Power Constrained Optimal Control of Wave Energy Converters

Habeebullah Abdulkadir and Ossama Abdelkhalik

***Abstract*—A control logic of a Wave Energy Converters (WEC) aims to convert wave power into electricity. For optimal energy conversion, most WEC control methods require reactive power which is power that flows from the WEC to the ocean, at times, in order to increase the overall harvested energy, over a period of time. The power take off (PTO) unit that has the capability of providing high levels of reactive power is usually expensive and complex. In this work, an optimal control analysis is presented in which the objective of the optimal control problem is to maximize the harvested energy while constraining the reactive power to not exceed a given threshold. The optimal control derivation is presented for the case of a solo WEC assuming irregular wave input. Low fidelity numerical simulations are presented comparing the proposed power-constrained Bang Singular Bang (PCBSB) control to the known Optimal Reactive Loading (ORL) control.**

***Index Terms*—Wave energy converter, Optimal control, Positive power control, Reactive power, Pontryagin Minimum principle.**

I. Introduction

ENERGY from the ocean is one of the least exploited renewable and sustainable energy sources, and possesses an enormous amount of untapped energy. The high cost of electricity generated using wave energy converters is one of the major drawbacks in the technology deployment [1, 2]. Over the years, several diverse set of WEC technologies and designs have been proposed, they face a number of common persistent design difficulties. A significant example is the implementation of power-take-off (PTO) control algorithms for effective power capture [3, 4]. This drawback is usually because many of the developed control algorithms require the PTO system to be capable of unlimited bidirectional power. This is when a WEC would put power into the ocean at times in order to harvest more energy over a longer period of time [5]. A PTO unit that can provide unlimited reactive power is usually complex and expensive. This study aims to minimize the need for reactive power, and hence enable the use of relatively simple PTO units, while still attempting to maximize the converted energy.

In general, a WEC would harvest most energy when its motion is in resonance with the incident waves. A control logic typically attempts to achieve this resonance. For instance, the Latching control approach, which was proposed by Budal and Falnes [6] [7] in the 1980s, works by locking the buoy at some moments to keep its motion in phase [8] [9]. This control does not need reactive power. The only power required is that needed to operate the latching mechanism. Reference [10] implements a latching type control on a floating wave energy converter in deep water. Actively controlled motion-compensated platform was used as a reference for power absorption and latching.

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A control approach may also change the natural frequency of the device to make it closer to the resonance conditions. Recent researches try to achieve energy harvesting maximization by formulation the problem as a constrained optimization problem [11, 12, 13]. These optimization based controllers generally optimize the absorbed power, which is the product of force and velocity [14]. Reference [15] presented a constrained and unconstrained optimal control formulation for a single-degree-of-freedom WEC device using Pontryagin's minimum principle; the control was tested in the cases of periodic and non-periodic excitation forces. The constraint considered in [15] is not on the power; rather it is on the maximum displacement of the buoy. The results in [15] showed that the optimal control is in one of two modes: singular arc and bang-bang and hence is referred to as the bang-singular-bang (BSB) control; the Numerical results show that the BSB control performs better than Bang-Bang (BB) control in terms of the amount of extracted energy, the singular arc solution was also obtained and presented in [16, 17]. Reference [18] presented a model predictive controller for the Wavestar wave energy converter that maximizes its power generation. Two other controllers (Optimal controller and Optimal gain controller) were implemented on the buoy wave energy absorber model to compare. This was in order to justify what prediction horizons are suitable for a finite horizon MPC and to see which of the controllers maximizes the power. Reference [19] presented an optimal control solution to maximize energy absorption while taking into account the actuator limits and the general limitations of a WEC design. The control for a single WEC device in [19] employs a moment-based phasor transform.

Optimal WEC control strategies with constrain the flow of power in one direction has researched as well, often called a passive control. [20] presented a control scheme based on MPC and meant to maximise the energy capture of a wave-energy point absorber and reduce the reactive power —be it electrical, mechanical or otherwise—associated with the control. This was achieved my modifying the objective function. [21] also presented two MPC control variations designed to only allow unidirectional power flow by setting nonlinear constraints applied to a two-body point absorber. Considerably more power than the linear passive control, thus proving to be a good option for unidirectional PTO systems. Passive optimal control has been presented in many forms over the years, the interest of this work is to investigate the improvement of a control method that allows for a limited amount of reactive power.

In this paper, an optimal control maximizing the harvested energy while eliminating the reactive power capability, or having limited amount of reactive power for single WEC with irregular excitation are derived rigorously using optimal control theory. The paper is organized as follows. In section II, a dynamic model of simplified WEC device is established, derivation of the proposed power-constrained control formulation is presented as a subsection. Section III show the dynamic model of an array of simplified WEC devices, the power-constrained control formulation is presented in the subsection. Section IV shows the simulation results. Conclusions are presented in Section V.

II. Dynamic modeling of a Solo WEC

A simple WEC model can be represented as a second order mass-spring-damper system, as shown in fig. 1. The motion of the floater is similar to the motion of ship which is more complex because of the anomaly shape.

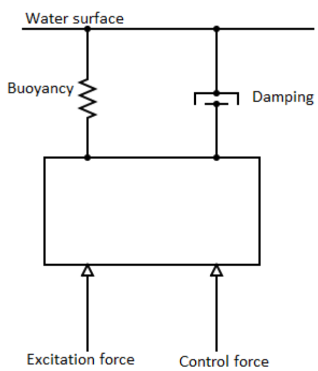

Fig. 1. Schematic of a simplified WEC device

If the wave height and motion are small, the motion of the floater is restricted to heave motion only, and the linear equation of motion for a 1-DoF (heave only) solo WEC is [22]:

$$m\ddot{x}(t) = \int_{-\infty}^{\infty} h_f(\tau)\eta(t-\tau,x)d\tau + f_s - u - \mu\ddot{x}(t) - \int_{-\infty}^{t} h_r(\tau)\dot{x}(t-\tau)d\tau \quad (1)$$

where x is the heave displacement of the buoy from the sea surface, t is the time, m is the buoy mass, u is the control force, and f_s is the hydrostatic restoring force due to buoyancy and gravity and it reflects the spring-like effect of the fluid. The pressure effect around the immersed surface of the floater is called the excitation force, f_e, where η is the wave surface elevation at buoy centroid and h_f is the impulse response function defining the excitation force in heave. The radiation force, f_r, is due to the radiated wave from the moving float, where μ is a frequency dependant added mass (representing the mass of fluid displaced by the body during operation), and h_r is the impulse response function defining the radiation force in heave, A reasonable state-space approximation for the radiation term can be achieved with a state-space model of order 4, as described in [23]. The dynamics of the motion of the floater can be written using Newton's law as shown in Eqn. (2).

$$m\ddot{x} = f_e + f_r + f_s - u \quad (2)$$

III. Optimal power-constrained control in irregular waves

Using a state space representation of this WEC system, let the state row vector be $x = [x, \dot{x}, \dot{x}, \vec{x}_r]^T = [x_1, x_2, x_3, \vec{x}_r]^T$, where x_1, x_2, x_3 and $\vec{x}_r$ are the displacement, velocity, time, and radiation states respectively. The optimal control problem, assuming irregular waves, can be formulated as follows: Maximize the harvested power, $u(t)x_2(t)$, or equivalently,

$$\text{Minimize } J((\vec{x}(t), u(t)) = \int_0^{t_f} -u(t)x_2(t)dt, \quad (3)$$

subject to the constraints on both control forces and power:

$$|u(t)| \leq \Upsilon, \quad (4)$$

$$u(t)x_2(t) \geq -\epsilon \quad (5)$$

where ϵ is the available reactive power, and subject to the constraints from the equations of motion:

$$\dot{x}_1 = x_2 \quad (6)$$

$$\dot{x}_2 = \frac{1}{m}\left(F_{ext}(x_3) - C_r\vec{x}_r - kx_1 - u\right) \quad (7)$$

$$\dot{x}_3 = 1 \quad (8)$$

$$\dot{\vec{x}}_r = \boldsymbol{A}_r\vec{x}_r + \boldsymbol{B}_r x_2 \quad (9)$$

where $\boldsymbol{A}_r$, $\boldsymbol{B}_r$, $\boldsymbol{C}_r$ are the radiation matrices used to approximate the radiation convolution term in Eqn. (1), and the radiation state vector $\vec{x}_r \in R^{4\times1}$. The dimensions of each these matrices are as follows: $\boldsymbol{A}_r$ is a 4×4 matrix, $\boldsymbol{B}_r$ is a 4×1 matrix and $\boldsymbol{C}_r$ is a 1×4 matrix. The above inequality constraint on power in Eq. (5) can be written in the form of an equality constraint by adding a positive slack variable α to be:

$$-u(t)x_2(t) + \alpha = \epsilon \quad (10)$$

Similarly, the above inequality constraint on control force in Eq. (4) can be written in the form of an equality constraint as:

$$u(t) + \beta = \Upsilon \quad (11)$$

where β is a positive slack variable. The Hamiltonian of the problem can then be written as:

$$H = -ux_2 + \lambda_1 x_2 + \frac{\lambda_2}{m}\left(F_{ext}(x_3) - C_r\vec{x}_r - kx_1 - u\right) + \vec{\lambda}_r\left(\boldsymbol{A}_r\vec{x}_r + \boldsymbol{B}_r x_2\right) + \gamma_1\left(-ux_2 - \epsilon + \alpha\right) + \gamma_2\left(u + \beta - \Upsilon\right)$$

where $\vec{\lambda}_r$ is the costate of the radiation states, γ_1 and γ_2 are the costates associated with the power constraint and the control constraint respectively. The necessary conditions of optimality can be found in several references such as [24, 25]; for the system in hand these conditions can be written as follows:

$$\dot{\lambda}_1 = \frac{k}{m}\lambda_2 \quad (12)$$

$$\dot{\lambda_2} = -\lambda_1 + u\,(1+\gamma_1) - \vec{\lambda}_r \boldsymbol{B}_r \quad (13)$$

$$\dot{\lambda_3} = -\frac{1}{m}\frac{\partial F_{ext}(x_3)}{\partial x_3}\lambda_2 \quad (14)$$

$$\dot{\vec{\lambda}}_r = \frac{\lambda_2}{m}\boldsymbol{C}_r - \lambda_r \boldsymbol{A}_r \quad (15)$$

The $H_u = 0$ stationary condition yields:

$$\frac{\lambda_2}{m} + x_2\,(1+\gamma_1) = 0 \quad (16)$$

The $H_{\gamma_1} = 0$ stationary condition yields:

$$-ux_2 - \epsilon + \alpha = 0 \quad (17)$$

The $H_{\gamma_2} = 0$ stationary condition yields:

$$u + \beta - \Upsilon = 0 \quad (18)$$

One way of solving the differential equations (12)–(15) is to convert them into algebraic equations using Laplace transform. Hence we can write the following equations in the S domain:

$$X_1(s) = X_2(s)/s \quad (19)$$

$$sX_2(s) = \frac{1}{m}\Big(F_{ext}(s) - C_r\vec{X}_r(s) - kX_1(s) - U(s)\Big) \quad (20)$$

$$\vec{X}_r(s) = \Big(s\boldsymbol{I} - \vec{A}_r(s)\Big)^{-1}\boldsymbol{B}_r X_2(s) \quad (21)$$

Solving for $X_2(s)$ by substituting Eq. (19) and Eq. (21) into Eq. (20), we get:

$$X_2(s) = \frac{(F_{ext}(s) - U(s))\,s}{ms^2 + C_r(s\boldsymbol{I} - A_r)^{-1}s\boldsymbol{B}_r + k} \quad (22)$$

Substituting $X_2(s)$ back into Eq. (19) and Eq. (21) to obtain X_1 and X_r :

$$X_1(s) = \frac{(F_{ext}(s) - U(s))}{ms^2 + C_r(s\boldsymbol{I} - \vec{A}_r(s))^{-1}s\boldsymbol{B}_r + k} \quad (23)$$

$$\vec{X}_r(s) = \frac{\Big(s\boldsymbol{I}_r - \vec{A}_r(s)\Big)^{-1}\boldsymbol{B}_r\,(F_{ext}(s) - U(s))\,s}{ms^2 + C_r(s\boldsymbol{I} - A_r)^{-1}s\boldsymbol{B}_r + k} \quad (24)$$

Transforming the co-state equations to the S domain:

$$s\lambda_1(s) - \lambda_{10} = \frac{k}{m}\lambda_2(s) \quad (25)$$

$$s\lambda_2(s) - \lambda_{20} = -\lambda_1(s) + U(s)(1+\gamma_1) - \vec{\lambda}_r(s)\boldsymbol{B}_r \quad (26)$$

$$s\vec{\lambda}_r(s) - \vec{\lambda}_{r0} = \frac{\lambda_2(s)}{m}\boldsymbol{C}_r - \vec{\lambda}_r(s)\boldsymbol{A}_r \quad (27)$$

where λ_{i0} is the initial value for λ_i, $i = 1, 2, r$. From Eq. (25) and Eq. (27), one can write:

$$\lambda_1(s) = \frac{\frac{k}{m}\lambda_2(s) + \lambda_{10}}{s} \quad (28)$$

$$\vec{\lambda}_r(s) = \left(\frac{\lambda_2(s)}{m}\boldsymbol{C}_r + \vec{\lambda}_{r0}\right)(s\boldsymbol{I} + A_r)^{-1} \quad (29)$$

Substituting the above equations into Eq. (26), we get:

$$\begin{aligned} s\lambda_2(s) - \lambda_{20} = &-\frac{\frac{k}{m}\lambda_2(s) + \lambda_{10}}{s} + U(s)(1+\gamma_1) \\ &-\left(\frac{\lambda_2(s)}{m}\boldsymbol{C}_r + \vec{\lambda}_{r0}\right)(s\boldsymbol{I} + A_r)^{-1}\boldsymbol{B}_r \end{aligned} \quad (30)$$

Solving Eq. (30) for λ_2, we can write:

$$\begin{aligned} \lambda_2(s) = &\frac{\Big(m\lambda_{20} + \vec{\lambda}_{r0}(s\boldsymbol{I} + A_r)^{-1}\boldsymbol{B}_r m\Big)\,s}{[ms^2 + \boldsymbol{C}_r(s\boldsymbol{I} + A_r)^{-1}s\boldsymbol{B}_r + k]} \\ &+ \frac{(msU(s)(1+\gamma_1) - m\lambda_{10})}{[ms^2 + \boldsymbol{C}_r(s\boldsymbol{I} + A_r)^{-1}s\boldsymbol{B}_r + k]} \end{aligned} \quad (31)$$

Recall from Eq. (16) that:

$$\lambda_2(s) = -m\,(1+\gamma_1)\,X_2(s) \quad (32)$$

Substituting Eq. (31) and Eq. (22) into Eq. (32), and then solving for $U(s)$ we get a solution of the form:

$$U(s) = U_1(s) + U_2(s) \quad (33)$$

where,

$$U_1(s) = \frac{N_1(s)}{D_1(s)}, \quad (34)$$

$$N_1(s) = \big(ms^2 + \boldsymbol{C}_r(s\boldsymbol{I} + A_r)^{-1}s\boldsymbol{B}_r + k\big)\,F_{ext}(s) \quad (35)$$

$$D_1(s) = \boldsymbol{C}_r(s\boldsymbol{I} + A_r)^{-1}s\boldsymbol{B}_r - \boldsymbol{C}_r(s\boldsymbol{I} - A_r)^{-1}s\boldsymbol{B}_r \quad (36)$$

and,

$$U_2(s) = \frac{N_2(s)}{D_2(s)}, \quad (37)$$

$$\begin{aligned} N_2(s) = &\Big((\lambda_{20} + \vec{\lambda}_{r0}(s\boldsymbol{I} + A_r)^{-1}\boldsymbol{B}_r m)s - \lambda_{10}\Big) \times \\ &\big(ms^2 + \boldsymbol{C}_r(s\boldsymbol{I} - A_r)^{-1}s\boldsymbol{B}_r + k\big) \end{aligned} \quad (38)$$

$$\begin{aligned} D_2(s) \;=\; &s^2(1+\gamma_1) \times \\ &\big[\boldsymbol{C}_r(s\boldsymbol{I} + A_r)^{-1}s\boldsymbol{B}_r - \boldsymbol{C}_r(s\boldsymbol{I} - A_r)^{-1}s\boldsymbol{B}_r\big] \end{aligned} \quad (39)$$

As can be seen above, the U_2 is a transient term that depends only on the initial values of the co-states and is independent from the excitation force. Hence, in computing the steady state solution, the U_2 term will be dropped.

The time domain steady state solution of Eq. (33) is referred to as the singular arc solution. This solution along with the stationary conditions in Eq. (17) and Eq. (18) constitute the power-constrained optimal control presented in this paper, and is here referred to as the power-constrained bang singular bang (PCBSB) control. The algorithm to implement the PCBSB control is presented in Algorithm 1; essentially the singular arc control u_{sa} is computed and is checked against the constraints in Eq. (17) and Eq. (18); if u_{sa} violates any of the constraints in Eq. (17) or Eq. (18), then the violated constraint is activated.

IV. Simulation Results

The device used in the simulation is a single-body cylindrical point absorber with a $2m$ diameter and $3m$ draught. The controls developed in Section III applied to the cylindrical buoy excited by an irregular wave. The results of simulation using the control developed is compared to the well known Optimal Resistive Loading (ORL) control. We consider a Bretschneider spectrum for the generation of the irregular waves with a peak period of $T)p = 6$ s, and significant wave height $H_s = 0.8222$ m. The wave direction is set to 0^0 for all simulation. Hydrodynamics BEM solver Nemoh is employed to compute the exact hydrodynamics for for the 256

Algorithm 1 PCBSB control Algorithm

procedure COMPUTE POWER-CONSTRAINED CONTROL FOR A SOLO DEVICE(u)
  Input: State the maximum available control Υ
  Input: State the available reactive power ϵ
  Output: control
  Calculate excitation wave for the current time
  Compute singular arc control from Eq. (33)
  $u = u_{sa}$
  Check control force constraint from Eq. (18):
  if $u_{sa} \geq \Upsilon$ **then**
    $u_{sa} = \Upsilon$
  else
    if $u_{sa} \leq -\Upsilon$ **then**
      $u_{sa} = -\Upsilon$
  Check power constraint. Compute α from Eq. (17):
  if $\alpha \leq \epsilon$ **then**
    $u = sign(x_2) \times \Upsilon$

equally spaced frequencies between $\omega = 0.1, ..., 3.50$rad/s. The maximum control force is $\Upsilon = 1e3$N. The PTO unit is assumed to have a limited reactive power capability with $\epsilon > 0$ W. The value of ϵ used in the simulation is $1e4$ W. The results are compared to that obtained using the benchmark Optimal Resistive Loading (ORL) control for comparison with a passive control methods.

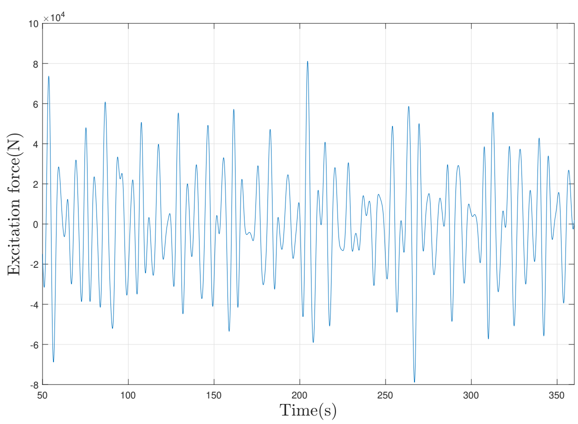

Fig. 2. Excitation force

The displacements and velocities of the buoy when Optimal ORL control and the presented PCBSB control are used to control the device are presented in Fig. 3 and Fig. 4 respectively. ORL controls are passive controls, while PCBSB exhibit some negative power. The plot of displacements is within reasonable limit for safe operation of the devices. However, when the buoy is allowed some reactive power as in the case of the PCBSB control, it appear to exhibit a higher displacement or velocity.

Fig. 5 shows the power generated by the buoy when the control algorithms were used. It should be noted that the power curve for the passive controls do not fall below the zero level, this translate to reactive power provide by the control. An important part of the control PCBSB is the ability

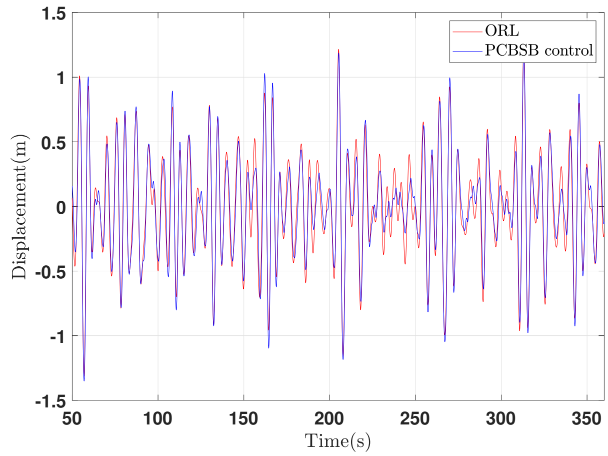

Fig. 3. Buoy heave displacements when using ORL and PCBSB

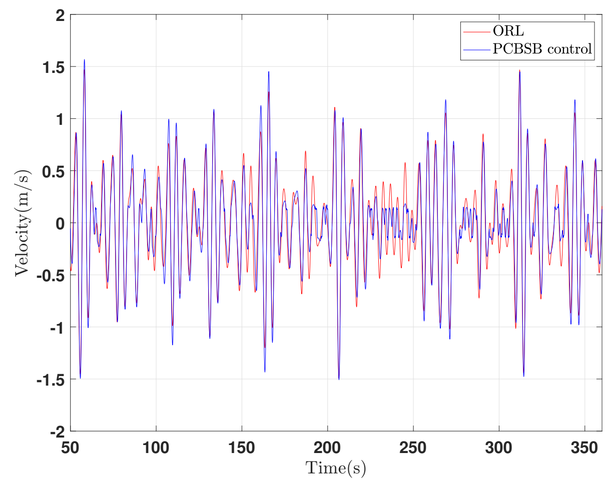

Fig. 4. Buoy heave velocity when using ORL and PCBSB

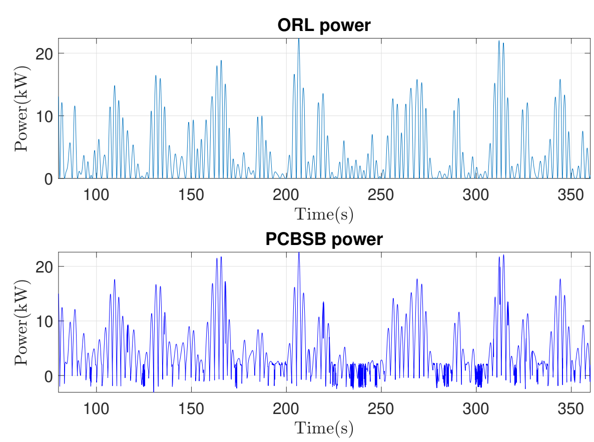

Fig. 5. Power extracted when using ORL and PPBSB controls.

to specify the amount of reactive power availed by the PTO unit of choice. The PCBSB control generates a higher amount of power than the passive control, as indicated on the power curve. It was observed that with increase in the amount of reactive power available, the overall power generation by the system increases. A theoretical maximum power of a device is usually achieved when the PTO is assumed to be able to give back to the ocean as much power as it takes from it, sometimes, even giving more overall power to the ocean than extracted.

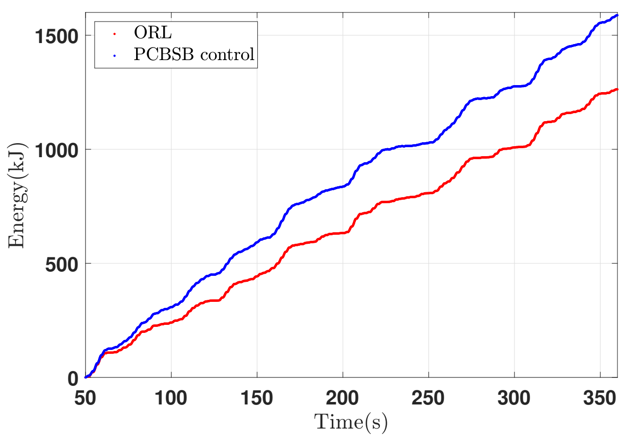

Fig. 6. The energy extracted over time when using ORL and PCBSB.

It is unsurprising that the overall energy harvested using PCBSB control is better. Due to the limited reactive power availed by the PTO, the overall performance of the control is not drastic. Fig. 7 shows the control force used by each of the control methods. A general maximum available control limit γ was applied on all the controls for fair comparison. The behaviour of the PCBSB is very similar to that of a Bang Bang control, both quickly switches between the maximum and minimum limits of the control force. This because the Singular arc solution of the optimal control often requires a high control force. The saturation limit on the control and the positive power constraint forces the control to be at the maximum allowable limit at most time, with the singular arc solution sparsely used.

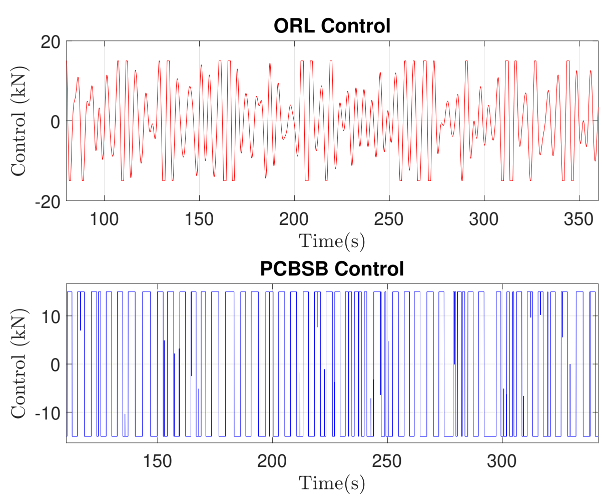

Fig. 7. The control force used by the PTO unit when using ORL and PCBSB.

V. Conclusion

A new control algorithm that attempts to maximize the power output from wave energy converters while constraining the use of reactive power to a limit specified by the PTO of choice is developed in this paper; here it is referred to as the Power Constrained Bang Singular Bang (PCBSB) control. This power control is derived analytically within the context of optimal control theory. Simulation results presented in this paper show that the overall performance of the control is significantly higher than a passive optimal control. The implementation of the presented control eliminates the complexity of designing a bi-directional PTO system needed to provide large amount of reactive power, we are otherwise able to use off the shelf PTO with limited bi-directional power capability. Future work will develop the positive power control for arrays of wave energy converters with an irregular excitation.

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