Optimal Constrained Control of Wave Energy Converter Arrays

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***Abstract*—This paper presents the development of an optimal constrained control for an array of wave energy converters (WECs). Most current WEC optimal control methods require reactive power; that is, a power flow from the WEC to the ocean, at times, in order to increase the overall harvested energy over a period of time. The power take off (PTO) unit that has the capability of providing bi-directional power is usually expensive and complex. In this work, an optimal control which maximizes the harvested energy while constraining the power flow direction is derived analytically for an array of WEC devices. Low fidelity numerical simulations are presented comparing the proposed control to the known Optimal Reactive Loading (ORL) control.**

***Index Terms*—Wave energy converter, Optimal control, Positive power control, Reactive power, Pontryagin Minimum principle.**

I. Introduction

The global attention in recent years has been brought to the correlation between climate change and rising levels of the carbon dioxide emissions. Ocean wave energy due to its vast and untapped potential is one of the important alternative energy sources will play a significant role in achieving carbon free energy production in the future. Similar to other means of generating renewable energy, such as wind turbines and solar panels, ocean wave energy converters are designed to be deployed in large arrays composed of many units [1].

A common WEC design is the Point absorber [2]; it is a typical style of a floating oscillating body, which usually utilizes a submerged or floating body to trap the oscillating force of the wave. This type of device, which is also modelled in this paper, has its dimensions small compared to the wave length of incoming waves and are very efficient when the incident wave frequency is close to the natural frequency. A WEC device would capture the highest possible energy when it is in resonance with the incident waves. Designers often attempt to design a device whose natural frequency will be close to the predominant wave condition of the deployment site.

Control methods are typically used to drive the device to achieve resonance with the ocean waves. Early control concepts like the Latching control approach, which was proposed by Budal and Falnes [3] [4] in the 1980s, works by locking the floater of the WEC at some moments to keep its motion in phase [5] [6], these types of control are referred to as passive control because they only require that some power be provided to operate the latching mechanism. Reactive controls on the other hand involves the use of power from the grid to drive the device at intervals. A major disadvantage of reactive controls is that; it often has unrealistically large force requirement than can not be provided by a realistic PTO system. The term reactive control is used to highlight the presence of a reactive power and therefore the necessity to give power into the system during part of the cycle. This requirement of a bi-directional power capacity has major implications on the effective practical implementation of reactive control [7].

Passive controls of WEC devices can also be achieved using modern optimal control theory (as many reactive controls). This is done by formulating the problem as an constrained optimization problem [8] [9] [10]. Optimization based controllers generally optimize the absorbed power, which is the product of force and velocity [11]. The control derived using optimization is often a form of reactive control, that is, it allows for the flow of power from the WEC to the ocean and they often do not take into account the inherent physical limitations of both the floater and the PTO units. However, the introduction of constraints can be used to enforce a limit on many aspect of the operation of the device [12][13][14][15]. In many of the WEC optimal control strategies, the constraints considered is often on the maximum displacement of the buoy and/or maximum control force available. A constraint on the power flow direction can however be used to limit or even hinder the flow of power to the ocean. In [16], two MPC control strategies were designed to only allow unidirectional power flow. This was achieved by setting nonlinear constraints applied to a two-body point absorber. The Bang-Bang control presented in [9] also only allowed unidirectional power flow. Another commonly known sub-optimal passive control is the resistive loading control [17], it is an approach where the instantaneous value of the PTO force is linearly proportional to the oscillating body speed, that is:

$$f_{pto} = -B_{pto} v(t)$$

where $B_{pto} > 0$ is the PTO damping coefficient. When the damping coefficient is tuned to its optimal value, the control is referred to as the Optimal Resistive Loading (ORL).

The extension of control strategies developed for a single device to an array of devices is not trivial, however, similar controls can be developed for implementation in an array of WECs. Control logic for an array seek to maintain constructive hydrodynamic interaction between the devices placed in a limited area, while also seeking to maximize the overall energy extracted from the array. A distributed MPC strategy for a highly-coupled clusters of sea wave energy converters was presented in [18], the array was partitioned into subsystems and each subsystem was controlled by a local MPC controller. It was found that the optimized power output increases as the degree of coupling increases between the devices in the array. Increases in power of up to 20 percent was achieved using realistic ranges of parameters with respect to the uncoupled case. Reference [19] presented a comparison of when a centralized controller for the devices in the wave farm is used with independently controlled devices in array. There was significant performance improvement from using global control of arrays. However, the challenge of unrealistic control requirements are also present in the control of arrays and are challenges to the implementation of control method which give optimal energy extraction.

This work aims to eliminate the need for reactive power in an array of WEC, this will be achieved by constraining the power in an optimal control formulation to always be positive, this control will hence enable the use of relatively simple PTO units, while still maximizing the converted energy. The control is derived analytically in the context of optimal control theory using Pontryagin's Minimum Principle. The paper is organized as follows. In section II, dynamic model for an array of simplified WEC device is established. The proposed Positive Power Bang-Singular-Bang (PPBSB) control formulation is presented in section III. Section IV shows the simulation results. Conclusions are presented in Section V.

II. System dynamics for a WEC array

A simple WEC model can be represented as a second order mass-spring-damper system. The equations of motion of a WEC array is an extension of that of a single WEC model presented in [20]. When there is more than one device placed in close proximity, the hydrodynamic interaction with one device will also have some effect on the adjacent devices. The radiation excited by one body will be dependent on velocity of the other device, V. This hydrodynamics interaction modeling will be done in frequency domain, then the analysis is then extended to the time domain state space formulation. The radiation process can be described by a frequency dependent impedance as

$$F_r(\omega) = Z(\omega)V(\omega) = (i\omega m_a(\omega) + b_r(\omega))V(\omega) \quad (1)$$

where $F_r(\omega)$ and $V(\omega)$ are the radiation force and WEC velocity in frequency domain. $Z(\omega)$ is the radiation impedance of the WEC with $m_a(\omega)$ as the added mass and $b_r(\omega)$ as the radiation damping. Regular wave excitation is computed from one frequency component, in which case Eq. (1) can be used for both time domain and frequency domain analysis. The impedance of the radiation excited by another body towards a first body can be written as:

$$Z_{ij} = i\omega m_{aij}(\omega) + b_{rij}(\omega) \quad (2)$$

The summation of the radiation force of the body and the force radiated from the other body gives total radiation force acting on a first body. The radiation equation representing the body i independent radiation and the inter-radiation force between the devices as

$$\begin{aligned} f_r(\omega) = (i\omega m_{aii}(\omega) + b_{rii}(\omega))v_{ii}(\omega) + \\ (i\omega m_{aij}(\omega) + b_{rij}(\omega))v_{ij}(\omega) \end{aligned} \quad (3)$$

The equation of motion for an array is written in compact matrix form in as:

$$-\omega^2(\boldsymbol{M}+\boldsymbol{M_a}(\omega))\mathbb{Z}+j\omega(\boldsymbol{B_r}(\omega))\mathbb{Z}+\boldsymbol{K_h}\mathbb{Z} = \mathbb{F}_{ex}+\mathbb{F}_c \quad (4)$$

In the Eq. (4), $\boldsymbol{M}$ and $\boldsymbol{M_a}(\omega)$ are the mass and added mass matrices of the array, $\boldsymbol{B_r}(\omega)$ is the radiation damping matrix, and $\boldsymbol{K_h}$ is the hydrostatic coefficient matrix of the array. Vector $\mathbb{Z}$ is the displacement vector of array. $\mathbb{F}_{ex}$ and $\mathbb{F}_c$ are the excitation force and control force vectors respectively.

The dynamics in Eq. (4) rewritten in time domain as:

$$(\boldsymbol{M} + \boldsymbol{M_a}(\omega))\ddot{\vec{x}} = \vec{f}_e - \boldsymbol{B_r}(\omega)\dot{\vec{x}} - \boldsymbol{K_h}\vec{x} - \vec{u} \quad (5)$$

where $\vec{x}$, $\dot{\vec{x}}$ and $\ddot{\vec{x}}$ are the position, velocity and acceleration vectors of the n-number of bouys in the array.

III. Positive-power constrained Optimal control formulation

In this section, the optimal constrained control for an array of WECs with regular excitation is derived using Pontryagin's Minimum Principle. WEC array equation of motion can be written in state-space form as:

$$\begin{Bmatrix} \dot{\vec{x}}_1 \\ \dot{\vec{x}}_2 \\ \dot{\vec{x}}_3 \end{Bmatrix} = A \begin{Bmatrix} \vec{x}_1 \\ \vec{x}_2 \\ \vec{x}_3 \end{Bmatrix} - \boldsymbol{B}\vec{u} + \boldsymbol{B}\vec{f}_e(x_3) + \begin{bmatrix} \vec{0}_{2n\times 1} \\ 1 \end{bmatrix} \quad (6)$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0}_{n\times n} & \boldsymbol{I}_{n\times n} & \vec{0}_{n\times 1} \\ [-[\boldsymbol{M}+\boldsymbol{M_a}(\boldsymbol{\omega})]^{-1}\boldsymbol{K_h}]_{n\times n} & [-[\boldsymbol{M}+\boldsymbol{M_a}(\omega)]^{-1}\boldsymbol{B_r}(\omega)]_{n\times n} & \vec{0}_{n\times 1} \\ \vec{0}_{1\times n} & \vec{0}_{1\times n} & \boldsymbol{0}_{1\times 1} \end{bmatrix}$$

$$B = \begin{bmatrix} \mathbf{0}_{n*n} \\ [\boldsymbol{M} + \boldsymbol{M_a}(\boldsymbol{\omega})]^{-1}_{n\times n} \\ \mathbf{0}_{1\times n} \end{bmatrix}$$

In this optimal control formulation, the power flow is constrained such that there is no power flow from the PTO to the ocean at any time. A minimization cost function formulated as:

$$Min : J((x(t), u(t)) = \sum_{n=1}^{N} \int_0^{t_f} \{-u_n(t)x_{2n}(t)\}dt \tag{7}$$

$$Subject\ to : Eq.\ (6) \tag{8}$$

and subject to: positive power constraints as follows

$$u_1(t)x_{21}(t) \geq 0,\ u_2(t)x_{22}(t) \geq 0$$

the positive power constraint can be rewritten as:

$$\vec{u}(t) \circ \vec{v}(t) \geq 0 \tag{9}$$

expressing the power constraint as a minimization problem and then converting it to an equality constraint, we have:

$$-\vec{u}(t) \circ \vec{v}(t) + \vec{\alpha} = 0 \tag{10}$$

where $\circ$ is the symbol of Hadamard vector products (an element wise vector multiplication function) and $\vec{\alpha}$ is a vector of slackness variable added to the inequality constraint to convert it to an inequality constraint. The other constraint is the physical limitation of the PTO units, this is achieved by setting limitation on the available maximum control force, $\vec{\Upsilon}$. This constraint can be expressed as:

$$|\vec{u}| \leq \vec{\Upsilon} \tag{11}$$

Since time affects the system dynamics and cannot be controlled, the WEC array dynamic system is considered as non-autonomous. For such dynamic system, the state vector is defined as:

$$\begin{aligned} \vec{\boldsymbol{x}} &= [\vec{x}_1, \vec{x}_2, x_3]^T \\ \vec{x}_1 &= [x_{11}, x_{12}, x_{13}, ..., x_{1n}]^T \\ \vec{x}_2 &= [\dot{x}_{11}, \dot{x}_{12}, \dot{x}_{13}, ..., \dot{x}_{1n}]^T \\ x_3 &= t \end{aligned}$$

Where $\vec{x}_1$ is the displacement vector of the array and $\vec{x}_2$ is the array velocity vector. The control force vector is defined as:

$$\vec{u} = [u_1, u_2, u_3, ..., u_n]^T \tag{12}$$

Where the length of the co-states λ vector is $2n + 1$, which is the same as the number of the constraint equations.

$$\vec{\boldsymbol{\lambda}} = [\vec{\lambda}_{1n}, \vec{\lambda}_{2n}, \vec{\lambda}_{3n}, ..., \lambda_n]^T \tag{13}$$

$$\vec{\alpha} = [\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n]^T \tag{14}$$

$$\vec{\gamma} = [\gamma_1, \gamma_2, \gamma_3, ..., \gamma_n]^T \tag{15}$$

The elements in the co-state vector $\vec{\lambda}_{1n}$ refer to the co-states relating the displacements of each device, and the elements in vector $\vec{\lambda}_{2n}$ refer to the co-states that correspond to the velocities of each device. Each element in $\vec{u}$ vector corresponds to the control force acting on the $n-th$ buoy. To solve this optimal control problem, we need to compute the Hamiltonian of the problem first [21] as:

$$\begin{aligned} H = -\vec{u}^T\vec{x}_{2n} + \vec{\lambda}_{1n}^T\vec{x}_{2n} + \frac{\vec{\lambda}_{2n}^T}{[\boldsymbol{M} + \boldsymbol{M}_a(\infty)]}(\vec{f}_e - \boldsymbol{B_r}(\omega)\vec{x}_2 \\ -\boldsymbol{K_h}\vec{x}_1 - \vec{u}) + \lambda_3 + \vec{\gamma}^T(\vec{u} \circ \vec{x}_2 + \vec{\alpha}) \end{aligned} \tag{16}$$

By computing the necessary conditions of optimality, we will find corresponding (x_1^*,x_2^*,x_3^*,fu^*,λ_1^*,λ_2^*,λ_3^*,γ^*) which satisfy the Euler-Langrange equation:

$$H_\lambda = \dot{x} \tag{17}$$

$$H_x = -\dot{\lambda} \tag{18}$$

$$H_u = 0 \tag{19}$$

$$H_\gamma = 0 \tag{20}$$

The necessary conditions for optimality are obtained as:

$$\dot{\vec{\lambda}}_{1n}^T = \vec{\lambda}_{2n}^T[\boldsymbol{M} + \boldsymbol{M}_a(\omega)]^{-1}\boldsymbol{K_h} \tag{21}$$

$$\dot{\vec{\lambda}}_{2n}^T = -\vec{\lambda}_{1n}^T + \vec{\lambda}_{2n}^T[\boldsymbol{M} + \boldsymbol{M}_a(\omega)]^{-1}\boldsymbol{B_r}(\omega) + \vec{u}^T + \gamma^T \circ \vec{x}_2^T \tag{22}$$

$$\dot{\lambda}_3 = -\vec{\lambda}_{2n}^T \frac{1}{[\boldsymbol{M} + \boldsymbol{M}_a(\omega)]} \frac{\partial \vec{f}_e(x_3)}{\partial x_3} \tag{23}$$

$$\vec{x}_{2n}^T + \frac{\vec{\lambda}_{2n}^T}{[\boldsymbol{M} + \boldsymbol{M}_a(\omega)]} + \vec{\gamma}^T \circ \vec{x}_{2n}^T = 0 \tag{24}$$

$$-\vec{u}^T \circ \vec{x}_2^T + \vec{\alpha}^T = 0 \tag{25}$$

The optimality condition is linear to the control $\vec{u}$. Thus, the WEC array optimal control problem with regular wave input is a singular arc. From Eq. 24;

$$\left(\vec{1} + \vec{\gamma}^T\right) \circ \vec{x}_{2n}^T + \frac{\vec{\lambda}_{2n}^T}{[\boldsymbol{M} + \boldsymbol{M}_a(\omega)]} = 0 \tag{26}$$

where $\vec{1}$ is is used as a notation for a 1×2 row vector of ones. Differentiating, we get:

$$\left(\vec{1} + \vec{\gamma}^T\right) \circ \dot{\vec{x}}_{2n}^T + \frac{\dot{\vec{\lambda}}_{2n}^T}{[\boldsymbol{M} + \boldsymbol{M}_a(\omega)]} = 0 \tag{27}$$

simplifying Eq. 27 and substituting into Eq. 21

$$\begin{aligned} &\Rightarrow -(\vec{1} + \vec{\gamma}^T) \circ \vec{x}_{2n}^T\boldsymbol{K}_h \\ &\Rightarrow -(\vec{1} + \vec{\gamma}^T) \circ \dot{\vec{x}}_{1n}^T\boldsymbol{K}_h \end{aligned} \tag{28}$$

Integrating Eq. 28

$$\vec{\lambda}_{1n}^{T} = -(\vec{1} + \vec{\gamma}^{T}) \circ \vec{x}_{1n}^{T} \boldsymbol{K}_h + C^T \tag{29}$$

Substituting Eq. 29 in to Eq. 22

$$\begin{aligned} \dot{\vec{\lambda}}_{2n}^{T} &= (\vec{1} + \boldsymbol{\gamma}^{T}) \circ \vec{x}_{1n}^{T} \boldsymbol{K}_h - C^T \\ &- \vec{\lambda}_{2n}^{T} [\boldsymbol{M} + \boldsymbol{M}_a(\omega)]^{-1} \boldsymbol{B}_r(\omega) \\ &+ (\vec{1} + \vec{\gamma}^{T}) \circ \vec{u}^{T} \end{aligned} \tag{30}$$

where C is a column vector of length n containing the constants of integration. Substituting for $\vec{u}$ in Eq. 30

$$\begin{aligned} \dot{\vec{\lambda}}_{2n}^{T} = (\vec{1} + \vec{\gamma}^{T}) \circ \vec{x}_{1n}^{T} \boldsymbol{K}_h - C^T + \vec{\lambda}_{2n}^{T} [\boldsymbol{M} + \boldsymbol{M}_a(\omega)]^{-1} \\ \boldsymbol{B}_r(\omega) + (\vec{1} + \vec{\gamma}^{T}) \circ [-(\boldsymbol{M} + \boldsymbol{M}_{\boldsymbol{a}}(\omega)) \dot{\vec{x}}_{2n} \\ - \boldsymbol{K}_h \vec{x}_{1n} - \boldsymbol{B}_r(\omega) \vec{x}_{2n} + \vec{f}_e]^T \end{aligned} \tag{31}$$

simplifying Eq. 31

$$C^T = -2(\vec{1} + \vec{\gamma}^{T}) \circ \vec{x}_{2n}^{T} \boldsymbol{B}_r(\omega) + (\vec{1} + \vec{\gamma}^{T}) \circ \vec{f}_e^{T} \tag{32}$$

Differentiating and simplifying Eq. 32

$$0 = 2(\vec{1} + \boldsymbol{\gamma}^{T}) \circ \dot{\vec{x}}_{2n}^{T} \boldsymbol{B}_r(\omega) + (\vec{1} + \vec{\gamma}^{T}) \circ \frac{\partial \vec{f}_e(x_3)}{\partial x_3}^{T} \tag{33}$$

simplifying and substituting the equation of motion, $\dot{\vec{x}}_2$:

$$\begin{aligned} 0 = 2[\vec{f}_e - \boldsymbol{B_r}(\omega)\vec{x}_2 - \boldsymbol{K_h}\vec{x}_1 - \vec{u}]^T \boldsymbol{B}_r(\omega) \\ [\boldsymbol{M} + \boldsymbol{M}_a(\omega)]^{-1} + \frac{\partial \vec{f}_3(x_3)}{\partial x_3}^{T} \end{aligned} \tag{34}$$

solving for $\vec{u}$

$$\begin{aligned} \vec{u}_{sa} = \vec{f}_e - \boldsymbol{B_r}(\omega)\vec{x}_2 - \boldsymbol{K_h}\vec{x}_1 - [\boldsymbol{B_r}(\omega) \\ [\boldsymbol{M} + \boldsymbol{M}_a(\omega)]^{-1}]^{-1} \frac{\partial \vec{f}_3(x_3)}{\partial x_3} \end{aligned} \tag{35}$$

Power condition $\vec{\alpha}$ is calculated as $\vec{\alpha} = \vec{u} \circ \vec{x}_2$ from Eq. 25. The control assumes knowledge of the excitation force and its derivatives at the current time. The sequence of implementation of the control is presented in Algorithm 1.

IV. Simulation Results

The devices used in the simulation are identical single-body cylindrical point absorber with a 4m diameter and 3m draught. The control developed in Section III is tested on an array of two identical heaving buoys. The devices in the array are aligned parallel to the direction of the freestream, the devices are placed 10m apart. The results of simulation in regular wave while controlled using the Positive Power Bang-Singular-Bang (PPBSB) control are compared with that obtained from an Optimal Resistive Loading (ORL). The regular wave is formulated with a peak period $T_p = 6$ s and significant wave height $H_s = 0.8222$ m. The hydrodynamic parameters as functions of the frequency was obtained from NEMOH boundary element method (BEM) matlab routine. The maximum control force availed by the PTO is $\Upsilon = 2e4$ N.

Algorithm 1 PPBSB control Algorithm for a WEC array

```
procedure IMPLEMENT CONTROL FOR AN ARRAY OF DEVICES(u⃗)
    Input: Maximum available control ϒ⃗,
    Output: Control
    Calculate excitation wave for the current time
    Compute singular arc control from Eq. (35)
    u⃗ = u⃗_sa
    Check control force constraint from Eq. (11):
    if u⃗_sa ≥ ϒ⃗ then
        u⃗_sa = ϒ⃗
    else
        if u⃗_sa ≤ −ϒ⃗ then
            u⃗_sa = −ϒ⃗
    Check power constraint. Compute α⃗ from Eq. (25):
    if α⃗ < 0 then
        u⃗ = sign(x⃗_2) × ϒ⃗
```

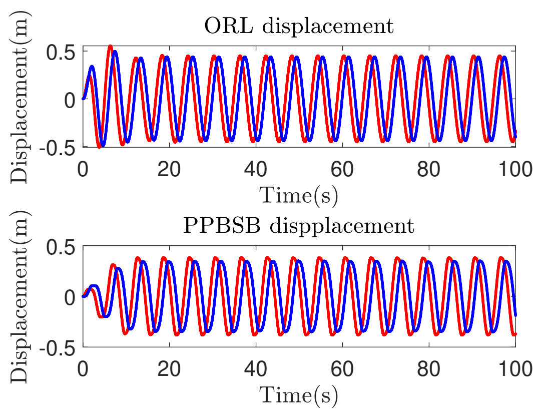

Fig. 1. Displacements of buoy-1 (red) and buoy-2 (blue) when using Optimal Resistive loading (ORL) and Positive power control (PPBSB).

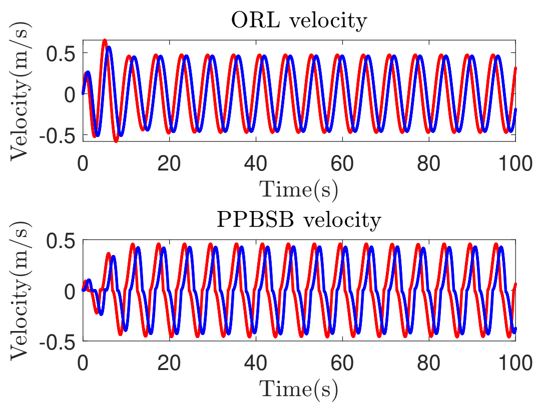

Fig. 2. Velocities of buoy-1 (red) and buoy-2 (blue) when using ORL and PPBSB controls.

The displacements and velocities of the buoys when Optimal Resistive Loading (ORL) control and PPBSB control algorithms were applied to control the array are presented in Fig. 1 and Fig. 2 respectively. Both control methods do not have negative power. The displacements are within reasonable limits for safe operation of the devices. The optimal PPBSB control being an optimal control maximizes the power extraction without necessarily having a higher displacement or velocity. PPBSB works on effectively utilizing the damping of the PTO to extract higher power from the wave. For this reason, it can be observed that the buoy displacements and velocities plots for ORL and PPBSB control are similar and close in magnitude.

Fig. 3 shows the power generated by individual buoys of the array when ORL and PPBSB algorithms were used to control the devices. The power from each array control case is then summed and presented as the total power in Fig. 4. Obviously, the PPBSB control algorithm gives better results with the highest power curve; while maintaining positive power only. This performance confirms the optimality of the PPBSB control over the ORL control.

It is unsurprising that the overall energy harvested using PPBSB control is better. Fig. 6 shows the control force used by the two control methods being considered. Comparing the ORL control and the PPBSB control, it is observed that the PPBSB control quickly switches between the maximum and minimum limits of the control force. This because the Singular arc solution of the optimal control often requires a high control force. The saturation limit on the control and the positive power constraint forces the control to be at the maximum allowable limit at most time, with the singular arc solution sparsely used.

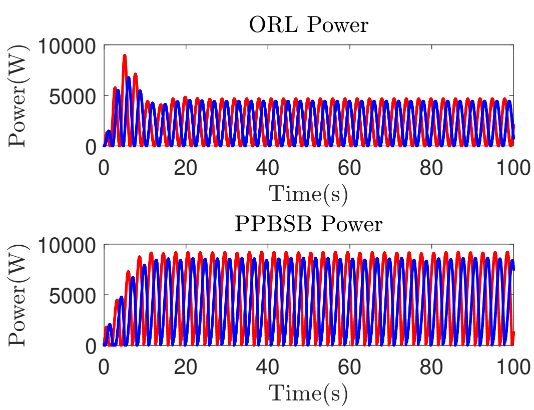

Fig. 3. The power extracted from buoy-1 (red) and buoy-2 (blue) in the array when using ORL and PPBSB controls.

The total energy from the WEC array using both control algorithm is presented in Fig. 5.

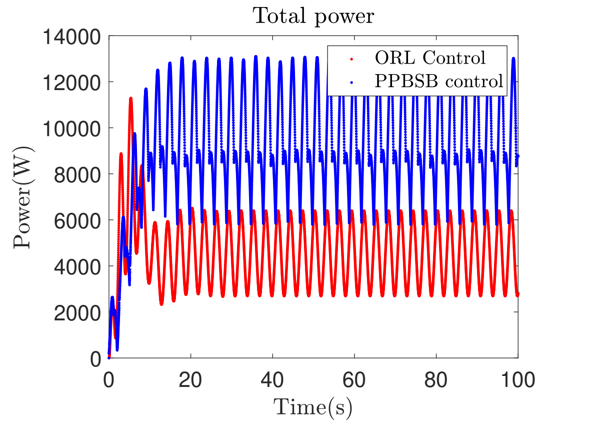

Fig. 4. The total power extraction by the devices in the array when using ORL and PPBSB controls.

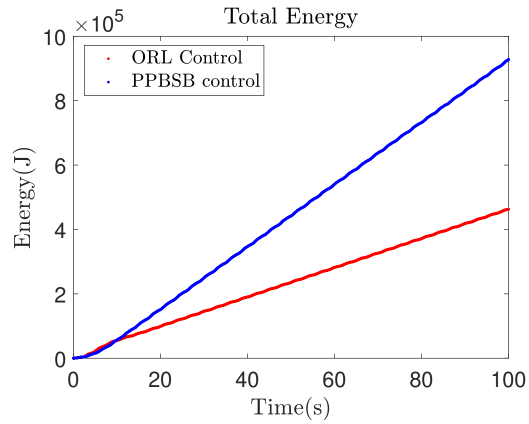

Fig. 5. The total energy extracted from the array over time by ORL and PPBSB controls.

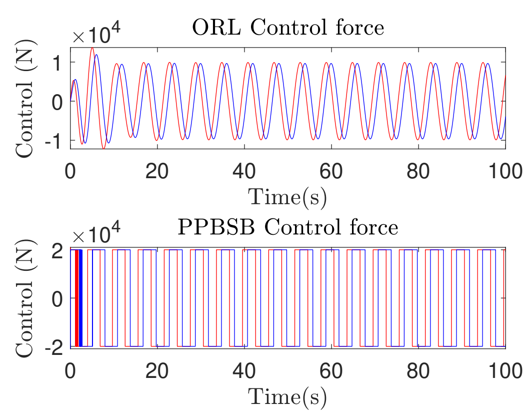

Fig. 6. The control force used by the PTO unit when using ORL and PPBSB controls.

V. Conclusion

A new control algorithm that attempts to maximize the power output from wave energy converters without the use of reactive power is developed in this paper; here, it is referred to as the Positive Power Bang-Singular-Bang (PPBSB) control. This positive power constrained control is derived analytically within the context of optimal control theory. Simulation results presented in this paper show that the overall performance of the PPBSB control significantly higher than that of the ORL control. The implementation of the positive power control eliminates the complexity of designing a bi-directional PTO system needed to provide reactive power. Future work will develop the positive power control for arrays of wave energy converters with an irregular excitation.

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