Optimal Control of Wave Energy Converters

Ossama Abdelkhalik, and Habeebullah Abdulkadir

***Abstract*—The control logic of a Wave Energy Converter (WECs) usually aims to increase the energy converted from waves. For optimal energy conversion, most WEC control methods require reactive power; that is a power flow from the WEC to the ocean, at times, in order to increase the overall harvested energy over a period of time. The power take off (PTO) unit that has the capability of providing reactive power is usually expensive and complex. In this work, an optimal control analysis is presented in which the objective of the optimal control problem is to maximize the harvested energy while constraining the power flow to be only in the positive direction; that is adding a constraint of no reactive power. The optimal control derivation is presented for the case of a solo WEC. Low fidelity numerical simulations are presented comparing the proposed control to the known Bang-Bang (BB) control and the Bang-Singular-Bang (BSB) control.**

***Index Terms*—Wave energy converter, Optimal control, Positive power control, Reactive power, Pontryagin Minimum principle.**

I. INTRODUCTION

IN 2016 the global ocean energy capacity was 536 MW; which is less than 0.03% of the total renewable energy, in the same year. One of the reasons ocean wave energy technology is not yet widely used is the relatively high cost of electricity generated using wave energy converters [8]. For an array of WECs, automatic control is one of the most pressing problems and least studied because of dynamic modelling difficulties. One of the challenges in the design of the control system for WECs is the need for reactive power. This is when a WEC would put power into the ocean at times in order to harvest more energy over a longer period of time [9]. A PTO unit that can provide reactive power is usually complex and expensive. This study aims to eliminate the need for reactive power, and hence enable the use of relatively simple PTO units, while still attempting to maximize the converted energy.

In general, a WEC would harvest most energy when its motion is in resonance with the incident waves. A control logic typically attempts to achieve this resonance. For instance, the Latching control approach, which was proposed by Budal and Falnes [5] [4] in the 1980s, works by locking the buoy at some moments to keep its motion in phase [2] [12]. This control does not need reactive power. The only power required is that needed to operate the latching mechanism. Reference [14] implements a latching type control on a floating wave energy converter in deep water. Actively controlled motion-compensated platform was used as a reference for power absorption and latching.

A control approach may also change the natural frequency of the device to make it closer to the resonance conditions. Recent researches try to achieve energy harvesting maximization by formulation the problem as a constrained optimization problem [11] [15] [3]. These optimization based controllers generally optimize the absorbed power, which is the product of force and velocity [19]. Reference [21] presented a constrained and unconstrained optimal control formulation for a single-degree-of-freedom WEC device using Pontryagin's minimum principle; the control was tested in the cases of periodic and non-periodic excitation forces. The constraint considered in [21] is not on the power; rather it is on the maximum displacement of the buoy. The results in [21] showed that the optimal control is in one of two modes: singular arc and bang-bang and hence is referred to as the bang-singular-bang (BSB) control; the Numerical results show that the BSB control performs better than Bang-Bang (BB) control in terms of the amount of extracted energy. Reference [18] presented a model predictive controller for the Wavestar wave energy converter that maximizes its power generation. Two other controllers (Optimal controller and Optimal gain controller) were implemented on the buoy wave energy absorber model to compare. This was in order to justify what prediction horizons are suitable for a finite horizon MPC and to see which of the controllers maximizes the power. Reference [7] presented an optimal control solution to maximize energy absorption while taking into account the actuator limits and the general limitations of a WEC design. The control for a single WEC device in [7] employs a moment-based phasor transform.

In this paper, an optimal control maximizing the harvested energy while eliminating the reactive power is derived rigorously using optimal control theory. The paper is organized as follows. In section II, a dynamic model of simplified WEC device is established. The proposed positive power control formulation is presented in section III. Section IV shows the simulation results. Conclusions are presented in Section V.

II. SYSTEM DYNAMICS FOR A SOLO WEC

A simple WEC model can be represented as a second order mass-spring-damper system, as shown in fig. 1. The motion of the floater is similar to the motion of ship which is more complex because of the anomaly shape.

If the wave height and motion are small, the motion of the floater is restricted to heave motion only, and the linear equation of motion for a 1-DoF (heave only) solo WEC is [6]:

$$
\begin{aligned}
m\ddot{x}(t) &= \int_{-\infty}^{\infty} h_f(\tau)\eta(t-\tau,x)d\tau \\
&+ f_s - u - \mu\ddot{x}(t) - \int_{-\infty}^{t} h_r(\tau)\dot{x}(t-\tau)d\tau \quad (1)
\end{aligned}
$$

where x is the heave displacement of the buoy from the sea surface, t is the time, m is the buoy mass, u is the

O. Abdelkhalik and H. Abdulkadir are with the Department of Aerospace Engineering, Iowa State University, Ames, IA 50010, USA, e-mail: (see https://www.aere.iastate.edu/ossama/).

Manuscript received July 26, 2021.

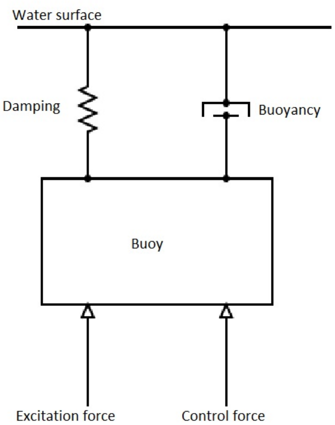


Fig. 1. Schematic of a simplified WEC device

control force, and f_s is the difference between the gravity and buoyancy force and it reflects the spring-like effect of the fluid. The pressure effect around the immersed surface of the floater is called the excitation force, f_e, where η is the wave surface elevation at buoy centroid and h_f is the impulse response function defining the excitation force in heave. The radiation force, f_r, is due to the radiated wave from the moving float, where μ is a frequency dependant added mass, and h_r is the impulse response function defining the radiation force in heave. The dynamics of the motion of the floater can be written as [10]:

$$m\ddot{x} = f_e + f_r + f_s + u \tag{2}$$

The hydrostatic force, f_s, reflects the spring like effect of the fluid. The hydrostatic force is proportional to the displacement:

$$f_s = -Kx \tag{3}$$

The excitation force, f_e, which can be decomposed as pressure effect around the immersed surface of the floater. In our paper the excitation force will be constructed using Fourier series:

$$f_e = \sum_{i=1}^{n} A_i sin(w_i t + \phi) \tag{4}$$

III. Positive-power constraint Optimal control formulation

The derivation presented in this section follows the standard process of deriving the necessary conditions of optimality in optimal control theory. The derivation below is different from that in reference [21] in that here there is a constraint on the PTO power to be always positive (no reactive power); this constraint was not considered in [21].

In the case of a single WEC device, which is the case in this paper, the objective function can be written as:

$$\text{Min} \quad J(u(t)) = \int_0^{t_f} \{-u(t)x_2(t)\}dt \tag{5}$$

Subject to the dynamics Eq. (1) and the positive power constraint:

$$u(t)x_2(t) \geq 0 \tag{6}$$

This constraint Eq. (6) is on the power, such that there is no reactive power supplied from the WEC PTO unit to drive the motion of the buoy at any time during the operation of the device. The Pontryagin's minimum principle [17] [16] is used to solve the optimal control problem. The positive power constraint can be re-written in the following form:

$$-u(t)x_2(t) \leq 0 \tag{7}$$

then converted to an equality constraint as:

$$-u(t)x_2(t) + \alpha = 0 \tag{8}$$

where α is a slack variable. Similar to the work done in [13] and [21], if we define x_1 as the position of the floater and x_2 as its velocity, then the equation of motion for the system in fig. 1 can be re-written in state space form as [20]:

$$\dot{x}_1 = x_2 \tag{9}$$

$$\dot{x}_2 = \frac{1}{m}(f_e(t) - cx_2 - kx_1 - u) \tag{10}$$

where f_e is excitation force, where the wave in this case is assumed a regular wave, and u is the control input. This system is non-autonomous because of f_e. We can replace the time variable from the equation of motion by defining another variable which is x_3. The state space of the system will become:

$$\dot{x}_1 = x_2 \tag{11}$$

$$\dot{x}_2 = \frac{1}{m}(f_e(x_3) - cx_2 - kx_1 - u) \tag{12}$$

$$\dot{x}_3 = 1 \tag{13}$$

Based on the equations of motion of the buoy, we need to formulate the optimal control problem as follow:

$$Min : J((x(t), u(t)) = \int_0^{t_f} \{-u(t)x_2(t)\}dt \tag{14}$$

Subject to: Eq. (11), Eq. (12),Eq. (13), and Eq. (6).

To start solving, we need to write out the Hamiltonian of the problem [1]:

$$\begin{aligned} H &= -ux_2 + \lambda_1 x_2 + \frac{\lambda_2}{m}(f_e(x_3) - cx_2 - kx_1 - u) \\ &+ \lambda_3 + \gamma(-ux_2 + \alpha) \end{aligned} \tag{15}$$

Based on the Hamiltonian, the necessary condition of the problem corresponding to (x_1^*,x_2^*,x_3^*,u^*,λ_1^*,λ_2^*,λ_3^*,γ^*) which satisfy the Euler-Langrange equation is derived as:

$$H_\lambda = \dot{x} \tag{16}$$

$$H_x = -\dot{\lambda} \tag{17}$$

$$H_u = 0 \tag{18}$$

$$H_\gamma = 0 \tag{19}$$

By solving for the necessary conditions for optimality, we obtain the equations below and also the state space equation:

$$\dot{\lambda}_1 = \frac{k}{m}\lambda_2 \tag{20}$$

$$\dot{\lambda}_2 = -\lambda_1 + \frac{c}{m}\lambda_2 + u + \gamma u \tag{21}$$

$$\dot{\lambda}_3 = -\frac{1}{m}\frac{\partial f_e(x_3)}{\partial x_3}\lambda_2 \tag{22}$$

$$x_2 + \frac{\lambda_2}{m} + \gamma x_2 = 0 \tag{23}$$

Eq. (23) can be further simplified as

$$\frac{\lambda_2}{m} + (1+\gamma)x_2 = 0 \tag{24}$$

$$-ux_2 + \alpha = 0 \tag{25}$$

Since $H_u = 0$, the solution in Eq. (24) does not yield an expression for the control u, which means the control is either on a singular arc or at its boundaries (limits).

From Eq. (24),

$$\frac{\dot{\lambda}_2}{m} + (1+\gamma)\dot{x}_2 = 0 \tag{26}$$

combining Eq. (20) and Eq. (24),

$$\dot{\lambda}_1 = \frac{k}{m}\lambda_2 \Rightarrow -k(1+\gamma)x_2 \Rightarrow -k(1+\gamma)\dot{x}_1 \tag{27}$$

Integrate Eq. (27)

$$\lambda_1 = -k(1+\gamma)x_1 + const. \tag{28}$$

Substitute Eq. (28) into Eq. (21)

$$\dot{\lambda}_2 = k(1+\gamma)x_1 - const. + \frac{c}{m}\lambda_2 + (1+\gamma)u \tag{29}$$

solving Eq. (12) for u and substituting in Eq. (29)

$$\begin{aligned}\dot{\lambda}_2 = k(1+\gamma)x_1 - const. + \frac{c}{m}\lambda_2 + (1+\gamma)[-m\dot{x}_2 \\ -cx_2 - kx_1 + f_e(x_3))]\end{aligned} \tag{30}$$

substituting Eq. (24) and Eq. (26) into Eq. (30) and simplifying

$$const = -2c(1+\gamma)x_2 + (1+\gamma)f_e(x_3)) \tag{31}$$

differentiating

$$0 = -2c(1+\gamma)\dot{x}_2 + (1+\gamma)\frac{\partial f_e(x_3)}{\partial x_3} \tag{32}$$

$$0 = -2c\dot{x}_2 + \frac{\partial f_e(x_3)}{\partial x_3} \tag{33}$$

substitute for $\dot{x}_2$ from Eq. (12)

$$0 = -\frac{2c}{m}\left[f_e(x_3) - cx_2 - kx_1 + u\right] + \frac{\partial f_e(x_3)}{\partial x_3} \tag{34}$$

Solving for u in Eq. (34), we can find the optimal control to be

$$u = f_e(x_3) - cx_2 - kx_1 - \frac{m}{2c}\frac{\partial f_e(x_3)}{\partial x_3} \tag{35}$$

From Eq. (25)

$$\alpha = ux_2 \tag{36}$$

Note that α is the the positive power variable.

$$\alpha = x_2\left[f_e(x_3) - cx_2 - kx_1 - \frac{m}{2c}\frac{\partial f_e(x_3)}{\partial x_3}\right] \tag{37}$$

If there is a saturation on the control, and the buoy is subject to oscillatory excitation forces, then it is possible to state that the optimal control can be defined as:

$$u = \begin{cases} u_{sa},\ H_u = x_2 - \dfrac{Fex}{2c} \ = \ 0, & \alpha \geq 0; \\ \Upsilon,\ H_u = x_2 - \dfrac{Fex}{2c} \ > \ 0, & \alpha < \ 0,\ x_2 > 0; \\ -\Upsilon,\ H_u = x_2 - \dfrac{Fex}{2c} \ < \ 0, & \alpha < \ 0,\ x_2 < 0; \end{cases}$$

where Υ is the maximum available control level, and u_{sa} is the singular arc control. If the states of the buoy at any given time does not satisfy the constraints, then the optimal control solution will switch to bang-bang solution.A bang-bang control uses the maximum control available at every time. The optimal solution will switch between the bang-bang mode and the singular arc mode.

IV. Simulation

The positive power control developed in Section III is simulated. There is a saturation limit on the control. The numerical parameters are chosen as follows: The mass of the buoy $m = 200000$ kg, the stiffness of hydrostatic force is $k = 120000$ N/m, and the damping coefficient is chosen to be $c = 100000$ Nm/s. The maximum control force is $\Upsilon = 100000$ N.

Fig. 2 and Fig. 3 presents the displacement and the velocity of the buoy when Bang-Bang, Bang-Singular-Bang, and Positive power control algorithms were applied to control the device.

As discussed earlier, a single degree of freedom oscillator will undergo its maximum movements when it is in resonance with the wave excitation force; however maintaining this resonance may require reactive power. This can be seen from the results in Fig. 2; it can be observed that the buoy displacement is largest when the Bang-Singular-Bang control is used; this BSB control requires reactive power. However, the displacement of the buoy when controlled using either the Bang-Bang (BB) control or the Positive power control (PBSB) is smaller than that of the BSB control. Note that both

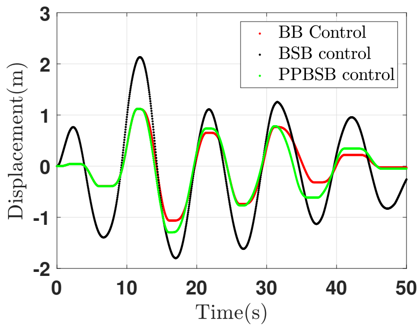


Fig. 2. Comparison of buoy heave displacement when using Bang-Bang control (BB), Bang-Singular-Bang control (BSB) and Positive power control (PPBSB).

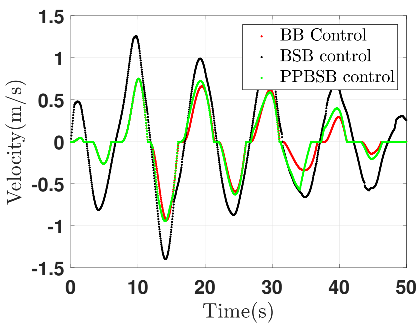


Fig. 3. Comparison of buoy heave velocity when using BB, BSB and PPBSB controls.

the BB and PPBSB controls do not require reactive power. It is observed that the PPBSB control causes the buoy to have slightly higher motion at some times.

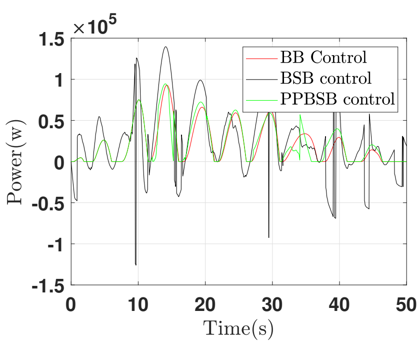


Fig. 4. The Power extraction by the PTO unit when using BB, BSB and PPBSB controls.

Fig. 4 shows the power generation by the three control algorithms. The BSB control gives better results with the highest power curve; the power curve below zero represents the reactive power that has to be provided by the PTO. The PPBSB control power curve is always above zero; same is the power curve of the BB control. Referring to Fig. 5, the BSB control clearly has the highest energy extraction as expected. The performance of the PPBSB control is better compared to BB control.

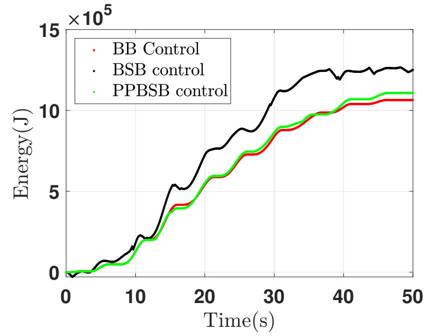


Fig. 5. The energy extracted over time by BB, BSB and PPBSB controls.

Fig. 6 shows the control force generated using each one of the three control methods. Comparing the BB control and the PPBSB control, it is observed that the BB control quickly switches between the maximum and minimum limits of the control force. This rapid switch may cause rapid degradation of the device and thereby reducing the overall effective lifespan. The PPBSB control algorithm is slightly better in that regard.

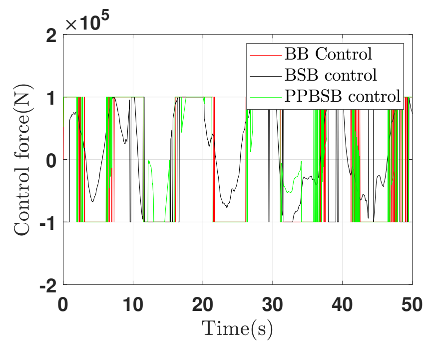


Fig. 6. Comparison of variation of control force applied when using BB, BSB and PPBSB controls.

V. Conclusion

A new control algorithm that attempts to maximize the power output from wave energy converters without the use of reactive power is developed in this paper; here it is referred to as the positive power control. This positive power control is derived analytically within the context of optimal control theory. Simulation results presented in this paper show that the overall performance of the positive power control is slightly better compared to the Bang-Bang control. The implementation of the positive power control eliminates the complexity of designing a bi-directional PTO system needed to provide reactive power. Future work will develop the positive power control for arrays of wave energy converters.

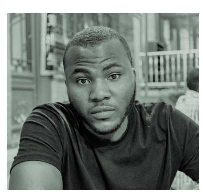
Acknowledgment

This paper is based upon work supported by NSF, Grant Number 2048413. The research reported in this paper is partially supported by the HPC@ISU equipment at Iowa State University, some of which has been purchased through funding provided by NSF under MRI grant number 1726447.

REFERENCES

[1] Jr Arthur E. Bryson. *Applied Optimal Control optimization,estimation and control*. Blaisdell, 1969.
[2] Aurélien Babarit and Alain H Clément. Optimal latching control of a wave energy device in regular and irregular waves. *Applied Ocean Research*, 28(2):77–91, 2006.
[3] Giorgio Bacelli and John Ringwood. Constrained control of arrays of wave energy devices. *International Journal of Marine Energy*, 34(0):e53 – e69, 2013. Special Issue Selected Papers - {EWTEC2013}.
[4] K. Budal. Theory for absorbtion of wave power by a system of interacting bodies. *Journal of Ship Research*, 21:248–253, 1977.
[5] K Budar and J Falnes. A resonant point absorber of ocean-wave power. *Nature*, 256(5517):478–479, 1975.
[6] W.E. Cummins. *The Impulse Response Function and Ship Motions*. David W. Taylor Model Basin Report.
[7] Nicolás Faedo, Giordano Scarciotti, Alessandro Astolfi, and John V Ringwood. Energy-maximising control of wave energy converters using a moment-domain representation. *Control Engineering Practice*, 81:85–96, 2018.
[8] J. Falnes. Wave-energy conversion through relative motion between two single-mode oscillating bodies. *Journal of Offshore Mechanics and Arctic Engineering*, 121(1):32–38, February 1999.
[9] Johannes Falnes. *Ocean Waves and Oscillating Systems - Linear Interactions Including Wave-Energy Extraction*, chapter 4. Cambridge University Press, 2002.
[10] Johannes Falnes. *Ocean waves and oscillating systems: linear interactions including wave-energy extraction*. Cambridge university press, 2002.
[11] Jørgen Hals, Johannes Falnes, and Torgeir Moan. Constrained optimal control of a heaving buoy wave-energy converter. *Journal of Offshore Mechanics and Arctic Engineering*, 133(1), 2011.
[12] J.C.C. Henriques, M.F.P. Lopes, R.P.F. Gomes, L.M.C. Gato, and A.F.O. Falcão. On the annual wave energy absorption by two-body heaving {WECs} with latching control. *Renewable Energy*, 45(0):31 – 40, 2012.
[13] P Kasturi and P Dupont. Constrained optimal control of vibration dampers. *Journal of Sound and vibration*, 215(3):499–509, 1998.
[14] Umesh A Korde. Latching control of deep water wave energy devices using an active reference. *Ocean engineering*, 29(11):1343–1355, 2002.
[15] Guang Li, George Weiss, Markus Mueller, Stuart Townley, and Mike R. Belmont. Wave energy converter control by wave prediction and dynamic programming. *Renewable Energy*, 48(0):392 – 403, 2012.
[16] Jack Macki and Aaron Strauss. *Introduction to optimal control theory*. Springer Science & Business Media, 2012.
[17] Lev Semenovich Pontryagin. *Mathematical theory of optimal processes*. CRC press, 1987.
[18] Mohsen N. Soltani, Mahdi T. Sichani, and Mahmood Mirzaei. Model predictive control of buoy type wave energy converter. In *The 19th International Federation of Automatic Control (IFAC) World Congress*, Cape Town, South Africa, August 24-29 2014. International Federation of Automatic Control (IFAC).
[19] David Wilson, Giorgio Bacelli, Ryan G Coe, Diana L Bull, Ossama Abdelkhalik, Umesh A Korde, and Rush D Robinett III. A comparison of wec control strategies. *Sandia National Laboratories Tech. Rep. SAND2016-4293*, 10:1431291, 2016.
[20] Z. Yu and J. Falnes. State-space modelling of a vertical cylinder in heave. *Applied Ocean Research*, 17(5):265–275, October 1995.
[21] Shangyan Zou, Ossama Abdelkhalik, Rush Robinett, Giorgio Bacelli, and David Wilson. Optimal control of wave energy converters. *Renewable energy*, 103:217–225, 2017.

Ossama Abdelkhalik is Associate Professor at Iowa State University. He received his PhD degree in Aerospace Engineering from Texas A&M University, and his BS and MS degrees from Cairo University in Egypt. Dr. Abdelkhalik's research is in the area of global optimization, dynamics, and control with applications to the wave energy conversion and space trajectory optimization. Dr. Abdelkhalik is Associate Fellow of AIAA and is associate editor of the AIAA Journal of Spacecraft and Rockets.



Habeebullah Abdulkadir received his MSc. degree in Aerospace Engineering at Beijing Institute of Technology, China in 2019. Since January 2021, he has been working toward his PhD degree at Iowa State University.