

Status and future of modeling of musical instruments: Introduction to the JASA special issue

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Status and future of modeling of musical instruments: Introduction to the JASA special issue^{a)}

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ABSTRACT:

Over the last decades, physics-based modeling of musical instruments has seen increased attention. In 2020 and 2021, the Journal of the Acoustical Society of America accepted submissions for a special issue on the modeling of musical instruments. This article is intended as an introduction to the special issue. Our purpose is to discuss the role that modeling plays in the study of musical instruments, the kinds of things one hopes to learn from modeling studies, and how that work informs traditional experimental and theoretical studies of specific instruments. We also describe recent trends in modeling and make some observations about where we think the field is heading. Overall, our goal is to place the articles in the special issue into a context that helps the reader to better understand and appreciate the field. © 2021 Acoustical Society of America. <https://doi.org/10.1121/10.0006439>

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I. INTRODUCTION

Musical instruments have been studied by scientists, engineers, and musicians for more than a century. (For comprehensive book-length reviews of the science of musical instruments see, e.g., Refs. 1 and 2.) As in many areas of science, this research can often be classified as being experimental or theoretical. A third way of conducting research—i.e., modeling—also plays an important role in the study of musical instruments. The purpose of this special issue of JASA, and the purpose of this article, is to describe recent progress made in the modeling of musical instruments and observe a few trends that suggest where the field is (in our opinion) heading.

This article is not intended to be an exhaustive review of the modeling field (for a few representative reviews we note Refs. 3–5). Instead, we will begin by going into a bit of detail for one instrument, the piano, that provides a nice example of how modeling has evolved and developed over several decades. Aspects of the evolutionary path followed in studies of the piano can be found in modeling work on other instruments, which we will then describe and discuss in much less detail with some examples on particular modeling aspects. As we review the work done on the piano, and then as we consider other instruments, we will return repeatedly to a number of central questions.

(1) *What does it mean to “model” an instrument?* A model of a musical instrument usually consists of a mathematical framework or algorithm designed to calculate the motion of one or more components of an instrument that

are involved in producing a musical tone. In many cases, this includes predictions for the sound pressure that would reach the ears of a listener. A model is usually based on a physical description of the instrument and involves the use of physics principles. For example, a model of a drum might be based on the physical dimensions and composition of the drumhead, and describe the vibrations of the drumhead using the laws of elasticity.⁶ Such a modeling approach is often termed “physical modeling.” Another common method is to use some measured properties of an instrument, such as the acoustic impedance of the bore of a trumpet,⁷ or the bridge admittance of a guitar, to construct an effective filter whose output contributes to the spectrum of a calculated tone.⁸ Such an approach to synthesizing a musical tone can also yield useful insights (depending on the questions being considered) and be appropriate when a real-time model is needed for a particular application.

(2) *What do we hope to learn about an instrument from modeling?* This is such a broad question that is hard to give an answer that will apply to all cases. In some cases, one might want to determine if a specific aspect of an instrument can significantly affect the nature or quality of the resulting sound. For example, one might want to determine if or how the material used to make a trumpet really matters. Or, considering a string, it can in general exhibit three (or more) different modes of vibration. Do they all contribute significantly to the tones produced by a particular instrument? Does the longitudinal string vibration really matter for a piano tone, or is the torsional motion of a violin string really perceptible? These kinds of questions can be addressed through modeling.

^{a)}This paper is part of a special issue on Modeling of Musical Instruments.

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(3) *How do we test or validate models?* This is an issue that is often not emphasized, with an implicit assumption that including more physics in a model is always better. If a model produces the sound pressure associated with a musical tone, one can compare the spectra and other properties of measured and modeled tones, conduct listening tests, or directly compare time-domain waveforms. For a string instrument, one can compare measured and calculated results for the motion of the string. Perfect agreement with a measured property is usually very challenging, due to complexities in both the modeling and the experimental measurement, and is, in many cases, not sought after.

(4) *How can modeling results inform instrument makers?*

This is an area in which modeling will (in our opinion) have a bigger role in the future. It is much easier for modeling to explore a large number of new different (virtual) instrument designs than for a maker to produce many different (real) instruments. In addition, modeling can sometimes yield insights that are quite difficult to obtain from experiments. For example, hypothetical changes to a piano soundboard or new rib designs for a guitar top plate can be explored by modeling and be of great value for an instrument maker.

(5) *How can modeling results be beneficial to instrumentalists?* Recent modeling attempts aim to shed light on the physics related to the interaction between the player and the instrument. This can offer insights into the effect of certain playing gestures to the generated sound. Furthermore, in combination with experimental measurements, it is possible to visualize how players try to control their instrument during performance (e.g., provide on-line information on the effect of the vocal tract while playing a wind instrument).

In Sec. II, we will describe in some detail how modeling of the piano has developed and been refined through the work of a number of different researchers. We will use this as an example of how and what modeling can teach us about an instrument. After that, we will mention aspects of modeling work on a variety of instruments, including the contributions to this special issue; this will allow us to explore phenomena that do not arise in the piano but are important in percussion instruments, woodwinds, brasses, and other cases. Finally, we will speculate about topics for which we expect to see significant attention and/or progress in the near future, especially towards model validation and musician-instrument interaction.

We wish to emphasize again that this article is not intended to be exhaustive, although we have tried to include ample references to help readers interested in probing more deeply into the field.

II. PIANO

The piano has been the object of many modeling studies and is a nice example of how work on a particular instrument has evolved as more and more fundamental physics has been incorporated into different aspects of the modeling. We now have what can be fairly termed a “full” model of the instrument. In our opinion, piano models can now produce what many would judge to be realistic tones and that can be useful for instrument makers in exploring new designs for real instruments.

Figure 1 shows a block diagram of a simplified piano model. This model is simplified as it omits the piano key and action, the lid, and the case, all of which have attracted the attention of modelers and all of which contribute to the sound produced. We will say more about those omissions below.

In general terms, the collision of a piano hammer sets the string into motion and also causes the hammer to rebound from the string. The string is suspended between two supports, one of which is quite rigid while the other is attached to a bridge that is itself rigidly connected to the soundboard. The string vibration causes the bridge and soundboard to vibrate, creating pressure waves (sound) that propagate into the surrounding air. Figure 1 gives a very simple representation of the “structure” of a piano. The hammers and strings are contained together in this figure to indicate that their motions are strongly coupled. The hammers and strings in Fig. 1 are linked to the soundboard through a double arrow to indicate that each component affects the motion of the other. This interaction is not as strong as the hammer-string coupling but is nevertheless important since when the motion of any individual string drives the soundboard, the motion of the soundboard then acts back on the entire system of strings producing, e.g., the familiar sympathetic sounds of many strings when the sustain pedal is depressed. The motion of the soundboard creates sound in the air but the effect of the sound pressure in the air has very little effect back on the soundboard so the arrow connecting the soundboard and air is directed only toward the box representing the air in Fig. 1.

Central to any piano model is, of course, the string. To be more accurate we should refer to the strings since most notes are produced by more than one string and there are 88 notes in a modern piano. Focusing initially on just a single string, the natural way to describe its motion is with the familiar wave equation for an ideal string, and that equation can be dealt with analytically in many cases. However, the presence of a number of effects including string stiffness, the nonlinear force-compression characteristics of real piano hammers, the fact that the hammer-string contact extends over a finite portion of the string, and the motion of the end of the string attached to the bridge requires that the hammer/



FIG. 1. Simple description of a lumped model of a piano. For more detail on the components of the instrument, see Ref. 9.

string portion of the model in Fig. 1 be solved computationally. A notable early work of that kind is due to Hiller and Ruiz,¹⁰ who described a finite difference-time domain algorithm for solving the wave equation. Hiller and Ruiz also considered different ways to excite the string including bowing and striking. While Hiller and Ruiz did not make any comparisons with the behavior of real piano strings, their approach set the stage for subsequent work on the hammer-string interaction and the motion of a string struck by a realistic hammer. Piano hammers consist of a wooden core covered with felt (a compliant material), so it is natural to use Hooke's law to describe the collision between a hammer and a string. However, early work showed that the force-compression characteristic of a real piano hammer follows an approximate power law of the form

$$F_h(z) = Kz^p, \quad (1)$$

where z is the compression of the hammer felt, K is an effective stiffness, and p is an exponent with a value in the range 2.5–4 for a typical hammer. Much early work focused on the excitation of string motion subject to the force in Eq. (1) given a piano hammer of specified mass and speed just prior to impact. There are many regimes to investigate; soft versus hard hammers (i.e., the values of K and p), the width of the hammer (along the string), and the position of the contact point. These and many other factors have been explored.^{11,12} These studies gave qualitative insights into how changes in the hammer properties affect the spectral composition of the string vibrations. Quantitative comparisons followed soon after^{13,14} as more rigorous analysis of the algorithms used to treat vibrations for stiff strings with damping were developed. Most of this work assumed that hammers can be described by Eq. (1). However, later work showed that losses in the felt leads to hysteresis in the force-compression characteristic¹⁵ so that the force depends not only on the compression z but also on the history of $z(t)$, an effect that is now usually included in modeling work.

As progress was made on hammer-string modeling attention began to turn to the soundboard. The simple model described in Ref. 16 used the equation of motion for a thin, orthotropic plate, including ribs, bridges, and the appropriate direction of the soundboard grain. The accuracy of that soundboard model, which was explored for both upright and grand piano geometries, was also confirmed through comparison with mechanical measurements on real soundboards.^{16–18} With this soundboard model it was then possible to attach strings as described in Ref. 13. The final step in filling out the model in Fig. 1 was to include the room air by solving the equations of linear acoustics¹⁹ subject to boundary conditions set by the vibrating surface of the soundboard. This resulted in a physical model of the piano that starts with the motion of a piano hammer and yields the sound pressure that would reach the ears of a listener.¹⁸ Judgements on the quality of the piano tones calculated using this early model are of course subjective, although it seems fair to say that they resemble those of a

fair-to-average quality piano. At the same time, that model does not contain many features and components of real pianos that are known to make quite audible contributions to the tone, leaving many avenues for further refinements which have been explored in recent years.

The models of piano strings, hammers, soundboards, and their interactions, and sound production described above and assembled in the model described schematically in Fig. 1 are all based on equations of motion that derive from Newton's laws of mechanics. However, there are still many approximations involved in these equations of motion and hence many routes for refinements and improvements, which have been the subject of subsequent work.

Regarding the strings, the early work^{13,14,18} assumed string vibrations along a single direction perpendicular to the plane of the soundboard. While that is the polarization excited initially from the collision with a hammer, the string termination at the bridge couples this with the other transverse polarization, leading to a variety of effects that have been studied experimentally and with semiquantitative models.²⁰ Including this coupling requires a more complete soundboard model but is conceptually straightforward.^{21,22} Longitudinal string vibrations are also present^{23,24} and have also been modeled.^{21,25,26} This again requires an appropriate description of the connection with the bridge, and also turns out to require the inclusion of nonlinear terms in the equations of motion.¹⁹ Experiments have given clear evidence that these string vibrations are audible in real tones.

Improvements in the modeling of hammers have addressed the need to explicitly include the motion of the piano action. Interestingly, classic work on the physiology of the player²⁷ discusses the importance of the player's touch in ways that suggest the importance of not just the velocity but also the acceleration of the piano hammer prior to contact with the string.²⁸ This in turn suggests a role for the piano action and specifically the bending of the hammer shank, which will in principle depend on how the piano key is depressed. This has been studied experimentally^{29,30} and through modeling.²² There is evidence for some bending of the hammer shank, although it is not clear that this has an audible effect on the tone. It is worth noting that modern hammers (i.e., since around 1850) have been made with rather large diameter maple shanks, a rather stiff wood, which would seem to indicate that piano makers wish to minimize bending. On the other hand, historical pianos (i.e., pre-1800 or so) used hammer shanks which were much smaller in diameter and which were therefore much more flexible.

As noted above, to take advantage of improvements in string modeling requires corresponding improvements in modeling of the bridge and soundboard. Recent soundboard models include three dimensional motion of the bridge (as compared to the earlier thin plate models), and also include tapered thicknesses for the ribs and board and the stress built into the board and ribs during assembly. These effects all lead to more complex (and in some cases nonlinear) equations of motion.

One direction that has not yet been explored in detail is the role of the case and rim.³¹ On the one hand, these are much thicker than the soundboard and are therefore expected to have much smaller vibration amplitudes. On the other hand, it is certainly easy to feel their vibrations by hand and the orientation of the lid is known to be important with regard to the directionality of the sound.

Regarding future developments about the piano, we expect that descriptions of the soundboard and other components will be made even more quantitative. At the same time, more work is needed to evaluate calculated tones. It will be especially important to elucidate the role played by various factors, such as hammer shank bending, longitudinal string vibrations, prestress of the soundboard, the design of the case, and more. There is no doubt that some or all these are audible but are they essential to produce a high quality tone? The answer to this question will be helpful for piano makers as they consider design improvements, and also to modelers who aim to develop real-time algorithms for applications such as electronic instruments. A more extensive and systematic approach to validations through listening tests (and other measures) will be essential for progress in these areas.

III. PHYSICS-BASED SOUND SYNTHESIS

In a similar fashion to the vibrating string, equations familiar from physics can be used as a starting point to model other musical instruments, subject to the required modifications. For example, the wave equation also applies to wave propagation inside a tube,¹ whereas using the Navier-Stokes equations one may also consider viscous effects.² Similarly, the Euler-Bernoulli framework can be used to describe the vibrations of a beam, while the Kirchoff-Love model tackles the case of thin plates.² In addition to such existing representations, it is often required to further extend physical models in order to capture the underlying physical phenomena that take place during sound production by complete musical instruments. Such additional elements are often possible to simulate accurately (as, e.g., in the case of nonlinear string vibrations) but there are several cases, especially in the presence of nonlinear interactions, where existing models fail to mimic the behavior of the system at hand. Besides the hammer impact described by Eq. (1), a typical example of nonlinear dynamics is the case of bow-string interaction. Although a significant amount of research has been carried out in that domain,³² and qualitative similarities between models and experimental measurements have been observed, it has been recently shown³³ that the numerical reproduction of a measured time-domain waveform (e.g., string velocity) cannot yet be achieved, leaving the authors of Ref. 33 to wonder whether any existing model can be relied upon to analyze bowed-string behavior.

Several such cases, where physical models still require improvement, are often related to nonlinear effects linked to the excitation mechanism of musical instruments and

involving phenomena such as turbulence, contact dynamics, amplitude-dependent parameters, and feedback-controlled excitation.³⁴ One particular example, related to collision modeling, may be used to illustrate the above problematic. In that case, the seemingly simple equation

$$f_c(x) = k_c h(x) x^\alpha, \quad (2)$$

where x is the displacement of an object, $h(x)$ denotes the Heaviside step function, k_c is a collision stiffness with α a collision exponent, and f_c the corresponding collision force, is still under investigation towards a numerically efficient and provably stable approximation. While classical approaches towards numerically solving an equation of motion of the form

$$m \frac{d^2x}{dt^2} = f_c(x) \quad (3)$$

have been used, alternative formulations diverting from this path have been proposed, e.g., by describing the system using Hamilton's equations³⁵

$$\frac{dx}{dt} = \frac{\partial H(x, p)}{\partial p}, \quad (4a)$$

$$\frac{dp}{dt} = -\frac{\partial H(x, p)}{\partial x}, \quad (4b)$$

where H is the total energy of the system and p the conjugate momentum, or by considering a quadratization of the potential energy V as $V = \psi^2/2$.³⁶

A. Numerical algorithms and issues

In the cases where models are proposed to numerically simulate such nonlinear effects, the challenge remains to formulate a suitable algorithm, bearing the required numerical properties. These are numerical stability, accuracy, and efficiency, aspects that have been studied in extreme detail within the field of numerical analysis, but also in relation to sound synthesis.^{5,37} Assuming that the proposed numerical solver is accurate enough for the required simulation—note that uncertainties in the material and environmental parameters are often large enough to only impose moderate requirements for the order of accuracy—a key issue regarding numerical solvers is that of stability. Especially in the presence of nonlinearities, a very effective method is to ensure numerical stability *via* energy-balanced models, an approach that has seen increased interest in recent formulations.^{21,35,38,39} This increased attention towards ensuring stability has led to the requirement that any new algorithms have to be proposed together with a proof of stability (or while stating any related limitations). Such proofs often manifest themselves in the form of a conserved energy-like quantity or a similar, numerically-defined structure.

Regarding the running-time required for such simulations, there are two application categories. Off-line simulations, where the results are obtained and analyzed after the

simulation is complete and real-time simulations, where it is possible to vary the system parameters during run time. This has implications to the choice of both the modeling paradigm and the solution algorithm. On the one hand, simplified geometries and minimal models may be used in combination with computationally efficient algorithms for real-time sound synthesis, including applications in virtual reality.⁴⁰ In that case, it is also necessary to consider the radiativity of the sound source,^{41–43} which poses several challenges both in terms of experimental measurements^{44,45} as well as to incorporate given directivity patterns to a room acoustics simulator.⁴⁶ On the other hand, when the underlying physics is the focus of a model-based study, more elaborate models are usually employed, involving heavy computations.^{47–51} The regular increase in computational capabilities results in a continued variation of the preferred algorithms.

For example, waveguide algorithms, which exploit the standing wave nature of waves in one dimension, were an attractive early choice for string synthesis.³ Over time, extensions to two and three dimensions were developed, along with generalizations to include many other physical effects. As computer power has increased, finite difference time domain (FDTD) algorithms have become extremely popular due to their ability to handle nonlinearities and to support analyses of stability and energy conservation, and because of extensive studies of such algorithms in the applied mathematics community. In addition, the incorporation of new physical effects is often more easily and intuitively done using the FDTD framework since the fundamental equations of motion are usually expressed as partial differential equations. From a numerical standpoint, wind instruments have posed severe challenges due to the inherent nonlinearities in the Navier-Stokes equations. Early work on wind instruments focused on the lattice-Boltzmann method (LBM)^{52–55} a qualitatively different approach. While LBM can be shown to be equivalent to an FDTD solution, the simplest LBM algorithms are quite efficient numerically but also have stability issues. These issues have been addressed in the most recent work,⁵⁶ though meanwhile FDTD solutions of the Navier-Stokes equations are now appearing and seem to be competitive.^{57–62} We therefore expect to see an increasing amount of work in modeling studies involving complex physical phenomena^{63,64} and detailed geometries.^{65,66}

B. Material properties

Another common difficulty across most physics-based synthesis attempts stems from the fact that material parameters (such as mass, density, stiffness, etc.) need to be included in the numerical models. These parameters are usually obtained from literature, where a range of values is given. Choosing the right value may be only accomplished by experimentally measuring the underlying parameter for the particular object that is used in a measurement. However, this is often impossible in practice either because

of the inability to identify material properties in a fully assembled instrument or because such properties vary anyway across different regions of the material (especially in the cases of wooden instruments or hand-crafted brass instruments). After all, the properties of the wood in the soundboards of different pianos are certainly not the same (for the bridges and ribs, etc.), yet a listener usually has no trouble in identifying different instruments as all being pianos.

Implications are critical in cases of (mostly wooden) bowed- and plucked-string instruments. Despite the fact that several researchers and instrument makers have studied the material properties of wood (as well as the effect of varnish), it remains practically impossible to *a priori* determine the “correct” parameters to use in a physical model. Similarly, for woodwind instruments, the viscothermal losses at the wall interior affect the behaviour of the instrument,⁶⁷ whereas in brass instruments the material is related to the vibrations of the walls of the resonator, which may couple with the acoustic field.^{68,69} On the other hand, experiments with flutists showed that for certain instruments such material properties have a negligible effect on an instrument’s tone⁷⁰ and that instrument quality depends instead on the efficacy of the manufacturing process.

In any case, regardless of the type of instrument, modeling results may be beneficial for instrument makers. They provide a first opportunity to, at least qualitatively, evaluate how certain changes in the construction of an instrument may affect the generated sound. Despite the uncertainties in the material properties of some instruments, a rough estimation of the effect of such changes may be obtained using physical modeling, before an instrument (or a series of instruments) need to be manufactured.^{71–73}

IV. MODEL VALIDATION

Another aspect that comes hand in hand with the formulation of a physical model is that of validating the model. Several modeling approaches are only proposed on the basis of numerical output, without any scrutiny regarding how accurately the model manages to capture real-world observations. There are a variety of reasons for such a lack of validation.

An extensive amount of effort is often devoted to the formulation of highly complex numerical solvers, leaving limited resources to compare the model output to experimental measurements.^{38,39,74} Judgement in such cases must rely on the plausibility of the model output, based on expectations and intuition, as well as on the analysis of numerical properties of the model. Alternatively, there is the option to assess the reliability of a physical model by comparing its output to “well established” numerical results⁷⁵ or by observing how the model output varies subject to parameter variation.⁷⁶

One obstacle towards obtaining high quality measurements for physical model validation stems from the fact that musical instruments are supposed to be played by humans

under certain conditions. Hence, reproducibility of measurements is often impossible to achieve. Furthermore, the use of microphones, sensors and other gauging devices may significantly affect the performance of musicians. Therefore, experimental setups offering minimal intrusiveness are under development for obtaining measurements under real-playing conditions.^{77,78} On the other hand, sophisticated artificial playing devices have been developed, in order to generate repositories of measurements under controlled laboratory conditions.^{79–82} This also offers the possibility to alter only one playing parameter at a time, without affecting other system control parameters, something that human players can rarely achieve. That is of particular value when it comes to comparing experimental measurements with physical models. Such experiments are also giving new insights into the role of the player in affecting the tonal properties and quality (which is touched on more in Sec. V).

An example of reliable numerical reproduction of measured signals obtained using an artificial player can be found in woodwind instruments. Using an artificial blowing machine, including an artificial tongue for the generation of realistic note transitions, mouthpiece pressure signals have been recorded that were subsequently resynthesised using a physical model.⁸² Furthermore, the fact that the artificial player was suitable for imitating a variety of articulation techniques, helped towards directing the physical model to resynthesise a clarinet concerto excerpt, as performed by a real musician.⁸³

In general, however, gathering suitable data for model validation requires sophisticated experimental setups, while it is still extremely difficult to isolate the physical effects under study. For example, while simulating the function of a brass instrument, one may simply switch the vibrations of the instrument walls on or off, in order to numerically assess their effect.⁸⁴ Achieving something similar in the lab has proven to be a quite challenging endeavor.⁶⁸ Similar difficulties are often encountered in relation to measuring flow inside a tube. The latter is also very demanding in numerical simulations, as computational fluid dynamics are often associated with long running times,^{59,85} a fact that hinders the generation of a large repository of simulated data that could be used to inform experimental results.

Such complexities in physical modeling and in experimental measurements alike often resulted in researchers reverting to qualitative approaches in order to validate the behavior of a physical model. Such validation may be achieved based on perceptual criteria, studies on the playability of a musical instrument, or other qualitative measures.^{86–88} For instance, regarding bowed-string instruments, uncertainties in the modeling of bow-string interaction, as well as the lack of sufficiently accurate measurements under artificial playing conditions, have averted researchers from obtaining a quantitative match between physics-based simulations and experiments. As specified recently,³³ a model that is fully based on underlying physical principles and is in adequate agreement with measured oscillations of bow-string interaction does not yet exist. In

such cases, empirical evidence may be used to enhance a model. This raises the question, to what extent such hybrid models may increase our understanding of the underlying physical phenomena. Meanwhile, it remains to be seen whether methods based on machine learning—that are already used for sound synthesis⁸⁹—will also be employed to analyze the function of musical instruments.

Furthermore, as mentioned previously, material properties are not always known in sufficient detail in order to allow physical models to be validated using experimental measurements. Although material parameters may be fine-tuned while running physical model simulations, such optimization attempts are only informative if the underlying model is reliable, a process which may result in a stalemate. In such cases, alternative approaches, such as carefully designed listening tests, could be used to substantiate the use of a certain physical model.

V. MUSICIAN-INSTRUMENT INTERACTION / PLAYABILITY

Finally, the development of reliable physical models as well as advances in computational and experimental capabilities, allow the transition from studying isolated musical instruments to analysing the interaction between musicians and their instruments. In earlier physical modeling studies, the interaction between the player and the instrument was usually modeled using idealised initial and boundary conditions, with the focus mainly lying on the oscillations of the instrument and not on the control exerted on it by the player. However, several experimental studies have been recently carried out on the musician-instrument interaction and some physical modeling attempts have pursued the simulation of this interaction. The possibility to validate such models is directly related to the artificial playing systems mentioned above. The repetitive and reproducible excitation offered by such devices is paramount for evaluating the performance of a physical model that includes the actions of the player. These actions are mostly located at the excitation mechanism of the instrument (fingers, embouchure, bow, mallet) and often involve an inherent nonlinearity, posing the numerical challenges mentioned previously. However, one should also consider additional damping or acoustic shadowing due to the presence of the player (a rather unusual example being the insertion of the player's hand inside a horn bell).

The development of such models may be beneficial to musicians in terms of enhancing their performance or adjusting the effort associated with certain actions. The possibility to correlate these actions to corresponding sound results can provide both instrumentalists and music teachers with valuable information regarding playing techniques.

VI. CONCLUSIONS AND OUTLOOK

While this is a short article, it contains a rather long list of references. Indeed, our reference list is far from being exhaustive, but will hopefully give the reader a segue into

the latest modeling work on many different instruments and on many different issues that are now attracting interest in the field. The diversity in the work described in the references and the JASA special issue shows very clearly that this is a vibrant field with many different types of work being pursued by many different researchers. We hope that this article and the special issue will be a valuable resource for both current and potential new practitioners in the field.

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