate a student had not "received" the knowledge properly, thus connecting to the first question in lower values indicated instructors believed mistakes indicate a flaw in understanding. Considering all these items together then, we interpreted this factor as representing beliefs about how students learn and thus called the composite *Transmissive beliefs about student learning*. Higher values on the composite variable corresponded to holding more teacher-centered beliefs about learning whereas lower values corresponded to holding more student-centered beliefs about learning.

The third factor consisted of the questions:

1. *From your perspective, students' success in Calculus I PRIMARILY relies on their ability to: 0 (solve specific kinds of problems) to 5 (make connections and form logical arguments)*,
2. *My primary role as a Calculus instructor is to: 0 (work problems so students know how to do them) to 5 (help students learn to reason through problems on their own)*,
3. Rate on a scale of 0 (*Strongly Disagree*) to 5 (*Strongly Agree*) the statement *In my teaching of Calculus I, I intend to show students how mathematics is relevant.*

Burn and Mesa (2015) had considered the first two items as instructors' beliefs about their role as teachers and students' studying. Both questions also have an underlying diptych in how one's conception of mathematics impacts student academic success. Higher values suggest a conception that mathematics is connected, and student success relies on students themselves working through those connections. Lower values suggest that student success is not dependent on students conceptualizing mathematics as connected. From this perspective, both items reference the internal connections in mathematics.

The last item, however, focuses on purveying a conception of mathematics as relevant. The relevance of mathematics can be thought of as seeing mathematics' connections to topics outside mathematics. With this last item considered alongside the previous two, we interpreted these items as reflecting instructors' beliefs about what conceptions they wanted to portray to their students (mathematics as internally and/or externally connected). Thus, we called the composite *Conceptions of mathematics as connected.* Higher values on the composite variable suggest that instructors sanction conveying a conception of mathematics as connected internally and externally. In comparison, lower values suggest instructors do not prioritize conveying such a conception.

The fourth factor consisted of:

1. *From your perspective, in solving Calculus I problems, graphing calculators or computers help students to: 0 (understand underlying mathematical ideas) to 5 (find answers to problems)*,
2. Rate on a scale of 0 (*Strongly Disagree*) to 5 (*Strongly Agree*) the statement *If I had a choice, I would continue to teach calculus*,
3. Rate on a scale of 0 (*Strongly Disagree*) to 5 (*Strongly Agree*) the statement *Familiarity with the research literature on how students think about ideas in calculus would be useful for teaching*.

Table 3 Model Summary

Data	R	R^2	SE	ΔR^2	ΔF	df1	df2	Significance level
CSPCC	.163	0.027	1.159	0.027	3.887	3	428	0.009

This factor seemed to include many disparate ideas, and thus this factor was a bit difficult to interpret. Furthermore, Cronbach's Alpha for this variable was only .34, and Giannoulis (n.d.) states, "scales that are multidimensional will cause alpha to be under-estimated if not assessed separately for each dimension." Given these uncertainties around whether the fourth factor captured a coherent construct, we did not carry forward the fourth factor into our CSPCC regression analysis.

CSPCC Regression Analysis *Expectations of student success, Beliefs about student learning,* and *Conceptions of mathematics,* together accounted for only 2.7% of the variance in the time spent lecturing. However, the overall multiple regression was statistically significant ($F[3, 428] = 3.89$, $p < .05$). These results are presented in Table 3.

As presented in Table 4, there were statistically significant effects of *Transmissive beliefs about student learning* on CSPCC instructors' time spent lecturing ($\beta_{learning} = .15$, $t = 3.08$, $p < .05$). Considering the positive beta value, the more CSPCC instructors believed in the effectiveness of teacher-centered instructional practices where students reproduce presented procedures, the more likely they were to spend time lecturing.

While the regression model is statistically significant, it was not practically significant. Cohen (1998) notes that models with R^2 values around .10 are considered to have a small practical effect, and our variance fell well below even this. To compound this issue, *Transmissive beliefs about student learning* was the only variable that was a significant predictor of time spent lecturing. Keith (2015, p. 62) notes that beta values between .10 and .25 are considered to have a moderate effect size, and so *Transmissive beliefs about student learning* had a moderate practical impact on instructor time spent lecturing.

AA Factor Analysis Table 5 displays the factor loadings for the AA data. The variables loading onto the first and second factors related to topics instructors felt they:

Table 4 Predictors of Time Spent Lecturing

Variable	B	SE	beta	*t*	Significance level	VIF
CSPCC data (N=432)						
Constant	4.192	0.056		75.149	0.000	
Expectations of student success	-0.121	0.075	-0.077	-1.604	0.109	1.024
Transmissive beliefs about student learning	0.255	0.083	0.150	3.078	0.002	1.042
Conceptions of mathematics as connected	-0.044	0.085	-0.025	-0.518	0.605	1.025

Table 5 Results of Factor Analysis of AA Variables with PROMAX Rotation

Variables	Factor Loading
Factor 1: Focus on fields and rings (α = .91)	
Rings	.84
Fields	.82
Field extensions	.66
Ring isomorphisms	.88
Ring homomorphisms	.90
Polynomial rings	.86
Factor 2: Focus on groups (α = .85)	
Groups and subgroups	.69
Group isomorphisms	.83
Group homomorphisms	.86
Quotient groups	.83
Lagrange's theorem	.69
Fundamental homomorphism theorem	.81
Factor 3: Facilitation authority (α = .71)	
I think lecture is the best way to teach	.63
I think lecture is the only way to teach that allows me to cover the necessary content	.62
I think students learn better when they struggle with the ideas prior to me explaining the material to them	-.80
I think students learn better if I first explain the material to them and then they work to make sense of the ideas for themselves	.74
Factor 4: Expectations of student success (α = .93)	
I think that all students can learn advanced mathematics	.94
I think all students can learn abstract algebra	.96
Factor 5: Personal influences on teaching (α = .56)	
Your own experiences as a student	.83
Your own experiences as a teacher	.83

0 (*would not cover*), 1 (*try to teach*), or 2 (*always teach*). The first factor consisted of rings, fields, field extensions, ring isomorphisms, ring homomorphisms, and polynomial rings. Those topics all relate to rings and fields, and thus we called the factor *Focus on fields and rings*. The second factor consisted of groups and subgroups, group isomorphisms, group homomorphisms, quotient groups, Lagrange's theorem, and the fundamental homomorphism theorem. Those topics all relate to groups, and thus we called the factor *Focus on groups*. Research teams argue that content plays an important role in determining "curricular elements" (Burn & Mesa, 2015, p. 45). Personal beliefs deriving from past experience influence decisions concerning what content should be covered. When time is constrained, it is often the case that certain content may need to be dropped in order to cover more pertinent content. Therefore, we interpreted these content coverage factors as signaling beliefs on what content instructors believe is important to cover. Higher values on the composite for either of these factors signaled instructors believed in covering these topics. In

comparison, lower values on the composites signaled instructors did not believe in covering these topics.

The third factor consisted of the following statements instructors rated on a 4-point scale of -2 (*Strongly Disagree*) to 2 (*Strongly Agree*):

1. *I think lecture is the best way to teach*,
2. *I think lecture is the only way to teach that allows me to cover the necessary content*,
3. *I think students learn better when they struggle with the ideas prior to me explaining the material to them*,
4. *I think students learn better if I first explain the material to them and then they work to make sense of the ideas for themselves*.

Higher values for questions 1, 2, and 4 center the teacher as the source of knowledge. Often in lecture classes, the instructor is viewed as "the more knowledgeable other" (Vygotsky, 1978). This "refers to someone who has a better understanding or a higher ability level than the learner, with respect to a particular task, process, or concept" (Galloway, 2001, p. 48). In this sense, the instructor is the one who controls the production of knowledge and shares, explains, and discusses ideas. Question 3 shifts this control though; higher values focus on students being the starting point as opposed to instructors. In classes using inquiry-oriented instruction, for example, instructors act as facilitators and allow students to engage with content, share their own ideas, and then discuss the ideas shared (Kuster et al., 2017). In this sense, students produce knowledge, and the instructor facilitates the activities and discussions. Looking at all these items together then, we interpreted these questions as reflecting a focus on who instructors believe should control knowledge facilitation, and thus we called the composite *Facilitation authority*. Higher values on the composite signaled beliefs that instructors should introduce, explain, and discuss ideas. In comparison, lower values signaled beliefs that students should be in charge of the production of ideas while instructors facilitate on the side.

The fourth factor consisted of the following statements instructors rated on a 4-point scale of -2 (*Disagree*) to 2 (*Agree*):

1. *I think that all students can learn advanced mathematics*
2. *I think all students can learn abstract algebra*.

We interpreted these questions as reflecting instructors' beliefs about students' learning abilities and academic success, paralleling the *Expectations of student success* factor in the CSPCC data. As discussed above, the CSPCC survey was designed to gain insight into faculty beliefs concerning "student success, their primary role as instructors, and students' approach to studying calculus I" (Burn & Mesa, 2015, p. 49). Mirroring these commonalities with the CSPCC variable, we called the composite *Expectations of student success*. Higher values for the composite variable corresponded to believing *more* in their students' academic abilities, a reversal of the

Table 6 Model Summary

Data	R	R^2	SE	ΔR^2	ΔF	df1	df2	Significance level
AA	.615	0.378	0.850	0.378	19.576	5	161	< 0.001

coding scheme for the *Expectations of student success* composite in the CSPCC data (where higher values corresponded to believing *less*).

The fifth factor consisted of items asking instructors to rate how influential instructors' experiences as: 1) students and 2) as teachers were on their teaching on a 3-point scale of 1 (*Not at all*) to 3 (*Very*). These reflect the personal experiences instructors believe impacted their teaching. It has been documented that instructors often teach the way they were taught (Oleson & Hora, 2014). It has also been determined that instructors' teaching practices are influenced by instructors' lives: (a) as an instructor, (b) as a student, (c) in non-academic roles, and (c) as a researcher (Oleson & Hora, 2014). Based on this research, it was reasonable that these two items loaded together. In turn, we called the composite *Personal influences on teaching*. Higher values on the composite corresponded to instructors believing more strongly that their experiences as a student and/or instructor have influenced their teaching. In comparison, lower values correspond to believing less in the influences of these prior experiences.

AA Regression Analysis *Focus on fields and rings, Focus on groups, Facilitation authority, Expectations of student success, and Personal influences on teaching* together accounted for 37.8% of the variance in the time spent lecturing. The overall multiple regression was statistically significant ($F[5, 161] = 19.58$, $p < .05$). These results are presented in Table 6.

As presented in Table 7, there were statistically significant effects of *Focus on groups, Facilitation authority*, and *Expectations of student success* on AA instructors' time spent lecturing ($\beta_{groups} = .17$, $t = 2.74$, $p < .05$; $\beta_{authority} = .49$, $t = 7.32$, $p < .05$; $\beta_{expectations} = -.15$, $t = -2.26$, $p < .05$). Thus, the more AA instructors believed in their students' capacity to learn AA, the less likely they were to spend time lecturing. By contrast, the more AA instructors believed in covering the various concepts associated with groups or believed in their role as the driving source for knowledge creation, the more likely they were to lecture.

Table 7 Predictors of Time Spent Lecturing

Variable	b	SE	beta	*t*	Significance level	VIF
AA data (N=167)						
Constant	2.569	0.066		39.064	0.000	
Focus on fields and rings	0.155	0.081	0.120	1.900	0.059	1.025
Focus on groups	0.226	0.082	0.174	2.741	0.007	1.039
Facilitation authority	0.710	0.097	0.489	7.317	0.000	1.154
Expectations of student success	-0.166	0.074	-0.149	-2.258	0.025	1.120
Personal influences on teaching	0.110	0.086	0.081	1.285	0.201	1.020

It is important to note that the regression model is both statistically and practically significant. Cohen (1998) notes that R^2 values around .30 are considered to have a medium effect size while R^2 values around .50 are considered to have a large effect size. Thus, our model has a moderate to large effect size. With regard to individual variables, our AA model had three statistically significant predictors of time spent lecturing. Keith (2015, p. 62) notes that beta values between .10 and .25 are considered to have a moderate effect size while beta values above .25 are considered to have a large effect size. By these standards, the beta values for *Focus on groups* and *Expectations of student success* had moderate but opposite effects on time spent lecturing, while *Facilitation authority* had a large effect on time spent lecturing.

Discussion

In this paper, we focused on a particular set of beliefs about teaching and learning mathematics and their impact on instructional practice, as measured by the self-reported percentage of class time spent lecturing. To better understand how contextual factors may be mediating this relationship, we looked at instruction in two very different undergraduate mathematics courses: Calculus I and Abstract Algebra. Taking these two sets of data, we ran two parallel models to identify beliefs that influence instruction. It was not our goal to combine the data and then identify belief items. Rather, we focused on running two parallel models from the two surveys to better understand which belief items acted similarly across the different contexts and which items were possibly context-specific.

In terms of our first research question — For Calculus I instructors, what is the relationship between beliefs and instructional practice? — we were able to identify and create three factors related to instructors' beliefs about teaching and learning of mathematics: *Expectations of student success, Transmissive beliefs about student learning,* and *Conceptions of mathematics as connected*. From these factors, only *Transmissive beliefs about student learning* was a significant predictor of time spent lecturing. The more CSPCC instructors believed in the effectiveness of teacher-centered instructional practices, the more likely they were to lecture.

In terms of our second research question — For Abstract Algebra instructors, what is the relationship between beliefs and instructional practice? — we were able to identify and create five factors related to instructors' beliefs about teaching and learning mathematics: *Focus on fields and rings, Focus on groups, Facilitation authority, Expectations of student success,* and *Personal influences on teaching*. However, only *Focus on groups* (as material to cover), knowledge *Facilitation authority* (resting with students versus with teachers), and *Expectations of student success* (in understanding AA material) were significant predictors of time spent lecturing. Thus, the more AA instructors believed in their student's capacity to learn, the less likely they were to lecture. The more AA instructors focused on covering the various concepts associated with groups, or the more they believed in their role as a purveyor of knowledge, the more likely they were to lecture.

To address our third research question — How might contextual factors mitigate the relationship between instructors' beliefs and instructional practice? — we now look across our analysis. We do so, first by considering how the belief factors were similar and different between the two contexts, and then by considering how the regression models differed.

About the Factors

In looking across the data sets, the variables of *Expectations of student success, Transmissive beliefs about student learning, Focus on groups,* and *Facilitation authority* were significant predictors of time spent lecturing in their respective data sets. A natural question then is whether there were similarities in meaning between the variables between the data sets and if similar variables impacted lecture in the same ways.

In looking at commonalities between variables, *Expectations of student success* appeared in both data sets, but the variable was statistically significant for the AA data but not the CSPCC data. In interpreting this variable in the AA data, the more instructors believed in their students' capabilities, the less likely they were to lecture. The difference in significance might result from the different questions subsumed under the variable in each data set, with the implication that the variable represented different constructs in different contexts. We argue against this notion. In the AA data, *Expectations of student* success consisted of belief questions about students' ability to learn abstract algebra or mathematics more generally. In the CSPCC data, the variable consisted of belief questions about final grade distributions and preparedness to take calculus. While both sets of questions literally ask something different, there is an underlying idea of students' mathematical capability. Thus, the construct is not radically different in meaning between the two data sets.

Given the similarities in these two expectations of student success constructs, and the differences in how powerful these constructs were in the respective regression models, it may be that there are contextual factors that are mitigating their effect. In particular, calculus instructors and abstract algebra instructors are likely to experience differences with regards to the external framing of the courses — with more structures around content, pacing, and coverage likely to be found in calculus where multiple sections of the same course are typically coordinated to ensure consistency between sections. For instance, with a tight pacing schedule developed outside the instructor's control, calculus instructors might not feel that they can adjust their teaching practice in ways that align with their beliefs about their students' abilities. Whereas in abstract algebra, with less external framing, these instructors may have a greater sense of agency to adjust their course and instructional practice to better match their expectations of success and ability.

Differences in course context related to student demographics could also be at play. Being a service course, fewer calculus students are likely to be mathematics majors than those in an abstract algebra course. An abstract algebra instructor may believe most of their students are mathematics majors and thus need to be able to do the kinds of mathematics they will see in the classroom. Instructors who then believe more in their students' abilities may then lecture less because they may couple the beliefs about student capacity with beliefs about what their student demographic needs to be able to do, thus leading to beliefs about having students engage in such actions in class. Calculus instructors may not see their students as needing anything more than information to use in their own majors. Thus, instructors may focus on getting those ideas across in whatever way they feel best accomplishes that, lecture and/or active learning, regardless of what they believe about their students' capabilities.

Transmissive beliefs about student learning from the CSPCC data and *Facilitation authority* from the AA data also appear to be very similar in their meaning (student-centered versus teacher-centered instructional beliefs) and in the items that loaded on each factor. For instance, belief questions about the efficacy of lecture appeared in both. Similarly, both included questions about students' role in the learning process, like struggling with ideas. The higher values for both these variables tended towards teacher-centered beliefs where an instructor is the driving force for knowledge production. Lower values corresponded to beliefs around student involvement in the production of knowledge. Given these similarities, it is noteworthy that both variables were statistically significant and behaved similarly for their respective data sets. In both cases, the higher instructors scored on these variables, the more likely they were to lecture. Given the meaning of the variables discussed previously, it would have been strange if this relationship was not the case.

However, there was a difference in practical significance, signifying different extents to which the variables, and the beliefs ascribed to those variables, impacted the instructor's self-reports of lecture. Looking at instructors who centered their role in the learning process (corresponding to higher values on *Knowledge facilitation authority* and *Transmissive beliefs about student learning*) across both groups, AA instructors lecturing practices were more likely to be impacted than calculus instructors' lecturing practices. Part of this difference may have to do with the questions subsumed under the composite variables. The *Knowledge facilitation authority* variable had two questions *directly* related to lecture (I think lecture is the best way to teach; I think lecture is the only way to teach that allows me to cover the necessary content) whereas the *Transmissive beliefs about student learning* variable only had one direct question (Calculus students learn best from lectures, provided they are clear and well-organized).

Another possible explanation lies in reframing the interpretation of the variables. In both cases, instructors who subscribed less to teacher-centered instruction were less likely to lecture. Comparing the difference in practical significance, however, AA instructors would spend less time lecturing than calculus instructors, even if both groups believe more in student-centered instruction. This suggests that abstract algebra instructors' student-centered versus teacher-centered beliefs have a greater impact on their practice than those of calculus instructors. Again, a possible explanation for this could lie in context differences. Abstract algebra instructors experience a greater degree of autonomy due to weaker external framing and greater job security. In turn, they may be able to act on their beliefs to a greater extent than their calculus instructor colleagues who have to contend with various external framing constraints (common syllabi, pacing guides, being a service course, etc.). Thus, our results suggest that the contextual constraints calculus instructors face may push other beliefs to moderate the impact of their student-centered versus teacher-centered beliefs.

About the Models

One aspect between data sets that differ greatly is the amount of variance explained by our models. For the CSPCC data, our model only explained 2.7% of the variance. By contrast, our AA data model explained 38% of the variance. There are notable differences in the data sets themselves that are likely to contribute to some of the differences we found. For instance, some of the factors indentified in the CSPCC data had quite low Cronbach's Alphas and the output variable used for that regression analysis was skewed, with over half of the respondents reporting that they lectured "very often". Regardless, the difference in the amount of variance explained by the two models remains compelling and we suspect the differences between these two models are capturing differences in the contexts of these courses.

The CSPCC data came from calculus instructors across the US. While institutions will vary, calculus generally serves as a service course for many other majors, such as STEM fields, business, economics, management, and social sciences (Selinski & Milbourne, 2015). On top of this service role, and perhaps because of it, calculus courses typically have greater external framing (e.g., having common finals, pacing, or content to cover (Rasmussen & Ellis, 2015). All this can create an external environment where instructors feel constrained to teach in certain ways and thus less able to act on their core beliefs. By contrast, abstract algebra imposes fewer external constraints on instructors (Johnson et al., 2017).

In addition to potential differences in the framing around these courses, there may also be significant differences in the populations of instructors who regularly teach these courses. For instance, 92% of the AA instructors were tenure-stream while only 47% of calculus instructors were, and thus there was a big difference in terms of job security between the two demographics. Taken together there are significant differences in the instructional context between these courses which may lead AA instructors to feel freer to act on their beliefs than those who teach Calculus. This interpretation is supported by our finding that student-centered versus teacher-centered instructional beliefs acted similarly between both data sets, but had a more practical impact on AA instructors' time spent lecturing than on CSPCC instructors' time spent lecturing.

This then has an interesting implication when juxtaposed to Johnson et al.'s (2017) analysis that AA instructors experienced internalized constraints. While it may be that AA instructors report feeling constrained, the lack of external constraints may still afford AA instructors enough leeway to act on their beliefs *in ways in which they are not conscious*. This would align with how some have conceptualized beliefs (Leatham, 2006) and be a significant affordance that calculus instructors may not experience due to the presence of external constraints. This affordance may subsequently explain the high amount of variance explained in the AA data despite Johnson et al.'s work suggesting AA instructors feel constrained. These differences in the CSPCC and AA results overall and on the impact of individual clusters of beliefs highlight the importance of looking at undergraduate mathematics instruction in context.

Conclusions

In our Abstract Algebra data, we were able to identify five belief factors that accounted for 37.8% of the variability in the self-reported amount of time spent lecturing. For this instructional context, with its relatively weak external framing, it is clear that beliefs about content, beliefs about how students learn, and beliefs about student ability are influential in determining instructional practice. However, in the Calculus data, the three belief factors we identified only accounted for 2.7% of the variability in the amount of time spent lecturing. Thus, in this instruction context, with its stronger external framing, we could not identify beliefs that impacted instructional practice.

There are some limitations to consider when looking at our findings. First, some of the Cronbach's Alphas found in the Calculus factors were quite low, even when only considering the factors that were retained in the model (ranging from .46 to .81). The lack of internal consistency in these factors may explain some of the low variance explained by the regression model. Additionally, we acknowledge that the factors identified from our analyses were only analyzed with relation to how much instructors reported lecturing. Not only was this a self-reported variable, but data on the amount of lecturing was also collected with two different questions on the two different surveys; with the AA instructors asked to provide an approximate percentage of class time spent lecturing and the calculus instructors being asked to rate, on a scale from 0 (Not at all) to 5 (Very often), the statement "*During class time, how frequently did you lecture*." The question posed to the AA instructors may provide more of an opportunity for nuance. In contrast, two calculus instructors who both lecture every day but for very different proportions of class may both indicate that they lecture "very often." The differences in the dependent variables for the regression models are a limitation of this post hoc analysis, along with the skewness from the calculus data, which may inhibit the power of the belief factors in the calculus model.

Even with these limitations, however, our analysis does provide evidence that *there is a contextual difference* in how beliefs impact instruction. The exploratory nature of our current research generates several areas for future work. First, given the differences in the two data sets, it is difficult to assess *how much* course context may be mediating the relationship between beliefs and instructional practice. While we maintain that the sheer numerical differences found in our regression analyses (with 2.7% of the variance explained in the CSPCC model vs 38% of the variance explained in the AA model) serves as evidence that course context maters, we acknowledge that those values would likely change if instructors had been given the same survey. Thus we argue that our study provides compelling evidence that context matters, and that it does need to be taken into account when investigating instructor's beliefs and their instructional practice, but we cannot offer precision as to the extent that it matters.

Second, more research needs to be done to establish *the reasons why this is the case.* For instance, in addition to differences in job title and the security therein, 68% of the calculus instructors in our study indicated having common final exams or common final exam components, whereas course coordination is uncommon in upper-division mathematics courses. The differences in job security could exacerbate this difference in instructional autonomy. However, these are only some of the contextual differences between calculus instructors' lived realities and abstract algebra instructors, and none of these contextual factors were included in our analyses. Future research is needed to investigate how these factors affect instruction differently in different contexts.

Appendix

Table 8 Descriptive Statistics for Variables of Interest

	N	Minimum	Maximum	Mean	Std. Deviation
CSPCC data					
Time spent lecturing	493	0	5	4.20	1.16
Expectations of student success	629	-1.51	3.07	.00	.80
Focus on skills and content	459	-2.05	2.13	-.02	.69
Conceptions of mathematics	637	-2.44	1.24	-.00	.69
AA data					
Time spent lecturing	219	0	4	2.64	1.09
Focus on fields and rings	181	-1.64	1	-.00	.82
Focus on groups	196	-4.91	.36	-.00	.78
Facilitation authority	211	-1.40	1.53	-.00	.73
Expectations of student success	216	-1.40	1.26	-.00	.97
Personal influences on teaching	218	-3.57	.58	.00	.86

Table 9 Investigation of Curvilinear Relationship Model Summary

Model	R	R^2	SE	ΔR^2	ΔF	df1	df2	Significance level
CSPCC data (N=432)								
1	.163	0.027	1.159	0.027	3.887	3	428	0.009
2	.224	0.050	1.149	0.024	3.529	3	425	0.015
3	.266	0.071	1.141	0.021	3.120	3	422	0.026
AA data (N=167)								
1	.615	0.378	0.850	0.378	19.576	5	161	0.000
2	.644	0.414	0.838	0.036	1.929	5	156	0.093
3	.646	0.417	0.849	0.003	0.161	5	151	0.976

Table 10 Investigation of Curvilinear Predictors of Time Spent Lecturing (CSPCC data)

Variable	b	SE	beta	t	Significance level
Model 1					
Constant	4.192	0.056		75.149	0.000
Centered teachers expectations of student success	-0.121	0.075	-0.077	-1.604	0.109
Centered concern of faculty to cover course content and impart basic skills first	0.255	0.083	0.150	3.078	0.002
Centered conceptions of mathematics	-0.044	0.085	-0.025	-0.518	0.605
Model 2					
Constant	4.361	0.085		51.091	0.000
Centered teachers expectations of student success	-0.060	0.085	-0.039	-0.709	0.479
Centered concern of faculty to cover course content and impart basic skills first	0.228	0.083	0.134	2.755	0.006
Centered conceptions of mathematics	-0.003	0.089	-0.002	-0.032	0.975
cSuccessExp_square	-0.054	0.063	-0.046	-0.850	0.396
cContentSkills_square	-0.273	0.090	-0.150	-3.044	0.002
cMathConception_square	-0.021	0.099	-0.010	-0.212	0.832
Model 3					
Constant	4.331	0.092		47.258	0.000
Centered teachers expectations of student success	-0.125	0.107	-0.080	-1.167	0.244
Centered concern of faculty to cover course content and impart basic skills first	0.104	0.138	0.061	0.757	0.450
Centered conceptions of mathematics	-0.267	0.132	-0.151	-2.026	0.043
cSuccessExp_square	-0.128	0.102	-0.108	-1.257	0.209
cContentSkills_square	-0.234	0.093	-0.128	-2.527	0.012
cMathConception_square	0.117	0.115	0.058	1.013	0.312
cSuccessExp_cube	0.052	0.050	0.110	1.042	0.298
cContentSkills_cube	0.097	0.083	0.095	1.160	0.247
cMathConception_cube	0.232	0.090	0.213	2.573	0.010

Table 11 Investigation of Curvilinear Predictors of Time Spent Lecturing (AA data)

Variable	b	SE	beta	t	Significance level
Model 1					
Constant	2.569	0.066		39.064	0.000
cFieldsandRings	0.155	0.081	0.120	1.900	0.059
cGroupsFocus	0.226	0.082	0.174	2.741	0.007
cRoleofTeacherandStudents	0.710	0.097	0.489	7.317	0.000
cPerceptionsofStudents	-0.166	0.074	-0.149	-2.258	0.025
cInfluenceofExperiences	0.110	0.086	0.081	1.285	0.201
Model 2					
Constant	2.405	0.142		16.929	0.000
cFieldsandRings	0.262	0.103	0.202	2.539	0.012
cGroupsFocus	0.141	0.156	0.109	0.909	0.365
cRoleofTeacherandStudents	0.735	0.098	0.506	7.499	0.000
cPerceptionsofStudents	-0.114	0.077	-0.102	-1.473	0.143
cInfluenceofExperiences	0.117	0.136	0.085	0.858	0.392
cFieldsandRings_square	0.202	0.111	0.143	1.827	0.070
cGroupsFocus_square	-0.018	0.050	-0.044	-0.357	0.722
cRoleofTeacherandStudents_square	-0.219	0.114	-0.124	-1.915	0.057
cPerceptionsofStudents_square	0.164	0.103	0.106	1.584	0.115
cInfluenceofExperiences_square	0.018	0.081	0.022	0.221	0.825
Model 3					
Constant	2.456	0.200		12.291	0.000
cFieldsandRings	0.282	0.186	0.218	1.514	0.132
cGroupsFocus	0.135	0.229	0.104	0.590	0.556
cRoleofTeacherandStudents	0.604	0.204	0.415	2.965	0.004
cPerceptionsofStudents	-0.043	0.196	-0.039	-0.221	0.825
cInfluenceofExperiences	0.104	0.156	0.076	0.664	0.508
cFieldsandRings_square	0.165	0.181	0.117	0.914	0.362
cGroupsFocus_square	-0.009	0.224	-0.021	-0.039	0.969
cRoleofTeacherandStudents_square	-0.244	0.120	-0.138	-2.026	0.045
cPerceptionsofStudents_square	0.138	0.112	0.090	1.229	0.221
cInfluenceofExperiences_square	-0.037	0.245	-0.045	-0.152	0.879
cFieldsandRings_cube	-0.032	0.157	-0.042	-0.203	0.839
cGroupsFocus_cube	0.001	0.040	0.010	0.024	0.981
cRoleofTeacherandStudents_cube	0.112	0.158	0.105	0.711	0.478
cPerceptionsofStudents_cube	-0.050	0.132	-0.068	-0.377	0.706
cInfluenceofExperiences_cube	-0.015	0.066	-0.059	-0.232	0.817

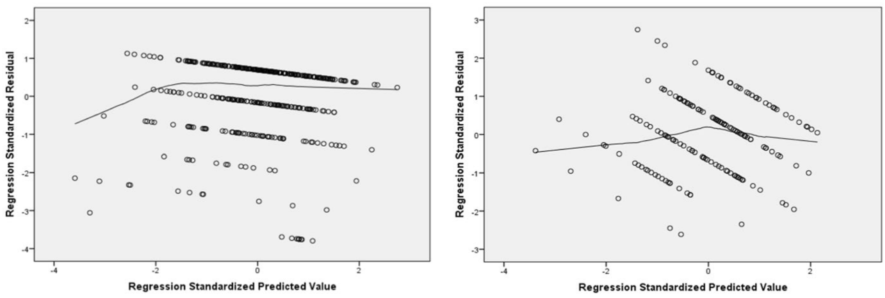

Fig. 1 Plot of standardized predicted values against standardized residuals for the CSPCC data (Left) and AA data (right). For the CSPCC data, the Loess line of best fit suggests linearity was not a reasonable assumption while the spread of the data suggests homoscedasticity was met. For the AA data, the Loess line of best fit suggests linearity was a reasonable assumption while the spread of the data suggests homoscedasticity was not met

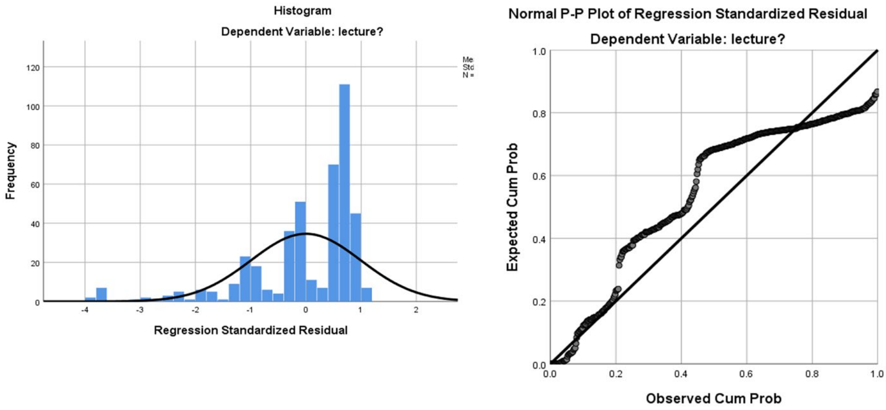

Fig. 2 Histogram of standardized residuals (Left) and P-P plot (right) for CSPCC data. Both suggest normality may be problematic

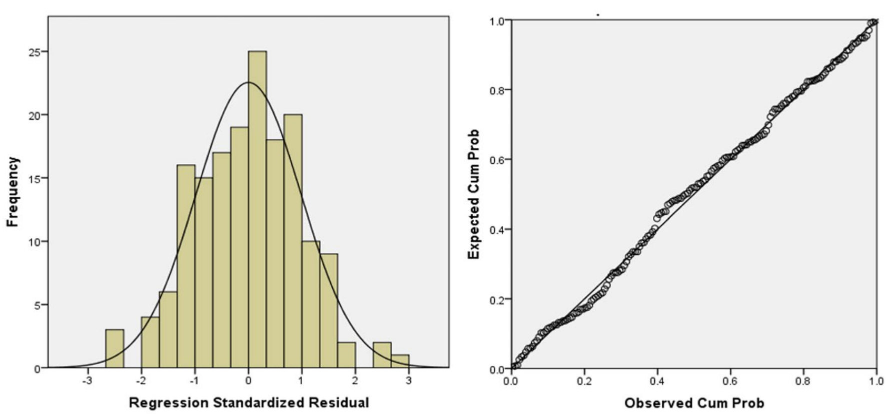

Fig. 3 Histogram of standardized residuals (Left) and P-P plot (right) for AA data. Both suggest normality was a reasonable assumption

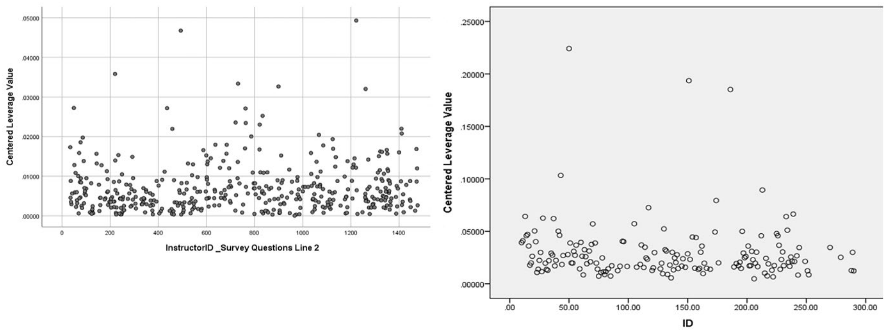

Fig. 4 Plot of centered leverage values against instructor ID for the CSPCC data (Left) and AA data (right). The distributions for both suggest some outliers, but more prominently in the CSPCC data

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Availability of Data and Materials The CSPCC data that support the findings of this study are available from the Mathematical Association of America, but restrictions apply to the availability of these data, which were used under license for the current study, and so are not publicly available. Data from the AA survey are available from the authors upon reasonable request.

Declarations

Conflicts of Interest The authors declare they do not have conflicts of interest.

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